MOODY'S ANALYTICS

MODELING METHODOLOGY

A Composite Capital Allocation Measure Integrating Regulatory and Economic Capital, and the Impact of IFRS 9 and CECL

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Abstract

In this paper, we propose a composite capital allocation measure integrating regulatory and economic capital. The approach builds upon the economic framework underpinning traditional RORAC-style business decision rules, allowing for an optimized risk-return tradeoff while adhering to regulatory capital constraints. The measure has a number of depictions, and it can be viewed as a weighted sum of economic and regulatory capital, as economic capital adjusted for a regulatory capital charge, or as regulatory capital adjusted for concentration risk and diversification benefits. Intuitively, when represented as economic capital adjusted for a regulatory capital charge, the adjustment can be represented as the additional top-of-the-house regulatory capital, above economic capital, allocated by each instrument's required regulatory capital. We show that the measure has ideal properties for an integrated capital measure. When regulatory capital is binding, composite capital aggregates to the institution's top-of-the-house target capitalization rate. We find the measure is higher than economic capital, but lower than regulatory capital for instruments with high credit quality, reflecting the high regulatory capital charge for this instrument class. Finally, we address how IFRS 9/CECL impacts the CCM and discuss the broader implications of the new accounting standards.

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Contents

1. Introduction

In a classic risk management setting, we measure credit portfolio risk using Economic Capital (EC), which reflects diversification, concentration, and other economic risks. With constraining regulatory capital requirements under Basel and stress testing, EC alone is insufficient when making business decisions. This paper proposes a Composite Capital Measure (CCM), a capital allocation metric integrating both economic and regulatory capital, that can be used to optimize the risk-return performance of a portfolio while adhering to regulatory requirements.

Our work extends Levy, Kaplin, Meng, and Zhang (2012) (LKMZ), who propose a unified decision measure that incorporates regulatory capital requirements into the traditional economic framework underpinning EVA and RORAC-style business decision measures. The LKMZ derivation adjusts return measures, which ultimately enter into RORAC and EVA, to reflect the implicit cost of regulatory capital constraints. In this sense, the adjustment proposed by LKMZ is performed in the return space. This paper reformulates the representation of the adjustment, allowing it to enter into the capital calculation, lending its application toward capital allocation. The measure can be interpreted either as EC adjusted for a regulatory capital surcharge, or as regulatory capital adjusted for concentration risks and diversification benefits.

The measure we introduce has a number of appealing characteristics. At the portfolio level, aggregated capitalization rate across instruments coincides with the institution's top-of-the-house capital, which aligns with regulatory requirements. At the instrument level, the measure accounts for both economic risk and capital redistribution needed to reflect the impact of each instrument's regulatory capital requirement. It also recognizes the degree to which regulatory capital is constraining, with regulatory capital playing a more prominent role as the constraint becomes more severe. Thus, there are two distinct effects, with (1) constraining top-of-the-house regulatory capital increasing capital above what economic capital requires, and (2) cross-sectional variation in instrument regulatory capital requirements resulting in a redistribution of capital, changing the relative appeal of each instrument. We call the first effect the deleverage effect and the second the redistribution effect.

Finally, we introduce an important modeling nuance that provides guidance on how loss allowance should be accounted for with the CCM. In particular, we observe that loss allowance serves the same purpose as capital does—as a reserve against potential future losses. From a decision metric perspective, loss allowance can be viewed as additional capital requirement; an increase in loss allowance decreases earnings, which, in turn, reduces available capital. With this in mind, we can extend the CCM to account for loss allowance, and relate it with the implications of moving from incurred loss accounting to IFRS 9 and CECL.

To summarize, when facing a constraining regulatory capital requirement, institutions should use CCM to allocate capital across different investments or business sectors. This practice enables meeting regulatory capital requirements, with capital allocation that guides them toward optimizing their risk return profile.

The remainder of this paper is organized as follows:

Section 2 formally introduces the CCM. Section 3 examines the impact of economic and regulatory capital on the CCM through a case study. Section 4 analyzes the impact of CECL/IFRS 9 on loss allowance and the CCM. Section 5 explores the impact of making strategic decisions using CCM-based RORAC. Section 6 concludes.

2. RegC-EC Composite Capital Measure

In traditional portfolio selection frameworks, institutions optimize their portfolio using risk-adjusted return decision measures such as Return on Risk Adjusted Capital (RORAC). The measures can be derived via mean-variance optimization, where the portfolio with the highest possible expected portfolio return-to-risk is chosen.[1] The problem changes in the presence of constraining regulatory capital requirements; the measures must be adjusted to reflect the implicit cost of the additional capital constraint. As discussed in the Introduction, LKMZ derive a Regulatory Capital Adjusted RORAC, where the return is adjusted to reflect the implicit cost of constraining regulatory capital. This section presents an alternative representation of the adjustment that enters into the capital metric in Theorem 1:

THEOREM 1

An institution optimizing its portfolio holdings of $\boldsymbol{N}$ risky assets under the constraint that the required regulatory capital does not exceed institution's available capital (represented by Equation (2)) can be formulated as follows:[2]

$$\max\left(\boldsymbol{w}'E(\boldsymbol{r}) + (1-\boldsymbol{w}'\mathbf{1})r_f - \frac{1}{2}\psi\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}\right) \tag{1}$$

s.t.

$$\boldsymbol{w}'\boldsymbol{RWC} \leq 1 \tag{2}$$

Where $\boldsymbol{w}$, $\boldsymbol{r}$, and $\boldsymbol{RWC}$ denote, respectively, the vector of holdings, normalized returns, and normalized required regulatory capital of $\boldsymbol{N}$ risky assets in the portfolio; $\mathbf{1}$ denotes a constant vector of ones; $\boldsymbol{\Sigma}$ denotes the assets' covariance matrix.[3] r_f denotes the borrowing cost, assumed to be constant and equal to the risk-free rate. Coefficient $\boldsymbol{\psi}$ captures the risk aversion level of the institution, not directly observable.

At the optimal, the following condition holds for any instrument $\boldsymbol{j}$, as it relates with the portfolio p:

$$\frac{E(r_j) - r_f}{CCM_j} + r_f = \frac{E(r_P) - r_f}{CCM_P} + r_f \tag{3}$$

where the composite capital measure CCM_j specifies the optimal capital allocation under the regulatory requirement for instrument $\boldsymbol{j}$, is a weighted sum of EC and RWC, with the weights being functions of DelR. It has two different but mathematically equivalent representations: either as EC adjusted for a regulatory capital charge:

$$CCM_j = EC_j + DelR \times RWC_j \tag{4}$$

Or as regulatory capital adjusted for concentration risk or diversification benefits:

$$CCM_j = RWC_j + \left(EC_j - (1 - DelR) \times RWC_j\right) \tag{5}$$

Where

$$DelR = \max\left(\frac{RWC_P - EC_P}{RWC_P}, 0\right) \tag{6}$$

Under this framework, the composite capital measure for the portfolio aggregates to the institution's top-of-the-house capital (i.e., regulatory capital):

$$CCM_P = RWC_P \tag{7}$$

PROOF

See Appendix B.

