MAY 2016

MOODY'S ANALYTICS

RESEARCH INSURANCE

Proxy Methods for Hedge Projection: Two Variable Annuity Case Studies

Authors
Aubrey Clayton PhD
Steven Morrison PhD

Moody's Analytics Research

Contact Us

Americas
+1.212.553.1658
clientservices@moodys.com

Europe
+44.20.7772.5454
clientservices.emea@moodys.com

Asia (Excluding Japan)
+85 2 2916 1121
clientservices.asia@moodys.co

Japan
+81 3 5408 4100
clientservices.japan@moodys.com

Overview

The challenge of projecting dynamic hedge portfolios for blocks of Variable Annuities (VA) with complex guarantees has proven to be extremely computationally demanding but also essential for obtaining hedging credit in reserves or capital calculations.

Our previous research has argued in favor of proxy function methods such as Least Squares Monte Carlo as alternatives to full nested stochastic calculations, and we have demonstrated the successful application of these methods for hedging in simple option examples including path-dependent options. This paper extends previous work by considering actual VA products with guarantees of the kind offered by insurers in North America and Europe.

By specifying the variables of the proxy functions in ways that reflect an understanding of the product features, we are able to design proxy functions that reproduce the complex behavior of market value and Greeks for these products, including path-dependency and discontinuous jumps. The proxy functions are shown to validate well compared to “brute force” nested stochastic calculations at a miniscule fraction of the computational overhead.

Table of Contents

1. Introduction

The typical requirements for an insurer to obtain credit for a "clearly defined hedging strategy" include calculating the hedging profit/loss over a projected run-off of assets and liabilities. In order to calculate the rebalanced hedge positions, both market value and sensitivity of value (i.e., "Greeks") required for the hedge must be known at any given simulation time step. For complex products, these calculations usually require "inner" Monte Carlo valuation using risk-neutral valuation techniques. Thus the stochastic problem of computing the run-off capital measure becomes a nested stochastic, or stochastic-on-stochastic, problem.

To understand the computational scope of such a problem, consider the following fairly typical scenario requirements:

1,000 run-off scenarios (real world scenario generator)

x 480 months (hedge rebalancing time steps)

x 10,000 valuation scenarios (risk-neutral scenario generator)

x 3 sensitivities (base value + 2 Greeks)

This single capital calculation therefore requires **14.4 billion** scenarios to be passed to the actuarial Asset Liability Management (ALM) or cash flow engine. For a large block consisting of millions of distinct policies, the simulation run-time quickly escalates out of control. As a practical approach, many insurers have sought to reduce run-time by imposing approximations on the liability portfolio (e.g., "model points") or unreasonable simplifying assumptions on the hedge portfolio, for example a reduction in the assumed rebalancing frequency.

As an alternative, beginning with the work of Bauer, Bergmann, and Reuss (2009) and Cathcart and Morrison (2009), drawing upon the American option pricing methods developed by Longstaff and Schwartz (2001), the use of proxy functions has emerged in recent years as a general approach for insurance problems requiring nested simulation. For example, proxy functions are now widely used by insurance firms in the projection of market consistent value of liabilities for the purpose of calculation of 1-year value-at-risk. These functions provide analytical approximations to market consistent value that can be evaluated quickly compared to a full risk-neutral simulation.

Previous research notes (Clayton, Morrison, Turnbull, and Vysniauskas 2013; Clayton et al. 2014) have illustrated the extension of proxy techniques to calculating Greeks of a simple put option and an exotic, path-dependent option. In this paper we demonstrate their application to real portfolios of Variable Annuities with complex guarantees.

Specifically, we consider products modeled closely on those offered by two insurers:

» "VA1": A flexible premium VA with guaranteed lifetime withdrawal benefit (GLWB) and annual ratchet, applied to a fund with a fixed asset allocation of equities and bonds.

» "VA2": A Variable Annuity with lifetime income benefit, underlying fund invested in equities and cash using a constant proportion portfolio insurance (CPPI) investment strategy, daily ratchet, and rollup assumption.

In each example, we define fitting scenarios, estimate market value and Greeks, and train proxy functions according to the methodology described in the previous research notes. The main innovation of the current work is that, among the risk factors, we include variables to describe individual policy characteristics. Therefore, the proxy functions we calibrate effectively describe the value and Greeks at the policy level, making them suitable for projection over future time steps as the composition of the aggregate portfolio changes.

2. Methodology

The process of designing and calibrating a proxy function generally involves four major steps:

- Identifying relevant risk factors that are allowed to vary and generating "fitting points" to fill out the risk factor space.
- Estimating "crude" value at each fitting point using a small number of inner scenarios.
- Fitting a proxy function through the crude estimates using regression or other function fitting techniques. For the purposes of these projects, we have used proxy functions based on artificial neural networks. While polynomial proxy functions are more commonly used in 1-year VaR applications, in our experience artificial neural networks are better able to handle the complex behavior we observe in multi-period Greek estimation.
- Validating the resultant functions at a relatively small number of "validation points," using a large number of inner scenarios to construct an accurate value at those points.

2.1 Path-dependency

As described in Clayton et al. (2014), for liabilities with path-dependent features the value at any given time will naturally depend on the path of economic risk factors up that that point. However, it may be the case that the relevant features of the path can be summarized in a small number of key variables that act as sufficient summary statistics of the path. For example, for a lookback option with payoff depending on the maximum attained value over the lifetime of the fund, the entire path of fund returns can be summarized by the "running maximum," that is, maximum value to date at a given point in the simulation. Similarly, for a VA guarantee, the path of fund returns may be fully captured by variables such as the current "moneyness" of the guarantee, meaning the current fund value relative to the income base from which guarantee cash flows are computed.

Exactly what variables are necessary to collapse the path down to a manageable number of dimensions will depend on the particular nature of the guarantees embedded in the product.

