Moody's ANALYTICS

B&H RESEARCH
JUNE 2013

Multi-year Projection of Run-off Conditional Tail Expectation (CTE) Reserves

Steven Morrison PhD
Craig Turnbull FIA
Naglis Vysniauskas
Moody's Analytics Research

Contact Us

Craig.Turnbull@moodys.com

Americas
+1.212.553.1658
clientservices@moodys.com

Europe
+44.20.7772.5454
clientservices.emea@moodys.com

Asia (Excluding Japan)
+85 2 2916 1121
clientservices.asia@moodys.co

Japan
+81 3 5408 4100
clientservices.japan@moodys.com

Overview

This paper describes and demonstrates how the more statistically sophisticated proxy fitting approaches previously developed for the one-year projection of market-consistent values can be naturally extended to produce full multi-year projections of CTE(70) run-off reserves. The capability to efficiently produce robust and accurate proxy functions for CTE(70) run-off reserve behavior across a wide range of multi-timestep, multi-risk-factor scenarios can significantly enhance firms' forward solvency projection analytics. We believe this can be extremely useful for firms to project their balance sheets and reserving and capital requirements as part of ORSA and other business planning requirements. The case study presented in this paper highlights how the proxy functions could be used to facilitate the medium-term stress testing, reverse stress testing and stochastic projection of the insurers balance sheet.

Contents

1. Introduction

Insurance firms' risk and capital assessment frameworks increasingly have computational needs that theoretically require 'nested simulation'. Such requirements arise wherever firms need to project their balance sheet and/or capital requirements within a set of future scenarios where the balance sheet and/or capital requirements themselves need to be estimated using simulation. Emerging global regulatory requirements such as Own Risk and Solvency Assessment (ORSA), along with general business planning requirements, have resulted in the need to project measures of value and risk over medium to long horizons over multiple time steps. For most firms the computational requirements associated with this type of nested simulation exercise are likely to be prohibitive given current model run-times.

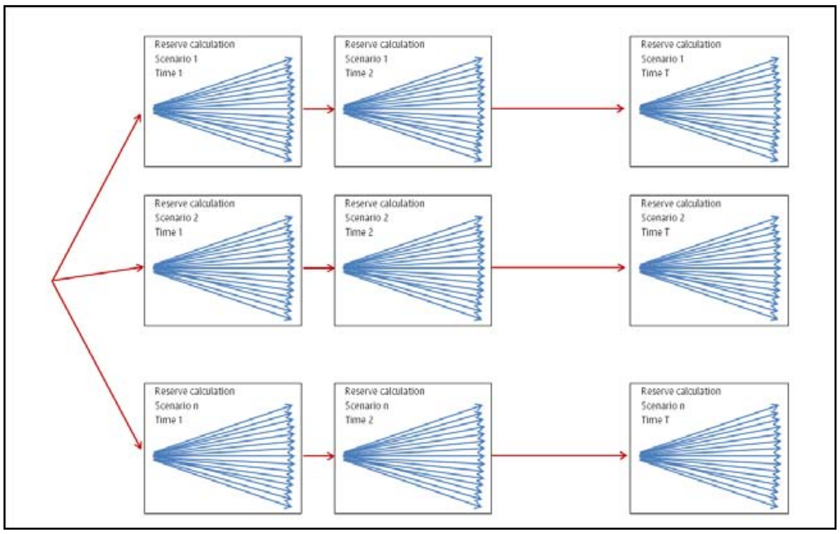
One particular area where the nested simulation challenge emerges is in the calculation of economic capital based on a 1-year Value-at-Risk measure. In this context, a number of 'proxy' methods have been developed to approximate the calculation of solvency requirements based on analytical formulae rather than simulation, with a resulting significant reduction in run-time. While previous work on proxy functions has focused on one year projection of market-consistent value, we have recently explored how the technique can be extended to one year projection of CTE run-off reserves (Morrison, Tadrowski, & Turnbull, 2013) and to multi-year projection of market-consistent value (Morrison, Turnbull, & Vysniauskas, 2013). This paper follows on from this research and considers the multi-year projection of the CTE run-off reserve. This topic is likely to be of particular interest to North American firms, where run-off CTE is widely used as a measure of both reserves and capital.

In Section 2 we provide a brief overview of the proxy function method as a means to avoid the computational demands of nested simulation. In particular, we discuss the use of Least Squares Monte Carlo as a robust and efficient method for calibrating the proxy function. Section 3 introduces a case study and applies the methodology described in Section 2 to produce proxy functions for the run-off CTE(70) reserve of a complex, path-dependent product over a 10-year projection horizon. The validation of the performance of the fitted proxy functions is presented in Section 4. Sections 5, 6 and 7 then go on to explore the applications of the multi-timestep proxy functions in areas such as stochastic balance sheet projection, reverse stress testing and stress and scenario testing.

2. An overview of proxy modeling and Least Squares Monte Carlo

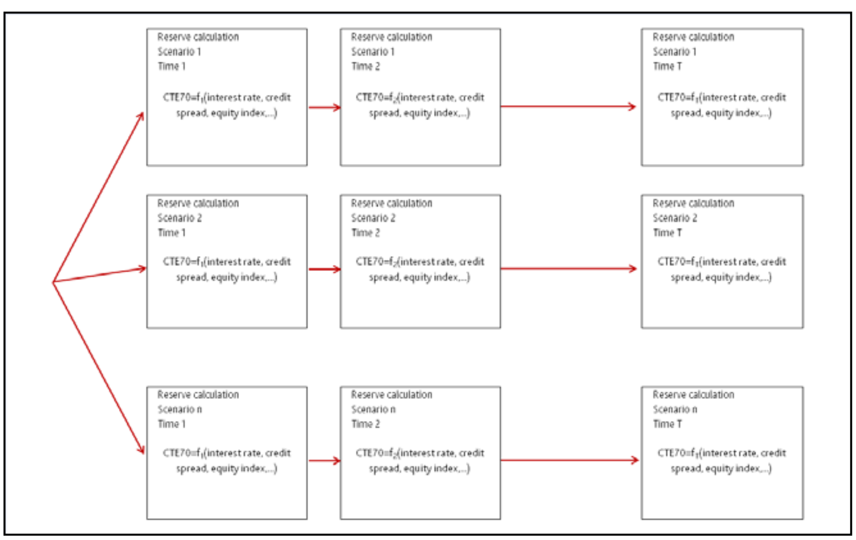
Figure 1 illustrates the projection of CTE run-off reserves[1] using a nested simulation approach. Firstly, a number of 'outer' scenarios are generated (indicated by red arrows). These might be deterministic or stochastic scenarios, and specify the paths of all relevant risk factors that impact on the firm's reserves. In each of these outer scenarios, at each future time horizon of interest, we calculate CTE run-off reserves using simulation. A number of 'inner' scenarios (indicated by the blue arrows) are used to generate future asset and liability cash flows, and hence the CTE reserves.

Figure 1: Projection of CTE run-off reserves over multiple time-steps using nested simulation



The vast majority of computational run-time in such a nested simulation exercise arises from the use of simulation to calculate reserves[2]. The idea of 'proxy' modeling is to replace the simulation based calculation of reserves with an analytical formula, or 'proxy function', as illustrated in Figure 2. Such formulae can be evaluated far faster than using a simulation based reserve calculation.

Figure 2: Projection of reserves over multiple time-steps using proxy functions



[1] Our focus here is on the projection of CTE run-off reserves but many of the ideas discussed here apply to projection of other quantities of interest, such as market-consistent value, CTE run-off capital, or 1-year VaR capital.

