Loss Given Default as a Function of the Default Rate

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Jon Frye
Senior Economist
Federal Reserve Bank of Chicago

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In a Nutshell

• Credit loss in a portfolio depends on two rates:
  – the portfolio's default rate (DR) and
  – the portfolio's loss given default rate (LGD).
  – At present there is a consensus model of DR but not of LGD.

• The paper compares two LGD models.
  – One is ad-hoc linear regression.
    ▪ LGD depends on DR (or on variables that predict DR).
  – A newly proposed LGD function has fewer parameters.

• The LGD function has lower MSE over a wide range of control variables.
  – "If you don't have enough data to reliably calibrate a fancy model, you can be better off with a simpler one."
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Topics

Definitions and consensus default rate model

LGD: role, research, and data

The LGD function

Comparing the LGD function and regression

Summary
Definitions

Define for a given loan:

D = 0 if the borrower makes timely payments, D = 1 otherwise
Loss = 0 if D = 0, Loss = EAD x LGD if D = 1
EAD = (dollar) Exposure At the time of Default, assumed = 1.
LGD = (fractional) Loss Given Default rate

PD = E [ D ], the Probability of Default
ELGD = E [ LGD ], Expected LGD
EL = PD x ELGD, Expected Loss rate

cDR = E [ D | conditions], Conditionally expected Default Rate
cLGD = E [ LGD | conditions], Conditionally expected LGD
cLoss = E [ Loss | conditions]; cLoss = cDR x cLGD

A given portfolio has a default rate (DR), a loss rate (Loss), and an LGD rate (LGD); Loss = DR x LGD.
Consensus Default Model

• A firm defaults when a variable crosses a threshold.
  – Robert Merton would say, "when the value of the firm's assets falls below the value of its liabilities."
    ▪ Or, "when the return on the assets of the firm is less than some threshold."
  – Assume that this variable has a normal distribution...

• This produces a probit model:

\[ cDR = \Phi[a + bY] = \Phi \left[ \frac{\Phi^{-1}[PD] - \sqrt{\rho}Y}{\sqrt{1 - \rho}} \right], Y \sim N[0, 1] \]

  – \( \Phi[\cdot] \) represents the standard normal CDF
  – \( Y = \) Systematic component of variable responsible for default
  – \( \rho = "Correlation", \) the square of sensitivity to \( Y \)
Vasicek, LogNormal, Data

Three distributions with Mean = 3.9%, SD = 3.6%
The Vasicek Distribution

• Has an explicit PDF, CDF, and inverse CDF.
  – They are functions of the normal CDF.

• The Vasicek distribution's parameters are PD and $\rho$.
  – PD = Probability of Default = mean
  – $\rho = "correlation"$ is monotonic with variance

• The reason $\rho$ is called correlation:
  – Suppose two firms are statistically identical:
    - The default of Firm 1 depends on $\sqrt{\rho} Y + \sqrt{1-\rho} X_1$
    - The default of Firm 2 depends on $\sqrt{\rho} Y + \sqrt{1-\rho} X_2$
    - The correlation between the variables responsible for default is $\rho$
LGD and Credit Loss

- Credit loss depends on **TWO** rates.
  - If DR and LGD were independent, that's one thing.
  - But risk is worse if both rates rise under the same conditions.

- To calibrate the credit loss distribution would involve:
  - ✔️ Modeling the default rate
  - ☐ Connecting the default rate and the LGD rate with math
    - Model cLGD and cDR jointly, or
    - Condition cLGD and cDR on the same underlying variables, or
    - Model cLGD directly conditioned on cDR such as done here
  - ☐ Calibrating the model of cLoss = cDR x cLGD

- This has rarely been attempted.
  - "LGD" papers do not calibrate credit loss models.
  - "Credit risk" papers often completely ignore LGD.
LGD Data and Research

Forever Banks don't define D or measure LGD
1982 Bond ratings are refined (B ⇒ B1, B2, or B3)
1980's Michael Milken
1990-91 First carefully observed high-default episode
1998 CreditMetrics model (assumes fixed LGD)
2000 Collateral Damage, Depressing Recoveries
2003 Pykhtin LGD model (has 3 new parameters)
2007 Basel II; banks collect data on D and LGD
2010 Modest Means: a simpler credit loss model
2012 Credit Loss and Systematic LGD Risk
2012 Altman's data on default and LGD
Two Words about LGD Data

• They are scarce.
  – Among all exposures, only those that default have an LGD.
  – This is a few percent of the data.

