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Fast Projection of Reserve and Capital Requirements with Proxy Functions

An emerging business requirement for North American insurers is the ability to project forward stochastic reserve and capital requirements under various planning scenarios to a specific future date. In this note we consider applying proxy functions to this task, using function fitting techniques described in a previous research note (Clayton and Morrison, 2018). We illustrate the power of the proxy approach to do basic stress-testing along with reverse stress-testing and sensitivity analysis for an example book of variable annuities and a backing asset portfolio.

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1. Introduction

An ongoing trend in the U.S. insurance sector in recent years – reflected most notably in the parallel development of principle-based reserving ("PBR") and risk-based capital ("RBC") regimes – has been the move towards reserve and capital requirements that include stochastic components. Typically these calculations involve projections of cash flows for liabilities and backing assets over the lifetime of the products in question using a prescribed stochastic model, with a relevant conditional tail expectation ("CTE") statistic of the present value of accumulated deficit distribution setting the required reserve or capital level. This practice has been present in the variable annuities market since the adoption of C-3 Phase II in 2005 and Actuarial Guideline XLIII ("AG-43") in 2008, and more recently it has become a regulatory requirement for all individual life products[1] issued after January 2017, as specified by NAIC Valuation Manual 20 ("VM-20").

Simultaneously, an emerging business need is the ability to *project* these requirements forward to some future date under a given planning scenario, either to forecast the firm's balance sheet in the near-term for such applications as stress-testing or capital optimization or to do longer-term projections for analysis of asset adequacy to meet promised liability cash flows ("cash flow testing"). This can be seen, in particular, in the requirements for the Own Risk and Solvency Assessment ("ORSA") under the NAIC's Solvency Modernization Initiative; the ORSA guidance manual, for example, includes the requirements for a "prospective solvency assessment" (TK-citation):

> *The insurer's prospective solvency assessment should demonstrate it has financial resources necessary to execute its multi-year business plan in accordance with its stated risk appetite. If the insurer does not have the necessary available capital (in terms of quality and/or quantity) to meet its current and projected risk capital requirements then it should describe the management actions it has taken (or will take) to remedy any capital adequacy concerns.*

Similar calculations will likely be required in the Canadian segregated funds market for dynamic capital adequacy testing ("DCAT"), in which companies must project asset requirements defined by CTE under various stress scenarios. The combination of these – stochastic calculations of reserve and capital requirements at future dates under various business planning scenarios – creates an inherently nested-stochastic problem, with total scenario requirements easily reaching into the hundreds of thousands or millions.

Our previous research notes have described a fast method of accomplishing the same projections using proxy function techniques, drawing on similar methods that have achieved great success in the realms of projecting market consistent value for 1-year value-at-risk capital calculations and American option pricing. Some necessary modifications to the function fitting algorithm have been made to account for the difference in probability measure and risk statistic being calculated: here, a CTE of a (typically) real-world distribution vs. an expected value of a risk-neutral distribution.[2]

In this note we illustrate the power of the proxy function approach by conducting an asset adequacy analysis for an example book of variable annuities with a portfolio of backing assets. After fitting proxy functions for a CTE(70) reserve and CTE(90) capital requirement, we use these to test whether projected assets will be adequate to meet projected reserve and capital requirements under various "narrative scenarios" of the coming 5 years of yield curves and equity market returns.

Next, we perform analyses that are only really possible with proxy functions: a "reverse stress test" to *find* the scenario(s) that *would* cause a reserve or capital breach in the future and a sensitivity analysis to quickly show the relationship between reserve/capital requirements and underlying economic risks. We also consider overlaying a probability model assumption for the economic risk variables defining the planning scenario to answer the question: *What is the probability that projected assets will be inadequate to cover future reserve and capital requirements?*

The latter investigations offer a glimpse of what the proxy methods can enable firms to accomplish. By replacing a fully nested-stochastic calculation with a simple function call, the approach allows for fast reassessment of reserve and capital positions under any change to underlying assumptions, even up to millions of times in a matter of seconds. Brute force scenario calculations, by contrast, would likely make this sort of analysis prohibitively costly.

1. Other than those lines of business exempt from the requirements after a stochastic exclusion test is performed. See TK.

2 See Clayton and Morrison (2018) for technical details. In particular, for the present note we employ the "quantile regression plus OLS" method described there.

2. Portfolio characteristics and scenario definitions

Our example block of liabilities consists of approximately 75,000 variable annuity policies with a heterogeneous mix of accumulation (GMAB), withdrawal (GMWB), and death benefit (GMDB) guarantees at various levels. The policyholder characteristics (issue age, policy anniversary dates, etc.) are likewise realistically mixed.

