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Efficient Asset allocation with Least Squares Monte-Carlo

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Overview

Over the past decades traditional asset allocation methodologies such as mean-variance optimization have been widely applied in the insurance industry. The benefits of such approaches for insurance portfolios have always been limited due firstly to the complex interactions between assets and liabilities and secondly to the estimation difficulties and inappropriateness of asset portfolio returns and variances as target measures for portfolio optimization.

In this paper, we analyze an asset optimization problem where measure of risk is defined by the capital required under Solvency II principles, and where the portfolio performance is defined by the Net Asset Value at time T=1. To overcome the technical challenges relating the assessment of such measures in a market-consistent environment, we used the Least Square Monte Carlo (LSMC) approach as implemented in the B&H Proxy Generator.

In the first section, we present how LSMC technique can be used to generate allocation dependent proxy functions for the market-consistent value of asset and liabilities at T=0 and T=1.

In the second section, we show how these proxy functions can be used to define an efficient asset allocation.

Presentation of the case study

By way of illustration we consider a book of with-profit insurance products with minimum guaranteed returns.

The initial asset allocation vector (sovereign bonds, corporate bonds, equity, property) is defined by
$A_{init} = (50\%, 20\%, 20\%, 10\%)$

The initial position of the balance sheet at T=0 is given in Table 1

Balance sheet (€)	T=0
Assets	110,000
Liabilities	-99,486
NAV	10,514

Table 1 Balance sheet for allocation A_{init} at T=0

The best estimate value of the liabilities has been calculated using 5000 simulations generated from the B&H Economic Scenario Generator.

We consider that the investment universe is defined by four different classes: 10 years Government bonds, 7 years BBB Corporate bonds, the DJ Eurostoxx50 equity index and a Pan-European property index. In addition, we assume that the asset portfolio is rebalanced every year to the starting asset allocation.

A change in asset allocation will not impact the value of Assets at T=0, but will impact the value of the liabilities at T=0 and hence the NAV at T=0 through the impact on the cost of options and guarantees underlying the product.

A change in the initial asset allocation will also impact future balance sheet positions due to the following dynamics:

» Changes in returns on invested assets will impact the total value of assets and the value of the liabilities following the impact on allocated bonuses.

» Rebalancing rules applied to move to the initial asset allocation will introduce a secondary impact on liabilities.

Any change in asset allocation will require calculating the liabilities based on 5000 trials. For many insurers a single stochastic run for liability valuation will take several hours. As a consequence, insurers are usually constrained to limit their ALM analysis to a reduced number of allocations. Such ALM analyses are usually based on trial and error and can be very time consuming. In the next section, we will show how these computational challenges can be bypassed by using the Least Square Monte Carlo approach.

Asset allocation impact on T=0 balance sheet

We now demonstrate how the least squares Monte-Carlo technique can fit, with a high level of accuracy, proxy functions replicating the market-consistent value of liabilities and the net asset value (NAV) of an insurance portfolio for any combination of asset allocation depending on proportions invested in different investment classes.

The proxy for the net asset value can be expressed as a polynomial function:

$$NAV_{t=0} = f\begin{pmatrix} equity\ weight, property\ weight, sovereign\ bonds\ weight, \\ corporate\ bonds\ weight \end{pmatrix}$$

For more details on the application of the LSMC technique within the B&H Proxy Generator please refer to our fact sheet[1].
To perform the fit of the proxy functions we use a total of 40,000 fitting scenarios.

Next, we run the full stochastic ALM model for the following asset allocation choices. We do this in order to validate the convergence of the fitting functions:

[1] http://www.barrhibb.com/documents/downloads/BH_Proxy_Generator_Factsheet.pdf

Validation scenario	Government bonds	Corporate bonds	Equity	Property
1	25%	25%	25%	25%
2	50%	20%	20%	10%
3	20%	20%	50%	10%
4	20%	20%	10%	50%
5	20%	50%	20%	10%
6	40%	20%	10%	30%
7	30%	10%	40%	20%
8	20%	30%	20%	30%
9	10%	40%	30%	20%
10	10%	20%	30%	40%
11	40%	40%	10%	10%
12	40%	10%	10%	40%
13	10%	40%	40%	10%
14	20%	20%	20%	40%
15	60%	10%	20%	10%

Table 2 Asset allocation for the different validation scenarios

By comparing full ALM runs with the results provided by the proxy function, we are able to see the quality of fit for both the NAV and market-consistent liabilities.