The linear form of CCM not only allows for convenient implementation, but it also provides intuitive interpretation of the measure. First, Equation (4) shows that CCM can be viewed as EC adjusted for a regulatory capital charge. The regulatory capital charge can be regarded as the additional top-of-the-house regulatory capital, above economic capital, allocated by each instrument's required

[1] The predetermined level of return variance is typically linked with the institution's risk appetite.

[2] Without loss of generality, we assume the available capital amounts to one unit.

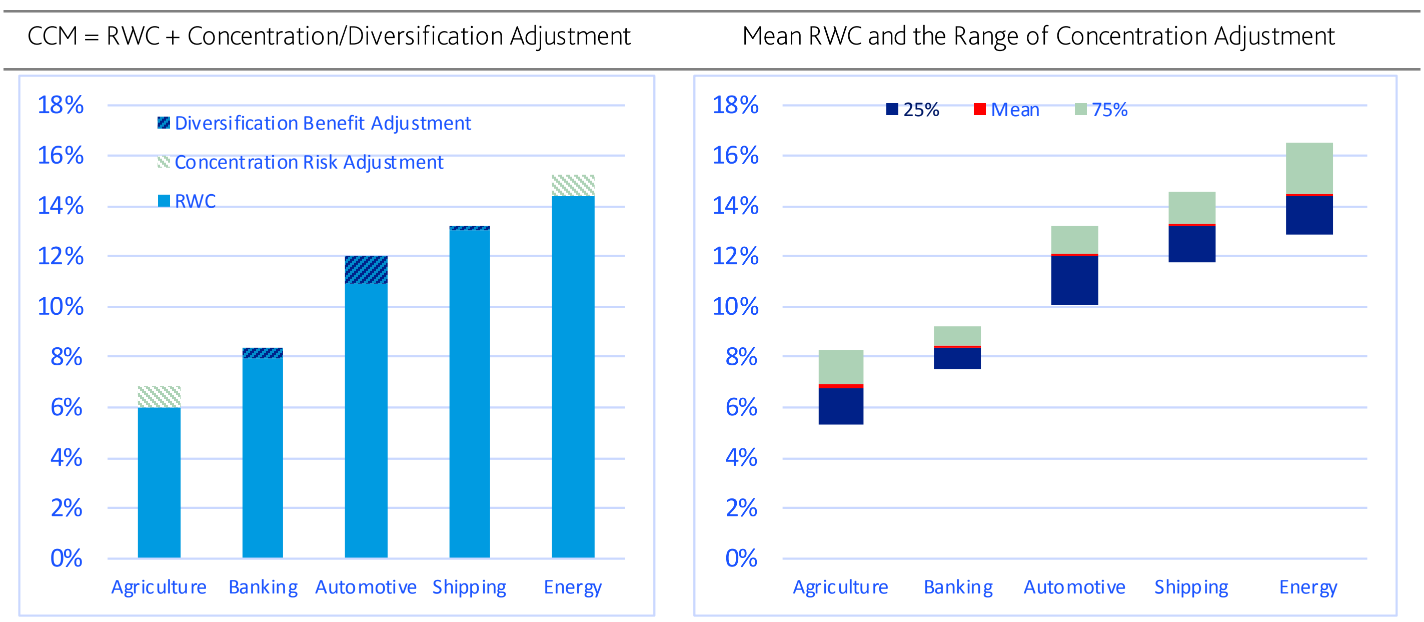
[3] We use bold font to imply that the variable is either an Nx1 column vector or an NxN matrix, differentiating it from a scalar variable.

regulatory capital. It is determined by two factors – instrument RWC, and top-of-the-house DelR — reflecting both the redistribution and deleveraging effects. The impact of both factors on CCM is consistent with economic intuition; CCM increases with an instrument's RWC, and weight on RWC increases with the degree to which regulatory capital is constraining (i.e., larger DelR).

Second, Equation (5) shows that CCM can also be represented as regulatory capital adjusted for the gap between instrument EC and the leverage-adjusted value of RWC.[4] A major difference between EC and RWC stems from EC accounting for concentration and diversification. Therefore, Equation (5) represents CCM as RWC adjusted for concentration and diversification. For visual illustration, the left panel in Figure 1 shows CCM for industries in a sample portfolio. We can see that the agriculture industry has a RWC of 6%. However, since the portfolio has a large holding and, thus, high concentration in the agriculture industry (a factor not accounted for by regulatory capital), the agriculture subportfolio receives a concentration adjustment on top of RWC, resulting in a CCM value roughly 2% higher than RWC. Similarly, the portfolio has relatively small exposure to the automotive industry. In addition, the borrowers from this industry happen to be less correlated with other borrowers in the portfolio. Consequently, the CCM of the automotive industry is lower than its RWC, reflecting the diversification benefit the automotive industry exposures bring to the overall portfolio. It is also noteworthy that, while the concentration and diversification adjustments can vary significantly across instruments, the adjustment center around zero, equating the top-of-the-house CCM with RWC. Figure 1, right panel, illustrates this finding, where we plot the mean instrument RWC for each industry, as well as the 25th and 75th percentile of instrument concentration, adjustment distribution.

Figure 2 reports CCM for several counterparties in a sample portfolio to demonstrate how their characteristics affect the concentration and diversification adjustment. For example, the exposure to CNB Financial has regulatory capital of 7.6%. However, its CCM is significantly higher at 9.4%. This occurs because CNB Financial's credit worthiness is very vulnerable to systemic economic shocks, captured by its high RSQ value, which measures the correlation between this particular borrower's credit migration with general economic conditions. Consequently, this exposure adds a relatively high degree of concentration risk to the portfolio, which raises its CCM above regulatory capital. The exposure to Prudential Financial also adds severe concentration risk to the portfolio, but for a different reason — the portfolio already contains a large exposure to Prudential Financial, so any marginal exposure to the borrower contributes materially to the portfolio concentration. In comparison to the two exposures mentioned above, some other exposures add diversification benefits to the portfolio. For example, the exposure to Valero Energy has low PD and very low RSQ and, thus, low CCM compared to regulatory capital.