2.2 Policy vs. portfolio

Complicating the issue described in the last section, the summary statistics describing the relevant features of the path to date may have different values from policy to policy within a portfolio of liabilities. For example, two policyholders may have different moneyness values (account value relative to income base) even despite a shared history of market returns, due to different fund allocations, inception dates, or account crediting rules ("ratchet" or "rollup" features). As a projection evolves, the mix of policies within the portfolio will naturally change, meaning that a small number of variables is unlikely to capture the full dynamics of the portfolio as a whole.

Instead, for the case studies presented here, we have taken the approach of designing proxy functions *at the policy level*, meaning the functions have policy-specific variables as inputs such as contract duration or time since last ratchet date. These are then combined with market risk variables (e.g., yield curves) that are common for all policies. The result is a single function that allows us to compute the value (or Greek) associated to each individual policy in the portfolio. At any given time, then, the portfolio value (or Greek) is computed as the sum over the portfolio:

$$V_{Portfolio}(t) = \sum_{i=1}^{N} V_{Policy}(\{policy\ variables(i,t)\}, \{market\ variables(t)\})$$

2.3 Handling discontinuities

Complex guarantees may give rise to discontinuities in the value (or Greek) with respect to any one of the underlying variables, particularly as other variables are held fixed. For example, a product with an annually-resetting ratchet applied to the guarantee (as in the VA1 example below) will show a discontinuity in the behavior of value vs. contract duration for a fixed moneyness value on either side of the ratchet date. This comes about for the simple reason that it means something very different to have, say, a fund value of $1.1M relative to a current income base of $1M immediately before a ratchet is applied (increasing the income base to $1.1M) vs. immediately afterwards, when there is still a chance the fund losses over the coming year will prevent the ratchet from kicking in.

However, the functional forms we use – either polynomials or artificial neural networks – are necessarily continuous with respect to any input variables. To address this, we have decomposed some variables, such as contract duration, into two separate components, for example

$$contract\ duration = number\ of\ ratchet\ periods + time\ since\ last\ ratchet$$

This allows the proxy function to assign significantly different values to policies with, say, $duration\ =\ 0.99$ and $duration\ =\ 1.01$, because those values now correspond to the points $(0, 0.99)$ and $(1, 0.01)$, which are "far apart" in risk-factor space.

Similarly, any policy asset allocation rules that apply differently to different "regions" of the risk-factor space, for example, a cap or floor on equity asset allocation (as in the VA2 example, below), may introduce discontinuities in the sensitivities with respect to underlying (i.e., delta). To allow for this kind of discontinuity, we include additional binary separating variables that take discrete values according to the region of the risk factor space a given point lies in.

Naturally, increasing the number of dimensions in the risk-factor space in this way generally comes at the expense of fitting quality, so we must choose explanatory variables carefully based on knowledge of the product features.

3. Case Study Results

In this section we present the results of applying the proxy function methodology described above in our two VA product examples.

3.1 VA1 – Fixed allocation with annual ratchet

For this exercise, we considered a block of Variable Annuities with the following product features:

» Flexible premium deferred VA in a waiting period

» Guaranteed lifetime withdrawal benefit (GLWB) with annual ratchet and deferral bonuses

» Three possible fixed fund allocations: conservative, moderate, and aggressive

Common market risk variables were:

» Yield curve level

» Yield curve slope

» Yield curve curvature

The policy-specific variables were identified as:

» Gender

» Asset allocation

» Issue age

» Contract duration

» Current moneyness of guarantee (defined as fund value / benefit base)

The first two policy-specific variables were discrete, requiring separate proxy function fits for each combination of gender and asset allocation. The results shown below reflect only the (female, conservative) combination, but other results were materially similar. The latter three variables were treated as continuous, with contract duration further separated into number of anniversary dates and time since last anniversary date, as described in the previous section. Additionally, policy rules (deferral bonuses, etc.) embedded in the product treated policyholders differently according to issue age in 5 year increments – that is, issue ages between 57 and 61 had one assumption, those from 62 to 66 had another, etc. – so indicator variables were used to label the issue age group for a given policy and allow the proxy function to separate their values.

For fitting purposes, we assumed an overall budget of 500,000 fitting points x 16 inner scenarios per fitting point. Note that these fitting points include policy variables as well as market variables, so the overall run-time for the fitting set was comparable to a single point-in-time valuation for a portfolio of 500,000 distinct policies using 16 inner scenarios (approximately 3 days on a desktop computer in this case). For each fitting point we calculated three estimates:

» Base market consistent value

» Market value under an equity "up" shock of 1%

» Market value under an equity "down" shock of 1%

The latter two were used to construct an estimate of policy dollar delta. All estimate values were normalized to be per $1M benefit base.

For validation purposes, we used a total of 825 points x 50,000 inner scenarios per point, chosen to demonstrate univariate stresses in each dimension (i.e., holding other variables constant). Comparing proxy function results against validation results shows excellent agreement for market consistent value and generally very good agreement for delta across all validation points.

Figure 1: Proxy vs. validation results (market consistent value)

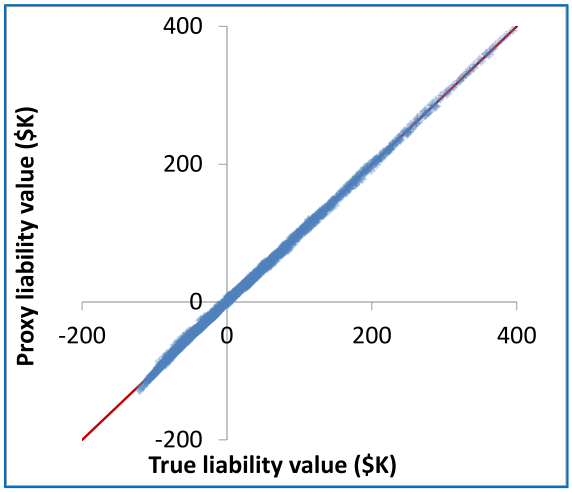
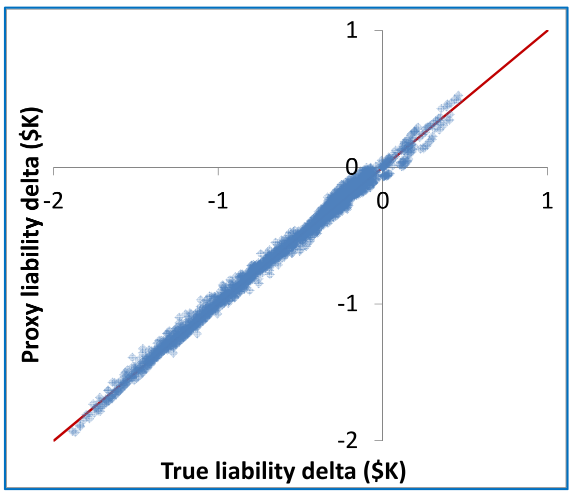