[2] The generation of outer scenarios is typically a relatively small overhead.

Such proxy modeling techniques have recently gained popularity in the context of projection of market-consistent values in order to calculate 1-year VaR economic capital. In this context, various techniques have been proposed for describing and calibrating the proxy function, with 'curve fitting' emerging as the most popular of these. By 2012, more than two-thirds of firms implementing a liability proxy function framework were using a curve fitting approach to their 1-year VaR economic capital modeling implementation (KPMG, 2012). The curve fitting approach typically describes the proxy as a polynomial function of the risk factors. Calibration of this polynomial involves selecting its coefficients so as to best fit the actual market-consistent value under a selection of stresses to the risk factors. In general, the actual market-consistent value is estimated using risk-neutral Monte Carlo simulation techniques, and the function is fitted using Least Squares regression techniques. This general calibration approach has therefore become known as Least Squares Monte Carlo (LSMC).

While the application of LSMC by insurance firms has to date focused on the one year projection of market-consistent value, we recently demonstrated that it can also be successfully applied to the one year projection of CTE (Morrison, Tadrowski, & Turnbull, 2013). The technique is illustrated in Figure 3 which shows three different ways to calibrate a proxy function for the CTE(70) of a 10-year equity put option at a one year horizon. We assume here that the only risk factor is the underlying equity index[3] and assume that the CTE(70) can be approximated by a cubic function of this equity index. The three calibration choices correspond to different numbers of 'outer' scenarios (describing stresses to the risk factors at the one year horizon) and 'inner' scenarios (used to estimate the CTE(70) conditional on each stress). Outer scenarios are chosen uniformly in the range 0.67 to 1.71 and inner scenarios are sampled from a Black-Scholes model with real-world calibration assumptions for equity volatility and the equity risk premium. In all three calibrations, the total number of 'inner' scenarios is 100,000 so that all calibration approaches have similar computational requirements.

In the example to the left, 4 outer scenarios are generated for the risk factors, with 25,000 inner scenarios per outer scenario being used to estimate the CTE(70)s. With such a large number of inner scenarios, we can expect the resulting CTE(70) estimates (shown by the green diamonds) to be statistically accurate. Furthermore, with 4 parameters to fit in the cubic function, we can choose these parameters so as to exactly fit the CTE estimates in each of the 4 outer scenarios. To validate the fitted function, we have analytically calculated the CTE(70) at 10 values of the equity index, chosen uniformly in the range 0.67 to 0.71 (indicated by the red square symbols). The fitted cubic function agrees closely with the analytic CTE(70)s at all 10 validation points.

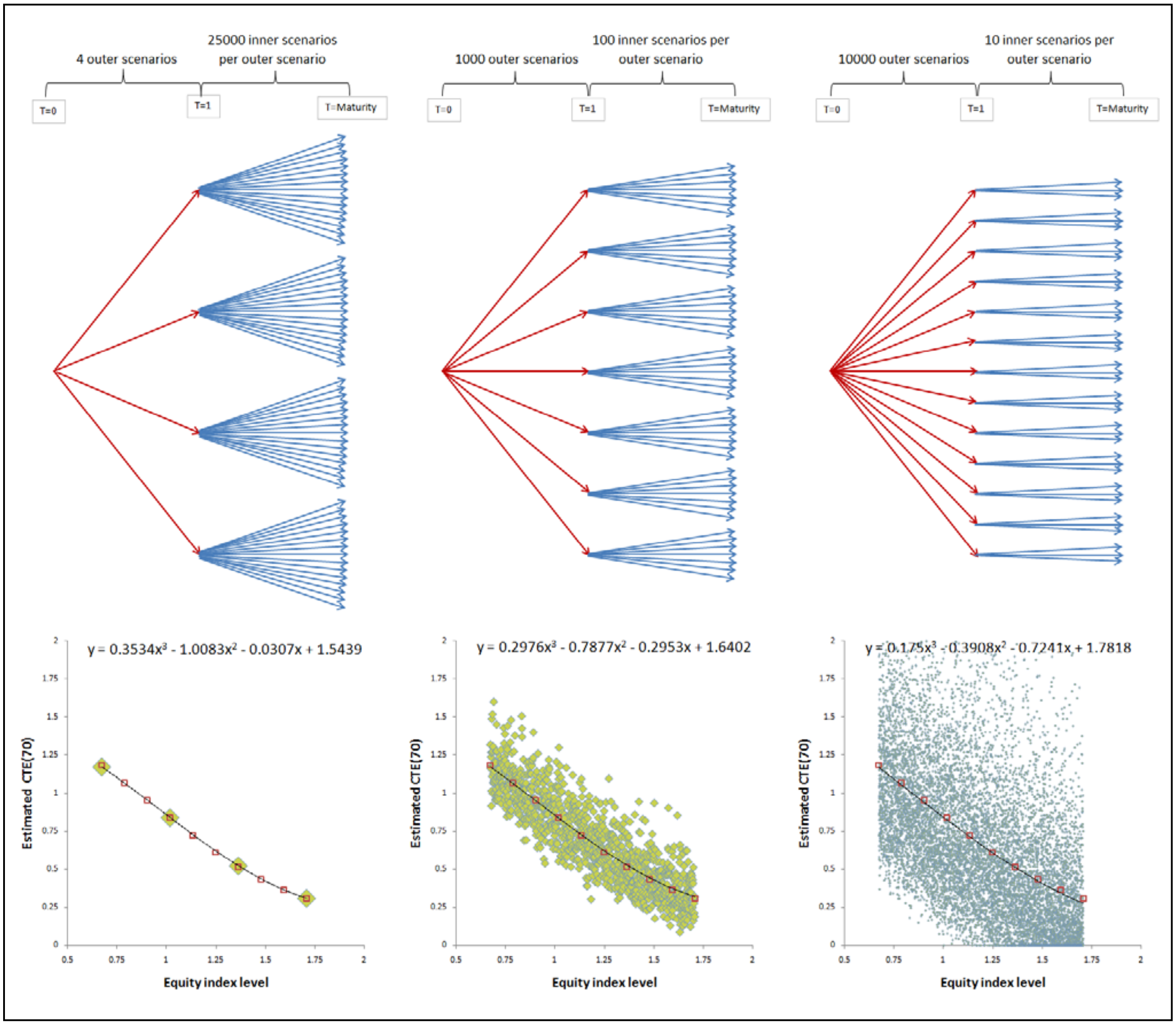
In the middle example, we have increased the number of outer scenarios from 4 to 1,000 and decreased the number of inner scenarios (per outer scenario) from 25,000 to 100 so as to retain the same overall computation time. Using 100 scenarios, the CTE estimates are far less statistically accurate, and the resulting statistical noise is apparent in the fact that the CTE estimates do not appear to fall on a smooth line but rather form a cloud of points. Nevertheless, we can observe some structure in the behavior of CTE as a function of equity index, with the CTE estimates tending to decrease with increasing equity index. This structure reveals itself in more detail when we perform a least squares fit through the cloud of points - although each individual CTE estimate may be noisy, the noise in different estimates is independent and so tends to cancel out when we perform a least squares fit. The resulting function is similar to that calibrated earlier, despite the different combination of outer/inner scenarios, and again validates well against the 10 analytical values.

Finally, in the example to the right, we have reduced the number of inner scenarios to 10[4], and so increased the number of outer scenarios to 10,000. Though the CTE(70) estimates here are individually extremely noisy, again this noise diversifies when we perform a least squares fit, and again the resulting cubic function again validates well against the 10 analytical values.