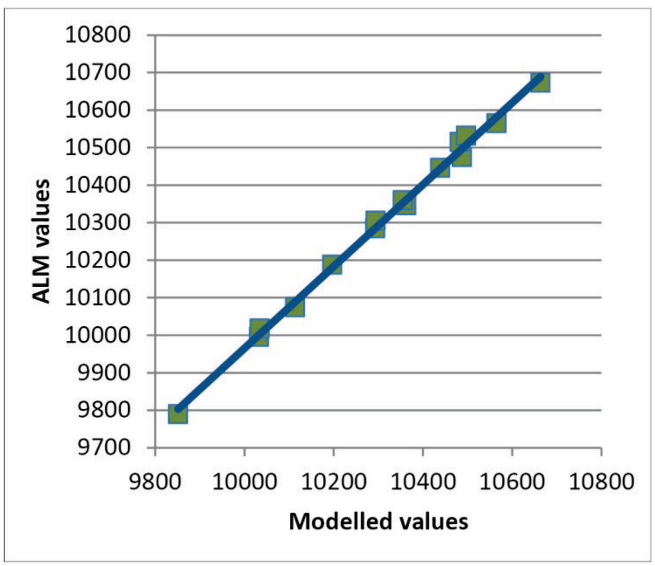


Figure 1 Out-of-sample validation for the proxy function of the NAV

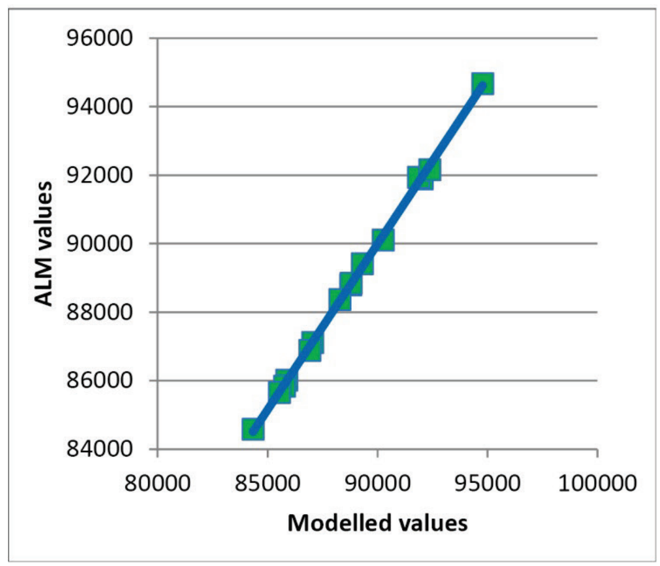
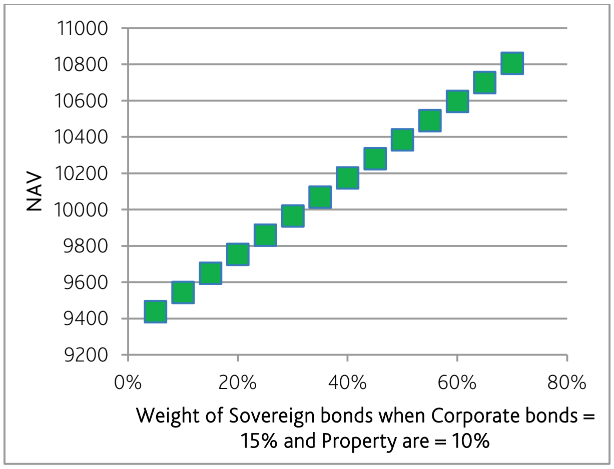


