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MOODY'S ANALYTICS

MODELING METHODOLOGY

Understanding GCorr® 2020 Europe CRE

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Abstract

Commercial real estate (CRE) exposures constitute a large share of credit portfolios held by financial institutions. For portfolio credit risk management, it is essential to understand how various exposures correlate with one another. In this paper, we provide an overview of the Moody's Analytics Global Correlation model (GCorr) for European CRE instruments, GCorr 2020 Europe CRE.

GCorr 2020 Europe CRE is estimated based on commercial real estate default data from Moody's Analytics Commercial Mortgage Metrics (CMM®) model. GCorr 2020 Europe CRE uses a factor model approach to describe the dynamics of the systematic and idiosyncratic components of European CRE credit risk. The systematic factors are modeled at geographical and property-type levels. This leads to a model that can more accurately capture CRE concentrations and correlations across 225 European CRE markets, defined by 18 countries, 27 cities or sub-regions, and the following five property types: hotels, industrial, multifamily housing, office, and retail.

The wide cross-sectional differences in correlations across markets highlight the importance of estimating the correlations at a granular level. Moreover, since the model provides both the inter- and intra-asset class correlations, it makes it possible to capture the concentration and diversification effects in a large credit portfolio containing European CRE and other exposures.

Contents

1. Introduction

Following the global financial crisis, European commercial real estate debt has gradually become an appealing asset class. Investors might expect higher returns on commercial real estate loans than on public market securities with comparable risk profiles. This expectation is mainly due to sourcing and execution complexity, as well as illiquidity of such instruments. In addition, adoption of conservative approach to collateralization, with average loan to value (LTV) in Europe at around 57% (compared to 80% prior to 2008), has resulted in a reduction of risks associated with investing in CRE debt.[1]

As a result, CRE debt represents a significant share of the credit portfolios held by financial institutions and specialized non-bank lenders. For example, in the UK, CRE loans comprised approximately 8.3% (£ 239 million) of all outstanding loans and approximately 51% of all loans to non-financial corporations (NFC) as of 31 December 2020, as shown in the right-side pane of Figure 1. The left pane displays the proportion of CRE loans to total loans to NFC in other European countries.[2]

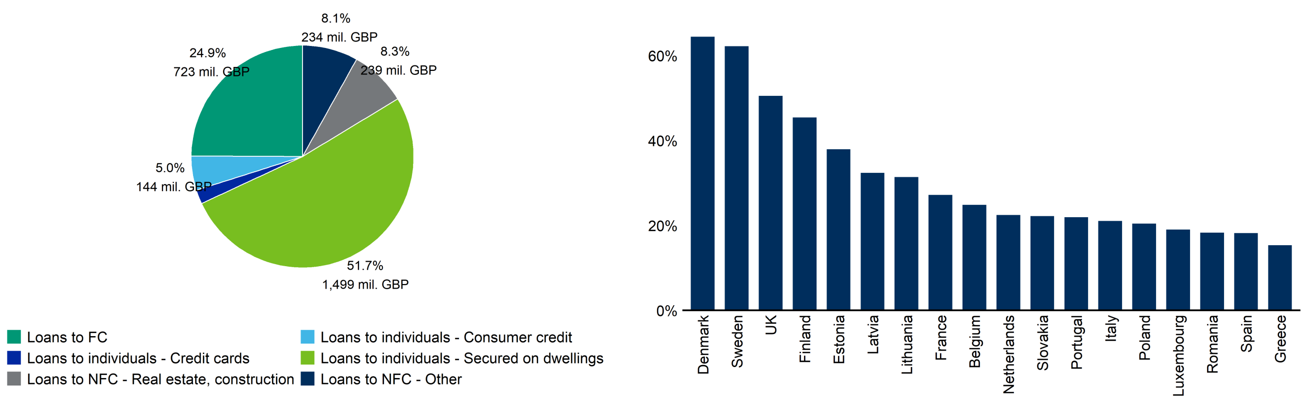

Figure 1: CRE Loans: Left Pane - Sterling Loans Outstanding Held by Monetary Financial Institutions in the UK as of 31/12/2020; Right Pane - CRE Loans to Total Loans to NFC as of 31/12/2020 (%)

Figure 2 illustrates how cross-asset default rates cluster around economic downturns.[3] The observed variation in default and delinquency dynamics, as represented by Moody's Analytics EDF™ (Expected Default Frequency) for corporates or CMM® probability of default (PD) metrics for CRE loans, establishes the importance of an integrated framework that captures cross-segment correlation effects both within an asset class and across asset classes in measuring portfolio credit risk.

In this paper, we discuss such an integrated approach, with an emphasis on modeling asset correlations for European CRE exposures. First, we describe the Moody's Analytics approach to modeling asset correlation in the context of portfolio credit risk modeling. Next, we explain how we estimate the asset correlation model for European CRE assets, known as GCorr Europe CRE, and its integration into the Moody's Analytics Global Correlation model (GCorr) framework. Finally, we present the key results of the GCorr 2020 Europe CRE model.

The Moody's Analytics approach to modeling asset correlations is to decompose a borrower's risk into systematic and idiosyncratic components. Pairs of borrowers within a portfolio are correlated through their exposures to systematic factors. Specifically, there are two sets of inputs that determine a pairwise asset correlation:

» The proportion of risk that is captured by the systematic factors, or R-squared values
» The correlations among the respective systematic factors (systematic factor correlations)

For a corporate borrower, we identify systematic factors using the borrower's weights to country and industry factors. For a European CRE borrower, we model systematic factors based on the borrower's geographical region (country, sub-region, or city) and property type. In the model, the combination of a property type and a region summarizes all systematic factors affecting a CRE market. With this setup, GCorr Europe CRE can statistically estimate CRE concentrations and correlations across 225 European CRE markets, defined by 45 geographical regions (country, sub-region, or city) and the following five property types:

[1] According to Man Institute. Research available at: `https://www.man.com/maninstitute/enduring-yield`

[2] See BoE Bankstat tables (data available at: `https://www.bankofengland.co.uk/statistics/tables`), and ECB Statistical Data Warehouse (data available at: `https://sdw.ecb.europa.eu/`).

[3] Source: Financial Conduct Authority – Mortgage lending statistics (June 2021); Moody's Analytics.

