Moody's Analytics

MODELING METHODOLOGY

Optimizing Assets under Basel III LCR Requirements

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Abstract

This paper describes a method to optimize assets under the Basel III Liquidity Coverage Ratio (LCR) requirements. We develop a framework that optimizes the trade-off between the risk and return of assets. We differentiate between assets held until maturity and those traded before maturity. We then derive expressions for the expectation and variance of returns of instruments held until maturity, with uncertainties in returns driven by credit risk. Although we developed the methodology in conjunction with the Basel III liquidity requirements, it is readily extendable to similar solvency requisites. We focus especially on High Quality Liquid Assets (HQLA) – a key component of the LCR – as these are particularly well-suited for optimization problems being, virtually by definition, easily sold/purchased in the market. Finally, we show the benefits of the optimization procedure, in terms of investment profitability and risk mitigation, via examples based on a portfolio of actively-traded HQLA.

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1. Introduction

This paper presents an approach for performing assets-side balance sheet optimization, while complying with the BCBS (2013) Liquidity Coverage Ratio (LCR) guidelines. In particular, we develop a framework for maximizing risk-adjusted returns under the LCR requirements and, based on this framework, propose a dynamic investment strategy.

Why focus on the LCR? A 2016 study conducted by the International Association of Credit Portfolio Managers (IACPM) and Oliver Wyman on the most binding constraints for financial resource management, has revealed the LCR as one of – if not *the* – most binding regulatory constraint for banks.[1] This situation represents a regime shift, with respect to the pre-crisis period, when funding was cheaper and not subject to heavy regulation. Moreover, a key component of the LCR are the High-Quality Liquid Assets (HQLA) which, being *liquid* by design, can be traded in the market with little cost, making them well-suited for portfolio optimization strategies.

What is the LCR? The Basel Committee on Banking Supervision (BCBS) defines the liquidity coverage ratio as the ratio of the *stock of high-quality liquid assets* to *total net cash outflows over the next 30 days*,

$$LCR = \frac{\text{Stock of HQLA}}{\text{Total net cash outflows over the next 30 calendar days}}. \tag{1}$$

The BCBS requires this ratio to be at least 100% on an ongoing basis and to be reported to supervisors at least monthly, with the operational capacity to increase the frequency to weekly or even daily during stressed situations. The purpose of the BCBS (2013) guidelines is to compel banks to hold high-quality liquid assets to cover net cash outflows over the next thirty calendar days, in order to "promote the short-term resilience of the liquidity risk profile of banks."

High-quality liquid assets are cash or other assets that can be monetised quickly, i.e. converted into cash, through sales and without significant loss in value. They should have low risk, low correlation with risky assets, and be diversified. The guidelines specify three separate levels of high-quality liquid assets:

- Level 1 assets can be included without limit and are not subject to haircuts;
- Level 2 assets cannot exceed 40% of the stock of HQLA after haircuts
  - Level 2A assets are subject to a 15% haircut;
  - Level 2B assets are subject to a 25% or 50% haircut. Level 2B assets cannot exceed 15% of the stock of HQLA.

The "Total net cash outflows over the next 30 calendar days" is the difference between expected cash outflows and expected cash inflows over the next 30-day period, under a given scenario. This difference is floored at 25% of expected cash outflows. The total expected cash outflows are calculated by multiplying the outstanding balance of various types of liabilities and off-balance sheet commitments by the rates at which they are expected to run-off or be drawn down. Each liability category can have a different rate. Analogously, the total expected cash inflows are calculated by multiplying the outstanding balance of various categories of contractual receivables by the rates at which they are expected to flow in under the given scenario. The rates depend on the instrument and transaction type.

The literature on the topic of balance sheet optimization under LCR constraints is not particularly broad. A first approach taken by Werne (2015) uses standard portfolio theory to optimize HQLA under LCR requirements, and emphasises the role of Level 2B assets – typically riskier and more profitable than Level 1 and 2A. The method assumes normally distributed returns and uses the Value-at-Risk as the measure of risk. However, the author does not provide details on the calculation of the expectation and variance of returns. A subsequent paper by Ihrig, Kumbhat, Vojtech, and Weinbach (2017) studies how a pool of banks have been managing their HQLA composition, following the LCR inception, and proposes to optimize a bank's liquid assets by considering seven HQLA

[1] See Khaykin, Koyluoglu, Elliott, and Spicer (2017).

components[2], focusing on the three largest: Excess Reserves, Treasury Securities, and Mortgage-backed Securities (MBS) of U.S. government-sponsored enterprises (or GSE MBS). They solve a mean-variance optimization problem, in which they include a risk tolerance parameter. The authors highlight potential additional liquidity management goals or constraints to be added to the problem, namely: business model, interest rate risk, leverage ratio, and risk-based capital requirement. They also make the point that an institution close to its leverage ratio would generally prefer Level 1 HQLA over Level 2 HQLA, as an additional $\$1$ of either type of assets affects the leverage ratio equally, but not the LCR, as the latter category is subject to haircuts in calculating HQLA. They also point out that risk-based capital requirements may actually be more binding for banks than the leverage ratio. Namely, if a bank's risk-based capital ratios are binding, Level 2 HQLA are less attractive, because such assets carry a non-zero risk weight. A bank effectively bound by a risk-based capital ratio likely leans toward increasing excess reserves and Treasury securities to meet its LCR, as those assets carry a zero-risk weight.

Our approach differs from Werne (2015) and Ihrig et al. (2017) in several aspects. In particular, we consider a more general framework, allowing for optimization of generic assets – not only HQLA. We derive precise equality and inequality constraints for HQLA composition in the "Stock of HQLA" and allow for HQLA in excess, i.e. not allowed to enter the "Stock of HQLA", to be included in the "Total net cash outflows over the next 30 calendar days" via their inflows. We recognize the difference between instruments held until maturity and those traded before their maturity and provide details on calculating expected returns and variances for the former type, under credit risk. Moreover, differently from Werne (2015), we do not rely on a particular distributional assumption on the instrument returns. Finally, we highlight the relevance of exposure limits in the optimization as an alternative (or complementary) way of measuring the exposures' risk.

Our framework follows an approach that maximizes risk-adjusted returns under LCR requirements. For instruments held until maturity, we consider *credit risk* – represented by LGD and PD term structure – as the source of risk, whereas, for instruments likely to be traded before their maturity, we implicitly consider *market risk* as the source of uncertainty. The problem we focus on can be phrased in the following way: *what assets configuration maximizes risk-adjusted expected return(s) over a certain horizon, given the current liabilities structure, while complying with Basel III LCR requirements?* We represent this problem as a linear (or quadratic) programming problem, for which efficient methods exist to solve it. Because we consider a given liabilities' structure, we introduce a *resources constraint*, which equals the market value of assets. This constraint corresponds to the situation in which we can purchase new assets only with the proceeds of the sale of assets currently held.

Based on this problem, we devise an investment *strategy* that entails rerunning the optimization procedure at each portfolio rebalancing date. The idea is that at each one of these dates, the portfolio manager searches for the optimal portfolio, considering asset returns from the *current* rebalancing date to the *next* one. We stress that, because of how HQLA are defined, this problem is particularly well-suited when considering HQLA, as these assets should:

1. Be held by the bank to cope with surges in outflows – suggesting, arguably, that these assets should be held for a material length of time and banks should avoid excessive volatility in their composition;[3]
2. Should be periodically monetised – meaning, a portion (or all) of these assets *must* systematically be sold in the market.[4]

Hence, when focusing on HQLA, it is reasonable to assume a sequence of rebalancing dates, not tightly spaced, at which the portfolio

[2] That is i) total reserve balances; ii) Treasury securities; iii) Ginnie Mae (GNMA) Mortgage-backed securities (MBS); (iv) non Government-sponsored enterprises (GSEs) agency debt; (v) GSEs debt; (vi) GSEs MBS; (vii) GSEs Commercial MBS.

[3] "The stock should be under the control of the function charged with managing the liquidity of the bank (eg the treasurer), meaning the function has the continuous authority, and legal and operational capability, to monetise any asset in the stock. Control must be evidenced either by maintaining assets in a separate pool managed by the function with the sole intent for use as a source of contingent funds, or by demonstrating that the function can monetise the asset at any point in the 30-day stress period and that the proceeds of doing so are available to the function throughout the 30-day stress period without directly conflicting with a stated business or risk management strategy. For example, an asset should not be included in the stock if the sale of that asset, without replacement throughout the 30-day period, would remove a hedge that would create an open risk position in excess of internal limits." BCBS (2013, Paragraph 33).

[4] "A bank should periodically monetise a representative proportion of the assets in the stock [of HQLA] through repo or outright sale, in order to test its access to the market, the effectiveness of its processes for monetisation, the availability of the assets, and to minimise the risk of negative signalling during a period of actual stress." BCBS (2013, Paragraph 30).

manager runs the optimization procedure, selecting the most profitable HQLA.

The problem specification allows assessing a new asset's impact on the portfolio. At each optimization, the user can decide which assets to include in the optimization and which positions to leave unchanged.

We show the advantages of implementing the ensuing methodology on a portfolio of actively traded assets.