[3] Risk-free interest rates, equity volatility and the equity risk premium are assumed to be fixed.

[4] 10 scenarios is the smallest number of scenarios we can use to estimate the CTE(70) using the standard statistical estimator of CTE.

Figure 3: Illustration of LSMC applied to the CTE(70) of a put option under three different outer/inner scenario combinations



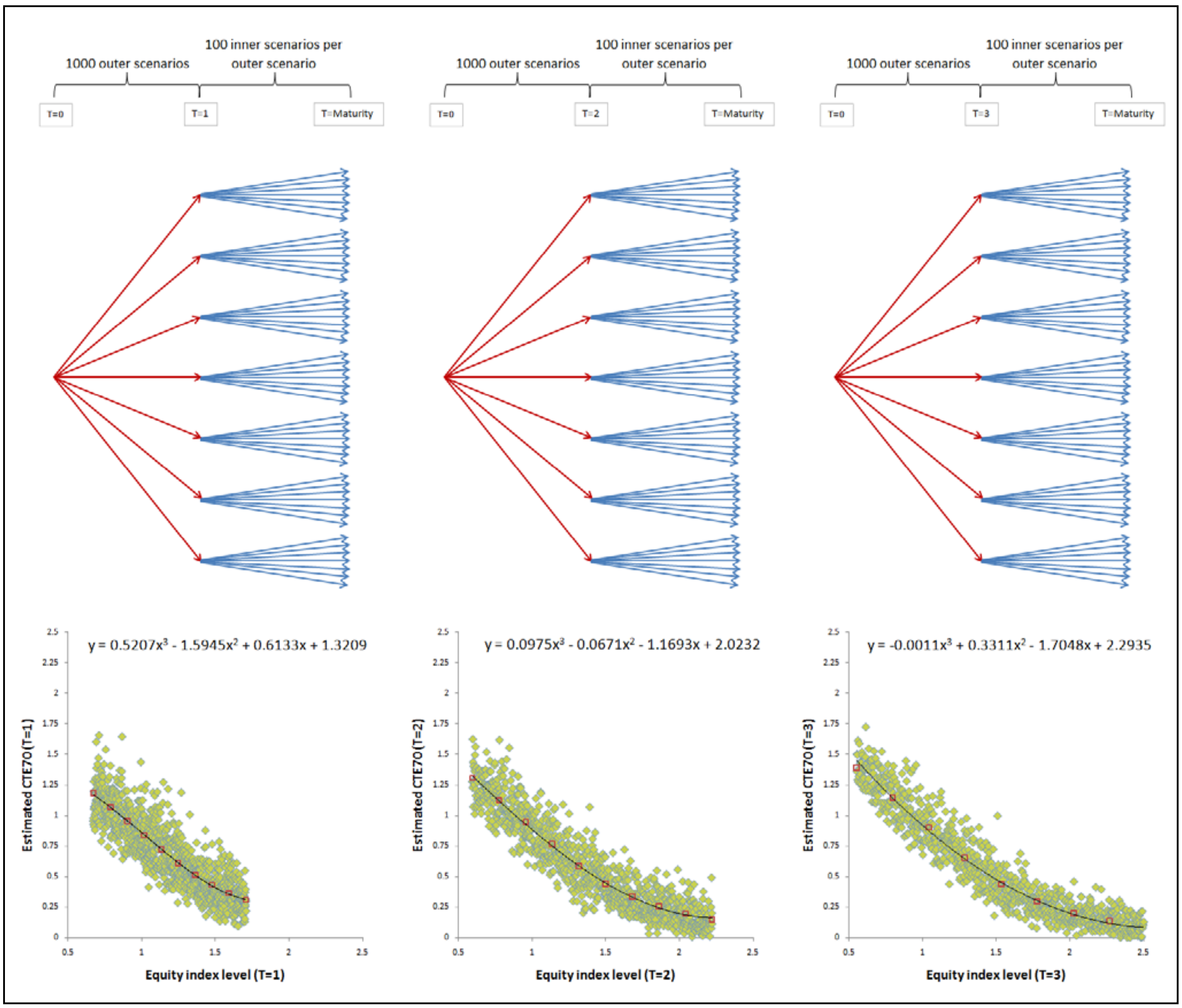
In the simple Black-Scholes example illustrated above (Figure 3), with a single risk factor, all three outer/inner scenario combinations considered appear to give similar qualities of fit. However, for more general higher dimensional problems, it turns out that certain outer/inner combinations produce more accurate proxies than others. More generally, we can ask, given a particular computational budget (i.e. total number of inner scenarios), how should one optimally[5] allocate this among outer scenarios? In the projection of market-consistent value, previous research indicates that generally the optimal scenario allocation is to choose a relatively small number of inner scenarios and large number of outer scenarios (Cathcart, 2012). Indeed, for the purpose of projection of market-consistent value the optimal allocation may be as few as one inner scenario per outer (or two inners per outer if antithetic variables are used). The intuition for this result is that the noise in inner scenario estimates is diversified away via the process of least squares fitting, and so it is relatively unimportant to achieve accurate estimates. Rather, it is better to have as many (noisy) estimates as possible as this ensures that we span the space of risk factors as widely as possible. This is particularly important as we consider more realistic high dimensional problems, where a relatively large number of outer scenarios are required to give a good coverage of the risk factor space.

[5] By optimal here, we mean the allocation that gives the most accurate out-of-sample validations.

In the calibration of proxy functions for CTE(70), our previous research (Morrison, Tadrowski, & Turnbull, 2013) indicates that the optimal number of inner scenarios is somewhat greater than that required in the calibration of proxy functions for market-consistent value. Firstly, the standard CTE estimator is only defined for a certain minimum number of scenarios. Furthermore, while estimates of market-consistent value are unbiased, the standard CTE estimator is biased for any finite number of scenarios (Manistre & Hancock, 2005), and bias, unlike noise, doesn't diversify away when we perform the least squares fit. However, the size of the bias decreases with an increasing number of scenarios, and indeed it asymptotically tends to zero. For the purpose of LSMC, we aim to increase the number of inner scenarios so that the bias is reduced to an acceptably low level, while retaining enough outer scenarios to span the risk factor space well. The resulting optimal number of inner scenarios will depend on the characteristics of the particular business being modeled, and we recommend that this is assessed on a case-by-case basis.

The above discussion focuses on development of proxy functions at a one year horizon. However, it is conceptually straightforward to generalise the approach to multiple years—we can simply repeat for each projection time of interest. Figure 4 illustrates this process for projection of the CTE(70) of the put option at one, two and three years.

Figure 4: Illustration of LSMC applied to the CTE(70) of a put option at years 1, 2, 3



In a previous paper (Morrison, Turnbull, & Vysniauskas, 2013), we discuss the methodology for multi-year LSMC in further detail, in the specific context of projection of market-consistent value. In that paper, we note that the approach illustrated in Figure 4 gives rise to computational requirements that scale approximately linearly with the number of required time-steps, and develop more efficient methods which allow calculations to be reused at different time-steps. Such methods rely on being able to use a single scenario to estimate market-consistent value, and so do not readily generalize to CTE statistics. We can, however, develop similarly efficient approaches in the projection of CTE. We will not explore such advanced approaches further here, but focus our attention on the basic method illustrated in Figure 4 for ease of illustration. However, it may be important to the reader to note that more advanced multi-timestep CTE fitting methods can be implemented that can provide similarly accurate fits and validation results whilst only requiring the fitting scenario budget of a single time-step fit.