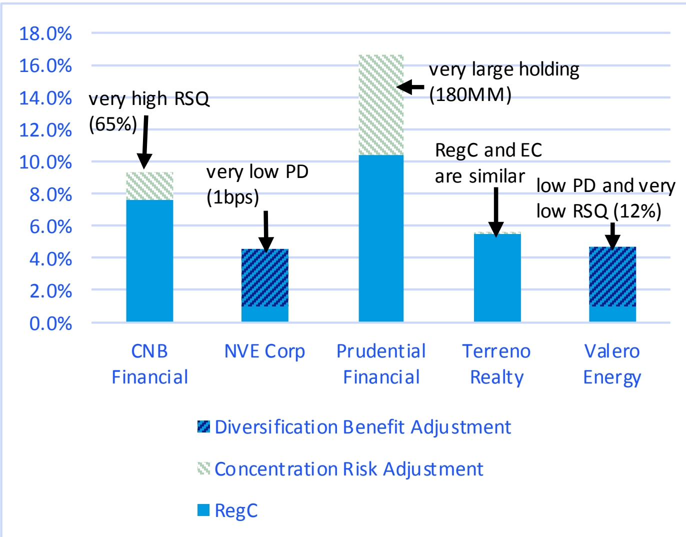
Figure 1 CCM as RWC Adjusted for Concentration Risk or Diversification Benefits.



[4] The leverage-adjusted RWC is a scaled version of RWC and is equal to EC when aggregated at the portfolio level.

Figure 2 Obligor-Level CCM Examples.

CCM = RWC + Concentration/Diversification Adjustment



Mean RWC and the Range of Concentration Adjustment

Borrower	CCM	RegC	Adjustment
CNB Financial	9.4%	7.6%	1.8%
NVE Corp	1.0%	4.6%	-3.6%
Prudential Financial	16.6%	10.4%	6.2%
Terreno Realty	5.5%	5.5%	0.0%
Valero Energy	1.0%	4.7%	-3.7%
…	…	…	…
Portfolio	7.9%	7.9%	0.0%

There are two points to consider when implementing the CCM. First, DelR, and, thus, relative weights on EC and regulatory capital reflect the degree to which regulatory capital is constraining, is institution-specific and can be time-varying. During a deteriorated credit environment for example, the average point-in-time (PIT) PD of a credit portfolio tends to increase significantly more than the through-the-cycle (TTC) PD. As a result, the RWA associated with a credit portfolio, typically measured based on TTC PD, is relatively low compared to the economic risk of the portfolio. In addition, the amount of additional regulatory capital buffer required above the regulatory minimum, such as the counter-cyclical buffer, is also likely to be smaller, which further loosens the overall regulatory capital constraint. DelR value can be calibrated as the relative difference between scenario-based, top-of-the-house economic and regulatory capital to capture this counter-cyclical pattern of regulatory capital constraint. Consequently, more weight is placed on EC during a deteriorated credit environment compared with a benign credit environment. The representation of weight through DelR is an appealing property that differentiates CCM from other integrated measures that typically do not provide theoretical justifications for the choice of weights on economic and regulatory capital.

Second, the derivation of Theorem 1 assumes DelR is determined by an institution's current top-of-the-house economic and regulatory capital. In reality, institutions hold more than their required minimum RegC, as suggested by Levy, Liang, and Xu (2017). Appendix C extends the framework to allow for an arbitrary (positive) value of DelR.

3. Impact of the Regulatory and Economic Capital on Composite Capital Allocation

This section further illustrates regulatory and economic capital's impact on composite capital allocation using a case study, which is based on a synthetic global loan portfolio with publicly listed obligors covering 61 industries and 72 countries.[5] We set r_f to be 0.02, λ to be 0.6. DelR is calculated as 13%. In addition, we compute instrument RWCs under Basel III advanced IRB with a 2% counter-cyclical butter.[6]

Figure 3, left panel, plots instrument CCM against EC in log scale with a 45-degree reference line (red). We can see that CCM is significantly greater than EC when EC is small. Its value converges towards EC as EC becomes larger. The observed pattern occurs due to the heterogeneous redistribution effect of the regulatory capital requirement on different instruments: safer instruments are reassigned with more capital; regulatory requirements are relatively more onerous for safer instruments than for riskier ones. Figure 3 illustrates the heavier regulatory burden on high credit-quality instruments, and shows that instrument RWC is generally higher than EC when EC is small and vice versa. Also note, even for risky instruments, CCM is always above EC, which occurs because the composite measure accounts for both types of capital.

Figure 3 CCM versus Other Capital Measures.

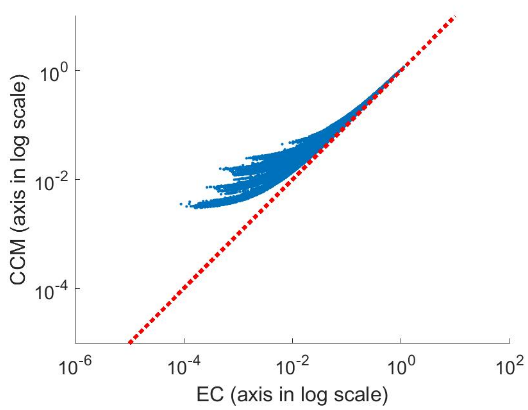
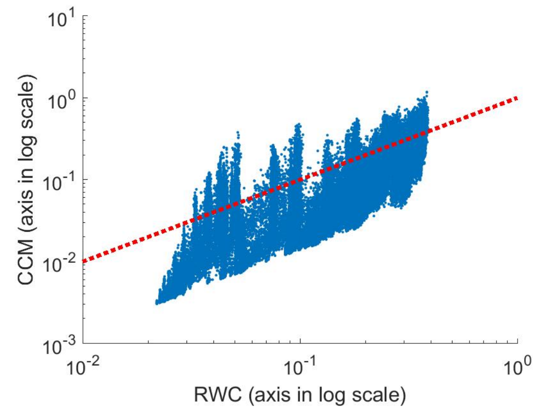
EC Versus CCM	Regulatory Capital (RWC) Versus CCM
	

Figure 3, right panel, plots instrument CCM against RWC. We can see that CCM is, in general, positively correlated with RWC. However, the correlation is low. Specifically, while CCM, on average, equals RWC, instruments with similar RWC can have a wide CCM range. This pattern resonates with that observed in Figure 1, right panel, and reflects the heterogenetic concentration/diversification effect associated with individual instruments.

[5] See Appendix A for more details regarding this sample portfolio.

[6] We use TTC PD and downturn LGD in the Basel III advanced IRB formula for RWA. The minimum capitalization rate is set as 8%. See Appendix A for details on we obtain how TTC PD and downturn LGD.