• They are noisy.
  – A single LGD is highly random. Most years have few defaults. In those years, portfolio average LGD is unavoidably noisy.

• Ed Altman (NYU) has a long data set on default and LGD.
  – It contains junk bonds numbering less than 1,000 most years.
    ▪ Ratings: Ba1, Ba2, Ba3, B1, B2, B3, Caa1, Caa2, Caa3, Ca, and C.
    ▪ Seniorities: Senior Secured, Senior Unsecured, Senior Subordinated, Subordinated, Junior Subordinated…
  – Despite this unobserved heterogeneity, the data give an idea…
Altman Bond Data, 1982-2011

Recovery Rate = 1 - LGD

-2.3 DR +.5

1990
2001
1991
2009
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2010
2011
The LGD Function

- The function is: \( cLGD = \Phi \left( \Phi^{-1}[cDR] - k \right) / cDR \)
  - This has one parameter, \( k \), called the LGD Risk Index.

- \( cLoss = cLGD \times cDR = \Phi \left( \Phi^{-1}[cDR] - k \right) \)

- Define \( k = \frac{\Phi^{-1}[PD] - \Phi^{-1}[EL]}{\sqrt{1-\rho}} \); \( cDR = \Phi \left( \frac{\Phi^{-1}[PD] - \sqrt{\rho} Y}{\sqrt{1-\rho}} \right) \),

- Substituting, \( cLoss = \Phi \left[ \frac{\Phi^{-1}[EL] - \sqrt{\rho} Y}{\sqrt{1-\rho}} \right] \)
  - In other words, if \( cDR \sim \) Vasicek [ PD, \( \rho \) ]
  - and if \( cLGD = \Phi \left( \Phi^{-1}[cDR] - k \right) / cDR \),
Instances of the LGD function

- Default
- LGD
- Formula
- Comparison
- $\Sigma$

The graph shows the conditionally expected LGD rate against the conditionally expected default rate for different values of $k$. The values of $k$ range from 0.20 to 0.93.
Features of the LGD Function

• Expresses a moderate, positive relation.
  – This seems like a more plausible starting place than the null hypothesis that there is no relationship at all.

• Has no new parameters to estimate.
  – Modelers already estimate PD, ρ, and EL.

• Is consistent with simplest credit loss model.
  – It can control Type I error in the context of credit loss.

• Depends principally on averages EL and PD.
  – Averaging is more robust than regression.
Comparison: Ground Rules

- This paper compares the predictions of the LGD function to those of linear regression.
  - Both methods use the same simulated default and LGD data.
    - Such data is free of real-world imperfections.
    - cLGD is simulated with a linear model, giving an advantage to regression.
  - Methods are compared by RMSE.
  - The data sample is kept short.

- Both LGD predictors need estimates of PD and $\rho$.
- In addition,
  - The LGD function needs an estimate of EL (= average loss).
  - Regression needs estimates of slope and intercept.
Comparison: Preview

• Using fixed values of control variables:
  – One simulation run is reviewed in detail.
  – 10,000 runs are summarized.
  – The LGD function outperforms regression.

• Using a range of values for each control variable:
  – Most variables have little effect on the result of the contest.
  – Two variables can change the result:
    ▪ the steepness of the relation that generates cLGD and
    ▪ the length of the data sample.
  – Different values of PD and EL don't materially change results.

• Using regression to attempt to improve the LGD function:
  – The attempt fails; supplementary regression degrades forecasts.
One Year of Simulated Data

- cDR has the Vasicek Distribution [PD = 3%, ρ = 10%].
- DR depends on Binomial Distribution [n = 1000, p = cDR].
- cLGD = a + b cDR = .5 + 2.3 cDR
  - Using a linear model gives an advantage to linear regression.
- LGD ~ N [ cLGD, σ² / (n DR)]; σ = 20%.
- Initial experiments involve 10 years of simulated data.
  - Banks have collected LGD data for about 10 years.
One Simulation Run