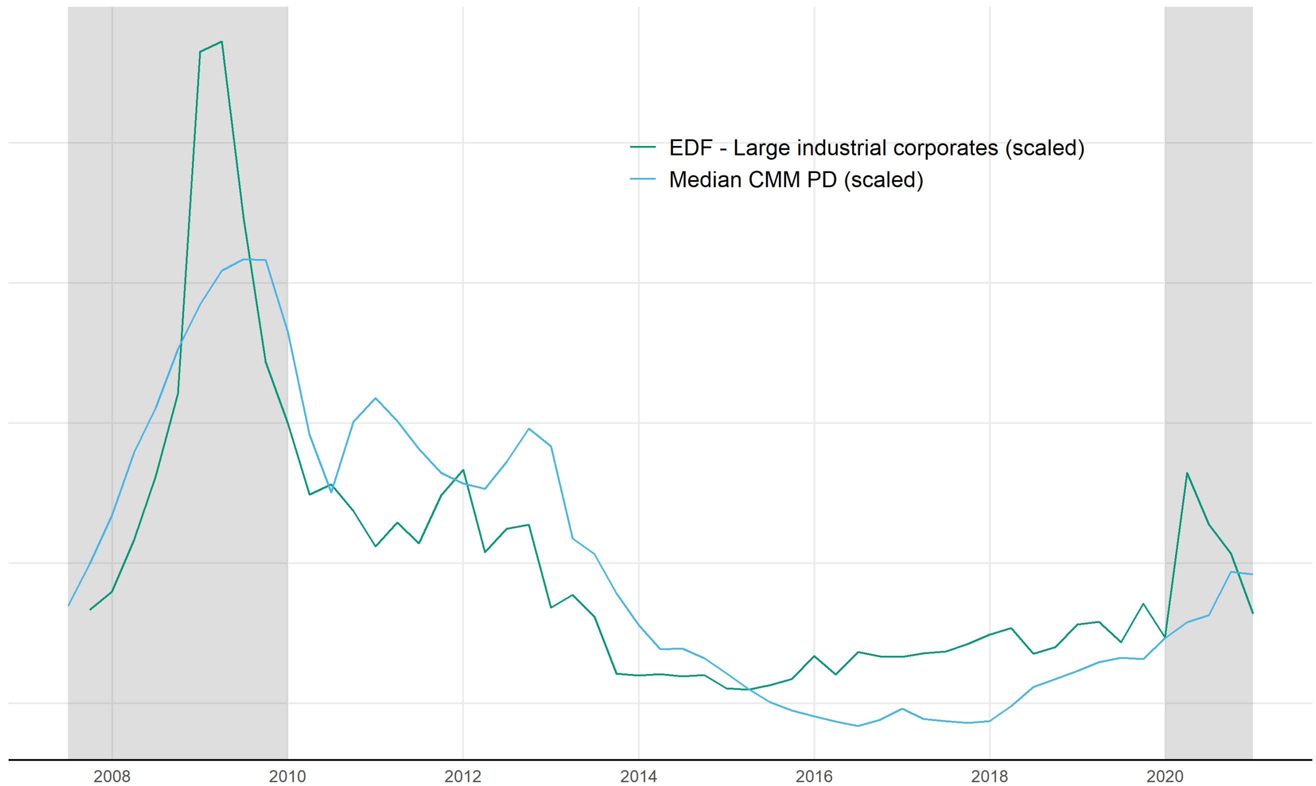

Figure 2: Default Rates of Corporate and CRE Exposures - United Kingdom

- Hotel
- Industrial
- Multifamily Housing
- Office
- Retail

These CRE factors are driven by macroeconomic variables, namely, Gross Domestic Product (GDP), Unemployment Rate (UR), and European CRE Index. With multiple CRE indices, it is possible to capture the cross-CRE class correlation and diversification, as well as the cross-CRE and non-CRE diversification.

The data used to calibrate the GCorr Europe CRE model comes from various sources. Our primary source is the Moody's Analytics CMM® Europe model. This model provides a probability of default (PD) estimate for CRE loans based on their attributes. For macroeconomic data, we obtain GDP, UR, and Europe CRE Index from Moody's Analytics Data Buffet®.

We then construct 45 factors representing European countries, sub-regions and cities, as well as five factors representing property types. The framework introduced in this paper improves CRE portfolio credit risk modeling with much more accurate correlation estimates within CRE exposures, as well as more accurate correlation estimates between CRE exposures and other types of exposures. Specifically, the model produces three types of correlations:

- Intra-CRE market correlation (the correlation between borrowers in the same European CRE market, for example, between borrowers located in Berlin, Germany)
- Inter-CRE market correlation (the correlation between borrowers in two different European CRE markets, for example, between borrowers located in Berlin, Germany and borrowers located in Paris, France)
- Correlation between European CRE and other markets (the correlation between borrowers in a European CRE market and borrowers in other segments such as non-European CRE, retail, or corporate)

This model's level of granularity provides a substantial improvement over existing models, which tend to be based on much coarser classifications. As illustrated in this paper, the cross-sectional variation in correlations across property types and regions can be substantial. This more granular parameterization enables substantially more accurate portfolio credit risk analysis. As we collect more sufficient and robust data internationally, similar correlation models can be built for other parts of the world.

The remainder of this paper is organized as follows:

Section 2 describes the Moody's Analytics modeling framework for asset correlations.
Section 3 describes the data used in the estimation and validation of GCorr 2020 Europe CRE.
Section 4 documents the estimation process.
Section 5 presents the estimated correlation parameters for GCorr 2020 Europe CRE.
Section 6 presents a validation of the correlation model.
Section 7 provides a benefit analysis for using a granular correlation model from a credit risk management perspective.
Section 8 provides concluding remarks.
Appendix A describes the methodology of systematic factors decomposition.
Appendix B provides a table listing the 45 distinct geographical regions and five property types included in the European CRE markets.

2. Moody's Analytics Portfolio Framework

Credit correlations include default correlations and credit migration correlations. Default correlation measures the extent to which the default of one borrower is related to that of another borrower; credit migration correlation measures the joint credit quality change, short of default, for two borrowers. We can infer the credit correlations of two borrowers by measuring their individual default probabilities and their asset correlation. The basic idea is intuitive: a borrower defaults when its asset value falls below the value of its obligations (in other words, its default point).

The joint probability of two borrowers defaulting during the same time period is simply the likelihood of both borrowers' asset values falling below their respective default points during that period. We can determine this probability by knowing the correlation between the two borrowers' asset values, and the individual likelihood of each borrower defaulting, as depicted in Figure 3.

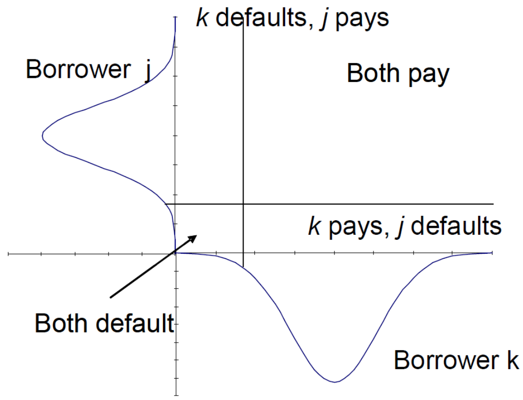

Figure 3: Joint Default Probability

With this setup, the joint distribution of the borrowers' asset values can be specified by the marginal distributions and a copula. Alternatively, the asset values dynamics can be captured by a factor model.

$$r_i = \sqrt{\rho_i}\phi_i + \sqrt{1-\rho_i}\epsilon_i \tag{1}$$

where:

- r_i is the *asset return*[4] of borrower i,
- ϕ_i is the systematic factor,
- ρ_i is the R-squared of borrower i — the proportion of risk that is captured by the systematic factor,
- ϵ_i is the idiosyncratic factor of borrower i.

The systematic factor ϕ_i (also called the custom index) represents the state of the economy during a particular period and summarizes all the relevant systematic risk factors that affect the borrower's credit quality. The variable ϵ_i represents the borrower-specific risk (the idiosyncratic event or shock) that affects the borrower's credit quality or ability to repay debt. While the shock in the systematic factor ϕ_i is the same for borrowers with the same custom index, the borrower-specific shock ϵ_i is unique to each borrower. By construction, the systematic factor ϕ_i is independent of the idiosyncratic factor ϵ_i, and both are distributed with a standard normal distribution. Two borrowers correlate with one another when both are exposed to correlated systematic factors (with potentially varying degrees). Mathematically, the correlation between the changes in credit quality measures for any two borrowers, both within and across asset classes, is equal to:[5]

[4] Asset return can be interpreted broadly as the variable that drives the credit quality changes of the borrower.
[5] The RiskFrontier® Monte Carlo simulation engine simulates correlated asset returns for each borrower i in a normalized space, where each asset return is distributed with a standard normal distribution. See Levy and Kumar (2021) for more details.

$$
\begin{aligned}
corr(r_i, r_j) &= corr(\sqrt{\rho_i}\phi_i + \sqrt{1-\rho_i}\epsilon_i, \sqrt{\rho_j}\phi_j + \sqrt{1-\rho_j}\epsilon_j) \\
&= \frac{cov(\sqrt{\rho_i}\phi_i + \sqrt{1-\rho_i}\epsilon_i, \sqrt{\rho_j}\phi_j + \sqrt{1-\rho_j}\epsilon_j)}{\sigma_{ri}\sigma_{ri}} \\
&= \frac{\sqrt{\rho_i}\sqrt{\rho_j}cov(\phi_i, \phi_j)}{1 \times 1} \\
&= \sqrt{\rho_i}\sqrt{\rho_j}corr(\phi_i, \phi_j)
\end{aligned} \tag{2}
$$