Figure 2: Proxy vs. validation results (delta)



Analyzing individual risk factors in isolation, we see that the proxy functions have captured some complex dependencies in the product value and delta. For example, for a policy with moneyness > 1 (implying a ratchet event is possible), we observe the discontinuity in policy value and delta on either side of the ratchet anniversary date:

Figure 3: Univariate validation results (value vs. contract duration)

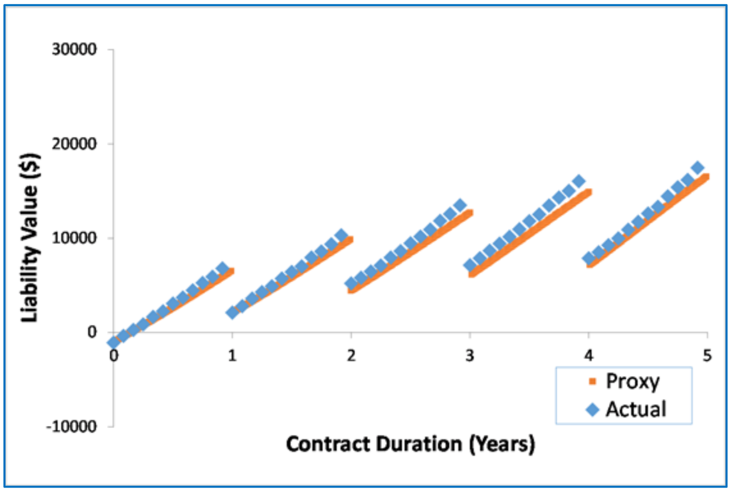
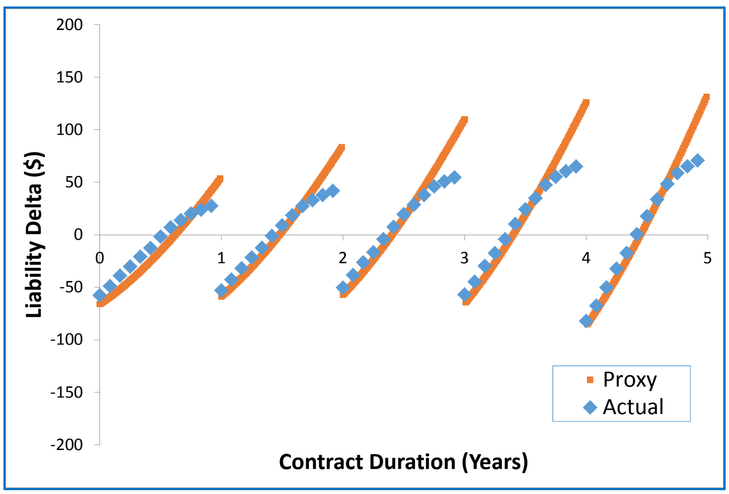
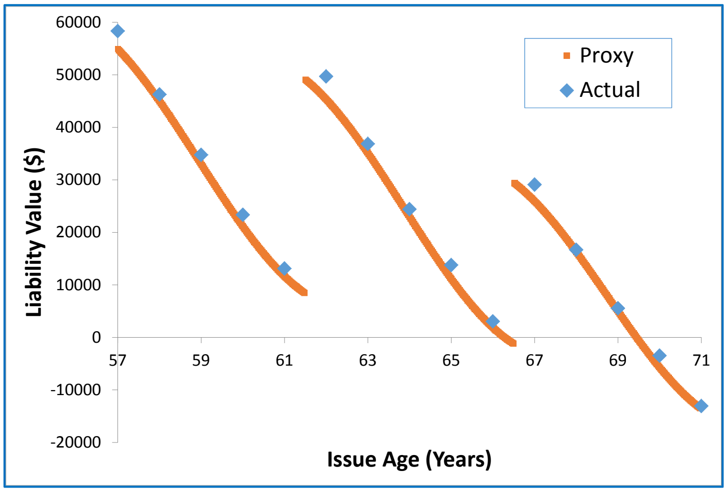


Figure 4: Univariate validation results (delta vs. contract duration)



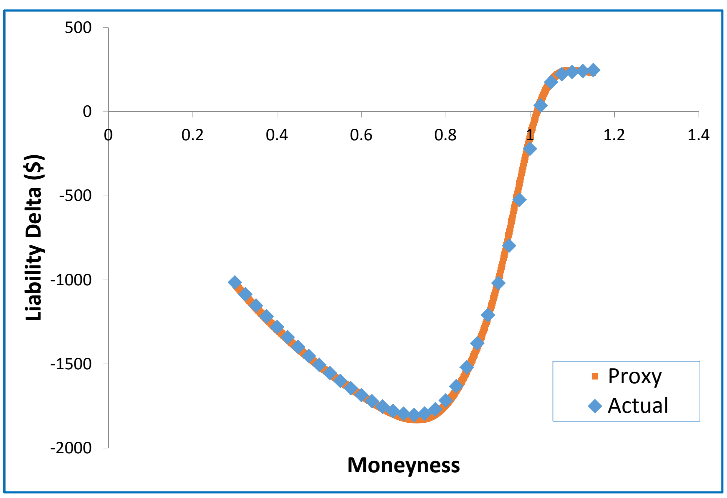
Similarly, by varying issue age only, we see the clear discontinuity arising from the assumed grouping of issue ages into 5-year increments:

Figure 5: Univariate validation (value vs. issue age)



We also observe an interesting feature of delta vs. moneyness for a policy close to its ratchet date: under ordinary circumstances, we would expect the policy delta to be negative, meaning (from the perspective of the policyholder) an increase in equity underlying decreases the value of the guaranteed minimum income from a fund invested in that equity index. Thus, the correct delta hedging position for the insurer is to short the index. However, close to ratchet date, an increase in the equity index brings a greater chance of the ratchet being applied and a corresponding increase to the benefit base; therefore, under these conditions the policy delta can reverse sign and become positive. Thus, for the insurer, the correct hedge position is long in the equity index. Here again the proxy function captures this complex dependency.

Figure 6: Univariate validation results (value vs. moneyness; duration = 0.8)



3.2 VA2 – Dynamic allocation with daily ratchet

For our second case study, we considered a Variable Annuity product with these features:

- Fund invested in a mixture of equities and cash, with minimum and maximum equity allocations of 10% and 100%, respectively.
- Daily reinvestment strategy using constant proportion portfolio insurance (CPPI) with multiplier = 5. Note that the CPPI strategy is managed for each individual policyholder, being based on their specific moneyness level.
- A daily ratchet applied during the accumulation phase along with a 5% per annum rollup (minimum crediting return).
- A fixed 5% annual withdrawal at each anniversary date during the withdrawal phase for a total of 20 years.

The market risk variables were:

- Yield curve level (defined as 20-year maturity par yield)
- Yield curve slope (defined as the difference between 20-year par yield and 1-year par yield)

Policy-specific variables were:

- Current moneyness of fund (defined as fund value / income base)
- Remaining accumulation period (accumulation phase)
- Remaining withdrawal period (withdrawal phase)

Policy lapses were assumed to be dynamic according to a logistic probability distribution applied to the moneyness variable.

An interesting difference between this and the previous example was that we did not need to consider asset allocation as a variable, because the CPPI investment strategy dictates the asset allocation as a function of moneyness. As in the VA1 case study, we separated remaining withdrawal period into number of withdrawals and time to next withdrawal to account for the discontinuity occurring at each withdrawal date. Also, we introduced binary indicator variables to identify those policies where the equity allocation had reached its minimum or maximum, since these regions naturally have different relationships to equity value.

We assumed a scenario budget of 40,000 fitting points x 20 inner scenarios per fitting point and estimated five quantities at each point:

- Base market consistent value (guarantee cost)
- Market value under a 1% equity up shock
- Market value under a 1% equity down shock
- Market value under a 1 basis point interest rate parallel up shock
- Market value under a 1 basis point interest rate parallel down shock

The equity up/down shocks were used to construct an estimate of policy dollar delta, and the interest rate up/down shocks were used to estimate policy DV01 (dollar value change per basis point change in yield). All estimates were normalized to a $1M income base.

3.2.1 Withdrawal phase

To get an overall assessment of the performance of the proxy function, we compare proxy vs. validation results from the cash flow model (using 5,000 risk-neutral scenarios) for a selection of risk-factor points distributed uniformly throughout the fitting space using Sobol sampling. We observe generally very good agreement for the guarantee cost and DV01 functions and good agreement for the delta function, with a few points of relatively large divergence.

Figure 7: Proxy vs. validation results (guarantee cost)

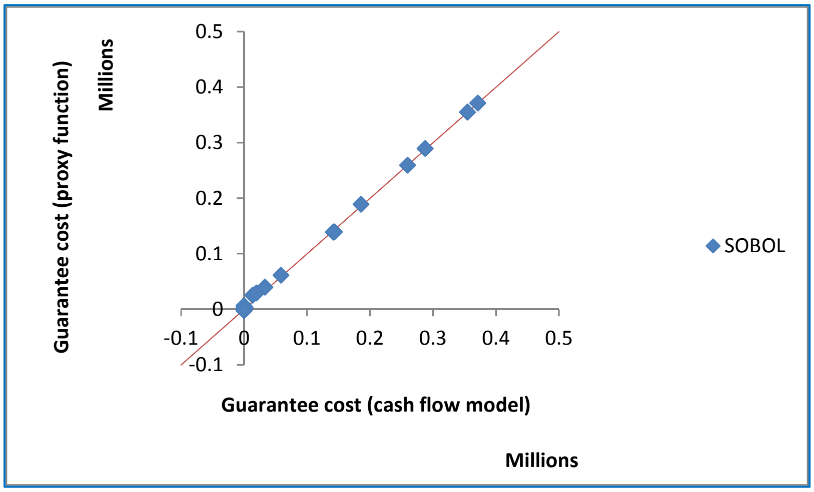


Figure 8: Proxy vs. validation results (DV01)