- Data Generator: $c_{LGD} = 0.5 + 2.3\ c_{DR}$
- 98th Percentile $c_{LGD} = 72\%$
- 10 Years Simulated Data
- LGD Formula: $k = 0.2276$
- Tail LGD by Formula = 66\%
- Linear Regression (not significant)
- Tail LGD by Regression Line = 86\%
- Default-weighted-average LGD
- Tail LGD by Default Wtd. Avg. = 60\%
10,000 Simulation Runs

- Default
- LGD
- Formula
- Comparison
- Σ

- Formula, $\text{RMS}_1 = 7.9\%$
- Regression, $\text{RMS}_1 = 11.6\%$

72.3%
Robustness

• The experiments so far have used a fixed set of values for the eight control variables:
  – Default side: PD = 3%, ρ = 10%, n = 1000
  – LGD side: a = .5, b = 2.3, σ = 20%
  – 10 years of simulated data; 98th percentile of cLGD

• The next experiments allow each variable to take a range of values.
Four Variables have Little Effect

- **Tail percentile**
  - Root mean squared error
  - Formula
  - Regression

- **Correlation**
  - Root mean squared error
  - Formula
  - Regression

- **# Firms**
  - Root mean squared error
  - Formula
  - Regression

- **SD of an LGD**
  - Sigma (standard deviation of LGD)
  - Formula
  - Regression

Default  LGD  Formula  Comparison  Σ
Two Variables that Affect Results

As the data sample extends, regression results improve.
Real-world data are autocorrelated, so improvement is slower than this.

The function outperforms only if it is not too far from the data generator.
The next slide shows the range \((.45 < b < 3.4)\) in a different style.
Where the Function Outperforms

\[ cLGD = 0.47 + 3.4 \times cDR \]

\[ cLGD = 0.5 + 2.3 \times cDR \]

\[ cLGD = 0.56 + 0.45 \times cDR \]

Range where LDG formula outperforms

Lines terminate at percentiles 2 and 98
Summary of Robustness Checks

• The LGD function outperforms ad-hoc linear regression as long as:
  – The data sample is short, and
  – There is a moderate positive relation between LGD and default.

• These conditions are believed to be in place in real-world LGD data.
From Junk Bonds to Loans

• So far, mean simulated LGD is greater than 50%.
  – That comes from Altman's regression line, .5 + 2.3 DR.
  – Loans tend to have *lower* LGDs than junk bonds.

• So far, mean simulated cDR equals 3%.
  – Loans tend to have *lower* PDs than this.

• The next experiments assume:
  – PD = 1% (and PD = 5% for comparison)
  – LGD ≈ 10% (as well as greater than 50%)
Where the Function Outperforms

PD = 1%, High LGD

PD = 5%, High LGD
LGD Function as Null Hypothesis

- The moderate positive relation of the LGD function can serve as a null hypothesis.
  - The residual is \((LGD_t - \Phi [ \Phi^{-1}[DR_t] - \hat{k}] / DR_t)\).
  - Regress this on \(DR_t\) and a constant.
  - If the regression is significant, use it to refine the prediction.

- Result: This tends to degrade function predictions,
  - if as before the function is not too wrong and data is scarce.
  - Reason: The function is approximately right, so significant regressions tend to point the wrong way.
When the data generator has moderate slope (.5 < b < 2), this *degrades* the forecast even if there are 50 years of data.
A Next-to-Last Word

- The conclusions made in this paper depend on particular values of control variables.

- In applied statistical work, the good-practice standard comparison is a statistical hypothesis test.
  - These are performed in "Credit Loss and Systematic LGD Risk"
    - Ideally, risk managers would perform tests as described there.
    - Realistically, few will follow through the technical difficulties.

- Still, this paper makes a point:
  - Unless the relation between LGD and default is steeper than people think, the LGD function produces better results on average than ad hoc regression on a short data set.
Summary

• A function states LGD in terms of the default rate.

• This paper compares its predictions to linear regression.
  – cLGD is generated by a linear model: cLGD = a + b cDR.
  – Statistical regression estimates the parameters poorly:
    ▪ Portfolio DR is random around cDR.
    ▪ Portfolio LGD is random around cLGD.
    ▪ Most important, the data sample is short.
  – The function outperforms for a good range of parameter values.

• Supplementary regression does not improve the function
  – in some cases even when 50 years of data are available.

• Until improvements are found, the LGD function appears to be a better practical guide than ad-hoc regression.
Questions?