We assume initial firm-wide assets of $1 billion backing these liabilities are invested in a mix of 25% equities and 75% in a 20-year duration bond index fund rebalanced quarterly.

At each future point in time, and conditional on an assumed future scenario detailed below, we define the required reserve as the CTE(70) and required capital as the CTE(90) of the distribution of present values of accumulated deficiencies over a stochastic run-off projection of assets and liabilities from that point until the end of the liabilities. That is, for each "outer" scenario we imagine an "inner" stochastic projection, in which we generate asset and liability cash flows to compute a deficit value and define reserve and capital requirements as tail statistics of this distribution (average of the worst 30% and worst 10%, respectively).

Our goal is to determine whether projected assets under the planning scenario are sufficient to cover the reserve and capital requirements. We consider this question at a particular horizon date 5 years in the future.

In projecting reserve and capital requirements at such a distant future date, we are faced with a naturally path-dependent problem: the particular path of the economic risk factors that define our planning scenario will have an effect both on the asset portfolio and on the in-force liability guarantees at that time, and therefore on the subsequent reserve and capital requirements. Moreover, the effects of each "outer" planning scenario are felt differently for each policy depending on the specific guarantees, and the risk metrics we seek – both CTEs – are non-additive, meaning we must account for the state of the whole portfolio simultaneously when projecting liability cash flows.

Therefore, to keep the number of "risk dimensions" from becoming too great, we limit the kinds of planning scenarios we can consider. Specifically, we allow planning scenarios with the following degrees of freedom:

» Change to the level and slope of the yield curve over the first quarter (described by the first two principal component shocks)

» A parallel shift to the yield curve over the remainder of the next 5 years, defined by the size of the shift and the period over which the shift takes place

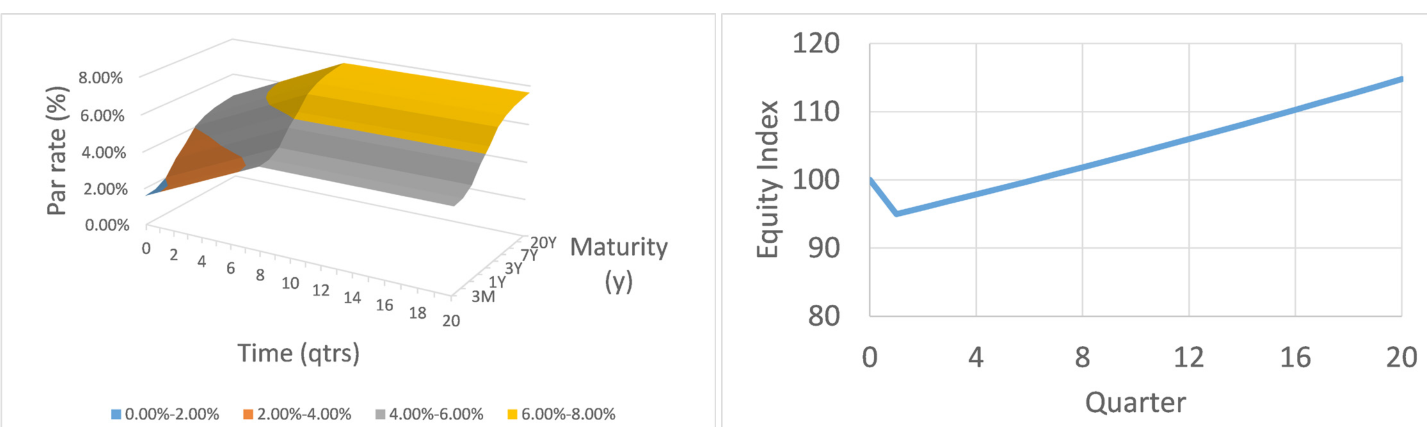
» The change in the U.S. equity index over the first quarter

» The subsequent returns of the equity index, assumed to be level for the remainder of the 5 year projection

In total then we have 6 risk factors (4 for yield curves and 2 for equities), any combination of which fully determines a planning scenario up to the horizon date. The charts below illustrate one such path.

For this exercise, we assume a total scenario budget of 100,000 fitting scenarios spread out over this 6-dimensional space (each with a single associated inner scenario result), and we fit proxy functions for the CTE(70) and CTE(90) of the PV deficit distributions.