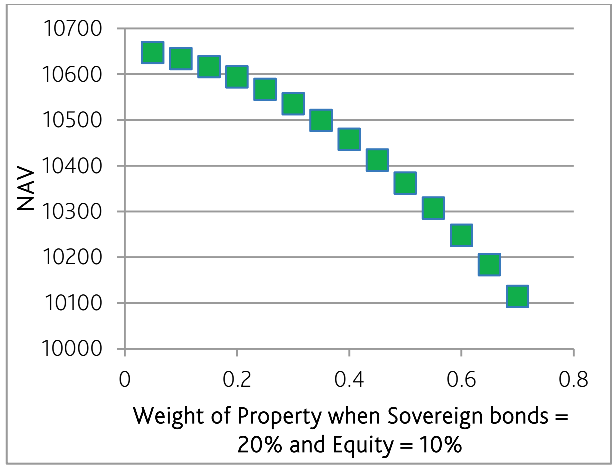
Figure 2 Out-of-sample validation for the proxy function of the Liabilities

The analysis above shows that even for very extreme variations in asset allocations, the fitted functions provide accurate estimations for both the liabilities and the net asset value.

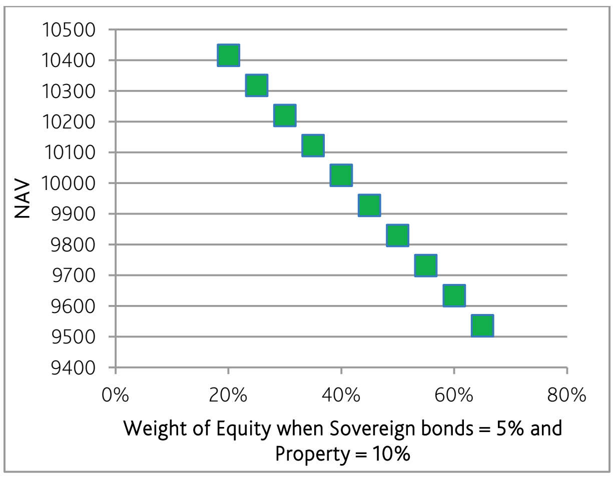
Once the function is validated, it provides an efficient tool to analyze the sensitivity of the net asset value to any changes in asset allocations as shown below.



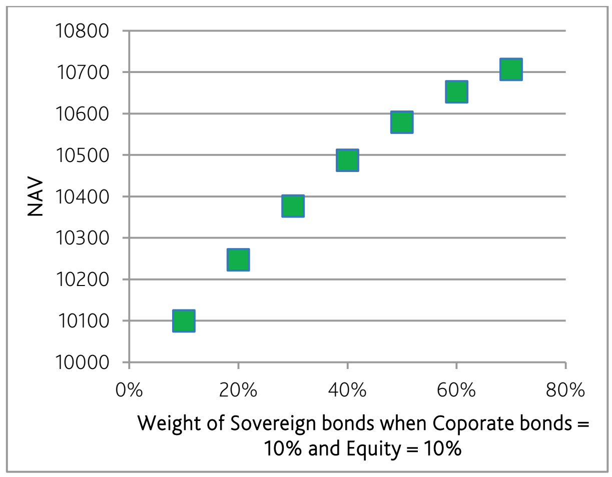
Impact on NAV of change in allocation when moving from Equities into Sovereign bonds



Impact on NAV of change in allocation when moving from Corporate bonds into Property



Impact on NAV of change in allocation when moving from Corporate bonds into Equities



Impact on NAV of change in allocation when moving from Property into Sovereign bonds

Figure 3 Variation of the asset allocation when moving 2 assets and fixing the 2 others

Figure 3 shows that, as we would expect, allocation in assets with low volatility reduces the cost of options and guarantees and this has a positive impact on the Net Asset Value. The interesting element here is that we can now obtain fast estimations of the NAV following changes in asset allocation.

Proxy fitting for T=1 Balance Sheet

The value of the liabilities at T=1 will depend on a larger number of parameters than the value of T=0, in addition to asset allocation, liabilities will depend on the market conditions realised at T=1.

This later component constitutes for many insurers the biggest challenge in the implementation of internal models as the derivation of a probability distribution for net assets requires the execution of several thousand simulations. Please refer to our previous research paper[2] for further details on this particular topic.

[2] http://www.barrhibb.com/documents/downloads/comparison_between_curve_fitting_and_least_squares_monte_carlo_techniques.pdf

In order to ensure that the LSMC technique converges with a high degree of accuracy, a number of mathematical conditions must be implemented with precision. For the purpose of this case study, we used the B&H Proxy Functions Generator to generate both liability proxies and the validation results.

