MOODY'S ANALYTICS

CASE STUDY

NOVEMBER 2022

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Building And Evaluating Risk-Aware Investment Strategies

Introduction

Investment managers who manage a portfolio of assets are often subject to restrictions. They must consider regulation, environmental and social guidelines, and the loss absorption capabilities of their sponsors. While many sponsors, for example pension funds or endowments, need to generate a certain level of income to meet their obligations, they also demand capital preservation. This dilemma is a major challenge for investment managers as the Capital Asset Pricing Model (CAPM) (Sharpe, 1964) states that higher expected return comes at the cost of more risk.

The investment industry offers various strategies to tackle this challenge, posing the question to investors: which strategy is best suited to meet their investment goals? Deriving the risk and return characteristics of the strategies solely from historically observable data, is error prone. Whereas advanced scenario simulation techniques cover historically observable and unprecedented market conditions and hence, offer a realistic view on the future risk and return characteristics. This helps investors pick the right strategy to achieve their overall objectives.

This paper considers a case study of using equity put-options to offer downside protection for the multi-asset portfolio. An advanced stochastic simulation model is utilized to evaluate the risk and return profile of the strategy.

Risk Measurement

'Risk' is a general term, with many quantifiable measures for the financial risk of an investment portfolio. The CAPM framework defines risk as the volatility of possible investment outcomes at maturity. The Value-at-Risk (VaR) is calculated from the portfolio return distribution – for example a $VaR_{0.95}$ is defined as the 5%-quantile of the return distribution. The Conditional Value-at-Risk (CVaR) is the mean of the returns below the VaR level. The CVaR is a subadditive risk measure like volatility, and can be optimized and constrained with convex and linear programming methods. Whereas VaR is relatively difficult to optimize. There are many more possible measures, but volatility, VaR and CVaR are commonly used to measure investment risk in absolute terms.

Each risk measure serves different use cases. CVaR is a popular choice to identify potential investment portfolio losses, because it accounts for higher moments of the return distribution. Hence, the quality of the CVaR measure is inversely linked to the estimation error of the return distribution. The return distribution is either derived from historical observations or from stochastic simulations.

Data challenges in risk estimation

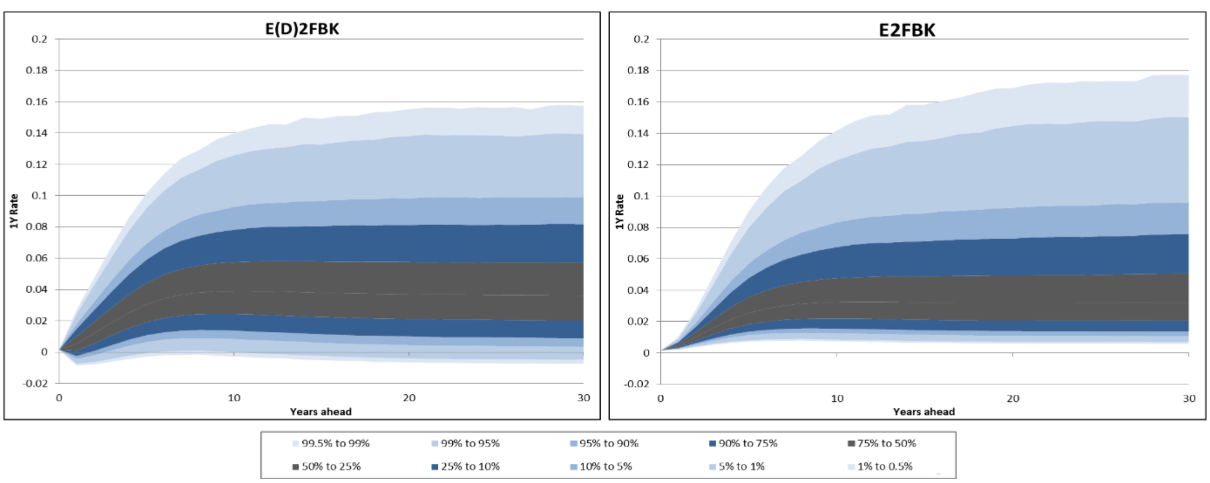
In his seminal work on Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) models in 1982, Robert Engle showed that the volatility levels of equity and fixed income markets are not stable over time. They tend to cluster instead. Shifts from low volatility clusters to high volatility clusters result in volatility jumps. Calibrations of the Heston model (1993) on equity option prices reveal a negative correlation of equity prices and volatility. Volatility jumps and severe negative equity performance, therefore come hand in hand, and shape asymmetric return distributions with long left tails.