Figure 1: Possible yield curve and equity evolution



3. Narrative scenarios

By choosing values for the planning scenario values described above, we are able to construct narrative scenarios of the possible economic future. Typically these are specified either by a regulator or by senior management. An example selection of narratives to consider for yield curves might consist of:

- Level yield curve
- Grade up/down
- Pop up/down 300 bps
- "Cap," meaning a pop up followed by a gradual decrease
- "Cup," meaning a pop down followed by a gradual increase

Similarly, possible equity narratives might consist of:

- Best estimate equity return
- Level returns of anywhere from -5% to +15% per annum
- Modest or sustained growth
- Q1 correction and a moderate recovery
- Q1 crash and a slow recovery

And so on, including combinations of yield curve and equity scenarios. Each narrative will need to be translated into precise risk factor values and then passed to the proxy functions, which then immediately report the required reserve and capital. For our example, we consider 104 total narratives made up of all combinations of 8 yield curve scenarios with 13 equity scenarios.

Projecting assets along the same paths allows us to quickly compare assets with reserve and capital requirements at the end of the planning scenario, which we summarize according to the Reserve and Capital Ratios:

$$Reserve\ Ratio = \frac{Available\ Assets}{Required\ Reserve};\ Capital\ Ratio = \frac{Available\ Assets}{Required\ Capital}$$

For example, ranking all the narratives by Capital Ratio shows the greatest risk is to an equities crash combined with low interest rates

Figure 2: Results of narrative scenarios for yield curves and equities (worst 12 shown):

Treasury Scenario	Equity Scenario	Terminal Assets ($MM)	Req. Reserve ($MM)	Reserve Ratio	Req. Capital ($MM)	Capital Ratio
Pop Down 300bp	Q1 Crash Slow Recovery	1,821.45	2,232.04	82%	$ 2,957.52	62%
Grade Down	Q1 Crash Slow Recovery	1,722.49	1,916.99	90%	$ 2,392.90	72%
Cup	Q1 Crash Slow Recovery	812.59	962.44	84%	$ 1,115.25	73%
Grade Up	Q1 Crash Slow Recovery	757.43	796.50	95%	$ 961.77	79%
Pop Up 500bp	Q1 Crash Slow Recovery	696.16	737.28	94%	$ 880.61	79%
Level	Q1 Crash Slow Recovery	1,122.15	1,216.18	92%	$ 1,408.88	80%
Pop Down 300bp	Level -5%	1,842.28	1,722.13	107%	$ 2,255.37	82%
Pop Up 300bp	Q1 Crash Slow Recovery	774.30	810.90	95%	$ 937.77	83%
Cap	Q1 Crash Slow Recovery	1,640.57	1,615.24	102%	$ 1,981.17	83%
Pop Down 300bp	Q1 Correction Moderate Recovery	1,906.75	1,495.25	128%	$ 2,095.76	91%
Pop Down 300bp	Level 0%	1,898.84	1,407.92	135%	$ 1,941.09	98%
Grade Down	Level -5%	1,743.32	1,408.32	124%	$ 1,752.02	100%

The pattern with respect to yield curves is somewhat surprising given the improved performance of the firm's fixed income assets in the low interest rate scenarios, but evidently this is more than offset by the increases to required reserves and capital in those environments.

4. Reverse stress-testing

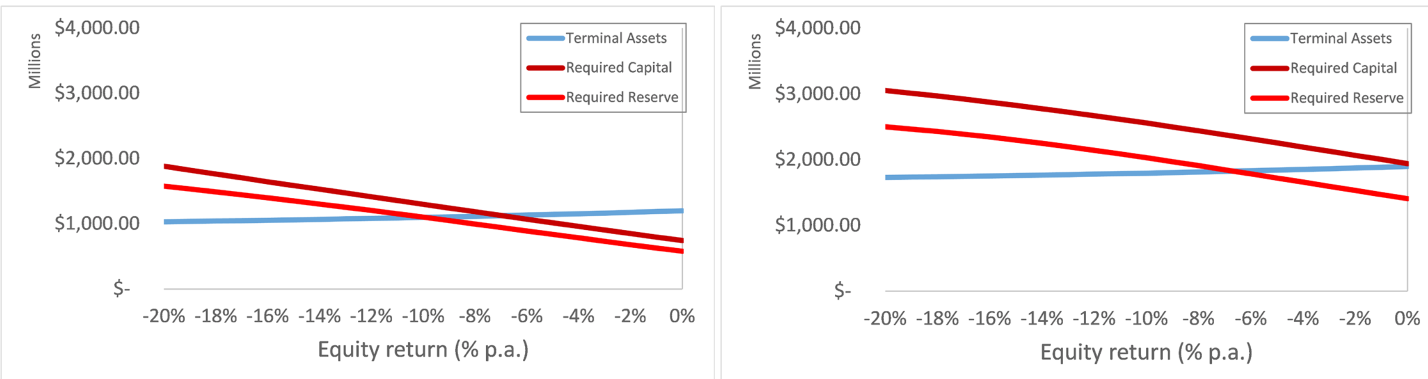
The preceding analysis of a manageable number of narrative scenarios is arguably possible with a "brute force" calculation approach. Even assuming a typical scenario budget of ~5,000 stochastic scenarios per narrative to accurately estimate CTE numbers, it would seem that we have not gained too much with the proxy method: perhaps a savings of approximately 80% (100,000 proxy fitting scenarios vs. 100 x 5,000 stochastic scenarios).