The exposure to market conditions will be measured against the following six market risk factors:

- Two factors representing the risk-free yield curve
  It is convenient to reduce the behavior of the yield curve to fewer variables. Here we have chosen the short rate and the mean reversion levels of the Libor Market Model.
- An equity index stress
- A property index stress
- A factor representing the changes in volatility of equity
- One factor representing the level of corporate credit spreads

Additionally, we have extended the number of risk factors in order to include the proportion invested in each of the asset classes—sovereign bonds, corporate bonds, equity and property.

The net asset value is then defined as a polynomial function that can be written as follows:

$$NV_{t=1} = f\begin{pmatrix} equity, property, yield\ curve\ short\ rate, yield\ curve\ mean\ reversion, \\ credit\ spread, equity\ weight, property\ weight, sovereign\ bonds\ weight, \\ corporate\ bonds\ weight \end{pmatrix}$$

We used 40,000 fitting scenarios to generate function fitting assets, liabilities and NAV. Scenarios have been generated using the SVJD equity model, LMM interest rate model and G2 corporate credit spreads and transition model as implemented in the B&H Economic Scenario Generator.

In order to perform validations, we compared the proxy function to accurate valuations provided by the stochastic ALM engine for the following selection of parameters:

Table 3 Asset allocation for the different validation scenarios

Validation scenario	Government bonds	Corporate bonds	Equity	Property	Equity index[3]	Short rate level[4]	Mean reversion level	Property index	Equity volatility	Credit[5]
1	14%	38%	10%	38%	1	0	-2	1	1	3.555
2	22%	23%	25%	30%	1	-2	0	1	1	3.555
3	10%	10%	20%	60%	1	0	2.5	1	1	3.555
4	32%	10%	21%	37%	1	-1.5	0	1	1	3.555
5	8%	30%	47%	15%	2	0	0	1	1	3.555
6	40%	40%	12%	8%	1.5	0	0	1	1.2	3.555
7	19%	20%	31%	30%	2	0	0	1	0.8	3.555
8	30%	30%	25%	15%	1	0	0	2	1	3.555
9	20%	20%	20%	40%	0.3	0	0	1	1	3.555
10	77%	7%	10%	6%	1	0	0	0.3	1	3.555
11	30%	25%	38%	7%	1	-1.5	1.5	1	1	3.555
12	20%	25%	25%	30%	2	2	2	1.5	1.1	6
13	21%	23%	28%	28%	0.5	-1.5	-1.5	0.6	0.9	2
14	40%	20%	30%	10%	0.4	1.5	-2	0.8	0.8	8
15	30%	40%	10%	20%	2	1.8	-2	2	1.2	1