3. Introducing the case study

The application of LSMC to a multi-year projection of CTE run-off reserves will be illustrated using a case study. We consider projection of a policy with a payout linked to the performance of a corporate bond portfolio, with a guarantee applied annually. The main assumptions are summarised below:

» Assume an annual return of max (fund return – 1.5%, 2%) is credited to the policy account,

i.e. Policy Account(t) = Policy Account(t-1) x (1 + max (fund return(t-1,t) – 1.5%, 2%))

and Policy Account(0) = Fund Value(0)

» The underlying fund is a diversified portfolio of US corporate bonds. The bonds are assumed to be invested with a credit mix of 70% A-rated and 30% BBB-rated, and with a term of 8 years. The bonds' credit rating and term are assumed to be re-balanced annually.

» The policyholder is assumed to exit the policy after ten years, and will receive the value of the policy account at that point.

» No allowance is made for tax, mortality, expenses or lapses.

We note that the payout on this policy is path-dependent. The annual return credited to the policyholder has a year-on-year guarantee of 2%, and so the payout at year ten depends not just on the final value of the underlying investment fund, but on each annual return over the 10-year period.

The starting CTE(70) reserve can be assessed using a set of real-world scenarios for the joint behavior of US interest rates and corporate bond returns at the valuation date. This was calculated using a standard B&H Economic Scenario Generator real-world calibration at end-December 2012. The CTE(70) for the policy payout (discounted at the fund return) was found to be 114% of the starting fund value, resulting in an initial reserve requirement of 14%. We now focus on the estimation of how this reserve will behave in future, using multi-timestep proxy functions[6].

Separate proxy functions were fitted at annual time-steps from year 1 to 9, using the approach illustrated in Figure 4. At each time-step, 1000 outer scenarios were chosen to span the space of risk factors, and 100 real-world inner scenarios per outer scenario were generated using the B&H Economic Scenario Generator. Our previous research (Morrison, Tadrowski, & Turnbull, 2013) indicates that around 100 inner scenarios are required in order for bias in the CTE estimates to be acceptably low for the particular product considered here. These scenarios were used to fit proxy functions depending on four risk factors:

» **Two factors representing the level of the risk-free yield curve**
In principle, the value of the policy depends on a large number of points on the yield curve. In practice, it is convenient to summarise the curve by a small number of variables. In this case study, we assume that the effect of the risk-free yield curve on the value of the policy can be well summarised by two risk factors: the yield curve 'level' (defined as the one year spot rate) and its 'slope' (defined as the ten year spot rate minus the one year rate).

» **One factor representing the level of corporate credit spreads**
Similarly to risk-free rates, in principle the policy value depends on a large number of credit spreads (of various maturities and credit ratings). However, for the credit model used here, we know that the level of credit spreads can be completely summarized by a single spread, and in this case study we choose the 10-year spot spread corresponding to a corporate BBB rating.

» **The credited policy account at the time of valuation**
The product payout depends on future fund returns, captured by the risk-free rate, rate volatility and spread curve risk factors described above. However it also depends on the path of past fund returns.

[6] In practice we develop proxy functions for the CTE(70) of the discounted policy payout, from which the reserve requirement is calculated by subtracting the current value of the fund.

At first sight this may appear problematic since, depending on the projection time, the value may depend on a large number of past fund returns. For example, the value of the policy in five years depends on five annual returns. This has the potential to significantly increase the dimension of the space of risk factors underlying the proxy function.

However, it should be noted that this path dependency is completely captured in a *single* factor: the current value of the policy account (i.e. the initial fund value rolled-up at the credited returns that have been accrued to the policy to-date using the credited return formula described above). At any future point in time, the current value of the policy account captures all required information about the impact of previous fund returns on the future payoff of the policy, and therefore on its CTE(70).

In summary, we assume that the CTE(70) of the discounted policy payout, at time *t*, can be written:

$$CTE(70)(t) = f_t(\text{yield curve level, yield curve slope, credit spread, policy account})$$

In all cases, proxy functions were selected and fitted using the B&H Proxy Generator.

4. Proxy function validation

Having fitted proxy functions, we validate these under a selection of out-of-sample scenarios for the risk factors. These scenarios were selected as follows. Firstly, 10,000 real-world scenarios were generated using the B&H Economic Scenario Generator, with a 2-factor Black-Karasinski model used for interest-rates, and G2 model used for corporate credit spreads and rating transitions. Here we have used a real-world calibration, initialised to risk-free US Treasury and corporate credit spreads at end-December 2012.

Validation of the proxy function in all 10,000 real-world scenarios would be a computationally costly exercise, requiring full nested simulation. Our approach to validation involves selecting a subset of representative 100 scenarios consisting of 50 'random' scenarios, and an additional 50 'adverse' scenarios, in which reserve requirements are expected to be relatively high. For each of these 100 validation scenarios, the 'actual' CTE(70) of the policy was estimated using 5,000 real-world scenarios and compared with that produced by the proxy functions.

Figures 5, 6 and 7 below show this comparison at years 1, 5 and 9 respectively.