4. CECL/IFRS 9 Loss Allowance Impact on Composite Capital Allocation

In the preceding discussion, we make the simplifying assumption that capital supply and demand is determined exogenously by only current available capital and required regulatory capital. In reality, both capital demand and supply are also affected by loss allowance and can be endogenous. Our simplifying assumption is reasonable when the level of loss allowance is low and when all assets in the portfolio are liquid.[7] However, as the new CECL/IFRS accounting rules are introduced, the level of loss allowance can increase significantly. In addition, institutions often hold many assets, typically loans, which do not have a liquid secondary market. Therefore, it is useful to relax our model assumptions and extend CCM to account for the presence of loss allowance.

Overall, the impact of CECL/IFRS 9 loss allowance rules on capital allocation is two-fold. First, higher level of loss allowance reduces available capital. Second, the increased volatility in loss allowance feeds into earnings volatility, which leads to higher capital demand under an illiquid market. We discuss in more detail these impacts, as well as how to adjust CCM to account for them in the following subsections.

4.1 Accounting for Loss Allowance Level with CCM

Intuitively, loss allowance serves the same purpose as capital does — reserves against potential future losses. From a decision metric perspective, loss allowance can be viewed as additional capital requirement; an increase in loss allowance decreases earnings, which, in turn, reduces available capital.[8] In Table 1, an institution considers raising cash to originate two loans, A and B. Both loans have a $10,000 notional and $800 required regulatory (Tier-1) capital. The loss allowance for Loan A and B is $200 and $500, respectively. To originate Loan A, the institution must raise at least $1,000 capital (and issue $9,000 debt) since, once the loan is originated, it must write-off $200 loss allowance from its available capital immediately, bringing its total available capital level to $800, just meeting the regulatory requirement. Similarly, to originate Loan B, the institution must raise $1,300 in capital. In general, the effective capital associated with each asset always equals its required regulatory capital plus loss allowance.

TABLE 1:

Relationship between Loss Allowance Level and Effective Capital

	LOAN A	LOAN B
Required Regulatory Capital (Tier 1)	$800	$800
Total Capital Raised (Effective Capital Required)	$1,000	$1,300
Loss Allowance	$200	$500
Available Capital (= Total Capital Raised – Loss Allowance)	$800	$800

Mathematically, the concept of effective capital can be integrated into the model framework for CCM by adjusting the regulatory capital constraint (Equation (2)) into

$$\boldsymbol{w}'\boldsymbol{RWC} \le 1 - \boldsymbol{w}'\boldsymbol{LA}, or\ equivalently, \boldsymbol{w}'(\boldsymbol{RWC} + \boldsymbol{LA}) \le 1 \qquad (8)$$

Where $\boldsymbol{LA}$ denotes the vector of normalized loss allowance for all instruments. The difference between Equation (8) and the original constraint Equation (2), is that we assume the addition of each risky instrument in the portfolio not only increases capital demand by increasing RWC, but reduces capital supply by introducing loss allowance. This condition essentially states that an institution must keep its effective (rather than minimum) required capital under a certain level when optimizing its portfolio. By replacing $\boldsymbol{RWC}$ with $\boldsymbol{RWC}^* = \boldsymbol{RWC} + \boldsymbol{LA}$, we can derive the formula for CCM accounting for loss allowance under the same framework as outlined in Section 2:

$$CCM_j^* = EC_j + \max\left(\frac{RWC_P^* - EC_P}{RWC_P^*}, 0\right) \times RWC_j^* \qquad (9)$$

When the portfolio is mean-variance optimized under the constraint of Equation (8), the following condition holds for all instruments:

[7] We use the liquidity assumption to ensure that institutions can freely sell assets at market value to raise capital, a necessary assumption for the single-period model described in Section 2.

[8] Note, we focus on Tier-1 capital. The case for Tier-2 capital is more complicated, as institutions can use some portion of loss allowance to serve as Tier-2 capital.

$$\frac{E(r_i) - r_f}{CCM_j^*} + r_f = \frac{E(r_P) - r_f}{CCM_P^*} + r_f \quad (10)$$

Equation (10) show that the CCM^* derived here is an optimal allocation measure of effective capital, with portfolio CCM^* equal to portfolio-effective capital. In practice, if an institution seeks to allocate regulatory capital only, it can simply subtract the loss allowance portion from CCM^* to arrive at the adjusted value of CCM:

$$CCM_j^{adj} = CCM_j^* - LA_j \quad (11)$$

4.2 Accounting for Credit Earnings Volatility with CCM

CECL and IFRS 9 require institutions to recognize lifetime, forward-looking expected loss for some or all assets not in default. These new accounting rules are expected to increase the volatility of loss allowance when compared with incurred loss accounting. This dynamic has important implications for capital management. The increase in loss allowance volatility feeds into earnings volatility, raising uncertainty in capital supply and the likelihood of capital breach in the future. In a very liquid market, an institution can simply sell its assets to raise capital in the event of capital breach. However, in reality, many instruments held by financial institutions, such as loans, do not have a liquid secondary market; selling these assets can result in material costs. In addition, capital breaches are more likely to occur during an economic downturn scenario, when the costs of raising capital can be significantly higher.

To limit the likelihood of a capital breach, institutions hold additional capital above the minimum required. The effective regulatory capital for the portfolio is, in general, higher than the minimum regulatory capital required plus loss allowance. Levy, Liang, and Xu (2017) explain how institutions can determine the exact amount of portfolio additional capital buffer and, thus, effective capital. They also propose methodologies to allocate the buffer to individual instrument's effective regulatory capital by examining the credit earnings volatility of the underlying portfolio. Using their effective regulatory capital as an input to calculate CCM, institutions implicitly account for the impact of credit earnings volatility on capital management. Interested readers should refer to Levy, Liang, and Xu (2017) for technical details, omitted here for brevity.