The remainder of the paper is organized as follows:

- Section 2 outlines the proposed optimization problems and addresses their solutions;
- Section 3 shows the methodology for estimating expected returns and their variance for instruments held until maturity;
- Section 4 presents some examples;
- Section 5 concludes;
- Appendix A shows details of the problems' definition;
- Appendix B provides a proof that the credit events considered in the calculation of the expected returns and variances form a partition of the probability space;
- Appendix C gives an alternative expression of the expected return and variance for instruments held until maturity;
- Appendix D reports key points from the Basel III guidelines.

2. Problem Specification

This section outlines the optimization framework. We first consider optimization of the "Stock of HQLA" and subsequently generalize to other assets. The difference between the two specifications is that in the former case, we can treat the LCR numerator and denominator as independent, but not in the latter. The reason being that the inflows from the "Stock of HQLA" are not allowed in the "Total net cash outflows over the next 30 calendar days", as specified in BCBS (2013, Paragraph 72). This trait simplifies the problem when focusing on the "Stock of HQLA".

2.1 Optimizing HQLA Composition

Banks look to maximize portfolio return, while minimizing risk and complying with regulatory requirements. Portfolio return is, however, uncertain, as it depends upon market price fluctuations (market risk) and on the ability of the counterparties to meet their financial obligations (credit risk). Arguably, the best a bank can hope for is to maximize the *expected* return on their portfolio(s).

We start by considering the following linear programming problem, in which we seek to maximize the expected return on the "Stock of HQLA", the numerator of expression (1), subject to the LCR constraints,

$$\begin{aligned}
\max_{\{a_i\}_{i\in\Lambda^H}} \sum_{i\in\Lambda^H} a_i\mu_i, \\
\sum_{i\in\Lambda^H} a_i = \text{Stock of HQLA pre-haircut}, \\
a_i \geq 0 \text{ for every } i \in \Lambda, \\
\sum_{i\in\Lambda_1} a_i \geq 0.6 \cdot \text{Total net cash outflows}, \\
\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_{2A}} (1-h_i)\, a_i \geq 0.85 \cdot \text{Total net cash outflows}, \\
\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\, a_i \geq \text{Total net cash outflows},
\end{aligned} \tag{2}$$

where μ_i,a_i, and h_i are the expected annualized return, "dirty" asset value,[5] and haircut, respectively, of asset i, Λ_ℓ is the set of level ℓ of high-quality liquid assets, with $\ell \in \{1, 2A, 2B\}$, Λ^H is set of all HQLA in the "stock of HQLA" with $\Lambda^H = \Lambda_1 \cup \Lambda_{2A} \cup \Lambda_{2B}$, "Total net cash outflows" is short notation for the "Total net cash outflow over the next 30 calendar days". Additionally, $h_i = 15\%$ for $i \in \Lambda_{2A}$ and $h_i \in \{25\%, 50\%\}$ for $i \in \Lambda_{2B}$.

Note The amount

$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\, a_i$$

corresponds to the "Stock of HQLA" and the condition

$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\, a_i \geq \text{Total net cash outflows}$$

[5] In this paper, we consider the "dirty" values of assets, that is the "clean" market price plus accrued interests. Note, BCBS (2013) does not specify whether value should be calculated using the "clean" or "dirty" price. However, given the interpretation of the "Stocks of HQLA" as assets that can be sold in order to use the proceeds to meet cash outflows obligations, using "dirty" prices is, arguably, justified. Moreover, the European Banking Authority seems to share this interpretation when reading the EU regulation, see `https://eba.europa.eu/single-rule-book-qa/-/qna/view/publicId/2013_154`.

corresponds to the requirement $LCR \geq 100\%$. The restriction

$$\sum_{i \in \Lambda^H} a_i = \text{Stock of HQLA pre-haircut}, \tag{3}$$

corresponds to a *fixed resources* constraint, that is the underlying assumption that liquid assets can only be purchased with the proceeds from the sale of other liquid assets. More specifically, the "Stock of HQLA pre-haircut" corresponds to the "dirty" market value of HQLA pre-haircut, which remains unchanged throughout the optimization.

Note Equation (3) holds under the assumption of zero bid/ask spread. The restriction $a_i \geq 0$ means the optimization only considers asset configurations in which all positions are long, and the last three inequality constraints come directly from BCBS (2013).[6]

The solution to Problem (2) is an "optimized" composition of "dirty" asset values $\boldsymbol{a}^* = \left[a_1^*, a_2^*, ..., a_{|\Lambda_H|}^*\right]'$. From this solution, we can deduce the optimal amount of position i by taking $n_i^* = \frac{a_i^*}{p_i}$, where p_i is the "dirty" price per unit of asset i, and n_i^* is the optimal number of instruments composing position i. From this, we can obtain the amount to sell/purchase as $\Delta n_i = n_i^* - n_i^0$, where $n_i^0 = \frac{a_i^0}{p_i}$ and a_i^0 is the value of asset i before the optimization. If $\Delta n_i > 0$ then the position in asset i should be increased, and vice versa.

In practice, users may want to keep some positions in HQLA fixed or not change them more than a certain amount. This can be achieved by adding equality or inequality constraints to the problem above. For instance, we can add the constraint $a_i = a_i^0$ to fix the amount of asset i, where a_i^0 is the value of the asset before the optimization, or we add $a_i \leq a_i^0\,(1 + x\%)$ and $a_i \geq a_i^0\,(1 - x\%)$ if the user wishes to change position i by no more than $x\%$. Exposure limits may be set in this fashion, for instance by the credit risk function of the bank. Section 4 provides an example of Problem (2) and show the role that exposure limits play.

There are several ways to estimate the expected returns μ_is. The optimization framework allows users to consider both instruments held until maturity and those traded before maturity. Section 3.1 provides details on the calculation of the expected returns for instruments held until maturity, exploiting the instrument's cash flows and PD-LGD term structures.

2.1.1 Repos and Reverse Repos

Repos are short-term funding contracts in which the party sells a security in exchange for money, with the promise to repurchase it at a future date. Under the unwinding principle, in the Basel III guidelines, repo contracts maturing within 30 days are to be unwound and the HQLA computed as if the party had the (will-be repurchased) security in their portfolio. In this case, the exchanged *security*'s *charateristics* are used in the "Stock of HQLA" calculation. Users may wish to keep the amount of these assets unchanged, namely, if a_{repo}^0 is the market value of the security underlying the repo contract at the time of the optimization, then the restriction $a_{repo} = a_{repo}^0$ can be added to the constraints of Problem (2).

In the case of a repo maturing after 30 days, the HQLA must be calculated on the *cash* received as part of the contract or the asset in which this was invested. In this case, the cash (Level 1 asset) or the security bought with it, can be included in the optimization problem. In particular, the cash can be reinvested in a different HQLA and sold before settling the repo contract.

Also, in the case of a *reverse repo*, the contract must be unwound for the purposes of the "Stock of HQLA" calculation, if maturing within 30 days.[7] Similar arguments to those outlined above on optimization, apply to reverse repos and collateral swaps, which are to be treated as repos or reverse repos, and similarly any other contract with similar form. [8,9]

[6] See Appendix A for a derivation of the problem specification and alternative representations.

[7] See Annex 1 in BCBS (2013).

[8] "Assets received in reverse repo and securities financing transactions that are held at the bank, have not been rehypothecated, and are legally and contractually available for the bank's use can be considered as part of the stock of HQLA." BCBS (2013, Paragraph 31).

[9] "Collateral swaps should be treated as repurchase or reverse repurchase agreements, as should any other transaction with a similar form." BCBS (2013, Paragraph 112).

2.2 Extension to all Assets

Section 2.1 considered changing the composition of assets that constitute the "Stock of HQLA". However, it might not be feasible to include all assets qualifying as Level 2 in the "Stock of HQLA" because of the BCBS (2013) restrictions, reported in Section 2.1. This section extends the optimization problem seen earlier to (virtually) all the assets in the balance sheet, including HQLA not allowed in the "Stock of HQLA". The difference from the previous optimization problem is that now the optimization affects the LCR numerator as well as the denominator. Namely, the "Total net cash outflows" will change depending upon the assets composition via their inflows.

From BCBS (2013, Paragraph 69) the "Total net cash outflows" can be expressed as

$$\text{Total net cash outflows} = \begin{cases} \text{Outflows} - \text{Inflows} & \text{Inflows} \leq 0.75 \cdot \text{Outflows} \\ 0.25 \cdot \text{Outflows} & 0.75 \cdot \text{Outflows} \leq \text{Inflows} \end{cases},$$

where Inflows $= \sum_{i \in \overline{\Lambda}^H} \iota_i + \sum_{i \in \overline{\Lambda}^R} \iota_i$, with ι_i denoting the inflow of asset i and $\overline{\Lambda}^H$ the assets that, although qualifying as Level 2, are not permitted to enter the "Stock of HQLA" because of the restrictions in Equations (17) and (18), $\overline{\Lambda}^R$ is the set of assets that do not qualify as HQLA, and "Inflows" denotes the sum of inflows.

Note Level 1 assets can comprise an unlimited share of the pool of HQLA at the LCR numerator.

Note The assets in $\overline{\Lambda}^R$ can only appear at the LCR denominator. Formally, we have $\overline{\Lambda}^H \cap \Lambda^H = \emptyset$ as an instrument used in the LCR numerator may not be considered in the denominator, and $\Lambda = \overline{\Lambda}^H \cup \Lambda^H \cup \overline{\Lambda}^R$ corresponds to all the assets.[10] However, the two sets $\overline{\Lambda}^H$ and Λ^H may contain the same *type* of instrument, e.g. the same sovereign bond.