[3] Multiplicative shock on the indices

[4] Multiplicative shock on the level of short rates and mean reversion

[5] Shock on the starting value of the spread CIR process

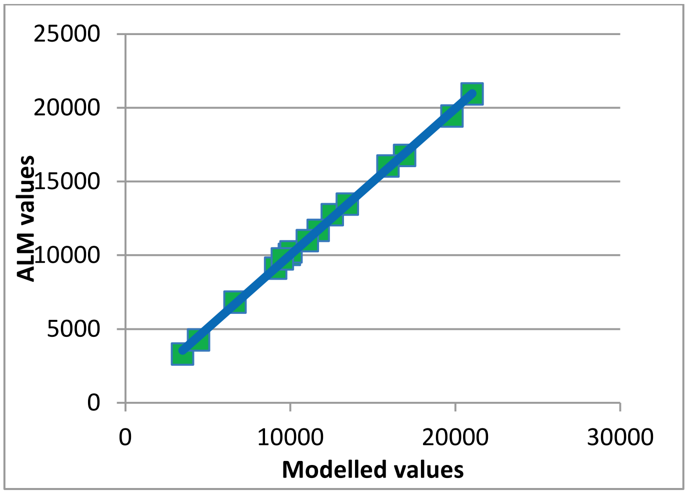


Figure 4 Out-of-sample validation for the proxy function of the NAV

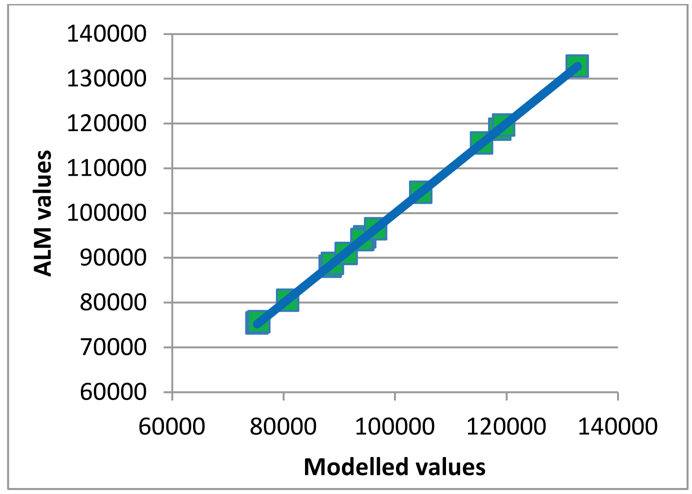


Figure 5 Out-of-sample validation for the proxy function of the Liability

Figures 4 and 5 show the validation results obtained at 15 different points including extreme variations in market risk factors and asset allocations. For each of these points the accurate estimation is based on 5000 market consistent simulations.

An accurate estimation of proxy functions offers multiple benefits:

- Ability to perform projections of market-consistent balance sheet and capital requirements under stress scenarios. This is part of the forward-looking capital assessment required under the ORSA component of Solvency II.
- Ability to obtain a probability distribution for any balance sheet item at T=1.

In this analysis we have used the probability distribution of Net Assets in order to calculate the SCR of this book of business.

The calculation of the distribution is based on 100,000 real world scenarios generated from the B&H Risk Scenario Generator software. To reduce the sampling error in the estimation of the 99.5th percentile to an acceptable level, we recommend using at least 100,000 trials.

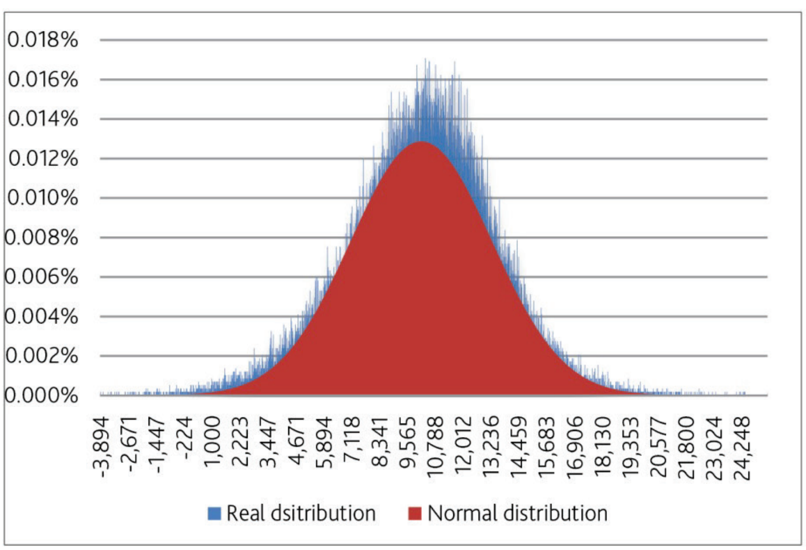


Figure 6 Distribution of the NAV of the initial allocation

Statistics	Values
Average	10,184
Kurtosis	0.354313
Skew	-0.18264
Median	10,313
Standard deviation	3101
75th percentile	8237
99.5th percentile	1499

Table 4 Statistics on NAV based on 100,000 real-world scenarios

The probability distribution of the Net Asset Value can be an effective tool to derive an objective and robust Risk Appetite Framework. It will also represent a powerful element in putting in place dynamic risk controls or to set risk based performance targets within an organisation.

Asset Optimization

Traditional asset allocations aim to maximize asset returns and minimize the volatilities of the same assets. Due to the complexities in the calculations mentioned in previous sections, the analysis is usually restrained to the investment portfolio without considering the implications on the Net Asset Value. As a consequence of these simplifications the impact of variations in asset allocations on the starting value of the balance sheet is ignored in the optimization problem.

