MOODY'S ANALYTICS

MODELING METHODOLOGY

Moody's CreditCycle
Reverse Stress Testing Capabilities

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Reverse Stress Testing Challenges

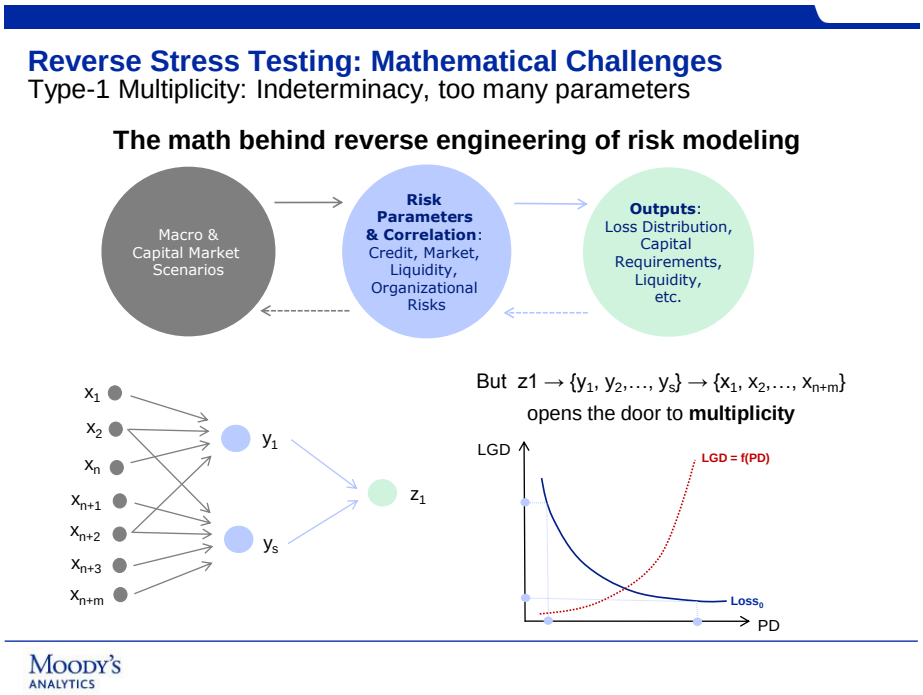
The Financial Services Authority defines reverse stress tests as "*tests that require a firm to assess scenarios and circumstances that would render its business model unviable, thereby identifying potential business vulnerabilities. Reverse stress-testing starts from an outcome of business failure and identifies circumstances where this might occur. This is different to general stress and scenario testing which tests for outcomes arising from changes in circumstances.*"

At its core, reverse stress testing proposes to "invert" the standard process, starting now from an "outcome" (business failure) with the aim of finding potential states-of-nature (scenarios) consistent with such an outcome. When implementing RST in a quantitative way, **multiplicity** emerges as a challenge. The same outcome (say, high expected losses or liquidity shortfall) could materialise under multiple combinations of risk factors (say, PDs, EADs and LGDs) and alternative macroeconomic scenarios.

We categorize the multiplicity into:

Type-1 multiplicity: Indeterminacy

In most RST frameworks we have a large number of risk and macroeconomic variables to match with a limited number of objectives/assumptions; i.e., more variables than equations. This indeterminacy issue can be resolved via a combination of (a) additional ad-hoc assumptions on some parameters (expert-judgment, market-wide assumptions, or values in line with regulatory guidelines), and/or (b) additional equations based on empirical findings, and/or (c) reducing the number of risk and macro factors that define a scenario while keeping its shape. The aim is to close the "degree of indeterminacy" to zero and end up with as many equations as unknowns.



Avenues to **"close" the degree of indeterminacy to zero** :

1) Additional equations.

2) Ad-hoc values for risk parameters: based on market information, expert judgment and/or regulatory guidelines.

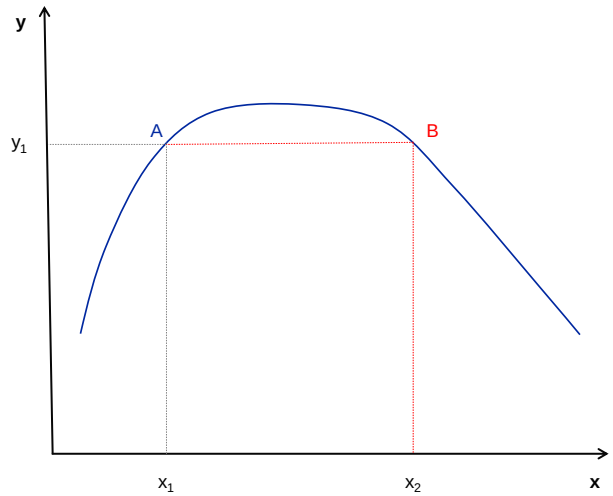
3) Lower the dimension of the scenario and risk parameter spaces: The use of **"factor analysis"**.