The optimization problem extended to all assets, becomes

$$\max_{\{a_i\}_{i \in \Lambda}} \sum_{i \in \Lambda} a_i \mu_i, \tag{4}$$

$$\sum_{i \in \Lambda} a_i - \text{Assets value pre-haircut} = 0,$$

$$a_i \geq 0 \text{ for every } i \in \Lambda,$$

$$\sum_{i \in \Lambda_1} a_i \geq 0.6 \cdot \text{Total net cash outflows},$$

$$\sum_{i \in \Lambda_1} a_i + \sum_{i \in \Lambda_{2A}} (1 - h_i)\, a_i \geq 0.85 \cdot \text{Total net cash outflows},$$

$$\sum_{i \in \Lambda_1} a_i + \sum_{i \in \Lambda_2} (1 - h_i)\, a_i \geq \text{Total net cash outflows},$$

$$\text{Total net cash outflows} = \text{Outflows} - \min\left(\text{Inflows}, 0.75 \cdot \text{Outflows}\right),$$

where "Assets value pre-haircut" corresponds to the "dirty" value of assets before any haircut is applied. Notice, the difference with respect to Problem (2), in which only the "Stock of HQLA pre-haircut" is kept constant.

Problem (4) can be solved by a two-step procedure. The first step is to solve the auxiliary problem

[10] See BCBS (2013, Paragraph 72) for the double counting prescription.

$$\max_{\{a_i\}_{i\in\Lambda}} \sum_{i\in\Lambda} a_i\mu_i, \tag{5}$$
$$\sum_{i\in\Lambda} a_i - \text{Assets value pre-haircut} = 0,$$
$$a_i \geq 0 \text{ for every } i \in \Lambda,$$
$$\sum_{i\in\Lambda_1} a_i \geq 0.6 \cdot \left(\text{Outflows} - \sum_{i\in\overline{\Lambda}^H} a_i\nu_i - \sum_{i\in\overline{\Lambda}^R} a_i\nu_i \right),$$
$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_{2A}} (1-h_i)\, a_i \geq 0.85 \cdot \left(\text{Outflows} - \sum_{i\in\overline{\Lambda}^H} a_i\nu_i - \sum_{i\in\overline{\Lambda}^R} a_i\nu_i \right),$$
$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\, a_i \geq \text{Outflows} - \sum_{i\in\overline{\Lambda}^H} a_i\nu_i - \sum_{i\in\overline{\Lambda}^R} a_i\nu_i,$$
$$\sum_{i\in\overline{\Lambda}^H} a_i\nu_i + \sum_{i\in\overline{\Lambda}^R} a_i\nu_i \leq 0.75 \cdot \text{Outflows},$$

where we have expressed $\iota_i = a_i\nu_i$ with ν_i representing the inflow over the next 30 days, per unit of asset value a_i. Expressing the inflows as linear functions of the asset values, allows us to keep using a linear programming framework. The second step is to solve

$$\max_{\{a_i\}_{i\in\Lambda}} \sum_{i\in\Lambda} a_i\mu_i, \tag{6}$$
$$\sum_{i\in\Lambda} a_i - \text{Assets value pre-haircut} = 0,$$
$$a_i \geq 0 \text{ for every } i \in \Lambda,$$
$$\sum_{i\in\Lambda_1} a_i \geq 0.6 \cdot 0.25 \cdot \text{Outflows},$$
$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_{2A}} (1-h_i)\, a_i \geq 0.85 \cdot 0.25 \cdot \text{Outflows},$$
$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\, a_i \geq 0.25 \cdot \text{Outflows},$$
$$0.75 \cdot \text{Outflows} \leq \sum_{i\in\overline{\Lambda}^H} a_i\nu_i + \sum_{i\in\overline{\Lambda}^R} a_i\nu_i.$$

The optimal $\boldsymbol{a}^*$ is then the solution from Problems (5) and (6) that achieves the highest value of the objective function $\sum_{i\in\Lambda} a_i\mu_i$. The constraint $\sum_{i\in\Lambda} a_i - \text{Assets value pre-haircut} = 0$ says that the market value of assets is kept unchanged. In particular, the underlying assumption is that an asset may be sold to purchase a different one with the proceeds of the sale.

Note In this framework, we are not changing the sources of funding nor their composition.

In practice, a bank's balance sheet may contain tens of thousands of assets, especially if we also consider retail positions. Consequently, we may wish to restrict the optimization to a subset of assets. The framework above allows us to do this by adding equality constraints to the problem, as mentioned in Section 2.1. The next section addresses this point further by considering pooling of assets.

2.2.1 Pooling of Assets

To reduce the dimensionality of the problem, one approach is to pool together assets with similar characteristics. There are several possible dimensions to consider, examples include: rating, riskiness/return, asset class, market segment (e.g. retail, wholesale, CRE, SME), type of instrument, time to maturity.

Once a pool has been determined, we can construct a representative synthetic position, with value $\overline{a} = \sum_{i \in P} a_i$, where P is the pool of assets with similar characteristics, and associated expected return $\overline{\mu} = \sum_{i \in P} \frac{a_i}{\overline{a}} \cdot \mu_i$ and variance $\overline{\sigma}^2 = \sum_{i \in P} \left(\frac{a_i}{\overline{a}}\right)^2 \cdot \sigma_i^2$, assuming no intra-pool correlation. Alternatively, one can specify an intra-pool correlation ρ_{pool} and set the variance of the pool to $\sum_{i,j \in P} \frac{a_i a_j}{\overline{a}} \cdot \sigma_i \sigma_j \rho_{pool}$.

2.3 Covariance Matrix

The problems illustrated in Sections 2.1 and 2.2 focus on the maximization of returns. In this section, we include in the optimization problem the variance of the portfolio, as measure of risk. One way of representing the problem is the following

$$\begin{aligned} &\max_{\boldsymbol{a}} \left(\lambda \boldsymbol{a}' \boldsymbol{\mu} - (1-\lambda) \boldsymbol{a}' \boldsymbol{\Omega} \boldsymbol{a}\right) \\ &\text{s.t. constraints}, \end{aligned} \tag{7}$$

where $\boldsymbol{a}$ represents a vector of asset values, $\boldsymbol{\mu}$ the vector of their expected annualized returns, $\boldsymbol{\Omega}$ the covariance matrix between annualized asset returns, $\lambda \in [0,1]$ a tuning parameter regulating the risk/return trade-off, and "constraints" corresponds to the restrictions introduced in the previous sections.

The objective function $\sum_i a_i \mu_i = \boldsymbol{a}' \boldsymbol{\mu}$, seen in the previous sections, is a special case of the function $\lambda \boldsymbol{a}' \boldsymbol{\mu} - (1-\lambda) \boldsymbol{a}' \boldsymbol{\Omega} \boldsymbol{a}$, with $\lambda = 1$. The choice of the parameter λ can be based on "trade-off analysis". In particular, a point at which a big change (an improvement) in one of the objectives can only be achieved by a big change (a deterioration) in the other objective, may be considered a good trade-off point. This would correspond to the point at which the tangent to the trade-off curve has slope equal to -1. See for instance Boyd and Vandenberghe (2004, Chapter 4) for more details on this point.

Section 3.2 shows how to compute the variance of returns for instruments held until maturity, under credit risk.

The correlations can be estimated in several ways. One approach is to retrieve times series of returns from a data providing service, such as Bloomberg™, and compute variances and correlations between the relevant annualized assets' returns. One drawback of this approach is that some instruments may not have been traded for long enough to have sufficient observations. Another approach is to use Moody's Analytics GCorr™ model for counterparties *assets* returns correlations to proxy *instrument value* returns correlations.[11,12] Using GCorr correlations to proxy instruments' correlations corresponds to making the following assumption

$$\begin{aligned} r_{V,t} &= \alpha + \beta r_{A,t}, \\ r_{A,t} &= \sqrt{RSQ}\phi_t + \sqrt{1-RSQ}\epsilon_t, \end{aligned} \tag{8}$$

where $r_{V,t}$ is the annualized return on the instrument's value, $\alpha, \beta > 0 \in \mathbb{R}$ are constants, $r_{A,t}$ is the assets return of the counterparty with $E\left[r_{A,t}\right] = 0$ and $Var\left[r_{A,t}\right] = 1$, $\sqrt{RSQ}$ is the sensitivity of the assets return to the systemic credit risk

[11] See Huang, Lanfranconi, Patel, and Pospisil (2012) for an overview of the model.