In this section we show how we resolve the optimization problem when Net Asset Value is defined as an indicator of an insurer's wealth. We will analyse separately the cases where the expected value of NAV and annual returns on NAV are chosen in the maximisation objectives. The riskiness of the NAV will be defined by the SCR calculated from a probability distribution.

The SCR at T=0 for a 1-year forward capital assessment will be given by $SCR_{t=1} = E[NAV_{t=1}] - q_{99.5\%}(NAV_{t=1})$.

Figure 7 shows NAV vs SCR when running simulations for different combinations of asset allocation.

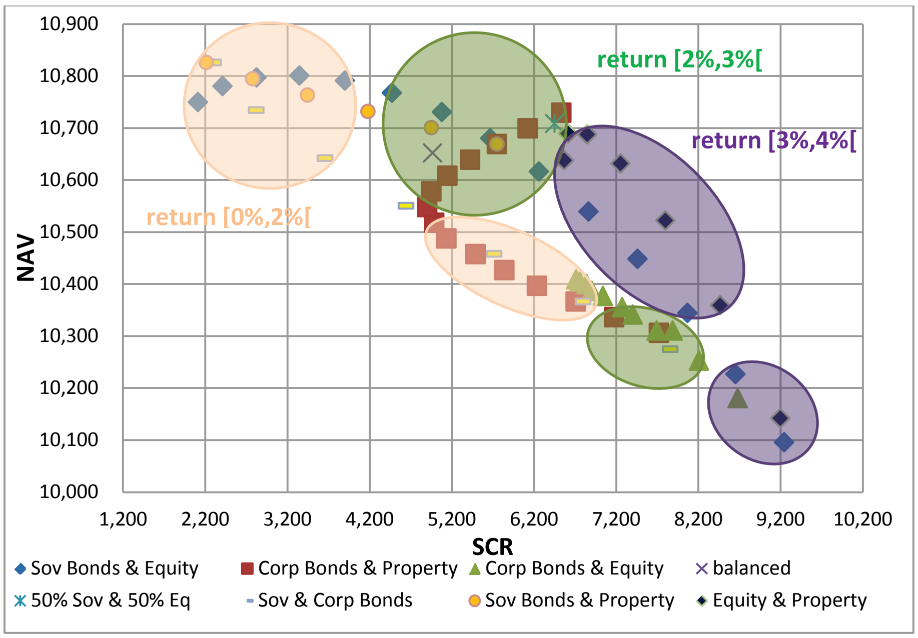


Figure 7 NAV at T=1 / SCR for variation of 2 assets allocation

These results show that NAV is maximized and SCR is minimized when the allocation is done in low risky assets. This is not surprising from shareholders point of view as the impact of the risk on a market-consistent balance sheet is measured through the cost of options and guarantees, and depending on the relative size of the liabilities compared to the NAV, the cost of options and guarantees will dominate the expected risk premium on invested assets.

In the configuration above, the optimization problem seems to be obvious; however a with-profit insurer is also constrained to deliver minimum returns on assets in order to be competitive compared to cash investments or compared to minimum rates offered by other insurers. This additional constraint requires the insurer to earn a minimum return on invested assets.
The coloured circles in Figure 7 represent a different range of asset returns.

When we target a maximization of NAV For instance and if we assume that target investment return is 3%, then the optimal asset allocation is given by *A = (25%,15%,50%,10%),* which corresponds to an expected NAV of 10,689 and SCR of 6,616.

An alternative approach to the optimization problem can be defined by maximizing returns on NAV instead of the absolute value of NAV.

Figure 8 shows the allocation diagram for returns on NAV vs SCR.