Figure 5: CTE(70) at year 1: Proxy vs actual value in 100 validation scenarios

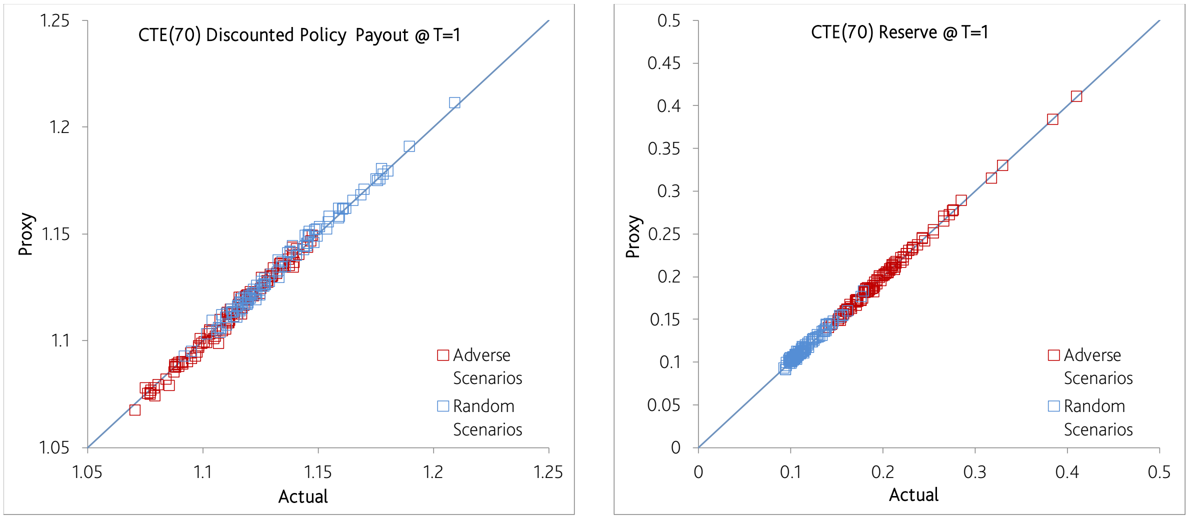


Figure 6: CTE(70) at year 5: Proxy vs actual value in 100 validation scenarios

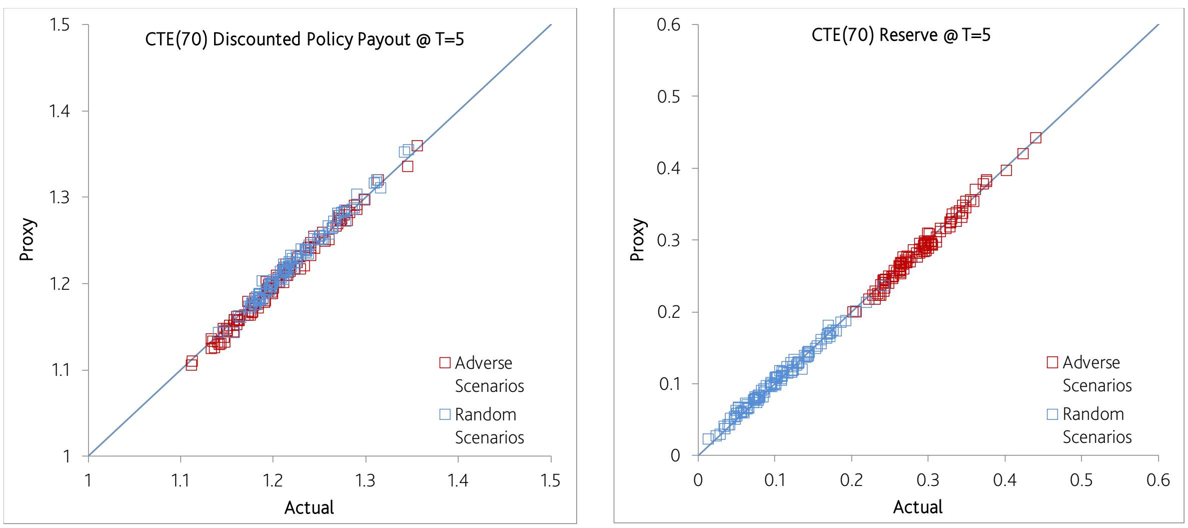
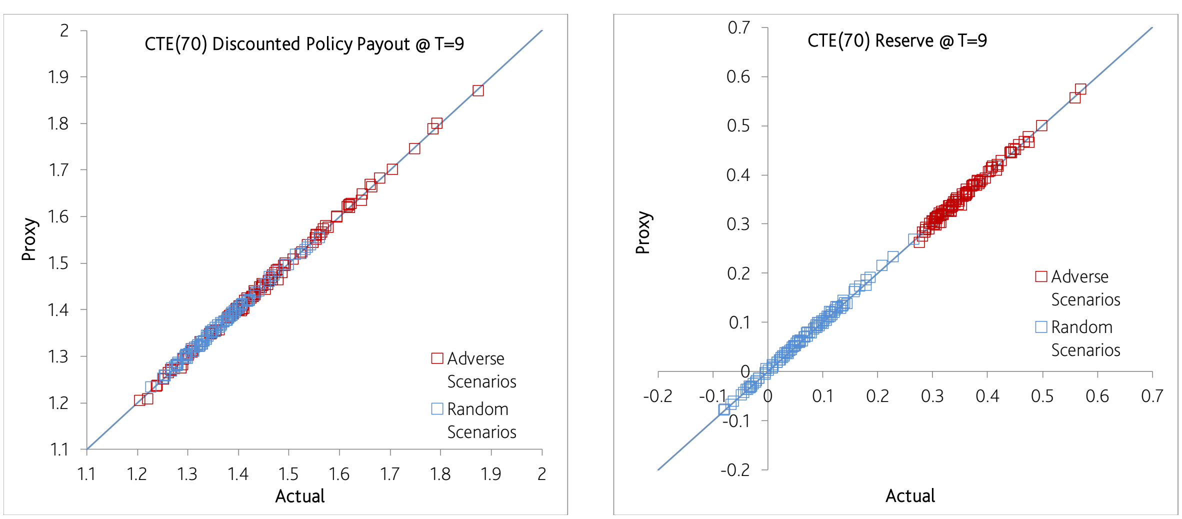


Figure 7: CTE(70) at year 9: Proxy vs actual value in 100 validation scenarios



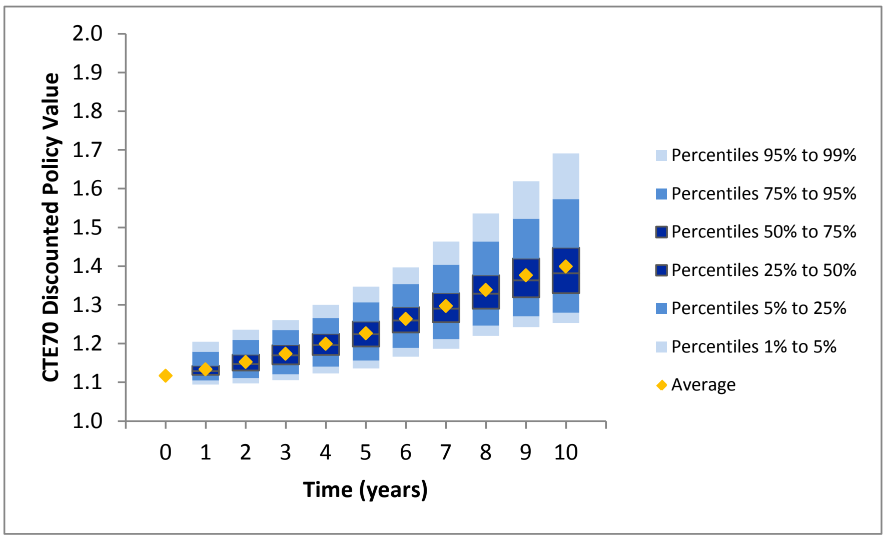
We can see that a good fit is obtained in most random and adverse scenarios, at all times considered. Having fitted and validated proxy functions for the CTE(70) at future points in time, we explore how these functions can be used in practice.

5. Proxy function applications: multi-year stochastic projections

The multi-timestep proxy function provides the means to efficiently implement multi-year real-world stochastic projections of the CTE(70) reserve requirement. This can provide the firm with probabilistic estimates of the likelihoods of different scenarios emerging over time, and a measure of the impact that these scenarios will have on the CTE(70) reserve requirement (with full allowance for the path-dependency and non-linearities that can be found in long-term product guarantees). Such analysis could form an important element of the forward projection of reserve requirements and economic capital, for example, for the purposes of ORSA.

Figure 8 below shows the probability distribution for the CTE(70) of discounted policy payout in each of the next 10 years, as implied by the multi-timestep proxy function and the standard end-2012 real-world calibration of the B&H Economic Scenario Generator (using 10,000 10-year scenarios).

Figure 8: Probability distributions for CTE(70) of discounted policy payout at every time-step



In Figure 8, the proxy function has been used to estimate the CTE(70) of discounted policy account payout in each of the 10,000 real-world scenarios in years 1-9. The time-0 value is calculated directly using a single set of real-world scenarios. The year-10 value is simply the product payout, and so can be calculated directly in each real-world scenario. From the crediting rate formula we know that the product payout at year 10 cannot be less than 1.02^{10} (1.22). The 1st percentile of the policy payout is 1.25 – even in a 1-in-100 scenario, there is at least one year where the fund earns more than the guaranteed minimum.

The upper tails of the above distributions are not necessarily the scenarios that cause most difficulty for the insurer— indeed, many of these scenarios will be where asset returns have been strongest and will actually be the most profitable for the insurer. To understand the insurer's risk exposure, we need to consider how the CTE(70) of the discounted policy payout is projected to behave relative to the asset portfolio value. Figure 9 shows the probability distributions produced for the CTE(70) reserve, defined as the CTE(70) of the discounted policy account payout net of the asset fund value. Again the proxy function has been used for the CTE(70) of discounted policy payout; the asset portfolio value can be directly calculated for each real-world scenario.