[12] By "counterparty's assets", we mean here the market value of the assets side of the balance sheet of the counterparty. The assets are related to credit risk, and in particular to default, because a counterparty's probability of default can be seen as the probability of its assets falling below a certain threshold, known as the default point and approximately corresponding to the liabilities level.

factor ϕ_t, and ϵ_t is the idiosyncratic shock to the assets return. Equation (8) shows a linear function between the return on the instrument's value and the assets return of the underlying counterparty. Because correlation is unaffected by affine transformations (with positive slopes), we have

$$\rho_{ij} = Corr\left(r_{V,t,i}, r_{V,t,j}\right) = Corr\left(r_{A,t,i}, r_{A,t,j}\right),$$

because

$$\begin{aligned} Corr\left(r_{V,t,i}, r_{V,t,j}\right) &= \frac{Cov\left(r_{V,t,i}, r_{V,t,j}\right)}{\sqrt{Var\left(r_{V,t,i}\right)}\sqrt{Var\left(r_{V,t,j}\right)}} \\ &= \frac{Cov\left(\alpha_i + \beta_i r_{A,t,i}, \alpha_j + \beta_j r_{A,t,j}\right)}{\sqrt{Var\left(\alpha_i + \beta_i r_{A,t,i}\right)}\sqrt{Var\left(\alpha_j + \beta_j r_{A,t,j}\right)}} \\ &= \frac{\beta_i \beta_j Cov\left(r_{A,t,i}, r_{A,t,j}\right)}{|\beta_i|\,|\beta_j|} \\ &= Cov\left(r_{A,t,i}, r_{A,t,j}\right), \end{aligned}$$

where we consider only the case $\beta_i, \beta_j > 0$.

Given correlations and variances of the assets returns, we can construct the matrix $\boldsymbol{\Omega}^*$. This matrix is symmetric but not necessarily positive semi-definite - a necessary condition for a matrix to be a covariance matrix. To address this issue, we take its spectral decomposition, $\boldsymbol{\Omega}^* = \boldsymbol{V}\boldsymbol{\Lambda}^*\boldsymbol{V}'$ where $\boldsymbol{V}$ is a matrix containing the eigenvectors of $\boldsymbol{\Omega}^*$ as its columns and $\boldsymbol{\Lambda}^*$ the corresponding eigenvalues on its main diagonal. We then replace the i-th eigenvalue e_i^* with $e_i = \max\left(e_i^*, 0\right)$ and obtain the following positive semi-definite covariance matrix $\boldsymbol{\Omega} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}'$, where $\boldsymbol{\Lambda}$ is a diagonal matrix with e_i as its (i,i) element.[13]

2.4 Portfolio Rebalancing Frequency and Net Cash Outflows

The solution to the problem specifications seen so far does not ensure that after the analysis date, i.e. the time of portfolio rebalancing, the LCR constraints will still be met. In particular, the requirement $LCR \geq 100\%$ may not be satisfied until the next portfolio rebalancing date. To mitigate this nuisance, we propose taking the maximum "Total net cash outflows" between the analysis date and the next rebalancing date. Namely, instead of using "Total net cash outflows" at the analysis date in the problem restrictions above, we propose using

$$\overline{\text{Total net cash outflows}} = \max_{t_0 \leq \tau \leq t_H} \left\{ \text{Total net cash outflows}^{\tau,\tau+30d} \right\},$$

where t_0 is the analysis date, that is the date of the optimization and portfolio rebalancing, t_H is the "rebalancing horizon", that is the date at which the next rebalancing will occur – or is expected to occur –, and "Total net cash outflows$_{\tau,\tau+30d}$" is the "Total net cash outflows over the next 30 days" as of time τ.

Similarly, if the "Total net cash outflows over the next 30 days" is a function of the assets, as presented in Section 2.2, the inflow per unit of asset value ν_i can be replaced with

$$\overline{\nu}_i = \max_{t_0 \leq \tau \leq t_H} \left\{ \nu_i^{\tau,\tau+30d} \right\},$$

where $\nu_i^{\tau,\tau+30d}$ is the inflow per unit of asset value as of time τ.

Note The method illustrated in this section reduces the probability of the LCR constraints being breached between rebalancing dates – however, it does not completely eliminate the possibility. This is because the market values of the assets may change between rebalancing date and the LCR restrictions concerning the share of "Stock of HQLA" of Level 2 assets may not be satisfied for the entire period.

[13] A symmetric matrix is positive semi-definite if and only if all its eigenvalues are non-negative.

2.5 Solving the Problem

The problem set forth in Section 2.1, and its extension in Section 2.2, is a *linear programming problem*. When adding a covariance matrix, as described in Section 2.3, we have a *quadratic programming problem*. Linear and quadratic programming problems are special cases of the larger class of *convex programming problems*, which are problems characterized by a convex objective function and convex (inequality and equality) constraints. There are several highly efficient software packages that perform convex optimization for most programming languages.

3. Expected Returns and Variances

3.1 Expected Returns

Section 2 introduced an optimization framework based on the maximization of expected returns, specifying only that they are expected annualized returns. This section provides more details about the calculation of these expectations. It is useful at this point to distinguish between instruments held until maturity and ones that are likely to be traded before their maturity.

We begin by outlining the expected return methodology for a cash flow-bearing instrument held until *maturity*. For these instruments, the uncertainty comes from the possible counterparty default. Denote with $\{CF_{\tau_1}, CF_{\tau_2}, \ldots, CF_{\tau_K}\}$ a sequence of cash flows that an instrument is expected to pay at times $\{\tau_1, \tau_2, \ldots, \tau_K\}$, with $\{FPD_{t_0,t_1}, FPD_{t_1,t_2}, \ldots, FPD_{t_{L-1},t_L}\}$ and $\{LGD_{t_1}, LGD_{t_2}, \ldots, LGD_{t_L}\}$ equally-spaced term structures of forward PDs and associated LGDs (to be applied to the notional of the instrument), with $t_L \geq \tau_K$.[14,15] Time t_0 corresponds to the analysis date and t_K to maturity date. Figure 1 gives a depiction of the possible paths that the credit risk process of the counterparty may follow. In particular, the north-east facing leaves of the tree represent the counterparty reaching default and an interruption of the process. The probability of a given path - that is a continuous segment that joins the starting point to one of the leaves – is given by the product of the probabilities of the branches that make up the path. In Appendix B, we show that the probabilities associated to the $L+1$ branches, and defined below, sum to one, forming a partition of the probability space.

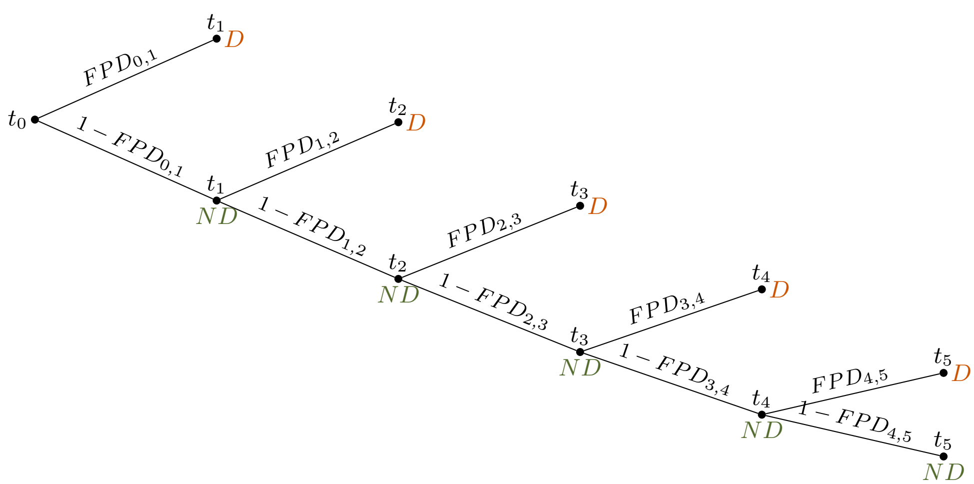

Figure 1: Five-tenor PD term structure. We have $L + 1 = 5 + 1 = 6$ possible paths.

The expected annualized return to maturity is

$$\mu = E_0 \left[\left(1 + \frac{\widetilde{V}_K - V_0}{V_0} \right)^{\frac{1}{\tau_K - t_0}} - 1 \right], \tag{9}$$

where V_0 is the dirty price of the instrument at analysis date t_0, $\widetilde{V}_K$ is the (random) value of the investment in the instrument at time τ_K, $E_0[\cdot]$ denotes an expectation based on time t_0 information, and

[14] By equally-spaced we mean $t_{i+1} - t_i = \Delta$ for any $i \in \{1, 2, ..., L\}$.
[15] Solutions that produce term structures of PDs and LGDs are CreditEdge™ and RiskCalc™.

$$\mu = \sum_{j=0}^{L} \left\{ s_j \cdot FPD_{t_j,t_{j+1}} \cdot \left(\left(1 + \frac{V_K^j - V_0}{V_0} \right)^{\frac{1}{\tau_K - t_0}} - 1 \right) \right\}, \tag{10}$$

$$V_K^j = \left(1 - LGD_{t_{j+1}}\right) N \left(1 + \hat{r}_{t_{j+1},\tau_K}\right)^{\tau_K - t_{j+1}} + \sum_{\tau \in \Theta | \tau \leq t_j} CF_\tau (1 + \hat{r}_{\tau,\tau_K})^{\tau_K - \tau}, \tag{11}$$

$$s_j = \prod_{\ell=1}^{j} \left(1 - FPD_{t_{\ell-1},t_\ell}\right),$$

where $\hat{r}_{a,b}$ is the annualized interest rate obtained from reinvesting the cash flow or recovery received at time a and held until time b, s_j is the survival probability up to t_j, V_K^j is the value at maturity of the instrument under scenario j, N is the face value of the instrument, $\Theta = \{\tau_1, \tau_2, \ldots, \tau_K\}$ is the set cash flow dates, and we use the conventions $FPD_{t_L,t_{L+1}} = 1$, $LGD_{t_{L+1}} = 1$, and $\prod_{\ell=a}^{b} x_\ell = 1$ if $b < a$.