Type-2 multiplicity: Properties on an inverse mapping

Even after we have closed the gap between equations and unknowns, a key challenge emerges when trying to reverse engineer a process: The inverse of a function may not behave as a function. Consider a stressed value of the risk factors x_0 that is mapped to a vector of outcomes y_0, such that $\Phi(x_0, y_0) = 0$. Mapping y_0 back (i.e., reverse engineering the process) could give us a value of x that is different from x_0: There may exist another vector x_1 that is consistent with the same outcome, $\Phi(x_1, y_0) = 0$. Some key properties of the mapping Φ will help us ensure that (at least locally) one can "safely" invert the process.

Reverse Stress Testing: Mathematical Challenges

Type-2 Multiplicity: Potential multiplicity in a reverse mapping even if $N = M$



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Reverse Stress Testing in Practice

To deal with type-1 multiplicity we propose the use of **"factor analysis"**. To ensure that these factors can still have a meaningful "interpretation" we link them to well-known macroeconomic series. The takeaway of this exercise is that intuition can still be kept on the nature of the scenarios while the underlying dimension of the indeterminacy has been dramatically reduced. The modeller now has a better chance of matching the number of equations to the number of risk and macro parameters. To handle type-2 multiplicity we make use of linear algebra techniques to provide structure on the stress-testing process that will allow us to carry RST exercises for linear models (and any nonlinear monotonic transformations of them—such as logarithmic and logistic).

Step 1: Factor Analysis to reduce the dimension of the parameter and scenario spaces

Examples of factor analysis on key US macroeconomic series

Reverse Stress Testing: US factor analysis

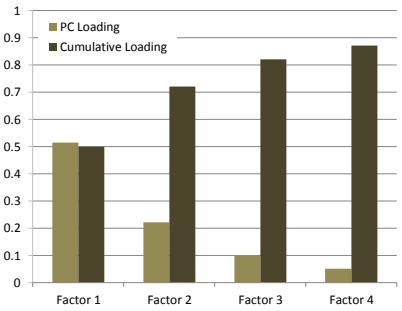
US factor analysis – 25 macroeconomic series

```
Factor analysis/correlation                    Number of obs    =     48
    Method: principal factors                  Retained factors =     18
    Rotation: (unrotated)                      Number of params =    297
```

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	12.61718	7.18730	0.5145	0.5145
Factor2	5.42988	2.99817	0.2214	0.7359
Factor3	2.43170	1.17792	0.0992	0.8350
Factor4	1.25379	0.39890	0.0511	0.8862
Factor5	0.85489	0.17976	0.0349	0.9210
Factor6	0.67512	0.18184	0.0275	0.9486
Factor7	0.49328	0.16899	0.0201	0.9687
Factor8	0.32429	0.12289	0.0132	0.9819
Factor9	0.20140	0.11619	0.0082	0.9901
Factor10	0.08521	0.01199	0.0035	0.9936
Factor11	0.07322	0.02267	0.0030	0.9966
Factor12	0.05055	0.01415	0.0021	0.9986
Factor13	0.03640	0.01441	0.0015	1.0001
Factor14	0.02198	0.01023	0.0009	1.0010
Factor15	0.01175	0.00626	0.0005	1.0015
Factor16	0.00550	0.00099	0.0002	1.0017
Factor17	0.00451	0.00398	0.0002	1.0019
Factor18	0.00053	0.00157	0.0000	1.0019
Factor19	-0.00104	0.00182	-0.0000	1.0019
Factor20	-0.00286	0.00089	-0.0001	1.0018
Factor21	-0.00375	0.00098	-0.0002	1.0016
Factor22	-0.00472	0.00228	-0.0002	1.0014
Factor23	-0.00700	0.00347	-0.0003	1.0011
Factor24	-0.01048	0.00672	-0.0004	1.0007
Factor25	-0.01720	.	-0.0007	1.0000

```
LR test: independent vs. saturated: chi2(300) = 2554.41 Prob>chi2 = 0.0000
```