Return distributions derived from historical observations only consider outcomes that were historically observable, and therefore involve the previously mentioned features by definition. However, the historical distributions do not account for the current levels of underlying risk factors, and data is scarce. For example, the historical distribution of interest rate changes contains sharp rises from single digit positive levels to double digit positive levels, and significant drops from midrange single digit positive levels to slightly negative rates. However, the form of the distribution of future interest rate changes is dependent on the current interest rate level. Therefore, applying the unconditional historical distribution of interest rate changes to measure the risk of fixed income instruments, is error prone.

Enhancing risk estimation with scenario simulations

Conditioned on the current risk factor levels, stochastic simulations can project meaningful asset price paths that have not been observed historically, with potentially more extreme outcomes. For example, from an economic perspective, it is more reasonable to consider significant rate increases, but only small rate decreases when interest rates are close to zero. In this scenario, the distribution of future rate changes would be right skewed rather than symmetric. Figure1 shows interest rate projections starting from a rate level close to zero.

Figure 1 E(D)2FBK is the proprietary Moody's Analytics Extended 2-Factor Black-Karasinski model with a displacement factor, allowing for negative rates. E2FBK is the Extended 2-Factor Black-Karasinski model.



To deliver meaningful insights, the Moody's Analytics Scenario Generator integrates modern technology and a suite of leading-edge stochastic models.

» Features of the interest-rate model:

a. Economy-specific business cycle equilibrium levels - (long term)

b. Mean reversion and convergence to ultimate target - (ultra-long term)

c. Short end of forward rate curve as a good predictor for short-term interest rates

d. Low rates tend to remain low for long periods

e. Capability to model negative rates

- The equity model has stochastic, mean-reverting volatility, which enables it to fit the whole volatility surface. It is also able to capture many more empirically observed features of equities such as rare events and volatility clustering. Large movements or jumps in equity returns tend to be negative and to be associated with large volatility, resulting in negative skew and changes in the volatility levels define the kurtosis. It models asymmetric correlations for equities across economies, with large falls in one having a contagion effect on others - this is the so-called tail correlation of negative events.
- The credit rating dynamics are represented by a Markov chain with transitions between ratings and defaults defined by a transition matrix calibrated to historical data. The Moody's Analytics Scenario Generator is extended by making both the credit spreads, and the transitions evolve stochastically.

Portfolio Optimization under Risk Budget Constraints

The following case study of a hypothetical investment manager, illustrates how powerful these capabilities can be for portfolio management. An institutional client wants to understand the long-term behavior and risks of their current portfolio. The investor would also like an assessment of their portfolio structure, to limit the expected loss to 10% with a 5% probability over a 10-year horizon. To keep it simple, the investment universe is constrained to global equity, European credit (constant maturity 10 years; BBB rating), and European government bonds (constant maturity 10 years; AAA rating).

With this information at hand, the most appropriate risk measure would be $CVaR_{0.95}$ (10years). Therefore, the goal function is

$$max\big(E(X)\big), s.t. E\big(X \big| X < F_X^{-1}(1-\alpha)\big) \geq \zeta_\alpha$$

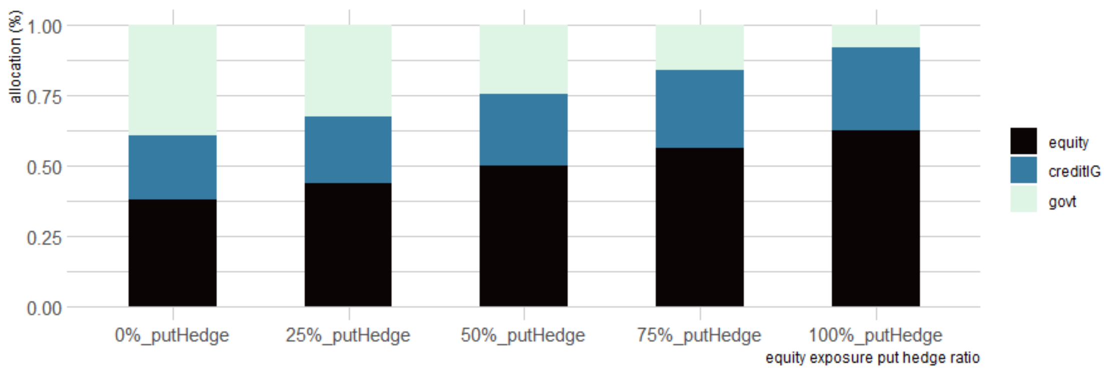
The portfolio return X is a random variable with distribution function F(X) and expected value E(X). ζ_α is the loss tolerance threshold (Expected Loss) at confidence level α.