5. Making Strategic Decisions Using CCM RORAC

Theorem 1 demonstrates that all instruments in an optimized portfolio facing regulatory capital constraints satisfy the following condition:

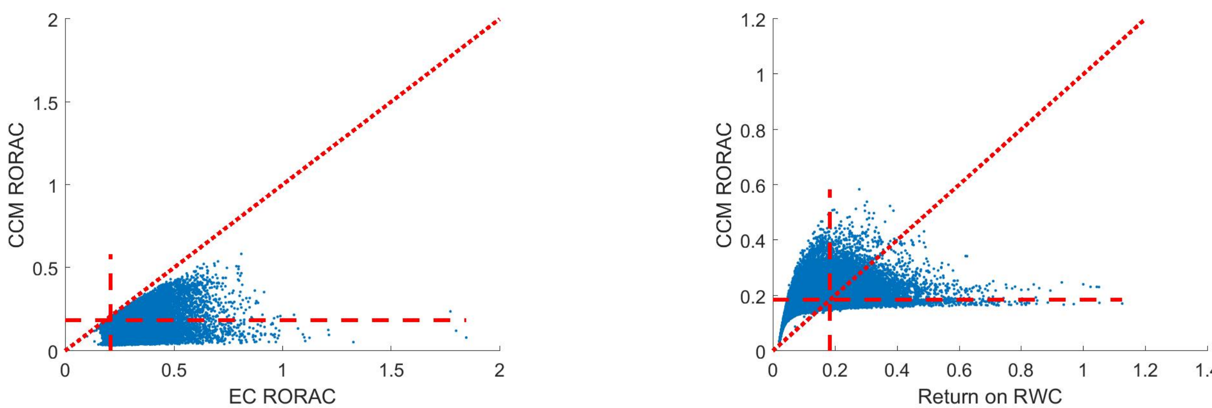
$$\frac{E(r_j) - r_f}{CCM_j} + r_f = \frac{E(r_P) - r_f}{CCM_P} + r_f \quad (12)$$

The left-hand side of Equation (12) shows a RORAC-style measure based on CCM, which has the same form as the traditional EC-based RORAC measure, with EC replaced by the CCM. Making business decisions under CCM RORAC ensures an institution maximizes the risk-return tradeoff of the portfolio while satisfying regulatory capital requirements. This section explores the impact of using EC-, RegC-, and CCM-based decision rules on our sample global portfolio.

Note that CCM RORAC has a close link with RegC-Adjusted RORAC proposed by LKMZ, as the two measures are derived from the same economic framework and thus inherently consistent. The only difference between the two lies at their focus on different dimensions of an asset's risk-return tradeoff; RegC-Adjusted RORAC focuses on adjusting the return dimension, while CCM adjusts the capital dimension. Consequently, they lead to the same business decision in practice. For example, XLMK show that the rank orders between instrument RORAC and RegC-Adjusted RORAC are very different in the IACPM portfolio.[9] XLMK reason that the implicit cost of regulatory capital requirement changes the expected returns of different instruments significantly. Instead of focusing on returns, we provide an alternative perspective by examining CCM RORAC.

Figure 4 CCM RORAC.

EC RORAC Versus CCM RORAC	Return on Regulatory Capital (RWC) Versus CCM RORAC



The left panel of Figure 4 compares CCM RORAC with EC RORAC.[10] We can see that while EC RORAC and CCM RORAC are positively correlated, the correlation is low; business decisions implied by CCM RORAC can be very different from those implied by EC RORAC. In addition, CCM RORAC is generally lower than EC RORAC, reflecting the fact that CCM is greater than EC. The right panel of Figure 4 compares CCM RORAC with return on regulatory capital. Again, the correlation between the two are very low. The most striking pattern observed in the figure is that instruments with very low return on RWC, typically those assets with high credit quality but low income, can have decent to even high CCM RORAC. This comparison suggests that CCM RORAC can help justify holding relatively high credit quality assets that are typically unattractive when viewed through a return on RWC lens.

[9] The IACPM portfolio is created by IACPM and ISDA and is used in their study on the comparison of credit capital models. A detailed description of this portfolio is provided in XLMK (2015).

[10] We use the same sample portfolio and settings as in Figure 3 to produce these figures.

TABLE 2

Top-Ten Exposures with the Highest CCM RORAC

ID	CCM RORAC RANK	EC RORAC RANK	RETURN ON RWC RANK	EDF	LGD	RSQ	MATURITY
L_W44013	1	82	4,521	0.03%	56%	34%	7.0
M_W54624	2	214	3,203	7.49%	61%	28%	6.9
L_W54624	3	251	3,533	7.49%	61%	28%	7.0
M_W58855	4	399	1,284	15.18%	61%	30%	5.4
S_W58855	5	248	5,306	15.18%	61%	30%	1.0
M_W54613	6	106	14,264	0.69%	57%	27%	5.9
L_W47866	7	97	15,467	0.04%	56%	39%	7.0
L_W58855	8	535	1,136	15.18%	61%	30%	7.0
M_W07520	9	347	3,513	8.96%	62%	16%	4.7
L_W54613	10	150	12,688	0.69%	57%	27%	7.0

Top-ten instruments (out of 92,397 in total) with the highest CCM RORAC in the sample portfolio. In all calculations, risk-free rate and λ are set to be 2% and 0.6 respectively.

The observations made from Figure 2 are consistent with the contents in Table 2, which reports RORAC measures for the top-ten instruments with the highest CCM RORAC in our sample portfolio.[11] It is striking that none of the instruments rank above 50 in EC RORAC or return on RWC, suggesting again only a loose correlation between CCM RORAC and other RORAC measures. In addition, we see that risky instruments with high PD and long maturities are, generally, favored. This finding is not surprising, considering that regulatory capital, accounted for in CCM, tends to be relatively less taxing for riskier assets when compared with EC. Nevertheless, the top performing instruments sometimes have very high credit quality, such as an annual PD of only three bps. This happens as the diversification benefit brought by the instrument is recognized in the CCM RORAC manifesting through low EC.

[11] We use the same sample portfolio and settings as in Figure 3.

6. Summary

This paper proposes a composite capital measure that integrates economic and regulatory capital based on the economic framework underpinning traditional RORAC-style business decision rules. We find that this measure can be represented as the weighted sum of economic capital and regulatory capital and captures both the top-of-the-house deleveraging and cross-sectional capital redistribution effects of regulatory capital requirements. We illustrate that CCM reflects the higher implicit cost imposed on instruments with high credit quality. Moreover, making business decisions according to CCM RORAC helps an institution obtain optimal risk-return tradeoff in the portfolio, while meeting regulatory capital requirements.

When the regulatory capital constraint is binding, CCM aligns with the organization's overall capitalization rate, allowing it to achieve the desired leverage. Moreover, capital allocation using CCM is optimal under the regulatory capital constraint, in the same sense as EC is optimal when no regulatory constraint is present.

Appendix A Description of the Sample Portfolio

PORTFOLIO OVERVIEW

Moody's Analytics creates a sample portfolio of floating rate term loans lent to 30,799 public-firm obligors across 72 countries and 61 industry sectors defined by Moody's. In this portfolio, each obligor is assigned three loans: one with a one-year maturity, one with a seven-year maturity, and a third with a maturity randomly assigned (with a uniform distribution) between one and seven years. At the aggregate level, loans of each maturity type account for approximately one-third of total exposure. The overall commitment amount of the portfolio is 150 billion USD with 41% exposure attributed to financial sectors.