Figure 9: Probability distributions for CTE(70) reserve at every time-step

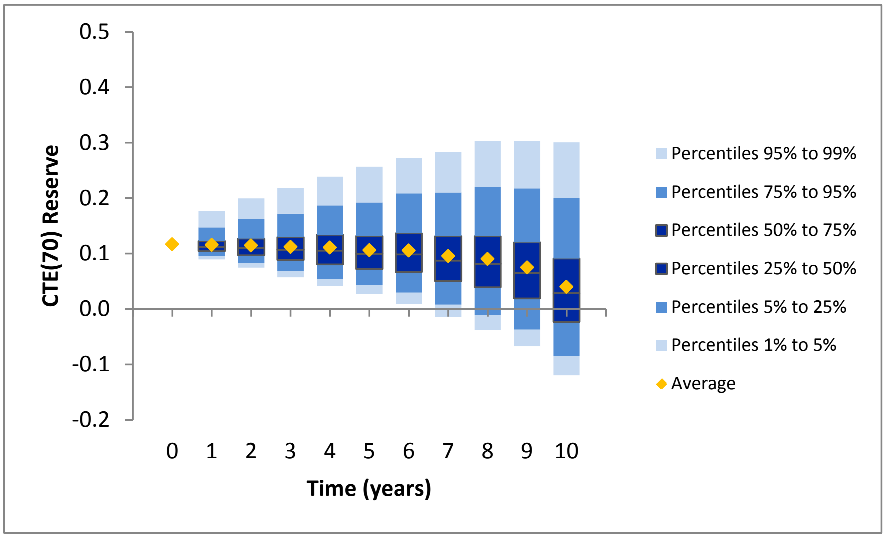


Figure 9 shows that the CTE(70) reserve value will, on average, fall over the life of the policy. This is intuitive – the product's guarantees are not hedged but are backed by assets that are invested in risky assets (corporate bonds). These assets, on average, earn a return which is higher than an annual guarantee rate or crediting rate. This results in assets, on average, growing faster than the CTE(70) of discounted policy payout. However, this effect can only take us so far: the downside risk in the asset portfolio, and, more importantly, the path-dependency in the liabilities, means that the policy payout exceeds the year-10 asset fund value in approximately 2 of every 3 real-world scenarios.

The real-world simulation output can also be used to identify the type of economic scenario that is associated with significant CTE(70) reserve or balance sheet deficits. The reverse stress testing concept will be addressed more fully in the following section, but we first show how the simulation output for the full probability distribution can be used to gain insight into the behavior of the reserve. Figure 10 jointly plots the simulated values for the CTE(70) reserve at year 5 together with the value for BBB-rated credit spreads that arose in the simulation. Figure 11 plots the same CTE(70) reserve, but this time with the 10-year Treasury yield.

Figure 10: Credit spreads and CTE(70) reserve at year 5

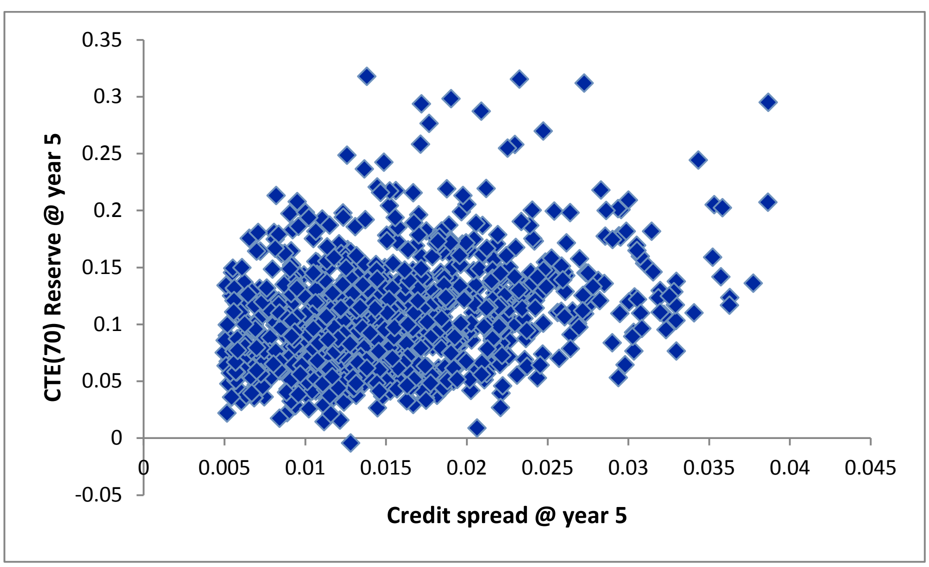
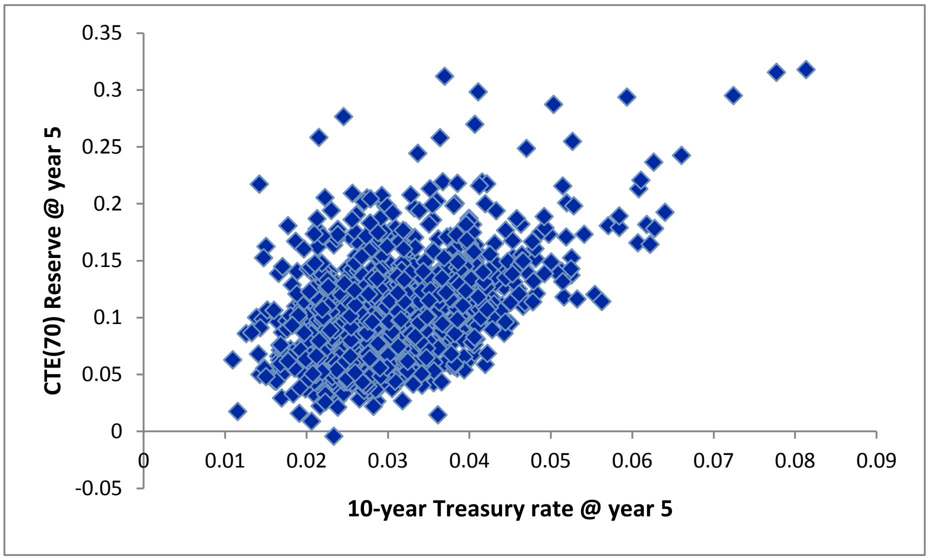


Figure 11: 10-Year Treasury rate and CTE(70) reserve at year 5



These scatterplots suggest that increases in 10–year treasury rates can be particularly problematic for the insurer. There is also a weaker exposure to increases in corporate credit spreads. Given the annual return guarantees in the product, this is unsurprising, as rising bond yields lead to losses on the underlying corporate bond fund.

6. Proxy function applications: reverse stress testing

The real-world simulation results presented above in Section 5 can be used to identify *reverse stress tests*, i.e. to find the scenarios that cause greatest stress to the insurer's balance sheet. As an example, we consider in this section how 5-year reverse stress tests can be identified from the real-world simulation output by ranking these simulations by their year-5 CTE(70) reserve and inspecting the scenarios with the largest deficits. In our illustrative example we only consider a single scenario. However, in a ranking of simulation output, there may be considerable variation in the simulated circumstances that have led to losses, so a 'real-life' implementation should consider a number of these scenarios in a reverse stress test study. Figures 12 and 13 below show the 5-year economic scenario path and balance sheet path produced by the worst ranking scenario in our example.