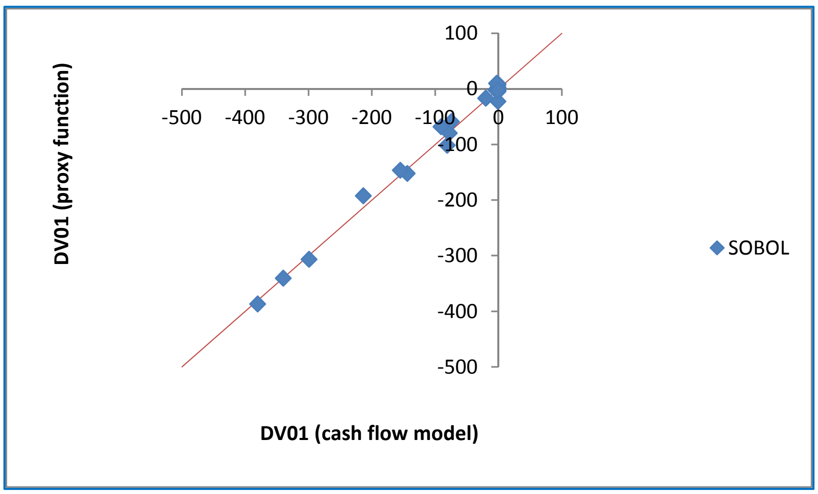
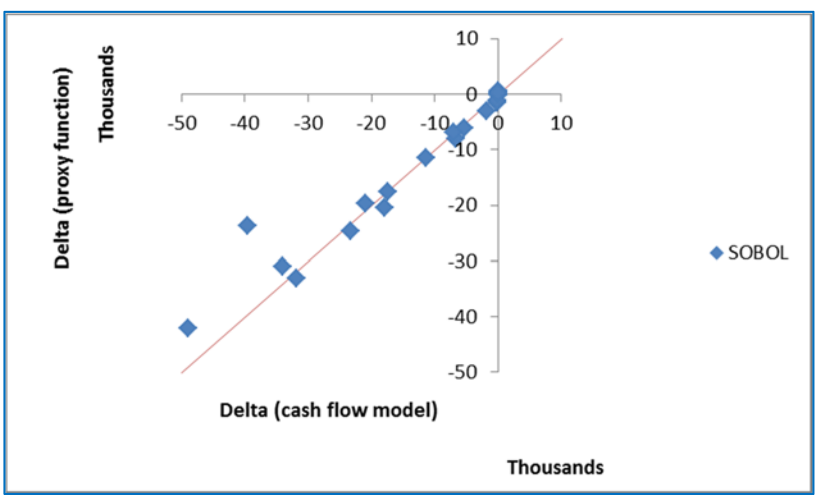
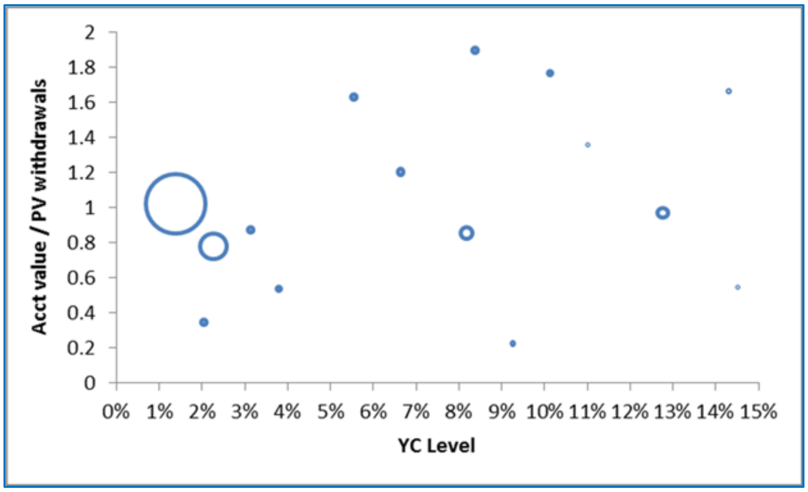


Figure 9: Proxy vs. validation results (delta)



Examining the delta proxy errors in detail shows that the largest errors occur for low yield curve levels and policies with fund value nearly equal to the present value of future withdrawals.

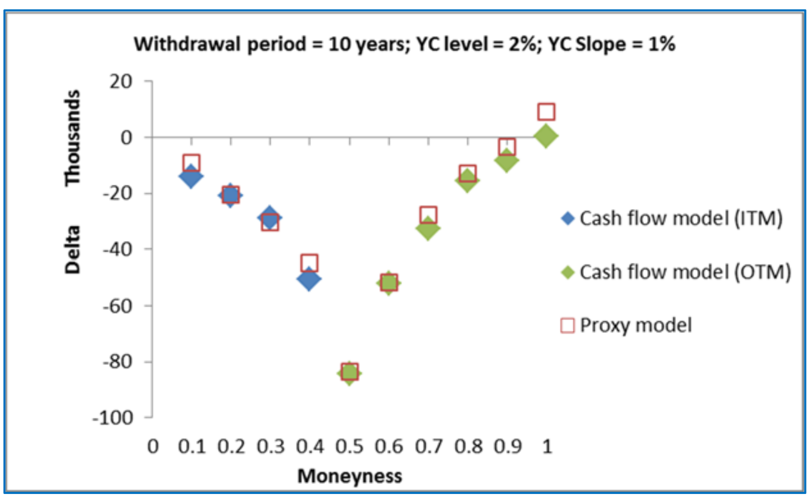
Figure 10: Size of delta proxy function errors as a function of moneyness and yield curve (bubble width proportional to absolute delta error)



This suggests a region of high curvature in the delta near this moneyness level, which the proxy function struggles to replicate, amplified in magnitude by the higher present value of withdrawals in a low yield curve environment. This is unsurprising given the CPPI rule that dictates the fund allocation as a function of account value relative to withdrawal benefit value.

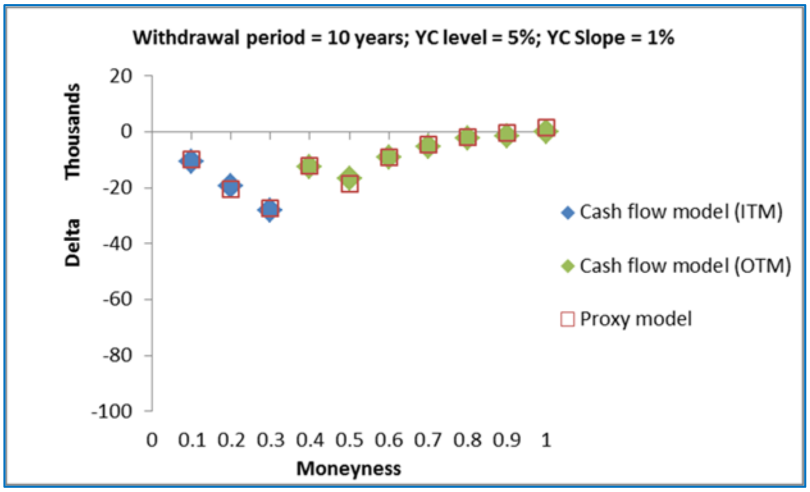
Isolating the moneyness variable by fixing yield curve and withdrawal period shows this high degree of curvature more clearly (here, the "ITM" policies are those with fund value < present value of withdrawals; "OTM" indicates fund value > present value of withdrawals):

Figure 11: Univariate validation results (delta vs. moneyness; low yield curve)



We also observe the change in delta profile for policies with equity allocations equal to the minimum or maximum; this manifests as a "wrinkle" in the center of the delta vs. moneyness chart.