If the underlying borrowers are part of the same market, and thus share the same systematic factor, the correlation equals the product of the square root of the two borrowers' R-squared values:

$$
\begin{aligned}
corr(r_i, r_j) &= corr(\sqrt{\rho_i}\phi_i + \sqrt{1-\rho_i}\epsilon_i, \sqrt{\rho_j}\phi_j + \sqrt{1-\rho_j}\epsilon_j) \\
&= \frac{\sqrt{\rho_i}\sqrt{\rho_j}cov(\phi_i, \phi_j)}{1 \times 1} = \sqrt{\rho_i}\sqrt{\rho_j}
\end{aligned} \tag{3}
$$

Equation 1 serves as the basis for the Monte Carlo simulation for the portfolio credit risk calculation. A factor model such as Equation 1 can be specified by two sets of parameters: the R-squared values of all borrowers and the correlations among systematic factors.[6]

For a corporate borrower, the systematic factors consist of 49 country factors and 61 industry factors. Using a parallel framework, the systematic portion of CRE credit risk is decomposed into geographic regions and property type factors. In particular, for the GCorr 2020 Europe CRE model, we describe the systematic risk of the exposures using geographical factors and property-type factors, driven by macroeconomic variables, as depicted in Figure 4.

[6] See Levy and Kumar (2021) for details regarding how the RiskFrontier Monte Carlo engine simulates correlated asset returns for various asset classes.

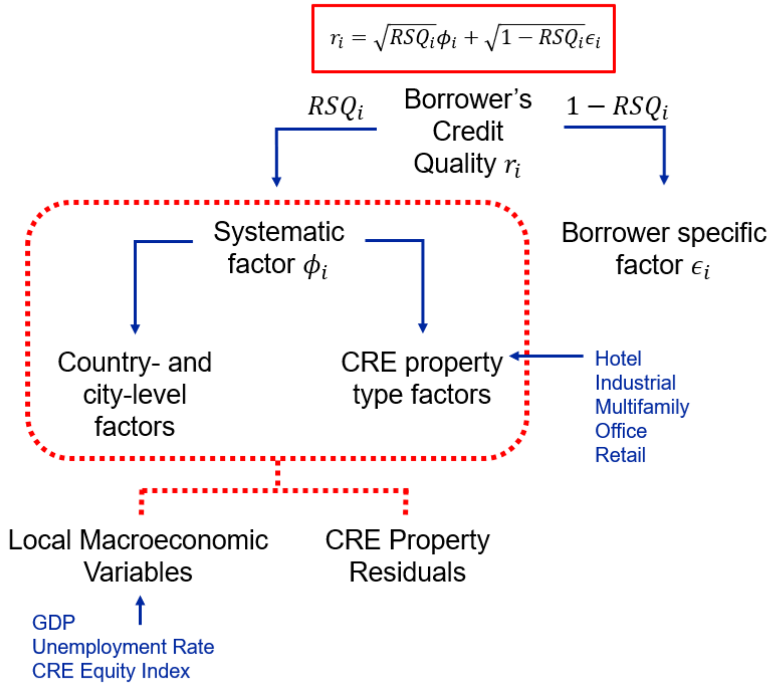

Figure 4: CRE Factor Model

Europe CRE factors have an explicit link to local economic conditions. The CRE factors that are driven by macroeconomic variables have a more economically intuitive interpretation and allow users to perform stress testing at the national level.

GCorr 2020 Europe CRE contains 225 unique composite factors, which are the combinations of one of the 45 geographical factors, plus one of the five property type factors. The 45 geographical factors are the general economic factors that drive the credit qualities of borrowers in these major regions. The five property type factors are the property-specific factors that reflect systematic shocks to the credit qualities of these types of exposures. We chose these specific factors after considering the factors that determine the credit quality of CRE loans, the granularity of the modeling framework, and data availability.

To estimate R-squared values, we examined the correlation patterns grouping borrowers by geographical region and property type. The model provides 225 different R-squared values for each possible combination of region and property type.

Next, we describe the data sources used in estimating the model.

3. Data

This section discusses the data used to model GCorr 2020 Europe CRE. We estimate the model using data for July 1999–December 2020 period.

We use various CRE datasets to estimate the model. In particular, we leverage the newly-updated[7] Moody's Analytics CMM® Europe model. CMM® Europe provides probability of default (PD) and loss given default (LGD) for commercial real estate properties in 18 countries, over 60 cities (sub-regions), and various property types and sub-types.[8]

In the CMM® framework, the underlying collateral value backing a CRE loan (in other words, the commercial property) is modeled as a stochastic process driven by both market-wide and idiosyncratic factors. Monte Carlo simulation is used to generate future paths of collateral net operating income (NOI) and market value. In the model, a CRE loan's credit event is doubly triggered by the collateral's financial condition at the time of default: both the sustainable NOI falls below the total debt service, and the property's market value falls below the total outstanding loan balance. In order to capture the actual observed borrower default behavior, CMM® is empirically calibrated to historical defaults. The various datasets used by CMM® to estimate collateral NOI and value are listed below.

- » Cushman & Wakefield—Provides national, city and submarket-level aggregate market statistics for office, retail and industrial.
- » MSCI's IPD real Estate Index—Provides national, city, and submarket-level aggregate market statistics across property types.
- » Trepp's Commercial Mortgage-Backed Securities (CMBS) Deal Library—One of the largest commercially-available databases in the CMBS universe. The Deal Library contains comprehensive information and history on the properties that serve as collateral within the CMBS transactions.
- » Moody's Analytics Data Buffet®—Source of historical time-series data for macroeconomic variables of interest, including GDP, unemployment rate, inflation, interest rate, and home prices.
- » Proprietary data and models used to derive historical NOI and forecasts; various published studies and reports.

We use two outputs from CMM®: the time series of default probabilities for typical properties in the European CRE markets and the NOI volatility parameters.

[7] Starting from January 2021, CMM® Europe switches data providers from Coldwell Banker Richard Ellis (CBRE) Econometric Advisors to Cushman & Wakefiled (C&W), offering expanded country, city, and property type coverage. CMM® Europe is constantly being updated to include additional submarkets.
[8] For more information on CMM®, please see Bao and Chen (2018).

4. Estimation of Correlation Parameters

As shown in Equation 2, the correlation between two borrowers is determined by each borrower's R-squared (ρ) and the correlation between the borrowers' systematic factors. For example, the asset correlation between two office borrowers located in Berlin, Germany is equal to ρ^{Office}_{Berlin}, and the asset correlation between a Berlin, Germany office and a Paris, France office borrower is calculated as follows:

$$\sqrt{\rho^{Office}_{Berlin}}\sqrt{\rho^{Office}_{Paris}}corr(\phi^{Office}_{Berlin}\phi^{Office}_{Paris}) \tag{4}$$

4.1 R-squared (ρ) Estimation

To estimate a commercial property's R-squared, we measure what proportion of the property's NOI variation is explained by the market-level NOI variation. Specifically, we estimate R-squared using the following formula:

$$RSQ_{GEO_k,PT_j} = \frac{\sigma^2_{M_{GEO_k,PT_j}}}{\sigma^2_{M_{GEO_k,PT_j}} + \sigma^2_{H_{GEO_k,PT_j}}} \tag{5}$$

where:

- $\sigma^2_{H_{GEO_k,PT_j}}$ represents the volatility of the idiosyncratic factor for geographical region k and property type j which is estimated from property-level NOI time series and available in CMM®
- $\sigma^2_{M_{GEO_k,PT_j}}$ represents the volatility of the market factor for geographical region k and property type j which is estimated from market-level NOI time series and available in CMM®[9]