However, the true power of the proxy method starts to become apparent if we use the narrative-functional relationship in *reverse*. That is, by setting a desired Reserve or Capital Ratio, we can use the proxy function to construct the narrative(s) that would achieve this end state, which naturally may have multiple solutions depending on which variables are allowed to change.

For example, in the figures below we show the relationship of projected assets, reserves, and capital against equity returns under two yield curve assumptions: one in which interest rates stay constant, and the other where interest rates pop down 300 bps and then are held fixed. We see in the former case that a sustained equity loss of approx. 8% would cause assets to be insufficient to cover capital requirements, whereas 10% losses would cause an insufficiency to cover reserves. In the latter case, the spread between capital and reserve requirements has grown and exposures to equity increased, such that even a 0% loss in equities would result in a capital breach and a sustained loss of approx. 7% would result in inadequate reserves.

Such narratives may not have been among those originally considered, but the proxy function approach quickly reveals their importance.

Figure 3: Assets/capital/reserve vs. equity returns (left with level yield curve; right with yield curve down 300 bps)



5. Sensitivity analysis

Furthering the kinds of analysis shown above, with the proxy functions in hand we can investigate all manner of relationships between reserves/capital and the economic variables. For example, we can equally well fix the equity returns at "best estimate" levels and use the proxy functions to show the behavior of assets, reserves, and capital vs. yield curve changes. Figure 4 below is one such example.

From this we see immediately that the firm is reasonably well hedged against parallel shifts to the yield curve across a wide range; decreases in asset value due to rising interest rates are met with commensurate decreases in reserve and capital requirements, and similarly for falling interest rates. Similar analyses could show the effects of a steeper or flatter yield curve and possibly suggest changes to the firm's asset positions to better reduce volatility of surpluses through time.

Adding another dimension to the analysis could likewise be revealing. For example, we may be interested in the question: *How do changes in the yield curve affect overall sensitivity to equity market returns, measured by Reserve and Capital Ratios?* The proxy function makes this an easy task. Figure 5 below show an analysis of the Reserve Ratio as a function of equity return and yield curve shift, shown as a heat map and a 3d graph:

Figure 4: Assets/capital/reserve vs. yield curve changes (parallel shift)

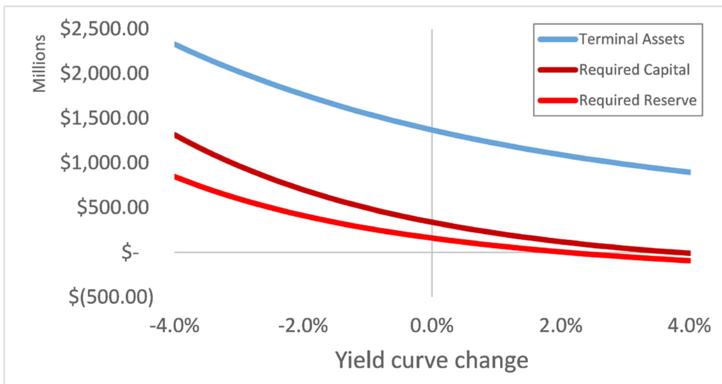
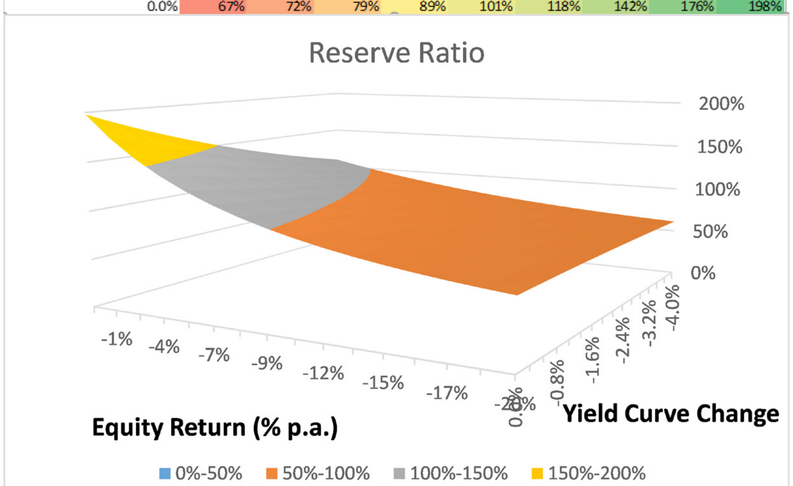