Equation (9) is an expectation of the investment's possible annualized returns at its maturity. Equation (10) expresses this expectation as a weighted average. All payments made by the counterparty underlying the instrument – cash flows and recovery – are cumulated until maturity at a given interest rate. For instance, if the counterparty defaults during the second time period, i.e. $\tau_D \in (t_1, t_2]$ with τ_D the time of default, the value of the investment in the instrument at time τ_K – maturity – is

$$\left((1 - LGD_{t_2}) N (1 + \hat{r}_{t_2,\tau_K})^{\tau_K - t_2} + \sum_{\tau \in \Theta | \tau \leq t_1} CF_\tau (1 + \hat{r}_{\tau,\tau_K})^{\tau_K - \tau} \right),$$

where the first summand represents the recovery collected at time t_2, $(1 - LGD_{t_2}) N$, and cumulated until time τ_K at the rate $\hat{r}_{t_{j+1},\tau_K}$, i.e. $\left(1 + \hat{r}_{t_{j+1},\tau_K}\right)^{\tau_K - t_{j+1}}$, and the second summand corresponds to the cash flows collected before default period – that is before t_1 – and cumulated until maturity. The probability of this event is simply $\mathbb{P}(\tau_D \in (t_1, t_2]) = (1 - FPD_{t_0,t_1}) FPD_{t_1,t_2}$, corresponding to survival until t_1 and default during $(t_1, t_2]$.

Note Using PD/LGD term structures with L tenors implies $L + 1$ possible outcomes at maturity τ_K. For instance, with $L = 3$ the $L + 1 = 4$ possible outcomes are: i) default in first period; ii) no default in first period and default in second period; iii) no default in first two periods and default in third period; iv) no default in all three periods.

Note An underlying assumption of the expression above is that recovered amounts are collected at the LGD tenor and reinvested until instrument maturity. Notice, a zero-coupon bond (ZCB) is a special case of the above, with $CF_{\tau_1} = CF_{\tau_2} = \ldots = CF_{\tau_{K-1}} = 0$ and $CF_{\tau_K} = N$.

Note In Equation (10), we only consider credit risk as source of uncertainty, that is we do not consider the volatility of the interest rates used for the reinvestment of the cash flows/recovery.

For instruments *traded* before their maturity, the return uncertainty comes from market fluctuations of the instrument's value. In this case, an alternative way of computing expected annualized return at horizon is using time series of observed prices. Namely,

$$\mu = E_0 \left[\left(1 + \frac{\widetilde{V}_H - V_0}{V_0} \right)^{\frac{1}{t_H - t_0}} - 1 \right],$$

where $\widetilde{V}_H$ represents the random value of the investment in the instrument at time t_H, and V_0 is the value of the asset at time t_0 (analysis date). Time t_H is the "rebalancing horizon", introduced in Section 2.4, and expressed as a fraction of a year, representing the horizon at which the user expects to (considers to) modify the position in the asset. In practice, for instruments not kept until maturity, μ should be provided by the market risk function of the bank.[16]

Appendix C provides an equivalent vector representation of the equation above.

In the next section, we show that a modification of Equation (10) allows us to compute also the variance of the returns for instruments held until maturity.

3.2 Variance of Returns

The variance of an instrument's annualized return, when held until maturity, can be obtained by exploiting the probabilities introduced in Equation (10), namely

$$\sigma^2 = Var_0\left[\left(1+\frac{\widetilde{V}_K - V_0}{V_0}\right)^{\frac{1}{\tau_K - t_0}} - 1\right] \tag{12}$$

$$= E_0\left[\left(\left(1+\frac{\widetilde{V}_K - V_0}{V_0}\right)^{\frac{1}{\tau_K - t_0}} - 1 - \mu\right)^2\right] \tag{13}$$

$$= \sum_{j=0}^{L}\left\{s_j \cdot FPD_{t_j,t_{j+1}} c_j^2\right\}, \tag{14}$$

$$c_j = \left(1+\frac{V_K^j - V_0}{V_0}\right)^{\frac{1}{\tau_K - t_0}} - 1 - \mu \tag{15}$$

$$V_K^j = \left(1 - LGD_{t_{j+1}}\right) N \left(1+\hat{r}_{t_{j+1},\tau_K}\right)^{\tau_K - t_{j+1}} + \sum_{\tau\in\Theta|\tau\le t_j} CF_\tau (1+\hat{r}_{\tau,\tau_K})^{\tau_K-\tau} \tag{16}$$

where $Var_0[\cdot]$ stands for a variance conditional on time t_0 information and the remaining variables are as defined in Section 3.1.

[16] There are several approaches to projecting returns, see for instance Fama and Bliss (1987), Cochrane and Piazzesi (2005), and Altavilla, Giacomini, and Costantini (2014).

4. Examples

This section presents a stilysed example of the HQLA optimization presented in Section 2.1. We consider a portfolio of ten traded instruments belonging to Levels 1, 2A, and 2B, in amounts that satisfy the LCR restrictions illustrated in Problem (2). We then run the following three exercises:

1. Maximization of HQLA expected return under the LCR constraints, i.e. Problem (2);
2. Maximization of HQLA expected return and minimization of their volatility, i.e. Problem (7), setting $\lambda = 0.9$;
3. Maximization of HQLA expected return under the LCR constraints and exposure limits, i.e. Problem (2) adding limits to the exposures.

Table 1 displays the name of the instruments, their "dirty" market price, yield to maturity, expected return, volatility, Moody's rating, time-to-maturity in years, one-year EDF value,[17] one-year CDS-implied EDF value,[18] and one-year LGD. The portfolio is as-of 20/08/2019. Market data was taken from Bloomberg and the PDs, i.e. EDF measures, and LGDs from Moody's Analytics RiskCalc and CreditEdge.[19]

INSTRUMENT	PRICE	YTM	EXP. RETURN	VOL. RETURN	RATING	TTM	EDF 1Y	CDS EDF 1Y	LGD 1Y
FRTR 0.5 05/25/2029 GOVT	109.24	-0.41	-0.33	0.01	Aa2	9.76		0.06	44.08
IRISH 1 05/15/2026 GOVT	109.67	-0.38	-0.30	0.01	A2	6.74		0.09	44.08
BOTS 0 07/14/2020 GOVT	100.09	-0.10	0.37	0.05	Baa3	0.90		0.79	44.09
NETHER 0.75 07/15/2028 GOVT	112.77	-0.63	-0.59	0.01	Aaa	8.90		0.03	44.08
PGB 2.875 10/15/2025 GOVT	121.15	-0.15	-0.12	0.00	Baa3	6.15		0.14	34.56
NESNVX 13/4 11/02/2037 REGS CORP	125.48	0.36	0.24	0.01	Aa2	17.49	0.01		44.99
IBM 2.85 05/13/2022 CORP	103.00	2.00	1.87	0.04	A2	2.73	0.05		36.15
FCAIM 3.75 03/29/2024 REGS CORP	113.78	1.00	0.86	0.02	Ba1	4.61	0.14		45.01
VW 4.125 11/16/2038 REGS CORP	135.93	2.05	-0.14	0.11	A3	19.24	0.06		49.18
NOVARTIS FIN. 14/26	114.53	-0.20	-0.16	0.00	A1	7.07	0.01		45.42

Table 1: Example of traded HQLA. The variables "YTM", "EXP. RETURN", "VOL. RETURN", "EDF 1Y", "CDS EDF 1Y", and "LGD 1Y" are expressed in percentage points. For instance FRTR 0.5 05/25/2029 GOVT has an expected return of -0.30% and an LGD of 44.08%.

We pick Level 1 instruments from European sovereigns and Level 2 instruments from corporates. We compute the expected returns of the instruments exploiting their PD and LGD term structures. For exposition purposes, we use a simpler approach than the one introduced in Section 3.1. Namely, we calculate expected returns as the expected yield to maturity, basing this expectation on the lifetime PD and LGD of the counterparty of the instruments, and analogously for the variance of the YTMs.[20]

Note The expected return of the instruments are slightly lower than their yield-to-maturity (YTM). The small difference between YTMs and expected returns is due to the credit risk – namely the counterparty's PD and LGD. Notice, the YTM are negative for all European sovereigns considered.

[17] See **https://www.moodysanalytics.com/-/media/products/edf-expected-default-frequency-overview.pdf** for an overview.

[18] See Dwyer, Li, Qu, Russell, and Zhang (2010) for an overview, available here **https://www.moodysanalytics.com/-/media/article/2016/cds-implied-edf-measures-fair-value-cds-spreads-at-a-glance.pdf**.

[19] See **https://www.moodysanalytics.com/product-list/creditedge** and **https://www.moodysanalytics.com/product-list/riskcalc** for an overview of CreditEdge and RiskCalc, respectively.