The systematic and idiosyncratic volatilities drive the stochastic process of log NOI changes in the CMM® model.[10] While it is preferable to define R-squared values as the ratio of systematic variance to total variance of total returns, the property-level capitalization rates are not currently available to us.[11]

The CMM® model provides idiosyncratic and market volatilities for 150 CRE market combinations. The R-squared estimation starts by using Equation 5 to calculate the raw R-squared values for the 150 markets with available data. Next, we regress the raw R-squared values onto geographical region and property type indicator variables using the following regression:

$$RSQ_{GEO_k,PT_j} = \beta_{GEO_k}C_{GEO_k} + \beta_{PT_j}C_{PT_j} + \epsilon_i \tag{6}$$

where:

- C_{GEO_k} is 225 x 45 matrix of dummy variables indicating geographical region
- C_{PT_k} is 225 x 5 matrix of dummy variables indicating property type

The regression allows us to determine 225 final R-squared values by using estimated beta parameters for each combination of 5 property types and 45 geographical regions.[12]

4.2 Estimation of Correlations among Systematic Factors

To measure the correlation between systematic factors ($corr(\phi_i, \phi_j)$) affecting two CRE borrowers, we define systematic credit risk factors ϕ_i directly in terms of macroeconomic variables, namely country-level GDP (GDP), country-level Unemployment Rate (UR), and Europe-wide CRE Index (CREI).

9 Because the market and idiosyncratic factor are orthogonal by construction, the total volatility is equal to the sum of the systematic and idiosyncratic volatilities.
10 For details on how the volatilities are estimated, see Bao and Chen (2018).
11 Total return on commercial property includes not only NOI, but also a capital appreciation component. However, based on the available data we observe that the additional variance from the pricing side is marginal over a long period. This variance characteristic occurs because pricing metric has a mean-reverting property, thus limiting its variance over a long horizon when compared to the NOI levels, which do not necessarily have such a mean-reverting property.
12 Final R-squared values are subject to further adjustments, such as trimming the outliers.

Mathematically, we assume the loan systematic factors of CRE loans follow Equation 7. We rescale the systematic factors to have a mean of zero and a standard deviation of one.

$$\phi_i = \beta_{i,GDP} * \phi_{GDP,C(i)} + \beta_{i,UR} * \phi_{UR,C(i)} + \beta_{i,CREI} * \phi_{CREI} + \epsilon_{PT(i),GEO(i)} \tag{7}$$

where:

- ϕ_i is the systematic factor for borrower i,
- $GEO(i)$, $PT(i)$ and $C(i)$ are the Geographical Region, Property Type and Country of borrower i respectively,
- $\phi_{GDP,C(i)}$, $\phi_{UR,C(i)}$ and ϕ_{CREI} are the macroeconomic variables for GDP, UR, and CRE Index respectively,
- $\epsilon_{PT(i),GEO(i)}$ are the residual latent factors representing region $GEO(i)$ and $PT(i)$,
- $\beta_{i,k}$ is the marginal impact of macroeconomic variable k on CRE loan systematic factor i,
- Regression R-Squared is the percentage of ϕ_i's variance that can be explained by $\phi_{GDP,C(i)}$, $\phi_{UR,C(i)}$ and ϕ_{CREI}.

The benefit of introducing macroeconomic variables into the CRE model is to attach economic interpretation to the factors. Using Equation 7, we can explain factor patterns in terms of the geographical region's economic performance.

It is worth noting that Equation 7 also contains residual terms $\epsilon_{PT(i),GEO(i)}$, which account for the fact that, while macroeconomic variables have a strong relationship with CRE default data, they do not fully explain the dynamics in the default data and their correlations.

We use negative change in quarterly probability of default from CMM® $(PD_t - PD_{t+1})$ and log returns of quarterly macroeconomic variables. We detrend certain time series (GDPs, CRE Index) to make sure they are stationary.

With the properly-transformed data, we then construct the systematic CRE factors and estimate the factor correlations using the following steps:

1. $\beta_{i,GDP}$, $\beta_{i,UR}$. $\beta_{i,CREI}$ and Regression R-squared estimation.
   For each pair of (GEO, PT), we regress the negative changes in CMM® PD on the log return of macroeconomic variables with various lags to obtain coefficients $\beta_{i,GDP}$, $\beta_{i,UNR}$. $\beta_{i,CREI}$, and regression R-squared values. For each candidate model, we check the stationarity of the residuals and assess the overall significance of the regression model plus the individual significance of each coefficient. Then, for each pair of (GEO, PT), we select from all the models estimated with the highest adjusted R-squared value and correct coefficient signs.

2. $\epsilon_{PT(i),GEO(i)}$ estimation.
   To capture regression residuals beyond what is explained by regional macroeconomic variables, we conduct principal component analysis of the residuals from the various CRE markets to get CRE-specific risk factors.

By now, we have successfully estimated all the elements needed to construct the Europe CRE factors. The remaining steps link the factors to the current GCorr 2020 model.

3. Constructing "raw" Europe CRE factors.
   We construct the 225 (45 geographical regions x 5 property types) raw CRE factors using macroeconomic variables and regression residuals, which are already linked to the GCorr matrix, using the coefficients from Step 1.

4. Decomposition of raw Europe CRE factors.
   We decompose the 225 raw Europe CRE factors into 45 geographical region and five property type factors. Step 3 gives us 225 Europe CRE factors, including all the combinations of geographical regions and property types. We break the 225 CRE factors down, in an additive way, 45 geographical region and five property type factors. Appendix A describes how we perform this factor decomposition.

5. Constructing covariance matrix.
   We calculate the covariances across the CRE factors, between the CRE factors and other factors, as well as between the CRE factors, macroeconomic variables and local macroeconomic variables as shown in 5. These covariances are implied by the link of the raw CRE factors to the rest of the matrix and the decomposition from Step 4.

In calibrating the model, we impose various economic conditions on the correlations. For example, no negative systematic factor

	GCORR CORPORATE	GCORR CRE	GCORR RETAIL	LOCAL MACROVARIABLES	NATIONAL MACROVARIABLES
GCORR CORPORATE	$\Sigma_{Corp,\ Corp}$				
GCORR CRE	$\Sigma_{CRE,\ Corp}$	$\Sigma_{CRE,\ CRE}$			
GCORR RETAIL	$\Sigma_{Retail,\ Corp}$	$\Sigma_{Retail,\ CRE}$	$\Sigma_{Retail,\ Retail}$		
LOCAL MACROVARIABLES	$\Sigma_{LMV,\ GCorr}$			$\Sigma_{LMV,\ LMV}$	
NATIONAL MACROVARIABLES	$\Sigma_{MV,\ GCorr}$			$\Sigma_{MV,\ LMV}$	$\Sigma_{MV,\ MV}$

Figure 5: Constructing the Expanded Correlation Matrix

correlations should exist across CRE borrowers and between CRE and other asset classes.[13] Also, the systematic factor correlation across property types and geographical regions preserves the rank ordering observed from the empirical data.

4.3 Asset Correlation

We can now put together the R-squared value and the custom index described in the previous section to generate asset correlations between two CRE loan borrowers. In the GCorr model, each CRE market is defined as a property type and geographical region combination. The asset correlation between borrowers i and j in CRE markets depends on the R-squared values of borrowers i and j and the correlations of their custom indices.

$$Corr(r_i, r_j) = \sqrt{\rho_i}\sqrt{\rho_j}Corr(\phi_i, \phi_j) \tag{8}$$

[13] We adjust factor correlations such that the modeled correlations are economically intuitive and sound. Specifically, the rationale for no negative correlations condition is that, while in the short run one can observe negative empirical correlations between credit quality of various assets, in the long run custom factors are driven by shared, broad macroeconomic factors. Therefore, in the long run, positive correlations between them can be expected.

5. GCorr 2020 Europe CRE Results Overview

This section presents key empirical results of GCorr 2020 Europe CRE model estimation. We first discuss R-squared estimates and then provide an overview of estimated asset correlations.