Figure 12: Univariate validation results (delta vs. moneyness; medium yield curve)



Similarly to the VA1 case study, we expect a discontinuity in guarantee cost, in this case on either side of a withdrawal payment. Here the explanatory variables we have chosen allow the proxy function to exhibit this discontinuous behavior.

Figure 13: Univariate validation results (guarantee cost vs. remaining withdrawal period)

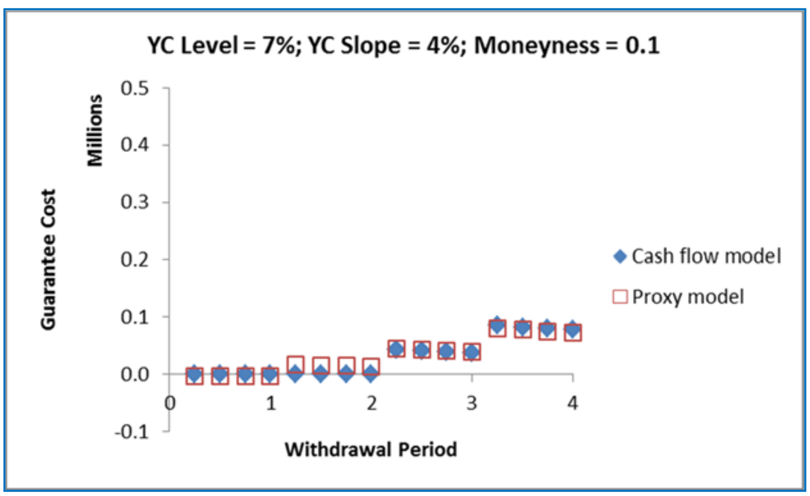
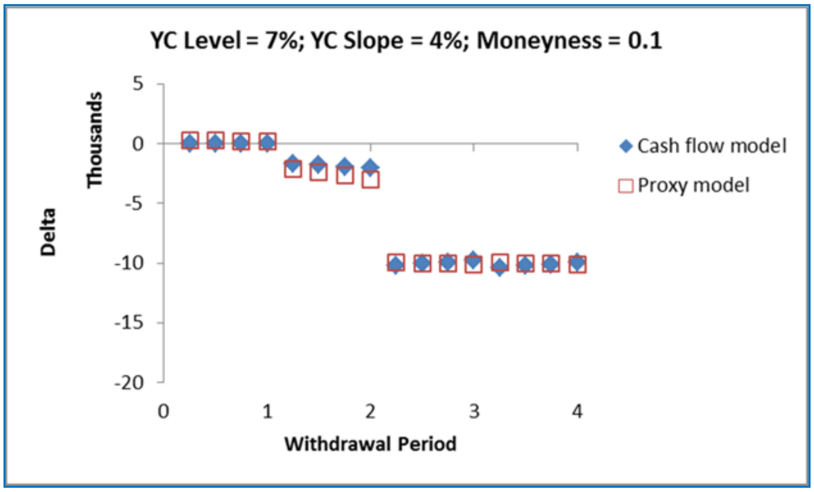
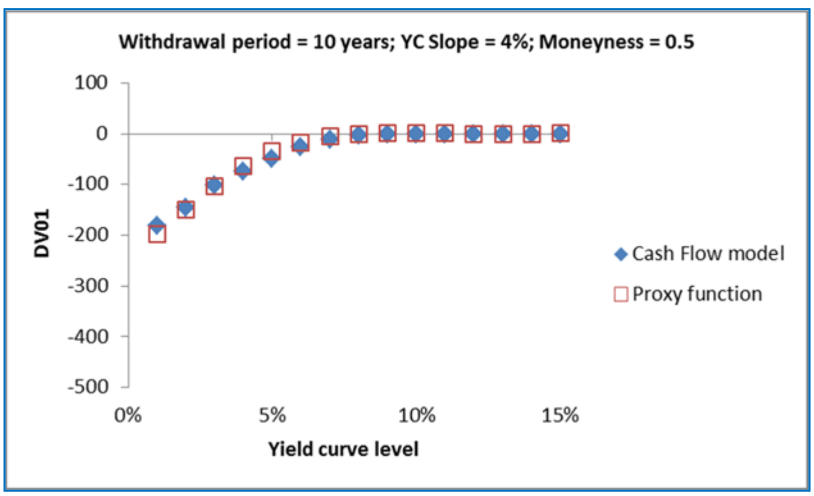


Figure 14: Univariate validation results (delta vs. remaining withdrawal period)



We also see a pronounced change in DV01 slope (i.e., policy convexity) as a function of the yield curve level. This suggests, for example, that duration hedging assumptions may break down as interest rates fall, depending on how the hedge portfolio is managed.

Figure 15: Univariate validation results (DV01 vs. yield curve level)



3.2.2 Accumulation phase

For policies in the accumulation phase, we observe similar results to the withdrawal phase. In particular, we see generally excellent agreement between the proxy and validation results for guarantee cost and DV01, with some pronounced errors in the delta.