OBLIGOR SELECTION

We select the 30,799 obligors as the entire universe of public companies (32,157 in total before exclusions) in the CreditEdge™ dataset as of 03/31/2014, excluding the following:

» Obligors whose total liability as of 03/31/2014 is less than $10 million USD

» Obligors whose one-year Expected Default Frequency (EDF™) value (EDF9) as of 03/31/2015 exceeds 20%.

» Obligors whose RSQ values are not available in Moody's Analytics GCorr™ 2014 universe.

COMMITMENT AMOUNT

In general, the commitment amount for a one-year loan is proportional to the underlying obligor's current liability, while the commitment amount for a seven-year loan is proportional to the obligor's long-term liability. [12, 13] The third loan whose maturity is between one and seven years has a commitment amount equal to 50% of the total commitment amount of the one-year and seven-year loans. The current and long-term liabilities are obtained as the actual values for each obligor as of 03/31/2014, with the following adjustments for different industries:

» Banks and S&L (NDY: N06); Insurance Companies (NDY: N29, N30): current and total liabilities reduced by 90% of total liability to account for non-debt liabilities, such as deposits and insurance claims.[14]

» Security Brokers and Dealers (NDY: N48): current and total liabilities reduced by 60% of total liability to account for non-debt liabilities such as accounts payable. [15]

» For all obligors, both current and long-term liabilities are floored below by $5 million USD.

INDUSTRY/COUNTRY WEIGHT AND RSQ SELECTION

GCorr 2014 values used.

PIT PD AND TTC PD

We use the EDF9 values of the underlying obligor as the instrument point-in-time (PIT) PD. We convert the PIT PD to a Moody's rating using the EDF9 to Moody's rating mapping table (produced by Moody's) as of 03/31/2015. We can convert a Moody's rating to a TTC PD using a mapping table based on a historical rating transition matrix estimated by Tsaig, Levy, and Wang (2010). Specifically, for any rating category, we take the transition probability from that rating to default as the TTC PD associated with the rating.

[12] The scaling factor used to convert liability to commitment amount is selected such that the total portfolio commitment amount is equal to $150 billion USD.

[13] The intuition behind this practice is that the average composition of the credit portfolios held by all banks should mimic the overall liability composition.

[14] For Banks and S&L, call report data shows that the average deposit to total liability ratio for U.S. banks is around 90%, with this ratio higher for smaller banks (around 95%) than for bigger banks (around 85%). For insurance companies, financial statements of companies such as AIG and Metlife show that the debt to total liability ratio is around 10%.

[15] Financial statements of firms such as Morgan Stanley and Goldman Sachs show that the debt to total liability ratio is around 40% for Security Brokers and Dealers.

TABLE 3:

Rating to TTC PD Mapping

RATINGS	TTC PD (%)
Aaa	0.01
Aa1	0.02
Aa2	0.02
Aa3	0.02
A1	0.03
A2	0.03
A3	0.03
Baa1	0.18
Baa2	0.18
Baa3	0.18
Ba1	1.20
Ba2	1.20
Ba3	1.20
B1	5.00
B2	5.00
B3	5.00
Caa1	19.23
Caa2	19.23
Caa3	19.23
Ca	19.23
C	19.23

Ratings to TTC PD mapping. Extracted from Table 1 in Tsaig, Levy, and Wang (2010).

LGD, DOWNTURN LGD, AND LGD VARIANCE PARAMETER SELECTION

LGD, downturn LGD, and LGD variance parameter for most obligors are extracted from Moody's LossCalc™ 3.0 as of 03/31/2015 under the assumption that instruments are senior unsecured loans. LGD related information of a small number of obligors (around 100) is not available in LossCalc 3.0. In these cases, we use the average values computed, based on corresponding variables of all other obligors.

INTEREST SPREAD

Marked to par spread over ZeroEDF rate with annual coupon payment is used. The ZeroEDF rate is assumed to be 2% with a constant term structure. The risk-aversion parameter λ is set to be 0.6.

SUMMARY STATISTICS

TABLE 4

Summary Statistics for Top Industry Exposures

INDUSTRY CODE	INDUSTRY NAME	WEIGHT	EDF	LGD	MATURITY	RSQ
N06	BANKS AND S&LS	12.7%	0.78%	55%	4.5	55%
N59	UTILITIES, ELECTRIC	5.9%	1.09%	44%	5.4	32%
N13	CONSTRUCTION	4.8%	2.31%	54%	3.1	36%
N51	TELEPHONE	4.1%	0.96%	51%	4.4	33%
N40	OIL, GAS & COAL EXPL/PROD	3.8%	3.05%	53%	4.7	41%
N05	AUTOMOTIVE	3.2%	1.76%	52%	3.4	40%
N29	INSURANCE - LIFE	3.1%	0.46%	52%	2.6	57%
N09	BUSINESS SERVICES	3.1%	1.07%	51%	3.4	30%
N46	REAL ESTATE	3.0%	0.87%	52%	4.6	34%
N25	FOOD & BEVERAGE	2.7%	0.57%	49%	3.8	31%
Portfolio Overall		**100%**	**1.09%**	**51%**	**4.0**	**39%**

Summary statistics of top-ten industries (by total commitment) in the portfolio. EDF, LGD, Maturity, and RSQ are average values weighted by commitment amount.

TABLE 5

Summary Statistics of Top Country Exposures

COUNTRY	WEIGHT	EDF	LGD	MATURITY	RSQ
USA	24.9%	0.76%	37%	4.5	36%
JPN	11.8%	0.46%	58%	3.4	43%
CHN	9.5%	1.88%	52%	2.7	40%
GBR	5.4%	0.50%	59%	4.0	38%
FRA	5.0%	0.32%	59%	3.8	39%
DEU	3.7%	0.52%	58%	4.0	36%
KOR	3.3%	1.99%	39%	3.5	39%
CAN	3.2%	0.77%	50%	4.2	40%
IND	2.7%	4.06%	60%	4.0	40%
HKG	2.7%	1.79%	37%	3.4	35%
Portfolio Overall	**100%**	**1.09%**	**51%**	**4.0**	**39%**

Summary statistics of top-ten countries (by total commitment) in the portfolio. EDF, LGD, Maturity, and RSQ are average values weighted by commitment amount.

Appendix B Proof of Theorem 1

THEOREM 1

Assume an institution wants to optimize its portfolio holdings of n risky assets under the constraint that the required RegC of the portfolio cannot exceed the institution's available capital (assumed to be 1):

$$\max\left(\boldsymbol{w}'E(\boldsymbol{r}) + (1-\boldsymbol{w}'\boldsymbol{1})r_f - \frac{1}{2}\psi\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}\right) \tag{13}$$

s.t.

$$\boldsymbol{w}'\boldsymbol{RWC} \le 1 \tag{14}$$

Where $\boldsymbol{w}$, $\boldsymbol{r}$, and $\boldsymbol{RWC}$ denote respectively the holding, normalized return, and normalized required regulatory capital vector of n risky assets in the portfolio; $\boldsymbol{\Sigma}$ denotes the covariance matrix of these assets. r_f denotes the borrowing cost, assumed to be constant. Coefficient ψ captures the institution's risk-aversion level.