5.1 R-Squared Values

In this section, we present the Europe CRE R-squared values estimated for GCorr 2020 Europe CRE model. GCorr 2020 Europe CRE R-squared values are estimated using volatility data from Moody's Analytics CMM® model.

Table 1 presents summary statistics of R-squared values by property type for GCorr 2020 Europe CRE. We observe that Hotel properties exhibit the largest R-squared values, followed by Multifamily Housing properties. The levels are significantly higher than levels observed for the other property types. Industrial properties exhibit the smallest R-squared values. In a context of the COVID-19 pandemic, we observe the highest fluctuations in credit events for Hotel property type due to the heavily decreased demand in the hospitality sector. At the same time, sustained demand for logistics space led to a relatively moderate increase in defaults caused by macroeconomic conditions for the Industrial property type. These observations are consistent with the R-squared rank ordering shown in Table 1.

Table 1: R-squared statistics for GCorr 2020 Europe CRE

PROPERTY TYPE	MIN	P25	MEDIAN	P75	MAX
Hotel	40.25%	54.71%	58.23%	62.06%	75.52%
Industrial	6.20%	18.31%	21.23%	24.40%	35.69%
Multifamily	27.45%	42.12%	45.68%	49.56%	63.21%
Office	10.93%	20.45%	22.75%	25.24%	34.11%
Retail	9.83%	22.43%	25.48%	28.78%	40.52%

5.2 Asset Correlations

The GCorr 2020 Europe CRE asset correlations among different hypothetical Europe CRE borrowers are presented in Figure 6. A hypothetical CRE borrower has 100% weight to a geographical region (country, sub-region, or city) and 100% to a property type (45 borrowers for each property type), and is assigned the R-squared value from GCorr 2020 Europe CRE. The correlations within a property type (for example, Hotels-Hotels) include 45 x (45 - 1) / 2 = 990 unique pairwise correlations. The cross-property type asset correlations include 45 x 45 = 2,025 unique pairwise correlations.

Each block in Figure 6 contains the histogram of the pairwise asset correlations, as well as the median asset correlation. The important observation is that for each property-type combination, there is significant cross-sectional variation across the geographical regions. Combinations within Hotels properties exhibit the largest asset correlation, followed by combinations within Multifamily Housing properties. The cross-property type combinations between Hotels and Multifamily Housing properties exhibit the largest asset correlation among cross-property type combinations.

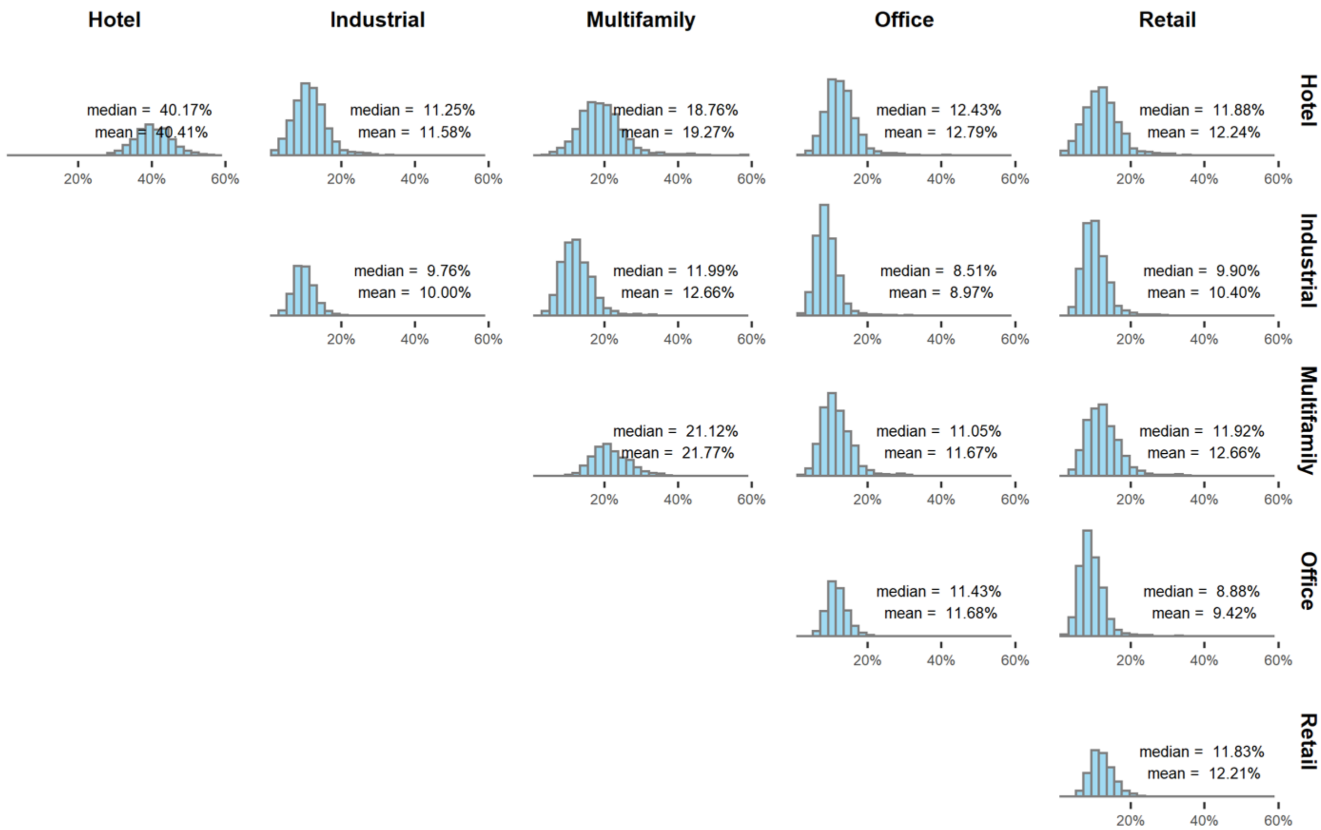

Figure 6: Distribution of asset correlations within Europe CRE borrowers by property type

Figure 7 shows the asset correlations between hypothetical Europe CRE borrowers and hypothetical corporate borrowers. A hypothetical CRE borrower has 100% weight to a geographical region and 100% to a property type (45 borrowers for each property type), and is assigned the R-squared value from GCorr 2020 Europe CRE. The hypothetical corporate firm assumes 100% weight to one of the 18 European countries and 100% weight to one of the 61 corporate industries. It uses the median R-squared value for the corresponding industry from GCorr 2019 Corporate. Each block in Figure 7 contains the histogram of up to 45 x 18 x 61 = 49,410 unique pairwise asset correlations,[14] as well as the median asset correlation. As in the case of correlations within CRE, there is significant cross-sectional variation across the geographical regions. Comparing the property types, Hotel and Multifamily Housing are the most correlated with Europe corporate.

[14] Note that R-squared values for some country-industry combinations are not available (in cases of smaller countries), thus the actual number of pairwise correlations might be smaller.