[20] In particular, we compute the expected return until maturity τ_K as $\mu_K = (1 - CPD_K)\, y_K^{Cumul.} + CPD_K \left(1 - \widetilde{LGD}_K\right) y_K^{Cumul.}$ where $y_K^{Cumul.}$ is the cumulative yield to maturity, that is $y_K^{Cumul.} = (1 - y_K)^{\tau_K - t_0} - 1$ and y_K is the yield-to-maturity as of time t_0. Similarly, we compute the variance as $\sigma_K^2 = (1 - CPD_K)\left(y_K^{Cumul.} - \mu_K\right)^2 + CPD_K \left(\left(1 - \widetilde{LGD}_K\right) y_K^{Cumul.} - \mu_K\right)^2$ where $\widetilde{LGD}_K = \dfrac{LGD_K \cdot N - V_0}{N - V_0}$, where N is the notional of the instrument and V_0, its "dirty" price at time t_0, and LGD_K the LGD at maturity, in terms of notional N.

Recall that, for the "Stock of HQLA" to include some Level 2B assets, which typically have higher returns than Level 1, there must be a high enough amount of Level 2A assets, which, in this case, is given by the instrument NESNVX 13/4 11/02/2037 REGS CORP.[21]

Table 2 shows aggregate statistics of the initial portfolio, that is, before any type of optimization. The "Stock of HQLA" amounts to €$82,000.00$, lower than the actual market value of HQLA, €102529.41 – this difference shows the effect of haircuts. The initial expected return is 0.09%. The table displays the expected return and standard deviation of the portfolio, relative to the exposures. That is, "Expected return" and "Volatility" correspond to

$$\mu^s = \frac{1}{\sum_{i\in\Lambda^H} a_i} \sum_{i\in\Lambda^H} a_i\mu_i,$$
$$\sigma^s = \frac{1}{\sum_{i\in\Lambda^H} a_i} \sqrt{\sum_{i,j\in\Lambda^H} \omega_{ij} a_i a_j \mu_i \mu_j},$$

where ω_{ij} is the covariance between asset i and j. In particular, we compute the variances as explained in Footnote 20 and set covariances as $\omega_{ij} = \rho\sigma_i\sigma_j$, for $i \neq j$, with $\rho = 0.1$.[22]

HQLA	NET OUTFLOWS	MKT VALUE	LCR	EXPECTED RETURN	VOLATILITY
82000.00	73800.00	102529.41	1.11	0.09	0.48

Table 2: Initial portfolio statistics, that is, before any type of optimization. The variables "HQLA", "NET OUTFLOWS", "MKT VALUE" are in Euros, the "LCR" is the ratio between "HQLA" and "NET OUTFLOWS", and "EXPECTED RETURN" and "VOLATILITY" are in percentage points; the expected return of the portfolio is 0.09%.

Tables 3 and 4 show the results of the first optimization exercise, i.e. the *linear programming problem*.

Note The expected return is now 0.95%, roughly ten times higher than the initial 0.09%, corresponding to a 955.6% increase. The portfolio volatility also increased, from 0.48% to 1.18%, corresponding to a 145.8% change. The "Stock of HQLA" – "HQLA OPT." in the table – amounts to €$82,629.71$, slightly more than the initial €$82,000.00$. On the other hand, the "Amount of HQLA pre-haircut", that is "MKT VALUE OPT." is the same as "MKT VALUE", the value before the optimization – this reflects the restriction

$$\sum_{i\in\Lambda^H} a_i = \text{Amount of HQLA pre-haircut},$$

in Problem (2). Comparing the columns "MKT VALUE" and "MLT VALUE OPT." in Table 3, i.e. the exposures before and after the optimization, we can see that the optimized portfolio is much more concentrated in three instruments, compared to the initial one.[23] The ever important maxim – and prudential principle – of "not placing all eggs in one basket" suggests that having high risk concentration is not ideal. The rest of the section addresses this point by first introducing the covariance matrix and then exposure limits into the optimization.

[21] See the HQLA restrictions reported in Section 1.

[22] The value of $\rho = 0.1$ is chosen for the sake of simplicity, to illustrate that even with modest correlations, the covariance matrix has an effect on the optimization. In practice, different instruments would have different correlations. For instance, sovereigns within the same "region", e.g. southern Europe, would have higher correlations than with the rest of the sovereigns. See for instance Giudici and Parisi (2017) for a study of the interrelations of sovereign risk in the Euro area.

[23] Note, "MKT VALUE OPT." would correspond to the vector $\boldsymbol{a}^*$resulting from the optimization, whereas "MKT VALUE" corresponds to the decision variables before the optimization, i.e. $\boldsymbol{a}^0$.

INSTRUMENT	LEVEL	SUBLEVEL	HAIRCUT	EXP. RET.	VOL. RET.	MKT VALUE	HQLA	MKT VALUE OPT.	HQLA OPT.
FRTR 0.5 05/25/2029 GOVT	1		0	-0.33	0.01	10000.00	10000	0.00	0.00
IRISH 1 05/15/2026 GOVT	1		0	-0.30	0.01	10000.00	10000	0.00	0.00
BOTS 0 07/14/2020 GOVT	1		0	0.37	0.05	10000.00	10000	62730.00	62730.00
NETHER 0.75 07/15/2028 GOVT	1		0	-0.59	0.01	10000.00	10000	0.00	0.00
PGB 2.875 10/15/2025 GOVT	1		0	-0.12	0.00	5000.00	5000	0.00	0.00
NESNVX 13/4 11/02/2037 REGS CORP	2	2A	15	0.24	0.01	23529.41	20000	0.00	0.00
IBM 2.85 05/13/2022 CORP	2	2B	50	1.87	0.04	4000.00	2000	39799.41	19899.71
FCAIM 3.75 03/29/2024 REGS CORP	2	2B	50	0.86	0.02	10000.00	5000	0.00	0.00
VW 4.125 11/16/2038 REGS CORP	2	2B	50	-0.14	0.11	10000.00	5000	0.00	0.00
NOVARTIS FIN. 14/26	2	2B	50	-0.16	0.00	10000.00	5000	0.00	0.00

Table 3: Optimized portfolio. Maximization of HQLA expected return under the LCR constraints.

HQLA OPT.	NET OUTFLOWS	MKT VALUE OPT.	LCR OPT.	EXP. RETURN	VOL. RETURN
82629.71	73800.00	102529.41	1.12	0.95	1.18

Table 4: Results of linear programming problem.

Table 5 shows the results of the *quadratic programming problem* as defined by Equation (7), with risk aversion parameter $1-\lambda = 0.1$. This parameter determines the risk/return tradeoff and reflects the propensity to risk of the institution. In this case, we place more importance on the expectation of returns rather than their variance. Notice, in this instance, the resulting HQLA portfolio is more diversified, compared to the results of the linear programming problem in Table 3. This reflects the presence of risk and diversification introduced via the covariance matrix. The volatility of the portfolio decreased to 0.08 from 0.48, a -500% change, at the expense of a reduction in the expected return, also down, to -0.14 from 0.09, a -255.6% change.

INSTRUMENT	LEVEL	SUBLEVEL	HAIRCUT	EXP. RET.	VOL. RET.	MKT VALUE	HQLA	MKT VALUE OPT.	HQLA OPT.
FRTR 0.5 05/25/2029 GOVT	1		0	-0.33	0.01	10000.00	10000	813.36	813.36
IRISH 1 05/15/2026 GOVT	1		0	-0.30	0.01	10000.00	10000	2903.80	2903.80
BOTS 0 07/14/2020 GOVT	1		0	0.37	0.05	10000.00	10000	0.00	0.00
NETHER 0.75 07/15/2028 GOVT	1		0	-0.59	0.01	10000.00	10000	2381.18	2381.18
PGB 2.875 10/15/2025 GOVT	1		0	-0.12	0.00	5000.00	5000	70570.00	70570.00
NESNVX 13/4 11/02/2037 REGS CORP	2	2A	15	0.24	0.01	23529.41	20000	767.27	652.18
IBM 2.85 05/13/2022 CORP	2	2B	50	1.87	0.04	4000.00	2000	0.00	0.00
FCAIM 3.75 03/29/2024 REGS CORP	2	2B	50	0.86	0.02	10000.00	5000	318.51	159.26
VW 4.125 11/16/2038 REGS CORP	2	2B	50	-0.14	0.11	10000.00	5000	0.00	0.00
NOVARTIS FIN. 14/26	2	2B	50	-0.16	0.00	10000.00	5000	24775.28	12387.64

Table 5: Optimized portfolio. Maximization of HQLA expected return and minimization of volatility, under the LCR constraints.

HQLA OPT.	NET OUTFLOWS	MKT VALUE OPT.	LCR OPT.	EXP. RETURN	VOL. RETURN
89867.43	73800.00	102529.41	1.22	-0.14	0.08

Table 6: Results of quadratic programming problem.

We now consider exposure limits instead of the covariance matrix. In particular, we solve Problem (2) including the restrictions

$$a_i \leq 1.5 \cdot \frac{\sum_{i \in \Lambda^H} a_i}{|\Lambda^H|} \text{ for all } i \text{ in } \Lambda^H.$$

That is, we do not allow more than $1.5 \cdot \frac{\sum_{i \in \Lambda^H} a_i}{|\Lambda^H|}$ to be allocated to a single asset. This corresponds to a *linear programming problem with exposure limits*. We now have an expected return of 0.42%, greater than the initial portfolio's 0.09% and the same volatility of 0.48%, see Tables 7 and 8. In practice, exposure limits might be provided by the credit risk function of the bank.