Analysis of Investment Strategies

Minimizing the sampling error between the estimated distribution (sample) and the true population distribution is key to measuring the portfolio risk under the CVaR-measure. Using the Moody's Analytics Scenario Generator, the investment manager can run forward-looking analyses. This allows quick and seamlessly comparison of various portfolios. The investment manager can then discuss the risk and investment impacts, and give the investor comfort that the appropriate decisions are being made to meet their overall objectives. As scenario simulations start from the current risk factor levels and project thousands of possible future risk factor paths, this approach gives much richer and meaningful information regarding future risk and return characteristics than back-testing (investigating how portfolios or investment strategies would have performed in the past).

For example, a portfolio is derived based on 10,000 scenario runs that maximizes the expected investment return while its $CVaR_{0.95}$ (10years)=0.1 (0%_putHedge in Figure 2). To analyze the effect of a put-option overlay on the portfolio risk and return characteristics, the portfolio optimization is re-run to derive strategic asset allocations (SAA) with partial put-option hedges for the equity exposures. The optimization covers a 10-year horizon and assumes that at the beginning of each year, the portfolio allocation is reset to the SAA and put-options maturing in one year at 10% out-of-the-money strike levels are bought. The investment strategy is simple, but it requires real-world equity path projections and nested implied volatility surfaces to calculate the costs of the put-options at the beginning of each year, and to account for them in the portfolio optimization and the strategy performance assessment. The portion of wealth that is invested into the SAA at the beginning of each year fluctuates due to variations in the put-option prices which are caused by year-over-year changes in the implied volatility levels.

Figure 2 Allocations for a risk budget of 10%, measured as the CVaR over 10 years at the 95%-confidence level. X%_putHedge indicates the equity put-option overlay notional relative to the equity notional; X=(put-option notional/ equity notional).



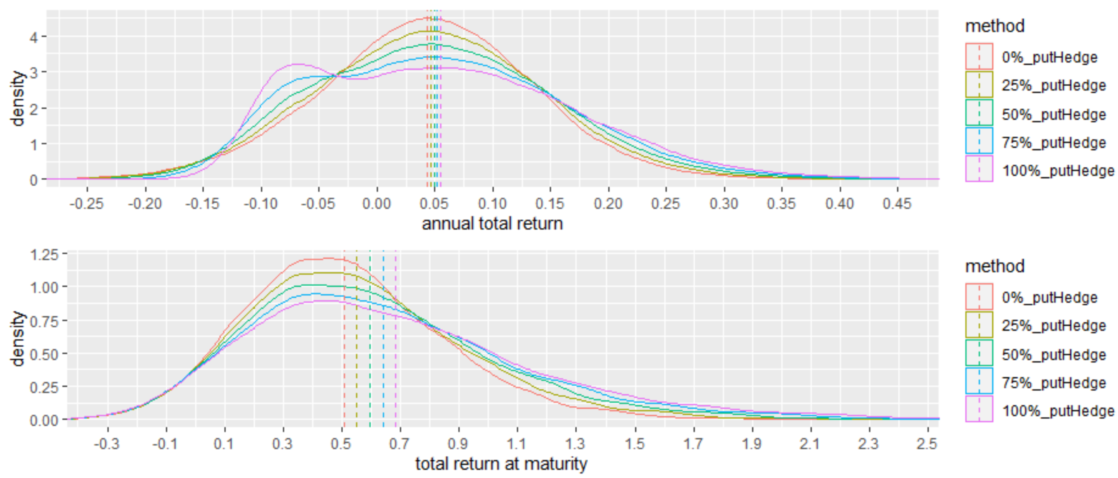
Source: Moody's Analytics calculations based on Moody's Analytics Scenario Generator projections (10,000 runs).

Using scenario simulations, SAAs are optimized for four different equity put hedge ratios – 25%_putHedge, 50%_putHedge, 75%_putHedge, 100%_putHedge. For example, "100%_putHedge" implies that all of the equity exposure of the SAA is hedged with the put-option overlay. Figure2 summarizes the strategic asset allocations including equity put-option overlays that maximize the expected return over 10 years and limit the $CVaR_{0.95}$ (10years)=0.1. The simulations reveal that the equity allocation could be raised from 38% to 63% (Figure2: 100%_putHedge) and still comply with the investor's loss tolerance over the 10-year horizon, if all of the equity exposure was hedged with the put-option overlay. The equity put-option overlay also frees up risk budget that allows an increase to the credit exposure in the portfolio by 5%-points while it reduces the portfolio duration significantly. This finding is meaningful as the Moody's Analytics Scenario Generator accounts for cross-asset class correlations.