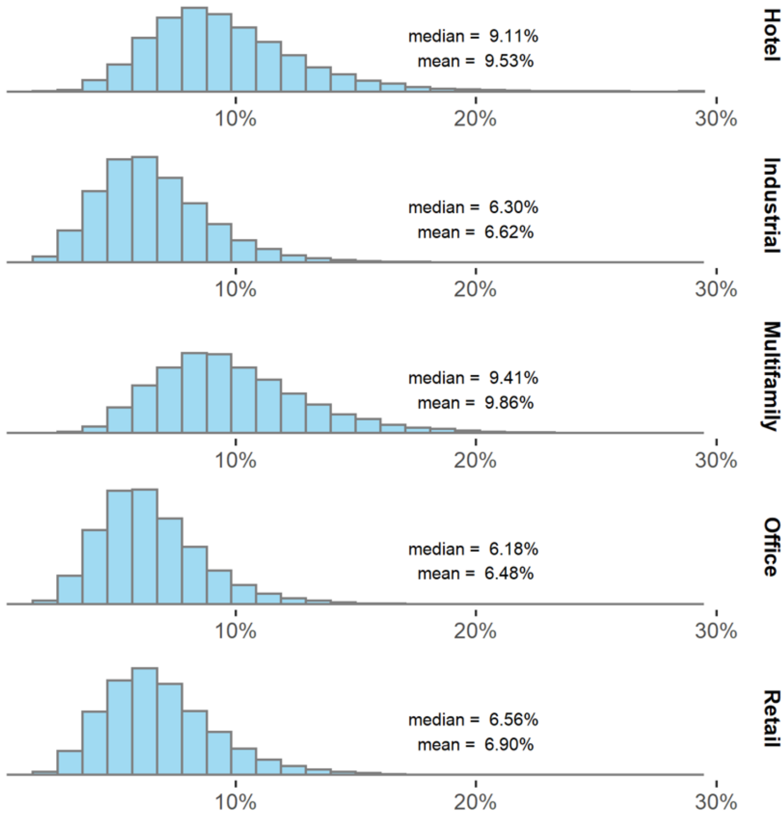

Figure 7: Distribution of asset correlations between Europe CRE and Europe Corporate borrowers by property type

The results highlighted above provide an overview of the core output of GCorr 2020 Europe CRE model; they are generally consistent with asset correlations produced by the previously developed GCorr CRE models. For example, we observe the range of average asset correlations of approximately 15%-34% in the case of the GCorr 2020 US CRE model and 15%-29% in the case of the GCorr 2020 Canada CRE model.[15] This is comparable to the range of 10%-40% shown in Figure 6. A somewhat broader cross-sectional variation is expected due to the larger systematic differences between European countries.

Next, we perform several exercises to further establish the validity of the results.

[15] See Pei and Jiang (2021b) and Pei and Jiang (2021a) for details.

6. Validation of Correlation Estimates

In this section, we present validation exercises for the GCorr 2020 Europe CRE model.

6.1 Comparison to Default-Implied Asset Correlations

To validate the correlations from GCorr 2020 Europe CRE, we compare the modeled asset correlation levels to those implied by correlations among delinquency rates. Although we use delinquency rates, the terms default and delinquency are interchangeable in the following discussion.

The joint default probability of borrower j with borrower k, denoted by JDF_{jk}, is given by:

$$JDF_{jk} = Prob(Borrower\ j\ defaults,\ Borrower\ k\ defaults) = N_2(N^{-1}(PD_j), N^{-1}(PD_k), \rho_{jk}) \tag{9}$$

Where

- N_2 is a bivariate standard normal distribution,
- N^{-1} is the inverse of a standard normal distribution,
- PD is the cumulative default probability,
- ρ_{jk} is the asset correlation between borrower j and borrower k.

For a pair of borrowers from different homogeneous pools, the realized default correlation for borrowers belonging to these two groups is given by:[16]

$$\rho^{default}_{Group\ j,\ Group\ k} = \frac{cov(DR_{Group\ j}, DR_{Group\ k})}{\sqrt{\mu_{Group\ j}(1-\mu_{Group\ j})}\sqrt{\mu_{Group\ k}(1-\mu_{Group\ k})}} \tag{10}$$

which can be related to the asset correlations as:

$$\rho^{default}_{Group\ j,\ Group\ k} = \frac{N_2(N^{-1}(\mu_{Group\ j}), N^{-1}(\mu_{Group\ k}), \rho_{jk}) - \mu_{Group\ j}\mu_{Group\ k}}{\sqrt{\mu_{Group\ j}(1-\mu_{Group\ j})}\sqrt{\mu_{Group\ k}(1-\mu_{Group\ k})}} \tag{11}$$

Where

- $DR_{Group\ j}$ is the default/delinquency rate for group j,
- $\mu_{Group\ j}$ is the empirical mean of default/delinquency rate series j,
- $DR_{Group\ k}$ is the default/delinquency rate for group k,
- $\mu_{Group\ k}$ is the empirical mean of default/delinquency rate series k,
- ρ_{jk} is the default/delinquency implied asset correlation between group j and group k.

Using Equations 10 and 11, we can calculate the default/delinquency implied asset correlation between two different homogeneous groups.

To validate the modeled asset correlations within Europe CRE, we use default rate time series from Moody's Analytics CMM® model to calculate the default implied asset correlations among different property types. We also use this methodology to calculate the default implied asset correlation between Europe CRE and Europe corporate defaults. To construct corporate default series, we create the following three default series based on size and industry sector:

- Large Europe Non-Financials
- Large Europe Financials
- Large Europe Combined

Table 2 shows the default implied asset correlation range within CRE borrowers and between CRE and corporate borrowers.

Comparing the default implied asset correlations shown in Table 2 to the modeled asset correlations depicted in Figure 6, we see that modeled asset correlations within CRE borrowers and between CRE and Corporate borrowers are in the same range as default implied

[16] For more details on default implied asset correlations, see Lee and Zhu (2008)

Table 2: Default-Implied Asset Correlation Range within Europe CRE Borrowers, between Europe CRE Borrowers and Europe Corporate Borrowers

	DEFAULT-IMPLIED
Within CRE borrowers	0.01%-22.25%
Between CRE and Corporate borrowers	0.04%-14.96%

correlations; they are also higher, on average. From a risk management perspective, it is reassuring that the modeled correlations are on the conservative side of the default-implied asset correlation levels.[17]

6.2 Validation of Projected Losses with Historical Scenarios at the National Level

This section presents several analyses illustrating levels and patterns in CRE credit portfolio losses produced by GCorr 2020 Europe CRE over historical, state-level economic scenarios. Our objective is to understand how different aspects of the modeling framework impact losses.

We run the exercise as follows: at the beginning of each quarter, we apply the realized macroeconomic variables during the following four quarters together with the input unconditional probability of default (PD) to generate the projected four-quarter cumulative loss using the GCorr model. We use a constant input PD for each portfolio and loss given default (LGD) = 100%.

As an example, we present results for two national CRE portfolios: Italy and UK. The portfolios are constructed using 12 x 5 instruments for UK (representing three major cities, eight sub-regions, nation, and five property types) and 6 x 5 instruments for Italy (five major cities, nation, and five property types). Figures 8 and 9 show the result of portfolio loss at the national level. We use national GDP, national UR, and Europe CRE Index.

As the results illustrate, our model projects loss dynamics consistent with those observed empirically. In particular, losses are higher during economic crises, such as the 2008 financial crisis. We also observe increased losses for Italy during the European debt crisis period. From a level perspective, the model predicts stressed losses at a similar level compared to those observed in the data.

[17] Note that the average default-implied correlations are affected by high values for Hotel and Multifamily property types, for which CMM® data is fairly limited. Focusing on the property type with the richest data (Industrial), we observe more comparable average levels of modeled and default-implied asset correlations (3.32% and 9.76% respectively).