Figure 16: Proxy vs. validation results (guarantee cost)

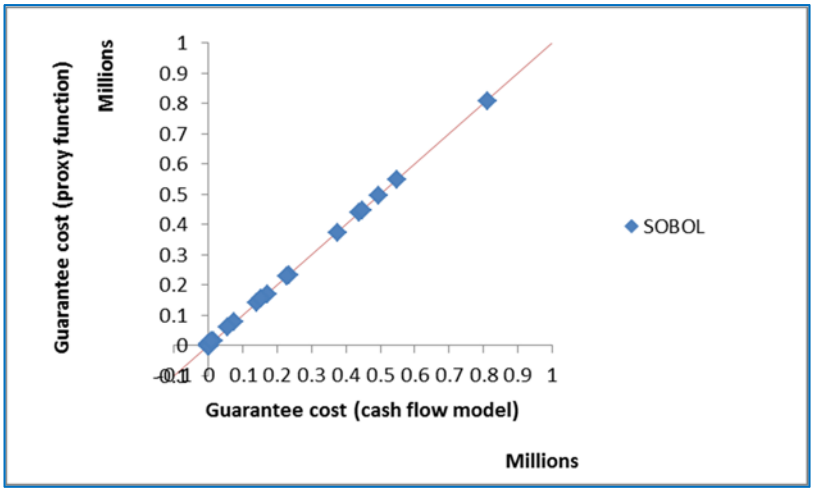


Figure 17: Proxy vs. validation results (DV01)

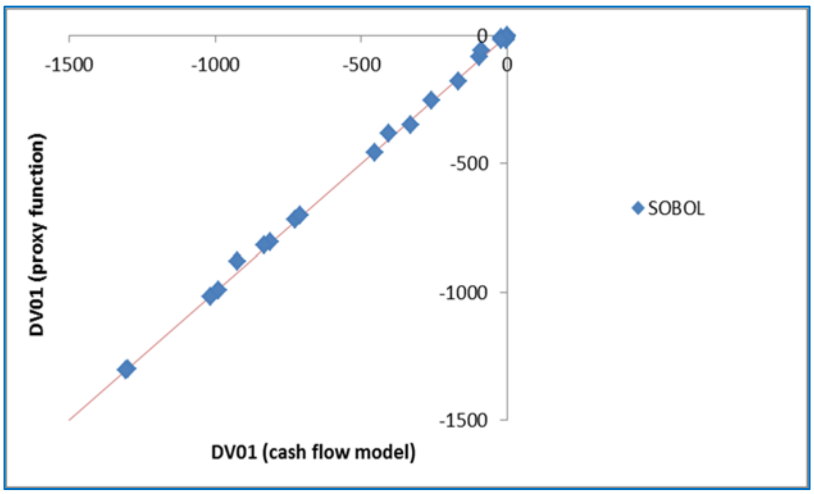
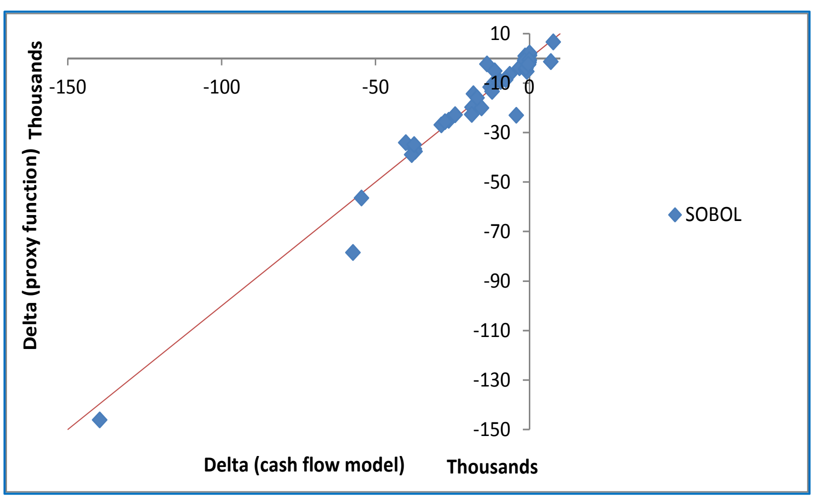


Figure 18: Proxy vs. validation results (delta)



The delta errors appear to be largest for policies near the moneyness threshold, consistent with a region of high curvature near this boundary.

Figure 19: Size of delta proxy function errors as a function of moneyness and yield curve

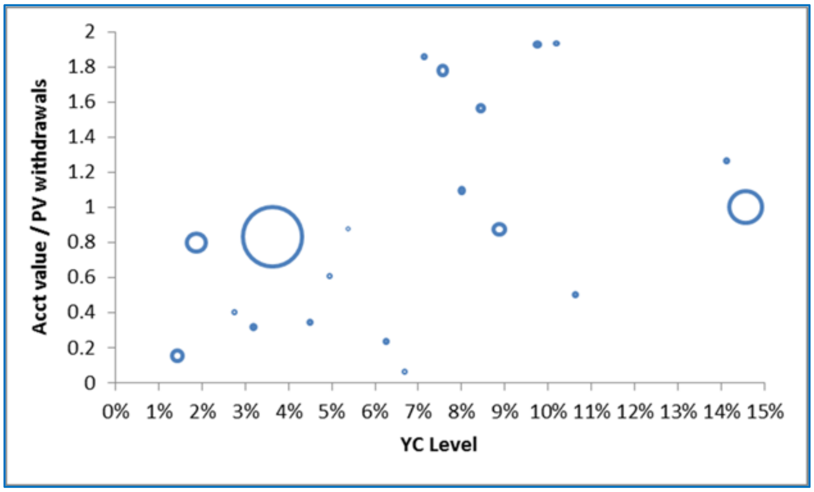
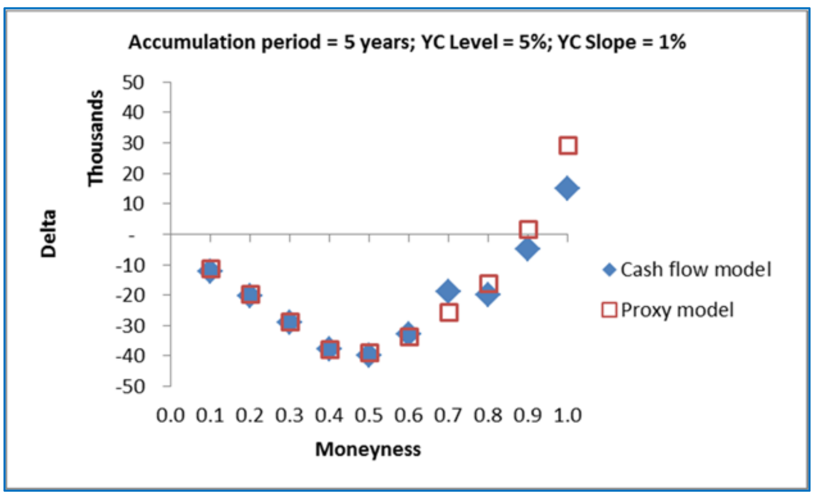