Hypothetical portfolio performance based on simulated market scenarios

Using scenario simulation techniques, investment managers can quickly assess the return distributions of multiple strategic allocations. This shows the investor how hedging equity exposure, with the discussed put-option overlay, can amplify expected returns in their portfolio while controlling the downside risk. Any number of alternative portfolios can then be analyzed and the impact holistically viewed over various time horizons.

Figure 3 Return distributions for strategic asset allocations combined with equity put-option overlays. Each asset allocation obeys a risk budget of 10%, measured as the CVaR over 10 years at the 95%-confidence level. Over the 10-year investment horizon (maturity), the strategic allocations are reset annually and equity put-options with 1-year maturity and 10% out-of-the-money strike levels are allocated. "X%_putHedge" indicates the put-option overlay notional relative to the equity notional; X=(put-option notional/ equity notional). The dashed vertical lines are the expected mean-returns of the strategies.



Source: Moody's Analytics calculations based on Moody's Analytics Economic Scenario Generator projections (10,000 runs).

With the help of scenario simulations, it is possible to analyze how hedging parts of the equity exposure with the put-option overlay, permits a higher equity allocation in the portfolio but limits the portfolio downside to the $CVaR_{0.95}$ (10years)-threshold (Figure 3 bottom graph, Table 2). Not only is it easier to show the investor how introducing equity put hedges can increase expected risk-adjusted returns over the 10-year investment horizon, but they can walk through a comparison of the risk-return profiles in the portfolios.

Table 1 Annual expected return statistics for strategic asset allocations combined with equity put-option overlays based on 10,000 simulation runs. "X%_putHedge" indicates the put-option overlay notional relative to the equity allocation; X=(put-option notional/ equity allocation).

	CVaR_0.995	CVaR_0.99	CVaR_0.95	1st Qu.	Median	Mean	3rd Qu.
0%_putHedge	-0.233	-0.211	-0.152	-0.016	0.045	0.044	0.104
25%_putHedge	-0.220	-0.201	-0.148	-0.020	0.046	0.046	0.111
50%_putHedge	-0.201	-0.185	-0.142	-0.026	0.047	0.049	0.118
75%_putHedge	-0.176	-0.164	-0.132	-0.033	0.047	0.052	0.126
100%_putHedge	-0.153	-0.144	-0.121	-0.039	0.047	0.055	0.132

Source: Moody's Analytics calculations based on Moody's Analytics Economic Scenario Generator projections.

Table 2 Expected return statistics for strategic asset allocations combined with equity put-option overlays at maturity based on 10,000 simulation runs. The investment horizon (maturity) is 10 years. "X%_putHedge" indicates the put-option overlay notional relative to the equity allocation; X=(put-option notional/ equity allocation).

	CVaR_0.995	CVaR_0.99	CVaR_0.95	1st Qu.	Median	Mean	3rd Qu.
0%_putHedge	-0.294	-0.243	-0.100	0.271	0.482	0.511	0.715
25%_putHedge	-0.294	-0.244	-0.099	0.285	0.513	0.550	0.773
50%_putHedge	-0.293	-0.245	-0.099	0.296	0.544	0.594	0.833
75%_putHedge	-0.295	-0.245	-0.099	0.312	0.574	0.641	0.901
100%_putHedge	-0.293	-0.245	-0.099	0.323	0.601	0.681	0.953

Source: Moody's Analytics calculations based on Moody's Analytics Economic Scenario Generator projections.

Table 1 summarizes the expected return statistics over the one year horizon. The additional insights can be used to discuss the loss reduction feature of the put-option overlay for the one year horizon. With the help of scenario simulations an in-depth analysis of the left tail of the return distribution for the 10-year investment horizon (Table 2) can be delivered. It can also help to explain to the investor that an increased allocation to growth assets, combined with a tail hedge such as an equity put-option overlay, may deliver the desired downside risk protection and boost the expected portfolio return.

Enhancing portfolio construction with stochastic simulations

Introducing the appropriate technology and analytical capabilities can extract valuable insights and help the decision-making process. For many portfolio managers today, the types of analyses in this case study are out of reach, or take a considerable amount of time and effort. This leaves less time to engage with clients, help grow the business and it is difficult for investment managers to make the right decisions or deliver the kind of transparency clients are looking for.

Technology can help bring more sophistication to a portfolio management process and allows investment managers to tailor their offering to their clients' needs.

Learn more about the Moody's Analytics Scenario Generator

The Moody's Analytics Scenario Generator delivers stochastic simulations for investment managers who want to assess how their asset portfolios might perform under various economic and market conditions. It helps the comparison of alternative asset allocations and analysis of investment strategies under stress conditions. Our proprietary simulation framework can offer a deeper understanding of portfolio risk profiles to help portfolio managers with portfolio construction decisions while adding meaningful value to long-term client relationships.

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