At optimal, the following condition holds for all instrument $\boldsymbol{i}$:

$$\frac{E(r_i)-r_f}{CCM_j} + r_f = \frac{E(r_P)-r_f}{CCM_P} + r_f \tag{15}$$

where the composite capital measure CCM_j, which specifies the optimal capital allocation under the regulatory requirement for instrument $\boldsymbol{j}$, is a weighted sum of EC and RWC with the weights being functions of DelR, if all instruments are priced at par. It has two different but mathematically equivalent representations: either as EC adjusted for a RegC charge (value normalized by instrument MTM here):

$$CCM_j = EC_j + DelR \times RWC_j \tag{16}$$

Or as RegC adjusted for concentration risk or diversification benefits:

$$CCM_j = RWC_j + \left(EC_j - (1-DelR)\times RWC_j\right) \tag{17}$$

Where

$$DelR = \max\left(\frac{RWC_P - EC_P}{RWC_P}, 0\right) \tag{18}$$

Under this framework, the dollar amount of composite capital measure for the portfolio aggregates to the dollar amount of institution's top-of-the-house capital (i.e., the maximum of RegC and EC):

$$\$CCM_P = \max(\$RWC_P, \$EC_P) \tag{19}$$

PROOF:

We first prove the following Lemmas:

LEMMA 1

Under the optimal solution of $\boldsymbol{w}$ under the setup of Theorem 1 and assume that the regulatory constraint is binding (i.e., Equation (14) achieves an equality), all instruments in the portfolio must obey the following relationship:

$$\frac{E\left(r_i - r_f - \phi \times RWC_i\right)}{EC_i} - r_f = \frac{E\left(r_P - r_f - \phi \times RWC_P\right)}{EC_P} - r_f \tag{20}$$

PROOF OF LEMMA 1:

Assume the risk aversion parameter ψ is known, we can solve the following LaGrange Multiplier problem:

$$L = \boldsymbol{w}'E(\boldsymbol{r}) + (1 - \boldsymbol{w}'\boldsymbol{1})r_f - \frac{1}{2}\psi\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} - \phi(\boldsymbol{w}'\boldsymbol{RWC} - \boldsymbol{1}) \quad (21)$$

First order condition:

$$\frac{\partial L}{\partial \boldsymbol{w}} = E(\boldsymbol{r} - \phi\ \boldsymbol{RWC}) - r_f - \psi\boldsymbol{\Sigma}\boldsymbol{w} = 0 \quad (22)$$

Clearly, we interpret $\boldsymbol{r} - \phi\ \boldsymbol{RWC}$ as the RegC adjusted return. For now, let us assume $\boldsymbol{\phi}$ is known, we can write out the condition for each individual assets $\boldsymbol{i}$:

$$E(r_i - \phi RWC_i) - r_f - \psi\sum_{j=1}^{N} w_j Cov(r_i, r_j) = 0 \quad (23)$$

It follows

$$E(r_i - \phi RWC_i) - r_f = \frac{1}{w_i}\psi\sum_{j=1}^{N} w_i w_j Cov(r_i, r_j) \quad (24)$$

Notice, $\sum_{j=1}^{N} w_i w_j Cov(r_i, r_j) = Cov(V_i, V_P)$ is the value at horizon covariance between instrument $\boldsymbol{i}$ and portfolio $\boldsymbol{P}$. Therefore,

$$E(r_i - \phi RWC_i) - r_f = \frac{1}{w_i}\psi Cov(V_i, V_P) = \psi\sigma_P RC_i \quad (25)$$

Rearranging, we have

$$\frac{E(r_i - \phi RWC_i) - r_f}{RC_i} = \psi\sigma_P \quad (26)$$

This is the regular Sharpe Ratio condition with the RegC adjusted return in the numerator. We can convert the condition to RORAC style condition:

$$\frac{E(r_i - \phi RWC_i) - r_f}{RC_i EC_P/\sigma_P} = \frac{\psi\sigma_P^2}{EC_P} \quad (27)$$

And

$$\frac{E(r_i - \phi RWC_i) - r_f}{EC_i} - r_f = \frac{\psi\sigma_P^2}{EC_P} - r_f \quad (28)$$

Here the adjustment is made on the numerator.

Since Equation (28) is true for the entire portfolio as well, we must have

$$\frac{E(r_P - \phi RWC_P) - r_f}{EC_P} - r_f = \frac{\psi\sigma_P^2}{EC_P} - r_f \quad (29)$$

Therefore,

$$\psi = \frac{E(r_P - \phi RWC_P) - r_f}{\sigma_P^2} \tag{30}$$

So that

$$\frac{E(r_i - \phi RWC_i) - r_f}{EC_i} - r_f = \frac{E(r_P - \phi RWC_P) - r_f}{EC_P} - r_f \tag{31}$$

Now we proceed to prove Theorem1.

We can rearrange Equation (31) and make the adjustment to appear on the denominator:

$$E(r_i - \phi RWC_i) - r_f = \frac{E(r_P - \phi RWC_P) - r_f}{EC_P} EC_i \tag{32}$$

And

$$E(r_i) - r_f = \frac{E(r_P - \phi RWC_P) - r_f}{EC_P} EC_i + \phi RWC_i \tag{33}$$

It follows

$$\left(E(r_i) - r_f\right) EC_P = \left(E(r_P - \phi RWC_P) - r_f\right) EC_i + \phi RWC_i EC_P \tag{34}$$

And

$$\left(E(r_i) - r_f\right) EC_P = \left(E(r_P - \phi RWC_P) - r_f\right) \left(EC_i + \frac{\phi EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)} RWC_i \right) \tag{35}$$

And

$$\frac{\left(E(r_i) - r_f\right)}{\left(EC_i + \frac{\phi EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)} RWC_i \right)} = \frac{\left(E(r_P - \phi RWC_P) - r_f\right)}{EC_P} \tag{36}$$

And

$$\frac{E(r_i) - r_f}{EC_i + \frac{\phi EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)} RWC_i} + r_f = \frac{1}{\frac{EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)}} + r_f \tag{37}$$

Define

$$CCM_i = EC_i + \frac{\phi EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)} RWC_i \tag{38}$$

And

$$CCM_P = \frac{\sum_{i=1}^{N} w_i CCM_i}{\sum_{i=1}^{N} w_i} \tag{39}$$

Equation (37) shows that at optimal, the ratio between excess return and CCM is constant across all instruments, and that CCM is a linear combination of EC and RWC. At this stage, the formula for CCM contains an unknown parameter ϕ. The value of ϕ can be deduced by recognizing that the optimal leverage the institution wishes to maintain is equal to the inverse of portfolio capitalization rate.