Figure 12: Economic scenario path in reverse stress test (worst ranking scenario for CTE(70) reserve at year-5)

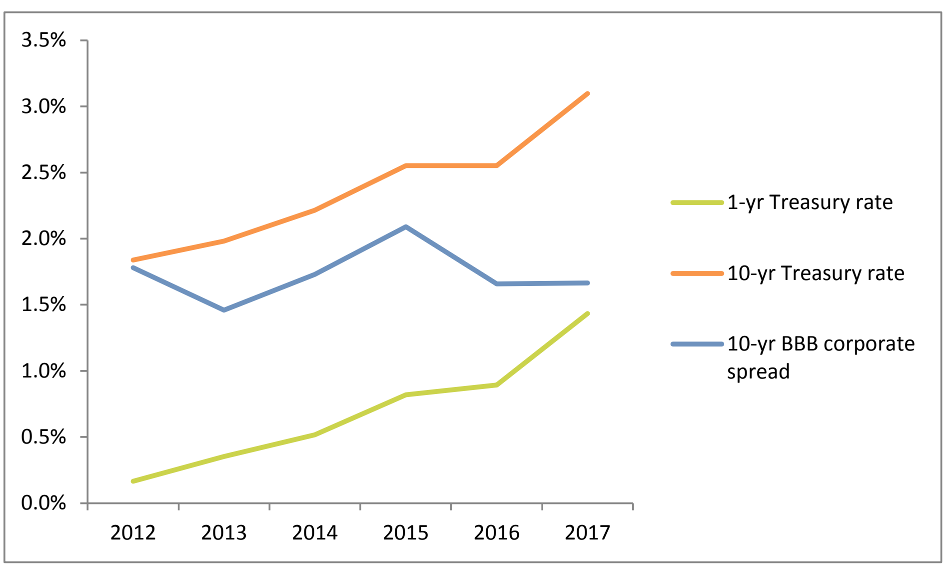
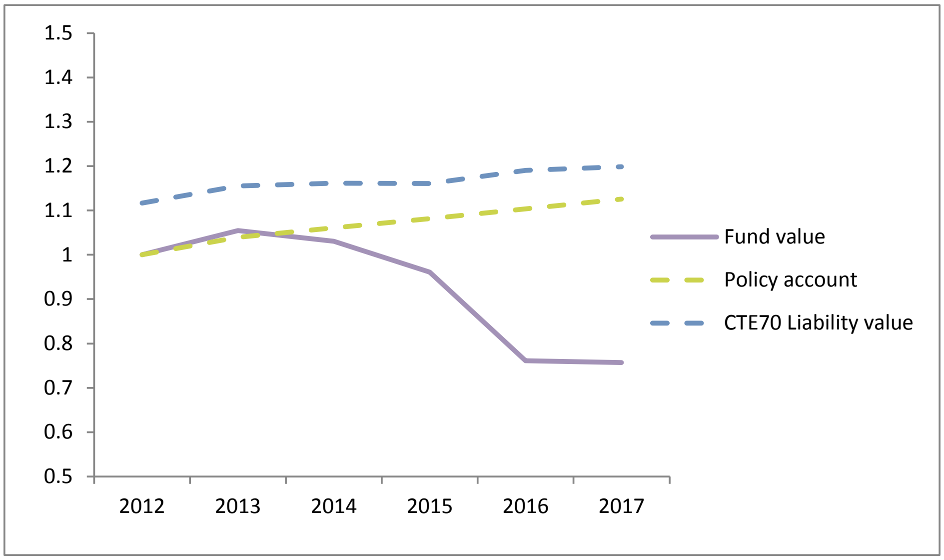


Figure 13: Asset and CTE(70) of discounted policy account payout behavior in reverse stress test



The economic scenario in Figure 12 is consistent with the insights we gained from the scatterplots in Section 5. Over the 5-year projection horizon considered for the reverse stress test, 10-year treasury rate increases by, approximately, 50% (from ~2% to ~3%). Figure 13 shows that the impact on the fund value is strongly negative, with the value after 4 years some 25% below the starting value. In addition, the particularly large negative return on the fund in year 4 arises due to a relatively large number of simulated credit defaults and transitions.

7. Proxy function applications: stress testing

The proxy function applications discussed in Sections 5 and 6 were focused on the use of multi-year real-world simulation modeling that had been enabled by the availability of the multi-timestep proxy functions. This section considers a different type of application – how proxy functions can be used to estimate the CTE(70) reserve and balance sheet associated with a number of multi-timestep deterministic stress test projections. It could be argued that the computational demands of acquiring CTE(70) estimates using full nested stochastic simulations within a handful of deterministic scenarios are sufficiently manageable as to make proxy functions redundant. However, we believe there are still significant benefits to the use of robust proxy functions in stress testing. In particular, proxy functions enable the impact of *what-if scenarios* to be measured immediately. For example, in the course of an ORSA forward solvency projection, it is likely a senior manager or regulator will ask how the balance sheet behaves if the stress test involved a slightly different path for credit spreads or interest rates or equities, etc. Proxy functions allow these questions to be answered in minutes instead of weeks.

As an example of how these macro stresses might be established, Moody's Analytics' economic experts regularly publish economic forecasts for a wide range of both global and regional economic variables under a range of alternative macro-economic scenarios. In 2012, these were[7]:

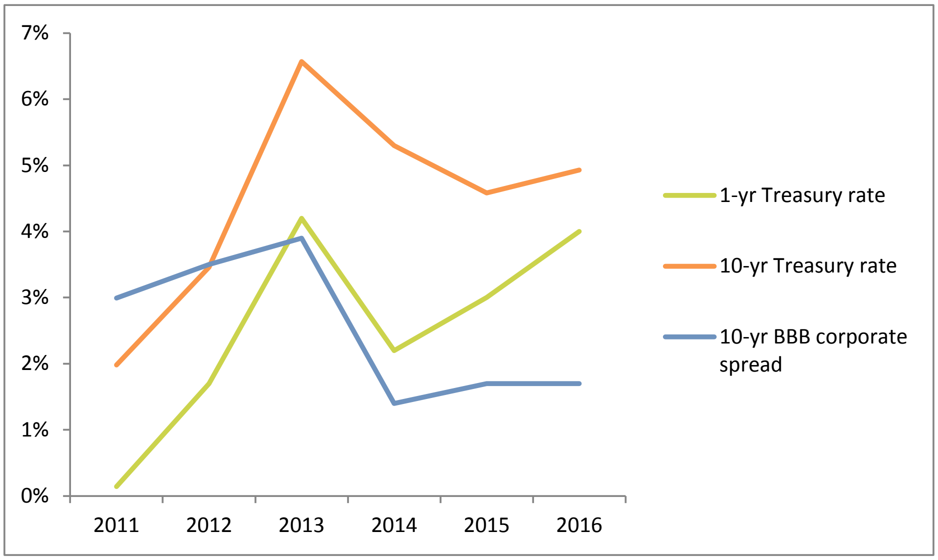
- Scenario 1: Stronger Near-Term Rebound;
- Scenario 2: Slower Near-Term Growth;
- Scenario 3: Double-Dip Recession;
- Scenario 4: Protracted Slump;
- Scenario 5: Below-Trend Long-Term Growth;
- Scenario 6: Oil Shock, Dollar Crash, Inflation.

In each of these scenarios, 5-year paths for a range of financial market and economic variables are specified, together with an estimate of the probabilistic severity of the scenario and an intuitive description of the background circumstances that determine

[7] See, for example, "U.S. Macroeconomic Outlook: Alternative Scenarios", Moody's Analytics, April 2012.

the scenario outcome. Figure 14 shows the 5-year paths[8] for 1-year Treasury rates, 10-year Treasury rates and BBB corporate bond spreads in the Oil Shock, Dollar Crash, Inflation Scenario.