Figure 20: Univariate validation results (delta vs. moneyness)



The functions of guarantee cost vs. moneyness and DV01 vs. yield curve level show a similar, but somewhat milder, profile as in the withdrawal phase.

Figure 21: Univariate validation results (guarantee cost vs. moneyness)

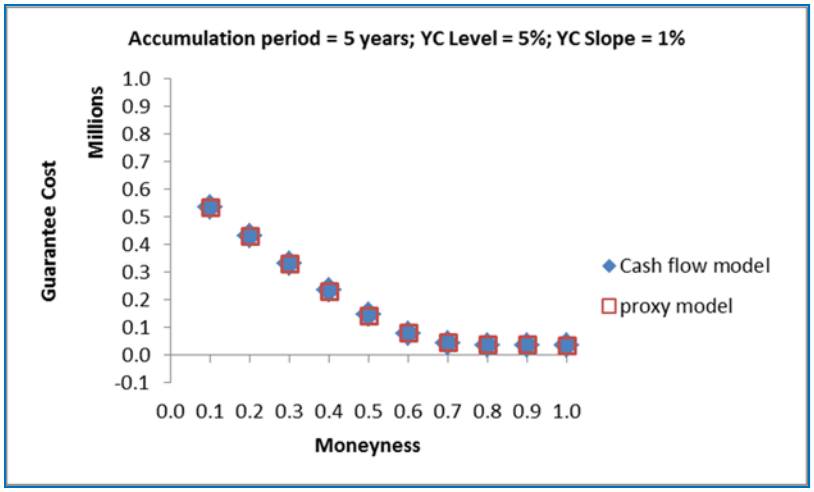
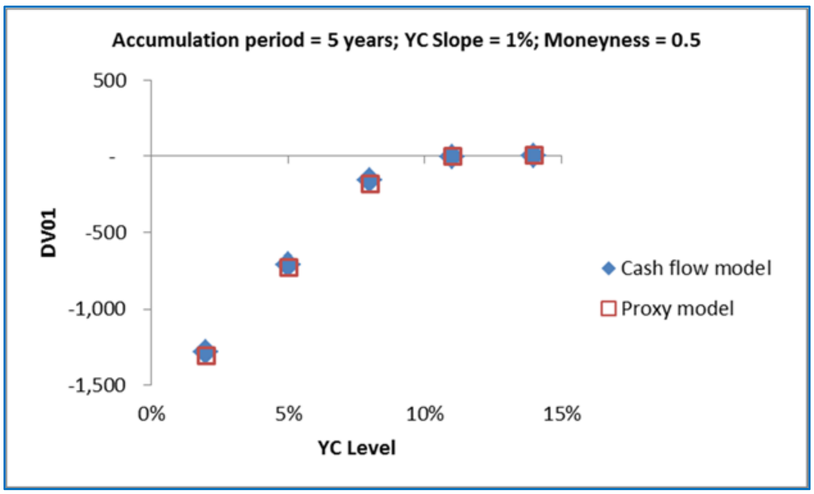


Figure 22: Univariate validation results (DV01 vs. yield curve level)



In all cases, the proxy functions perform well at capturing the relevant dependencies and demonstrate features in line with an understanding of the product structure.

4. Conclusions

We have extended the previous application of proxy function techniques from simple option test examples to portfolios of actual Variable Annuity products, illustrating this application for two products with very different guarantees and product features and for different sensitivities (delta and rho). In both examples, the proxy functions are seen to validate well against full Monte Carlo calculations. By combining market risk variables with policy-specific variables and special discrete variables to separate regions of the risk-factor space, we have shown that proxy functions are capable of reproducing the kinds of complex, path-dependent functional behavior exhibited by these products. Thus, the proxies are suitable for use in stochastic projection of dynamic hedges of the kind required for obtaining capital credit for such a hedging program, as an alternative to computationally infeasible nested stochastic calculations.

References

Bauer, Daniel, Daniela Bergmann, and Andreas Reuss. "Solvency II and nested simulations–a least-squares Monte Carlo approach." *Proceedings of the 2010 ICA congress* (2010).

Cathcart, Mark and Steven Morrison. "Least Squares Monte Carlo Simulation." *Contingencies* (2011).

Clayton, Aubrey, Steven Morrison, Craig Turnbull, and Naglis Vysniauskas. "Proxy Functions for Projection of Variable Annuity Greeks." *Moody's Analytics* (2013).

Clayton, Aubrey, Steven Morrison, Craig Turnbull, and Naglis Vysniauskas. "Multi-Period Modeling of Greeks Using Least Squares Monte Carlo: An Exotic Option Case Study." *Moody's Analytics* (2014).

Longstaff, Francis A., and Eduardo S. Schwartz. "Valuing American Options by Simulation: A Simple Least-Squares Approach." *Rev. Financ. Stud. Review of Financial Studies* 14.1 (2001): 113-47.