INSTRUMENT	LEVEL	SUBLEVEL	HAIRCUT	EXP. RET.	VOL. RET.	MKT VALUE	HQLA	MKT VALUE OPT.	HQLA OPT.
FRTR 0.5 05/25/2029 GOVT	1		0	-0.33	0.01	10000.00	10000	3519.30	3519.30
IRISH 1 05/15/2026 GOVT	1		0	-0.30	0.01	10000.00	10000	15379.40	15379.40
BOTS 0 07/14/2020 GOVT	1		0	0.37	0.05	10000.00	10000	15379.41	15379.41
NETHER 0.75 07/15/2028 GOVT	1		0	-0.59	0.01	10000.00	10000	0.00	0.00
PGB 2.875 10/15/2025 GOVT	1		0	-0.12	0.00	5000.00	5000	15379.40	15379.40
NESNVX 13/4 11/02/2037 REGS CORP	2	2A	15	0.24	0.01	23529.41	20000	15379.41	13072.50
IBM 2.85 05/13/2022 CORP	2	2B	50	1.87	0.04	4000.00	2000	15379.41	7689.71
FCAIM 3.75 03/29/2024 REGS CORP	2	2B	50	0.86	0.02	10000.00	5000	15379.41	7689.71
VW 4.125 11/16/2038 REGS CORP	2	2B	50	-0.14	0.11	10000.00	5000	6733.58	3366.79
NOVARTIS FIN. 14/26	2	2B	50	-0.16	0.00	10000.00	5000	0.09	0.05

Table 7: Optimized portfolio. Maximization of HQLA expected return under the LCR constraints and exposure limits.

HQLA OPT.	NET OUTFLOWS	MKT VALUE OPT.	LCR OPT.	EXP. RETURN	VOL. RETURN
81476.25	73800.00	102529.41	1.10	0.42	0.48

Table 8: Results of linear programming problem with exposure limits.

Note In all three exercises, the HQLA market value remains unchanged. This represents a situation in which we sell some HQLA and purchase alternative ones with the proceeds.

5. Conclusions

Our framework optimizes return on assets under Basel III Liquidity Coverage Ratio (LCR) liquidity requirements. We adopt a mean-variance approach, providing closed-form expressions for the expectation and variance of returns for assets held until maturity, under credit risk. We complement the theoretical exposition by illustrating three optimization exercises based on traded High Quality Liquid Assets (HQLA), and highlight the benefits of the approach in terms of profitability and risk management. Moreover, we show how, particularly in absence of a covariance matrix, it is possible to include exposure limits to avoid concentration risk. The methodology represents a tractable way of performing assets-side balance sheet optimization taking into account LCR requirement and the institution's risk aversion.

A. Problem Representation

The BCBS (2013) guidelines impose the following restrictions on HQLA,

$$\sum_{i\in\Lambda_{2a}} (1-h_i)\,a_i + \sum_{i\in\Lambda_{2b}} (1-h_i)\,a_i \le 0.40 \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right), \tag{17}$$

$$\sum_{i\in\Lambda_{2b}} (1-h_i)\,a_i \le 0.15 \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right), \tag{18}$$

$$\text{Total net cash outflows} \le \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right),$$

which imply the following restrictions

$$\sum_{i\in\Lambda_1} a_i \ge 0.6 \cdot \text{Total net cash outflows}, \tag{19}$$

$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_{2a}} (1-h_i)\,a_i \ge 0.85 \cdot \text{Total net cash outflows},$$

$$\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i \ge \text{Total net cash outflows}.$$

The optimization problem of interest can be written as

$$\max_{\{a_i\}_{i\in\Lambda^H}} \left(\sum_{i\in\Lambda^H} a_i\mu_i\right), \tag{20}$$

$$\sum_{i\in\Lambda^H} a_i - \text{Stock of HQLA pre-haircut} = 0,$$

$$a_i \ge 0 \text{ for every } i \in \Lambda,$$

$$\sum_{i\in\Lambda_{2a}} (1-h_i)\,a_i + \sum_{i\in\Lambda_{2b}} (1-h_i)\,a_i \le 0.40 \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right),$$

$$\sum_{i\in\Lambda_{2b}} (1-h_i)\,a_i \le 0.15 \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right),$$

$$\text{Total net cash outflows} \le \left(\sum_{i\in\Lambda_1} a_i + \sum_{i\in\Lambda_2} (1-h_i)\,a_i\right).$$

Substituting restrictions (19) into Problem (20) yields the formulation in Problem (2).

Note The LCR numerator does not depend on the denominator, insofar as the amount of resources invested in the "Stock of HQLA" is kept constant during the optimization, that is, if part of the HQLA are not traded for non-HQLA instruments – or HQLA at the numerator – as, in that case, the net cash outflow changes as well. Furthermore, we focus on long positions, hence $a_i \geq 0$ for every $i \in \Lambda$.

B. Probabilities of Outcomes for Expected Returns

In this section, we show that, with n tenors, there are $n+1$ possible outcomes at maturity of the instrument, and that the probabilities associated with these events sum to one, partitioning the probability space. That is, we show that $\sum_{j=0}^{n} p_j = 1$, where p_j is the probability of the jth outcome.

From Section 3.1, we have

$$p_j = \begin{cases} s_j \cdot FPD_{t_j,t_{j+1}} & \text{if } j < n \\ s_j & \text{if } j = n \end{cases},$$

where $s_j = \prod_{\ell=1}^{j} \left(1 - FPD_{t_{\ell-1},t_\ell}\right)$ with the convention $\prod_{\ell=a}^{b} x_\ell = 1$ if $a > b$.

Define $P_n = \sum_{j=0}^{n-1} s_j \cdot FPD_{t_j,t_{j+1}} + s_n$.[24] For $j < n$, p_j is the probability of not defaulting until tenor t_j and subsequently defaulting, and s_n is the probability of the counterparty never defaulting.

We show by induction that $P_n = 1$ for any $n \in \mathbb{N}$. For $n = 1$, we have

$$\begin{aligned} P_1 &= \sum_{j=0}^{0} s_j \cdot FPD_{t_j,t_{j+1}} + s_1 \\ &= FPD_{t_0,t_1} + (1 - FPD_{t_0,t_1}) \\ &= 1, \end{aligned}$$

$\prod_{\ell=1}^{0} \left(1 - FPD_{t_{\ell-1},t_\ell}\right) = 1$ by the above specified convention. We now show that if the statement holds for n, then it holds for $n+1$ (induction hypothesis). We can write

$$\begin{aligned} P_{n+1} &= \sum_{j=0}^{n} s_j \cdot FPD_{t_j,t_{j+1}} + s_{n+1} \\ &= \sum_{j=0}^{n-1} s_j \cdot FPD_{t_j,t_{j+1}} + s_n - s_n + s_n FPD_{t_n,t_{n+1}} + s_n \cdot \left(1 - FPD_{t_n,t_{n+1}}\right) \\ &= \underbrace{\sum_{j=0}^{n-1} s_j \cdot FPD_{t_j,t_{j+1}} + s_n}_{P_n = 1} \underbrace{- s_n \left(1 - FPD_{t_n,t_{n+1}}\right) + s_n \cdot \left(1 - FPD_{t_n,t_{n+1}}\right)}_{=0} \\ &= 1, \end{aligned}$$

which completes the proof.

[24] Note, this definition of probabilities is equivalent to the one introduced in Section 3.1 as $\sum_{j=0}^{n-1} s_j \cdot FPD_{t_j,t_{j+1}} + s_n = \sum_{j=0}^{n} s_j \cdot FPD_{t_j,t_{j+1}}$ when imposing $FPD_{t_n,t_{n+1}} = 1$. Also, $P_n = \sum_{j=0}^{n} p_j$.

C. Alternative Representation of Expectation and Variance of Returns for Instruments held until Maturity

In this section, we show how the expression for μ introduced in Equation (10) can be expressed more compactly in vector notation as

$$\mu = \boldsymbol{p}' \cdot \boldsymbol{f}_{\mu}(\boldsymbol{v}),$$

where $\boldsymbol{f}_{\mu} : \mathbb{R}^{L+1} \to \mathbb{R}^{L+1}$ defined by $x_i \mapsto \left(\frac{x_i}{V_0}\right)^{\frac{1}{\tau_K - t_0}} - 1$, with x_i the i-th component of $\boldsymbol{x} \in \mathbb{R}^{L+1}$, L being the number of tenors of the PD/LGD term structures, and

$$\boldsymbol{v} = \boldsymbol{\ell} + \boldsymbol{c},$$

$$\begin{aligned}
\boldsymbol{p} &= \boldsymbol{s} \odot \boldsymbol{\phi} \\
&= \underbrace{\begin{bmatrix} 1 \\ s_1 \\ \vdots \\ s_{L-1} \\ s_L \end{bmatrix}}_{\boldsymbol{s}} \odot \underbrace{\begin{bmatrix} FPD_{t_0,t_1} \\ FPD_{t_1,t_2} \\ \vdots \\ FPD_{t_{L-1},t_L} \\ 1 \end{bmatrix}}_{\boldsymbol{\phi}} \\
&= \begin{bmatrix} s_0 \cdot FPD_{t_0,t_1} \\ s_1 \cdot FPD_{t_1,t_2} \\ \vdots \\ s_{L-1} \cdot FPD_{t_{L-1},t_L} \\ s_L \end{bmatrix},
\end{aligned}$$