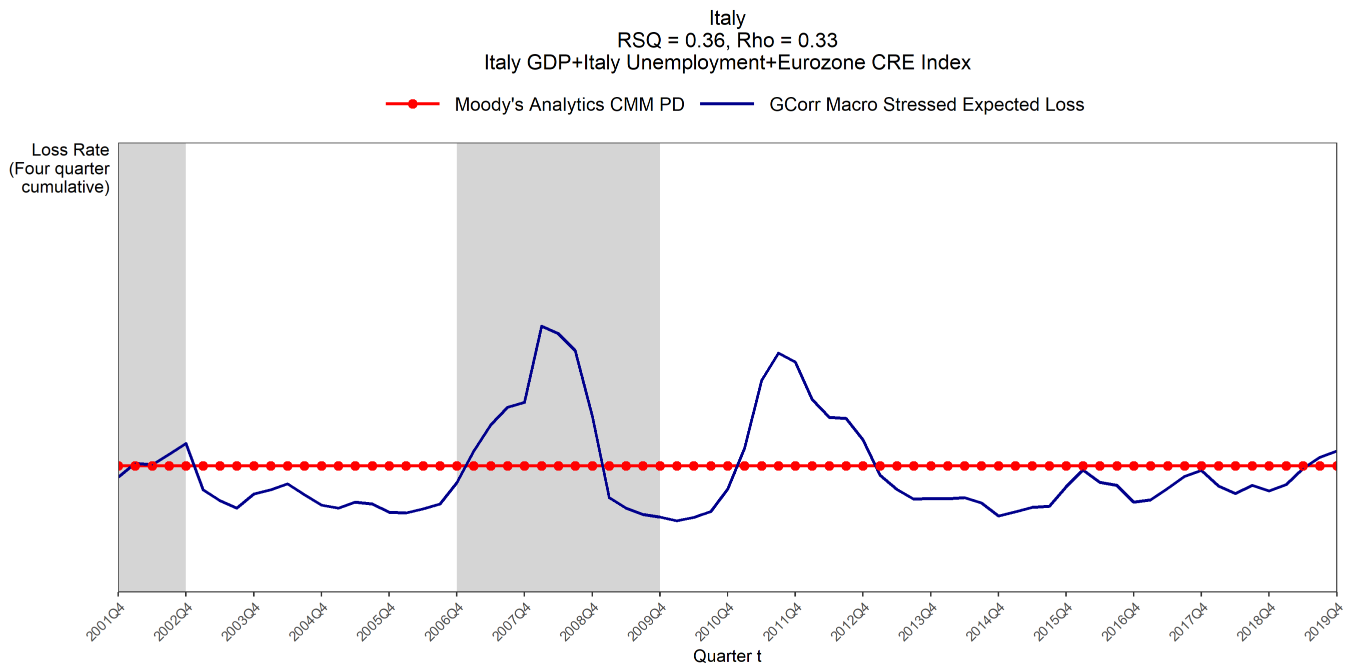

Figure 8: Italy national portfolio loss dynamics with constant input PD and LGD = 100%

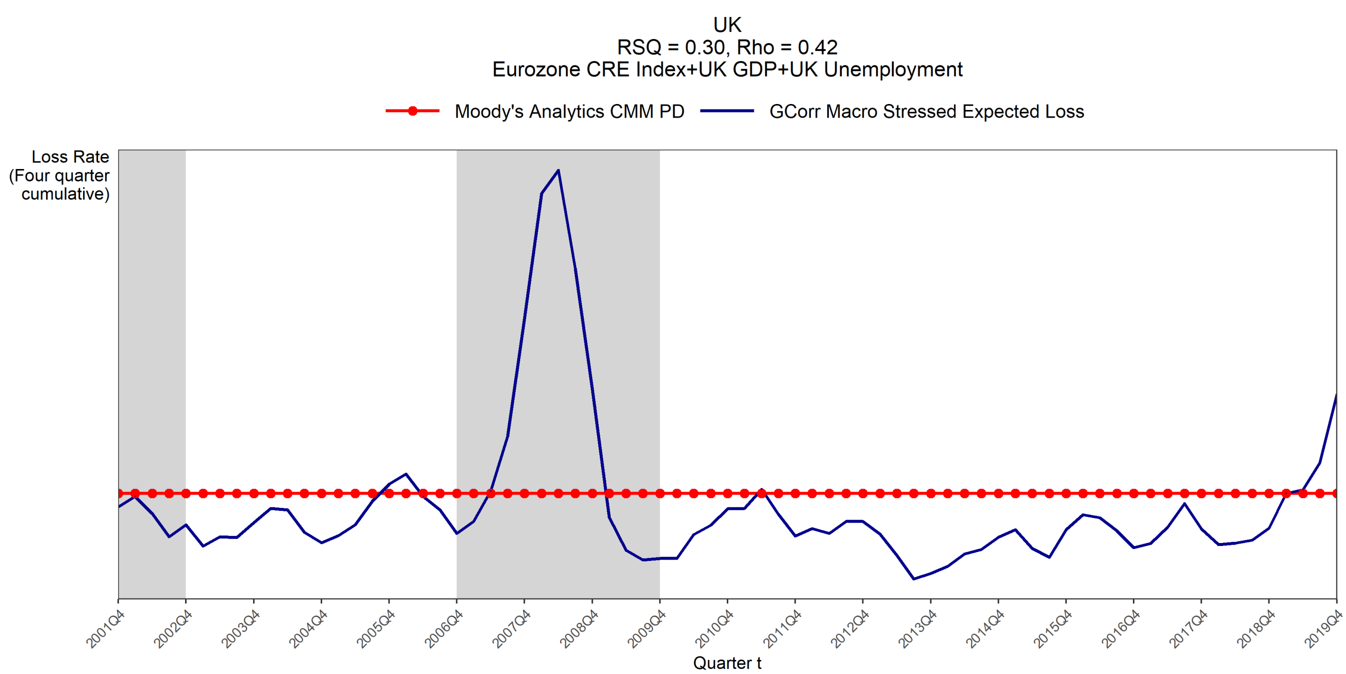

Figure 9: UK national portfolio loss dynamics with constant input PD and LGD = 100%

7. Utilizing a Granular Correlation Model for Credit Risk Management

In this section, we analyze the benefits of using a more granular correlation model from a credit risk management perspective. In particular, we look at a sample CRE portfolio to evaluate the impact of our model in measuring portfolio credit risk. The sample portfolio includes 5,000 Europe CRE accounts, with a total of EUR 5 billion in commitment. We allocate the accounts to CRE indices of 45 regions (18 European countries, 27 cities or sub-regions), and five property types. Table 3 presents the RiskFrontier® output after running the portfolio with the following two setups:

» Run 1: Map all CRE instruments to industry N46 (Real Estate). We allocate CRE accounts in this setup to industry N46 and one of the 18 European countries. In this model, a custom index for all CRE factors is determined based on geographical location only. For this run, we use GCorr 2020 Corporate.

» Run 2: CRE-only using CRE indices. We allocate CRE accounts to 18 European countries, 27 sub-regions or cities and five property-type factors. For this run, we use GCorr 2020 Europe CRE.

Note, R-squared values are the same between Run 1 and Run 2.

Table 3: RiskFrontier Outputs for CRE Portfolio Analysis (in EUR)

	USING GCORR 2020 CORPORATE		USING GCORR 2021 EUROPE CRE	
	Amount w.r.t. EL	% MTM Exposure	Amount w.r.t. EL	% MTM Exposure
Commitments	5,000,000,000		5,000,000,000	
Book Exposure	5,000,000,000		5,000,000,000	
MTM Exposure	5,000,000,000		5,000,000,000	
Total Spread Revenue	47,139,500	94.3 bp	47,139,500	94.3 bp
Expected Loss	12,438,908	24.9 bp	12,438,908	24.9 bp
Expected Spread Revenue	34,700,592	69.4 bp	34,700,592	69.4 bp
Unexpected Loss	116,656,170	233.3 bp	84,898,820	169.8 bp
Capital	664,557,385	1329.1 bp	416,593,655	833.2 bp
Expected Shortfall	779,474,377	1558.9 bp	477,839,395	955.7 bp
Sharpe Ratio	29.75%		40.87%	
RORAC	5.28%		8.39%	

As expected, Run 1 leads to a higher capital number at 1,329 basis points. In Run 1, we map all CRE accounts to a single industry and 18 countries. In this case, the ability to identify any concentration pockets that exist in the portfolio is reduced. By using a granular model in Run 2, we can more accurately identify concentrations and see a diversification benefit within the CRE portfolio, with economic capital reduced to 833 basis points. Table 3 also suggests the corporate factors cannot fully capture the co-movement of CRE indices.