Figure 5: Reserve Ratio as a function of equity returns and yield curve changes

YC_Change/Equity Return	-20%	-17%	-15%	-12%	-9%	-7%	-4%	-1%	0%
-4.0%	61%	64%	68%	72%	78%	85%	94%	104%	110%
-3.5%	62%	66%	70%	75%	81%	89%	99%	111%	118%
-2.9%	63%	67%	71%	77%	84%	93%	105%	119%	127%
-2.4%	65%	68%	73%	79%	88%	98%	111%	127%	137%
-1.9%	65%	69%	75%	82%	91%	102%	117%	137%	148%
-1.3%	66%	70%	76%	84%	94%	107%	124%	147%	161%
-0.8%	67%	71%	78%	86%	97%	111%	131%	158%	175%
-0.3%	67%	72%	79%	88%	100%	116%	138%	169%	190%
0.0%	67%	72%	79%	89%	101%	118%	142%	176%	198%



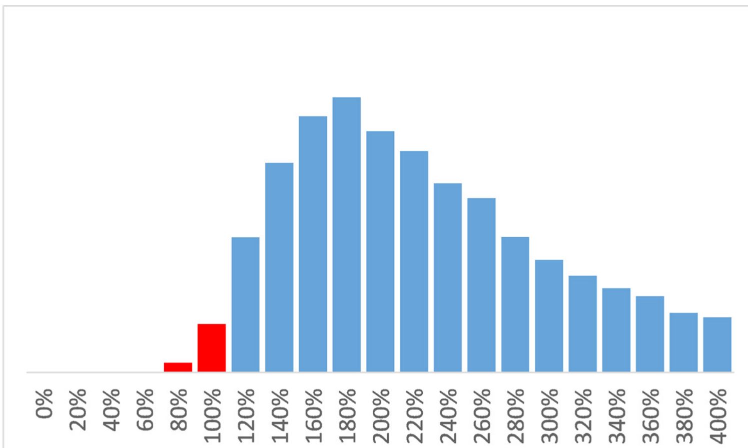
6. Probabilistic analysis

Thus far we have consider the planning scenarios to be "deterministic" in the sense that they are specified without reference to an underlying probability model. However, there is of course no reason we cannot add probabilities into the mix. With a stochastic model for the economic risk drivers, we can sample planning scenarios from their own distribution and answer questions such as: *What is the probability of available assets being less than required capital at year 5?*

For example, shown below is a histogram of the Capital Ratio over 10,000 samples from a simple mean-variance-covariance model of the 6 economic variables comprising our planning scenarios:

We can discern the overall shape of the distribution and estimate probability of asset inadequacy – here on the order of about 2%. We could in principle even extend this to see the effect that *changing* stochastic model assumptions has on this probability, or many other creative investigations. Under any assumption, the proxy function would allow extremely fast recalculation of the company's reserve and capital positions in each scenario, millions of times if needed.

Figure 6: Capital Ratio histogram based on stochastic scenario model (N=10,000)



7. Summary

We have shown examples of some of the many possible uses proxy functions can have within the framework of projecting stochastic reserves and capital defined by CTEs. With a considerable savings of scenario run-time over full nested-stochastic runs, the proxy function approach allows an insurer to quickly recalculate its reserve and capital position in a variety of possible future economic environments. This can be used for stress-testing the company balance sheet, testing asset and liability management decisions, finding risk tolerance limits, cash flow testing, and many other similar related applications.

As a bonus, the proxy function approach allows for analyses that would be essentially impossible under normal procedures, namely using the functional relationship given by the proxy function to identify stress scenarios of importance, perform sensitivities with respect to key risk drivers, or to do quick stochastic projections. As the requirements for projecting stochastic reserves and capital become increasingly common for U.S. insurers, we expect the proxy function method to play a key role in making these calculations computationally tractable.

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