Figure 14: ECCA macroeconomic stress test scenario – Oil shock, US Dollar crash, Inflation



This scenario produces very significant increases in the Treasury curve over the first two years of the stress test (the 10-year Treasury rate triples over the first two years). After this shock period, Treasury yields then revert to a more stable position. Credit spreads fall over the 5-year projection, presumably because high inflation is substantially eroding the nominal value of firms' debt. Figure 15 shows the projected asset values and CTE(70) of the discounted policy payout. Again, the CTE(70) estimates are projected using the proxy function developed in Section 3.

Figure 15: Asset and CTE(70) of policy account payout projection using multi-timestep proxy function – ECCA Oil shock / USD crash / inflation scenario

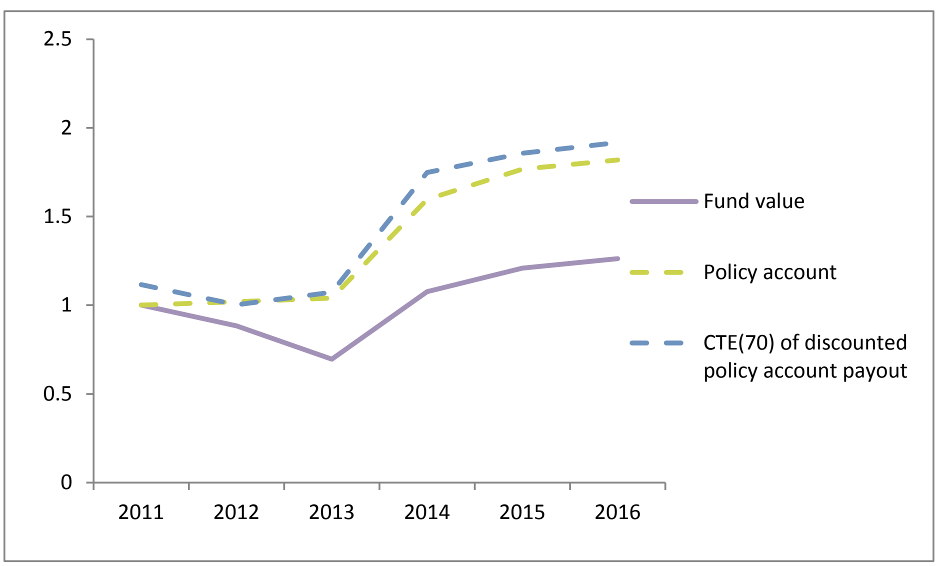


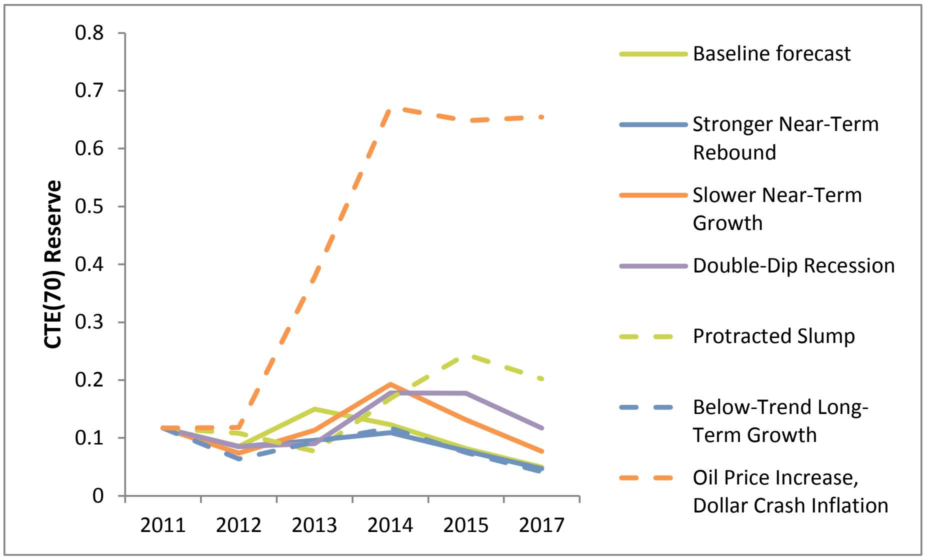
Figure 15 shows that the exceptional increases in bond yields over the first two years of the stress test projection result in a fall in the asset fund of more than 30% by the end of year 2. The product receives its 2% minimum guaranteed return in each of these

[8] Note that these stress tests were specified with end-2011 start points, and so have a different start date to the simulation output presented in Section 5.

years, so that the gap between the asset portfolio value and the credited policy account is 35% of the starting fund value. Fund returns are very strong in the subsequent three years, but, interestingly, the gap between the asset value and the liability value keeps on growing. This is because the credited return is applied to the size of the credited account, which is proportionally 50% bigger than the asset portfolio value after two years. So, even after the 150 basis point haircut, the credited account grows faster than the asset portfolio.

This is an interesting example of one of the types of path-dependency that exists in this form of product. The result is financial Armageddon, as highlighted in Figure 16. This shows the CTE(70) reserve requirement projection over all 6 of the above stress test scenarios, as well as the baseline forecast.

Figure 16: Asset and liability projection using multi-timestep proxy function – 7 ECCA scenarios



Interestingly, the 5-year projection of the CTE(70) reserve in the Oil Shock, Dollar crash scenario lies well outside the 5-year probability distribution in Figure 9. The stochastic analysis in Section 5 suggested that the 99th percentile value of the CTE(70) reserve at year 5 was 0.25, whereas this stress test produced a value after 5 years of around 0.65. The stochastic model implies the probability of such an extreme path for interest rates emerging is extremely low. This is a good example of how stress testing and stochastic simulation can provide alternative insights into the measurement of risk. It is ultimately a matter of judgment whether this tells us that the stress testing is identifying feasible event risks that are beyond the capabilities of the stochastic model to identify; or whether the stress test scenario is vanishingly improbable.

8. Conclusions

This paper has described and demonstrated how the more statistically sophisticated proxy fitting approaches previously used in one-year projection of market-consistent values can be applied to produce full multi-year projection of CTE(70) reserves. The quality of fit, as measured using out-of-sample validation testing, was shown to be consistently high throughout the ten-year projection horizon considered in our case study.

The capability to efficiently produce robust and accurate proxy functions for CTE(70) reserve behavior across a wide range of multi-timestep, multi-risk-factor scenarios can significantly enhance firms' forward solvency projection analytics. We believe this can be extremely useful for firms to project their balance sheets and reserve requirements as part of ORSA and other business planning requirements. The case study presented in this paper highlights how the proxy functions could be used to facilitate medium-term stress testing, reverse stress testing and stochastic projection of the insurer's balance sheet.

References

Cathcart, M. J. (2012). Monte Carlo Simulation Approaches to the Valuation and Risk Management of Unit-Linked Products with Guarantees. Heriot-Watt University PhD Thesis.

KPMG. (2012). *Economic Capital Modeling in the Inurance Industry.*

Manistre, J., & Hancock, G. (2005). Variance of the CTE estimator. *North American Actuarial Journal* .

Morrison, S., Tadrowski, L., & Turnbull, C. (2013). *One-year Projection of run-off conditional tail expectation (CTE) reserves.* Moody's Analytics.

Morrison, S., Turnbull, C., & Vysniauskas, N. (2013). *Multi-year projection of market-consistent liability values.* Moody's Analytics.