We also look at the portfolio runs of a stand-alone CRE portfolio and a combined CRE, retail, and corporate portfolio to evaluate the impacts of our model in measuring portfolio credit risk. Table 4 displays capital with respect to expected loss and presents the RiskFrontier® output after running the portfolio with the following setups:

» Run 1: CRE-only portfolio using CRE indices. We allocate CRE accounts to one of the 45 regions (18 European countries, 27 cities or sub-regions) and five property types. For this run, we use GCorr 2020 Europe CRE model.

» Run 2: Corporate-only portfolio using corporate indices. We allocate corporate accounts to 57 countries and 61 industries. For this run, we use GCorr 2020 Corporate model.

» Run 3: Retail-only portfolio using retail indices. We allocate retail accounts to one of the 18 regions (seven European countries and one of the 11 regions of the UK) and four retail product types. For this run, we use GCorr 2020 Europe Retail model.[18]

» Run 4: Combined CRE corporate and retail portfolio. Portfolio comprised of 1/3 CRE, 1/3 corporate, and 1/3 retail exposures (in terms of book commitment). For this run, we use GCorr 2020 model combined with GCorr 2020 Europe CRE model.

[18] Please refer to Ozkanoglu and Astakhov (2020) for additional details.

Table 4: RiskFrontier Output for Combined Corporate, Retail, and CRE Portfolio Analysis (in EUR)

Portfolio	Commitment Amount	Stand-Alone Capital		Combined Capital		Diversification Benefit
CRE Only	5,000,000,000	416,593,630	833.2 bp	366,747,726	733.5 bp	-11.97%
Corporate Only	5,000,000,000	146,149,507	292.3 bp	59,739,435	119.5 bp	-59.12%
Retail Only	5,000,000,000	131,188,836	262.4 bp	111,516,956	223.0 bp	-15.00%
Total	15,000,000,000	693,931,973	462.6 bp	538,004,117	358.7 bp	-22.47%

The stand-alone retail portfolio leads to a higher capital requirement of EUR 416.6 million. The CRE part of the combined portfolio only needs EUR 366.7 million and generates approximately 12% reduction in capital requirement, when allocating capital based on risk contribution. Corporate and retail parts of the combined portfolio also show a significant reduction in capital requirement, about 59% and 15% respectively compared to the stand-alone portfolios.

As Table 4 shows, GCorr 2020 Europe CRE more accurately captures the diversification benefit provided by including various asset classes in a combined credit portfolio.

8. Conclusion

Credit portfolio risk assessment and management require a reasonably accurate model that accounts for correlations within and across different asset classes. Moody's Analytics GCorr framework provides the means for estimating such correlations for a wide range of assets, including corporate, CRE, and retail exposures. For credit institutions with substantial European CRE exposures, Moody's Analytics has developed the GCorr 2020 Europe CRE correlation model which provides an integrated and granular framework for measuring the portfolio risk of such entities.

In GCorr 2020 Europe CRE, asset correlations between retail borrowers are determined using two inputs: correlation between systematic factors, which represent the state of the economy and summarize all the relevant systematic risk factors that affect the borrower's credit quality, and R-squared values, which measure the proportion of risk that is captured by the systematic factor (as opposed to idiosyncratic risk). To estimate these correlations, we use data from Moody's Analytics CMM® Europe model which covers July 1999—December 2020, and includes the impact of the COVID-19 pandemic. Methodology for estimation of these inputs provides clear economic intuition for the resulting asset correlations by implicitly linking those to empirical default rates observed in the European CRE market, and to macroeconomic variables such as GDP, Unemployment Rate, and CRE Index. This approach enables model users to perform stress testing with real macroeconomic scenarios.

Further applications of the model lie in the estimation of various portfolio risk statistics, such as required economic capital within Moody's Analytics RiskFrontier® framework. We find that the GCorr 2020 Europe CRE model, which provides correlation estimates for European CRE exposures at a granular level, is better positioned to identify concentrations and capture co-movement of retail indices than the coarser model. Utilizing GCorr 2020 Europe CRE for estimation of economic capital requirements in a portfolio with substantial European CRE exposures provides significant diversification benefit.

Finally, the methodologies outlined in this paper can also be combined with institution-specific data to create a custom correlation model focused on particularities of a portfolio.

Appendix A. Decomposing Systematic Factors

In GCorr 2020 Europe CRE, the model covers 45 geographical regions and five property types, spanning 225 Europe CRE markets. To reduce dimensionality, smooth out model noise, and simplify the framework, we introduce the additive structure into the GCorr CRE framework:

$$\phi_{r,p} = \phi_r + \phi_p$$

where

- » $\phi_{r,p}$ represents the 225 "raw" CRE factors for each combination of 45 geographical regions and five property types,
- » ϕ_r represents the 45 decomposed geographical factors,
- » ϕ_p represents the five decomposed property factors.

For example, the return of Berlin Office properties is regressed onto a 100% weight on Berlin and 100% weight on Office. The decomposition can be achieved through different approaches. In GCorr 2020 Europe CRE, we apply the following methodology.

Correlation Optimization

Given that the factors are estimated for the correlation model, one way of decomposing the factors is to ensure that the decomposed factors imply a similar level of correlation as the original, undecomposed factors. Mathematically, assuming the factor follows the structure $r_{r,p} = r_r + r_p$, the decomposed r_r and r_p can be found by minimizing the distance function:

$$\{r_r, r_p\} = argmin_{r_r, r_p} \sum_{i,j} \left(cor(r_{r_i} + r_{p_i}, r_{r_j} + r_{p_j}) - cor(r_{r_i, p_i}, r_{r_j, p_j})\right)^2$$

The optimized solution can be obtained with a numerical method. A common approach is conjugate gradient, where we start from an initial guess, and move the variables toward the optimized number based on the gradient of the distance function iteratively.

Appendix B. List of Property Types and Geographical Regions

Table 5 shows the 45 distinct geographical regions and five property types included in Europe's CRE markets. Each European CRE market in the model is defined by the combination of these factors (for example, "Berlin Hotel" market will have weight of one on factor CREGDE002 and weight of one on factor CREPEU01).

Table 5: Geographical Regions and Property Types

Factor Code	Region	Factor Code	Region
CREGAT001	AUT Nation	CREGGB007	GBR North East
CREGBE001	BEL Nation	CREGGB008	GBR Scotland
CREGCH001	CHE Nation	CREGGB009	GBR South East
CREGCZ001	CZE Nation	CREGGB010	GBR Wales
CREGDK001	DEN Nation	CREGGB011	GBR West Midlands
CREGDE002	DEU Berlin	CREGGB012	GBR Yorkshire and the Humber
CREGDE003	DEU Dusseldorf	CREGIE001	IRL Nation
CREGDE004	DEU Frankfurt	CREGIT002	ITA Bologna
CREGDE005	DEU Hamburg	CREGIT003	ITA Genoa
CREGDE006	DEU Munich	CREGIT004	ITA Milan
CREGDE001	DEU Nation	CREGIT001	ITA Nation
CREGDE007	DEU Stuttgart	CREGIT005	ITA Rome
CREGES001	ESP Nation	CREGIT006	ITA Turin
CREGFI001	FIN Nation	CREGLU001	LUX Nation
CREGFR002	FRA Lille	CREGNL001	NLD Nation
CREGFR003	FRA Lyon	CREGNO001	NOR Nation
CREGFR004	FRA Marseille	CREGPL001	POL Nation
CREGFR001	FRA Nation	CREGPT001	PRT Nation
CREGFR005	FRA Nice	CREGSE001	SWE Nation
CREGFR006	FRA Paris		
CREGGB002	GBR Bristol	Factor Code	Property Type
CREGGB003	GBR East Midlands	CREPEU01	Europe Hotel
CREGGB004	GBR East of England	CREPEU02	Europe Industrial
CREGGB005	GBR London	CREPEU03	Europe Multifamily
CREGGB006	GBR Manchester	CREPEU04	Europe Office
CREGGB001	GBR Nation	CREPEU05	Europe Retail

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