where $\odot$ is the Hadamard (or elementwise) product between vectors,

$$\boldsymbol{\ell} = \begin{bmatrix} (1 - LGD_{t_1})\, N\, (1 + \hat{r}_{t_1,\tau_K})^{\tau_K - t_1} \\ (1 - LGD_{t_2})\, N\, (1 + \hat{r}_{t_2,\tau_K})^{\tau_K - t_2} \\ \vdots \\ (1 - LGD_{t_L})\, N\, (1 + \hat{r}_{t_L,\tau_K})^{\tau_K - t_L} \\ 0 \end{bmatrix},$$

$$
\begin{aligned}
\boldsymbol{c} &= \begin{bmatrix} 0 \\ \sum_{\tau\in\Theta|\tau\leq t_1} CF_\tau (1+\hat{r}_{\tau,\tau_K})^{\tau_K-\tau} \\ \vdots \\ \sum_{\tau\in\Theta|\tau\leq t_{L-1}} CF_\tau (1+\hat{r}_{\tau,\tau_K})^{\tau_K-\tau} \\ \sum_{\tau\in\Theta|\tau\leq t_L} CF_\tau (1+\hat{r}_{\tau,\tau_K})^{\tau_K-\tau} \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \kappa_{t_0,t_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{t_0,t_1} & \kappa_{t_1,t_2} & \cdots & 0 \\ \kappa_{t_0,t_1} & \kappa_{t_1,t_2} & \cdots & \kappa_{t_{L-1},t_L} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix}, \\
\kappa_{t_j,t_{j+1}} &= \sum_{\tau\in\Theta|t_j\leq\tau\leq t_{j+1}} CF_\tau (1+\hat{r}_{\tau,\tau_K})^{\tau_K-\tau}
\end{aligned}
$$

where the remaining variables are as defined in Section 3.

Note The amount $\kappa_{t_j,t_{j+1}}$ corresponds to the value at maturity of the cash flows collected between tenors t_j and t_{j+1} – this formulation allows to express the equations above in terms of *time buckets*.

Moreover,

$$
\boldsymbol{s} = \begin{bmatrix} 1 \\ 1 - FPD_{t_0,t_1} \\ \vdots \\ (1-FPD_{t_0,t_1})\,(1-FPD_{t_1,t_2})\cdots\left(1-FPD_{t_{L-2},t_{L-1}}\right) \\ (1-FPD_{t_0,t_1})\,(1-FPD_{t_1,t_2})\cdots\left(1-FPD_{t_{L-1},t_L}\right) \end{bmatrix}.
$$

The approach to compute the variances in Equation (12) is similar to the one just illustrated, with the only difference lying in the function applied to $\boldsymbol{v}$, namely

$$
\sigma^2 = \boldsymbol{p}^{'} \cdot \boldsymbol{f}_\sigma(\boldsymbol{v}),
$$

where $\boldsymbol{f}_\sigma : \mathbb{R}^{L+1} \to \mathbb{R}^{L+1}$ defined by $x_i \mapsto \left(\left(\frac{x_i}{V_0}\right)^{\frac{1}{\tau_K - t_0}} - 1 - \mu\right)^2$.

D. Basel III Highlights

In this section, we highlight some core concepts from the BCBS (2013) Liquidity Coverage Ratio guidelines.

KEY EXCERPTS FROM GUIDELINES	REFERENCE
"The objective of the LCR is to promote the short-term resilience of the liquidity risk profile of banks."	[Par. 1]
"The Committee has developed the LCR to promote the short-term resilience of the liquidity risk profile of banks by ensuring that they have sufficient HQLA to survive a significant stress scenario lasting 30 calendar days."	[Par. 14]
"This standard aims to ensure that a bank has an adequate stock of unencumbered HQLA that consists of cash or assets that can be converted into cash at little or no loss of value in private markets, to meet its liquidity needs for a 30 calendar day liquidity stress scenario."	[Par. 16]
"Given the uncertain timing of outflows and inflows, banks are also expected to be aware of any potential mismatches within the 30-day period and ensure that sufficient HQLA are available to meet any cash flow gaps throughout the period"	[Par. 16]
"The total net cash outflows for the scenario are to be calculated for 30 calendar days into the future."	[Par. 17]
"The standard requires that, absent a situation of financial stress, the value of the ratio be no lower than 100% (ie the stock of HQLA should at least equal total net cash "outflows) on an ongoing basis because the stock of unencumbered HQLA is intended to serve as a defence against the potential onset of liquidity stress."	[Par. 17]
"During a period of financial stress, however, banks may use their stock of HQLA, thereby falling below 100%, as maintaining the LCR at 100% under such circumstances could produce undue negative effects on the bank and other market participants."	[Par. 17]
"In order to qualify as "HQLA", assets should be liquid in markets during a time of stress and, ideally, be central bank eligible."	[Par. 23]
"Assets are considered to be HQLA if they can be easily and immediately converted into cash at little or no loss of value."	[Par. 24]
HQLA characteristics: "High credit standing of the issuer and a low degree of subordination increase an asset's liquidity. Low duration, low legal risk, low inflation risk and denomination in a convertible currency with low foreign exchange risk all enhance an asset's liquidity. [...] The pricing formula of a high-quality liquid asset must be easy to calculate and not depend on strong assumptions. The inputs into the pricing formula must also be publicly available. In practice, this should rule out the inclusion of most structured or exotic products. [...] The stock of HQLA should not be subject to wrong-way (highly correlated) risk. [...] Being listed increases an asset's transparency. [...] Low volatility: [...] Volatility of traded prices and spreads are simple proxy measures of market volatility."	[Par. 24]
"A bank should periodically monetise a representative proportion of the assets in the stock through repo or outright sale, in order to test its access to the market, the effectiveness of its processes for monetisation, the availability of the assets, and to minimise the risk of negative signalling during a period of actual stress."	[Par. 30]
"Assets received in reverse repo and securities financing transactions that are held at the bank, have not been rehypothecated, and are legally and contractually available for the bank's use can be considered as part of the stock of HQLA."	[Par. 31]

"The stock should be under the control of the function charged with managing the liquidity of the bank (eg the treasurer), meaning the function has the continuous authority, and legal and operational capability, to monetise any asset in the stock. Control must be evidenced either by maintaining assets in a separate pool managed by the function with the sole intent for use as a source of contingent funds, or by demonstrating that the function can monetise the asset at any point in the 30-day stress period and that the proceeds of doing so are available to the function throughout the 30-day stress period without directly conflicting with a stated business or risk management strategy. For example, an asset should not be included in the stock if the sale of that asset, without replacement throughout the 30-day period, would remove a hedge that would create an open risk position in excess of internal limits."	[Par. 33]
"There are two categories of assets that can be included in the stock. Assets to be included in each category are those that the bank is holding on the first day of the stress period, irrespective of their residual maturity."	[Par. 46]
"The 40% cap on Level 2 assets and the 15% cap on Level 2B assets should be determined after the application of required haircuts, and after taking into account the unwind of short-term securities financing transactions and collateral swap transactions maturing within 30 calendar days that involve the exchange of HQLA. In this context, short term transactions are transactions with a maturity date up to and including 30 calendar days."	[Par. 48]
"Total expected cash outflows are calculated by multiplying the outstanding balances of various categories or types of liabilities and off-balance sheet commitments by the rates at which they are expected to run off or be drawn down. Total expected cash inflows are calculated by multiplying the outstanding balances of various categories of contractual receivables by the rates at which they are expected to flow in under the scenario [...]"	[Par. 69]
"Banks will not be permitted to double count items, ie if an asset is included as part of the "stock of HQLA" (ie the numerator), the associated cash inflows cannot also be counted as cash inflows (ie part of the denominator)."	[Par. 72]
"A bank should assume that maturing reverse repurchase or securities borrowing agreements secured by Level 1 assets will be rolled-over and will not give rise to any cash inflows (0%). Maturing reverse repurchase or securities lending agreements secured by Level 2 HQLA will lead to cash inflows equivalent to the relevant haircut for the specific assets. A bank is assumed not to roll-over maturing reverse repurchase or securities borrowing agreements secured by non-HQLA assets, and can assume to receive back 100% of the cash related to those agreements."	[Par. 145]
"The LCR should be used on an ongoing basis to help monitor and control liquidity risk. The LCR should be reported to supervisors at least monthly, with the operational capacity to increase the frequency to weekly or even daily in stressed situations at the discretion of the supervisor. The time lag in reporting should be as short as feasible and ideally should not surpass two weeks."	[Par 162]
"while the LCR is expected to be met on a consolidated basis and reported in a common currency, supervisors and banks should also be aware of the liquidity needs in each significant currency."	[Par 173]
"A "significant counterparty" is defined as a single counterparty or group of connected or affiliated counterparties accounting in aggregate for more than 1% of the bank's total balance sheet, although in some cases there may be other defining characteristics based on the funding profile of the bank."	[Par. 191]
"A currency is considered "significant" if the aggregate liabilities denominated in that currency amount to 5% or more of the bank's total liabilities."	[Par. 211]
"A "significant instrument/product" is defined as a single instrument/product or group of similar instruments/products that in aggregate amount to more than 1% of the bank's total balance sheet."	[Par. 193]

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