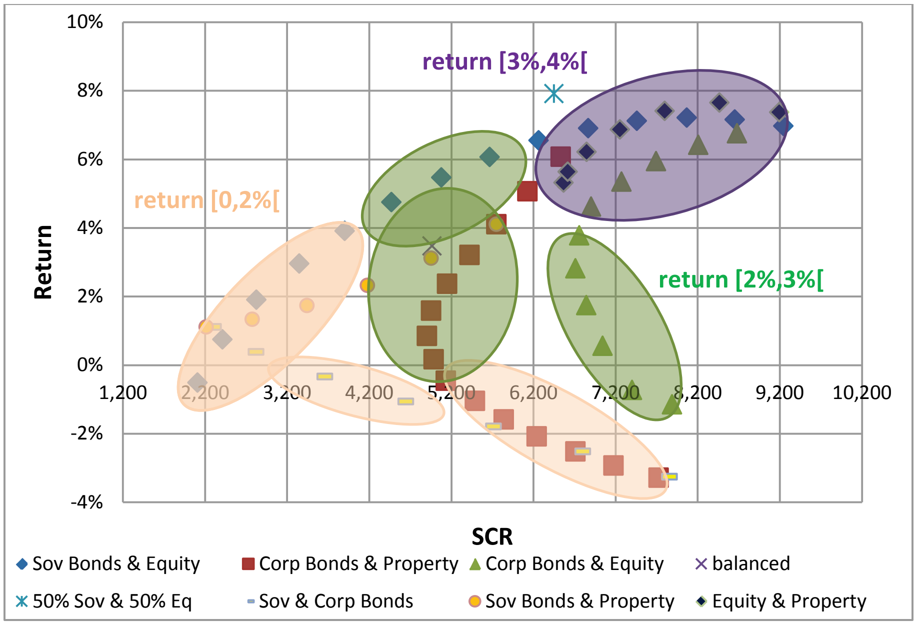


Figure 8 Return on NAV / SCR for variation of 2 assets allocation

When looking at returns, instead of absolute value of NAV, the shape of the allocations have more similarities with the Markowitz efficient frontiers. The optimal portfolios are very different than the situation where we maximized absolute value of NAV as optimal portfolios tend now to be those having the lowest starting NAV.

Table 5 shows optimal asset allocations under different optimization approaches when the minimum investment return for policyholder bonuses is set at 3%.

	NAV vs SCR	Return on NAV vs SCR	Return on invested asset vs SCR	Return vs Variance of invested assets
Optimal asset allocation	10%,10%,20%,60%	25%,15%,50%,10%	5%,15%,70%,10%	10%,10%,40%,40%
NAV at T=0	10,118	9,859	9,438	9,949
NAV at T=1	10,689	10,539	10,095	10,631
SCR	6,616	6,866	9,245	7,255

Table 5 Optimal asset allocation when minimum expected return on invested asset of 3% for policyholder bonuses.

The table above shows that there can be significant variation in assets allocated depending on the measures considered for the risk/return couple. The choice of NAV vs SCR provides in this case the highest NAV and the lowest SCR compared to other optimization approaches. An optimization based on Return on investments vs SCR provides the worst portfolio with the lowest value of NAV and the highest SCR.

Table 6 shows optimal asset allocations under different optimization approaches when the minimum investment return for policyholder bonuses is set at 2.5%.

	NAV vs SCR	Return on NAV vs SCR	Return on invested asset vs SCR	Return vs Variance of invested assets
Optimal asset allocation	40%,10%,40%,10%	50%,0%,50%,0%	40%,10%,40%,10%	35%,15%,40%,10%
NAV at T=0	10,076	9,923	10,076	10,069
NAV at T=1	10,726	10,708	10,726	10,680
SCR	5,614	6,452	5,614	5,669

Table 6 Optimal asset allocation when minimum expected return on invested asset of 2.5% for policyholder bonuses.

We observe that a reduction in bonus rates lead to an increase in the proportion invested in government bonds given that a lower risk premium is required. Again, we also observe that the optimization based on NAV vs SCR provides the best allocation with this highest NAV and the lowest SCR.

Conclusion

An asset optimization focusing only on the distributional characteristics of an investment portfolio will often not lead to an optimal portfolio from the perspective of value creation for a life insurance firm. The use of simplifying assumptions which ignore asset-liability interactions is often driven by computational challenges in the valuation of the insurer's market-consistent balance sheet. In this paper we extended the use of LSMC technique beyond its usual application to the SCR calculation. The same single functional form can be used to maximize the net assets of an insurance firm for a wide range of asset allocations. The flexibility of the functional forms gives the ability to integrate a large number of constraints such as regulatory/economic capital or targeted minimal bonuses for policyholders. The internal model hence becomes an effective decision-making tool allowing the management to assess the appropriateness of each strategic action under the numerous interdependent criteria that exist within an insurance company. The ability to embed the internal model within the decision process also gives a good proof on the use test criteria required under Solvency II for the validation of internal models.