Specifically, notice that the optimal weights under RegC constraint can be solved from Equation (22):

$$\boldsymbol{w} = \frac{E(\boldsymbol{r} - \phi\, \boldsymbol{RWC}) - r_f}{\psi} \boldsymbol{\Sigma}^{-1} \tag{40}$$

And the optimal leverage (same as total asset size since equity size is 1) is assumed to be

$$\boldsymbol{w}'\boldsymbol{1} = \left(\frac{E(\boldsymbol{r} - \phi \boldsymbol{RWC}) - r_f}{\psi} \boldsymbol{\Sigma}^{-1} \right)' \boldsymbol{1} = \frac{1}{\max(RWC_P, EC_P)} \tag{41}$$

Notice, if there is no RegC requirement, the optimal weights are instead

$$\boldsymbol{w} = \frac{E(\boldsymbol{r}) - r_f}{\psi} \boldsymbol{\Sigma}^{-1} \tag{42}$$

And the optimal leverage is assumed to be

$$\boldsymbol{w}'\boldsymbol{1} = \left(\frac{E(\boldsymbol{r}) - r_f}{\psi} \boldsymbol{\Sigma}^{-1} \right)' \boldsymbol{1} = \frac{1}{EC_P} \tag{43}$$

Combining Equations (41) and (43), we have

$$\frac{\left(\left(E(\boldsymbol{r} - \phi\, \boldsymbol{RWC}) - r_f \right) \boldsymbol{\Sigma}^{-1} \right)' \boldsymbol{1}}{\left(\left(E(\boldsymbol{r}) - r_f \right) \boldsymbol{\Sigma}^{-1} \right)' \boldsymbol{1}} = \frac{EC_P}{\max(RWC_P, EC_P)} \tag{44}$$

A reasonable parameterization for ϕ is that

$$\frac{E(r_P - \phi\, RWC_P) - r_f}{E(r_P) - r_f} = \frac{EC_P}{\max(RWC_P, EC_P)} \tag{45}$$

Solve for ϕ, we have

$$1 - \frac{\phi\, RWC_P}{E(r_P) - r_f} = \frac{EC_P}{\max(RWC_P, EC_P)} \tag{46}$$

And

$$\phi = \frac{\left(1 - \frac{EC_P}{\max(RWC_P, EC_P)}\right) \left(E(r_P) - r_f \right)}{RWC_P} \tag{47}$$

Now plug Equation (47) into Equation (38), we have

$$CCM_i = EC_i + \frac{\frac{\left(1 - \frac{EC_P}{\max(RWC_P, EC_P)}\right)\left(E(r_P) - r_f\right)}{RWC_P} EC_P}{\left(E\left(r_P - \frac{\left(1 - \frac{EC_P}{\max(RWC_P, EC_P)}\right)\left(E(r_P) - r_f\right)}{RWC_P} RWC_P\right) - r_f\right)} RWC_i \tag{48}$$

It follows that

$$CCM_i = EC_i + \left(1 - \frac{EC_P}{\max(RWC_P, EC_P)}\right) RWC_i = EC_i + \max\left(1 - \frac{EC_P}{RWC_P}, 0\right) RWC_i \tag{49}$$

And the weighted sum of CCM is

$$\begin{aligned} \sum_{i=1}^{N} w_i CCM_i &= \sum_{i=1}^{N} w_i EC_i + \max\left(1 - \frac{EC_P}{RWC_P}, 0\right) \sum_{i=1}^{N} w_i RWC_i = \sum_{i=1}^{N} w_i \left(EC_P + \max\left(1 - \frac{EC_P}{RWC_P}, 0\right) RWC_P\right) \\ &= \max(RWC_P, EC_P) \sum_{i=1}^{N} w_i \end{aligned} \tag{50}$$

Also note, Equation (45) implies that

$$\frac{EC_P}{E(r_P - \phi\, RWC_P) - r_f} = \frac{\max(RWC_P, EC_P)}{E(r_P) - r_f} \tag{51}$$

Plugging Equations (49) and (51) into (37), we have

$$\frac{E(r_i) - r_f}{EC_i + \frac{\phi EC_P}{\left(E(r_P - \phi RWC_P) - r_f\right)} RWC_i} + r_f = \frac{1}{\frac{\max(RWC_P, EC_P)}{E(r_P) - r_f}} + r_f \tag{52}$$

Notice, $RWC_P = CCM_P$ as indicated by Equation (50), and by the definition of CCM according to Equation (38) we have

$$\frac{E(r_i) - r_f}{CCM_i} + r_f = \frac{E(r_P) - r_f}{CCM_P} + r_f \tag{53}$$

Appendix C Calculating CCM with an Arbitrary Value of DelR

In practice, an institution may wish to use different DelR from that implied in Equation (6). These situations usually occur when the institution uses a scenario-based DelR, such as the DelR calculated under a severely adverse CCAR scenario. In these cases, a more general form of CCM can be derived.

In theory, given any of the two values of EC, RWC, and DelR, the value of the third variable can be deduced. This is how we obtain the value of DelR in Equation (6) — through the given values of EC and RWC. Naturally, when both values of DelR and RWC are given, we can deduce the implied value of EC by simply rewriting Equation (6) into:

$$\$EC_P^* = (1 - DelR) \times \$RWC_P \tag{54}$$

Intuitively, EC_P^* measures how much overall credit risk — including concentration and diversification risks — the institution is actually willing to tolerate while meeting the regulatory requirement as implied by the chosen value of DelR. For an arbitrary input of DelR, the value of $\$EC_P^*$ is generally different from $\$EC_P$. To calculate CCM, we need to adjust $\$EC_P$ to $\$EC_P^*$ as one of the direct inputs to ensure consistency with the input values of DelR and RWC. To do this, we first must adjust EC allocated to each individual instrument:

$$EC_j^* = EC_j \times \frac{EC_P^*}{EC_P} = EC_j \times (1 - DelR) \times \frac{RWC_P}{EC_P} \tag{55}$$

We then generalize the equation for CCM with the value of EC_P^* and EC_j^* instead of EC_P and EC_j under Equation (4):

$$CCM_j = EC_j^* + DelR \times RWC_j \tag{56}$$

Following the same logic outlined in the proof of Theorem 1 in Appendix B, we can show that the resulting CCM value would again sum to $\$RWC_P$ at the portfolio level.

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