US Factor Analysis – Top 4 Factors



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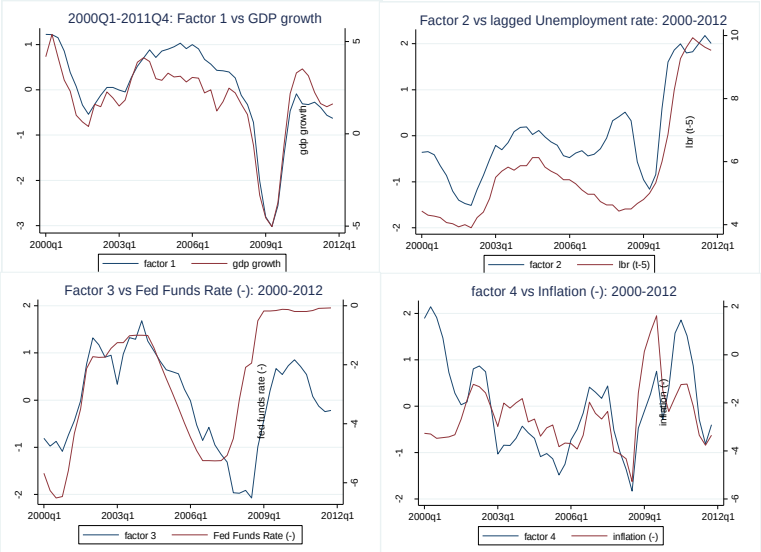
Reverse Stress Testing: US factor analysis



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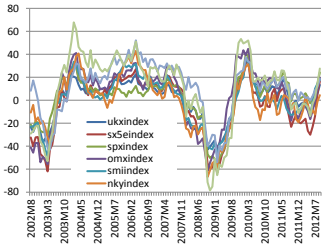
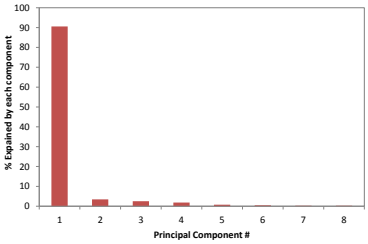
Examples of factor analysis on key market and credit risk parameters

Reverse Stress Testing: Correlation of Equity Indexes



Because of the high cross-correlation, we can reduce the modelling space from the 8 original variables to 1 (unobserved) principal component…

Variability Explained, %



One single principal component explains 91% of the original variability

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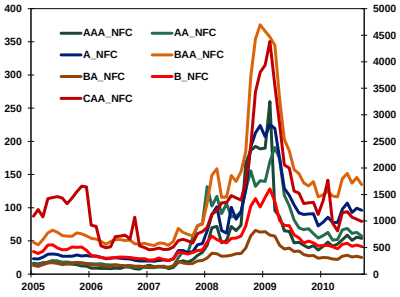
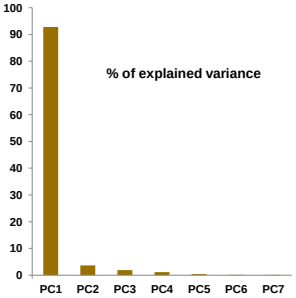
Reverse Stress Testing: CDS Spreads

Use **factor analysis** for forecasting **CDS spreads**:
Global Non-Financial Corporates for Moody's rating buckets (Aaa, Aa, A, Baa, Ba, B, Caa)





Source: Moody's Analytics, Capital Market Research Group

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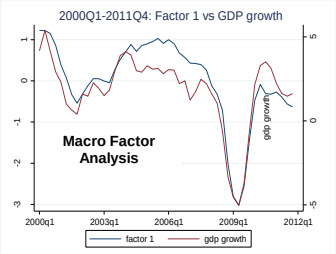
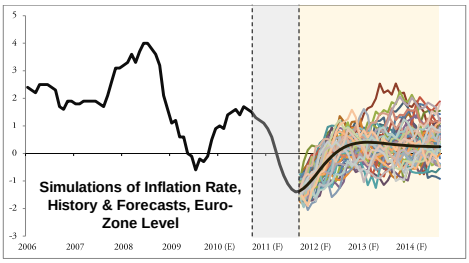
Step 2: From Factors to Scenarios

Mapping factor realizations or specific risk parameter levels to macro scenarios can be done through simulation, scenario-development, or sensitivity-analysis. All these techniques can be implemented in Moody's Credit Cycle and are described in the next section of the document.

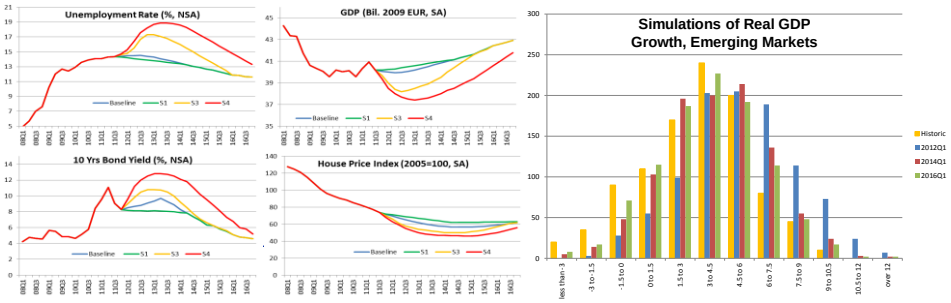
Reverse Stress Testing: Portfolio Simulations

Linking Factors' Realizations to Macro Scenarios





Standard Alternative Scenarios

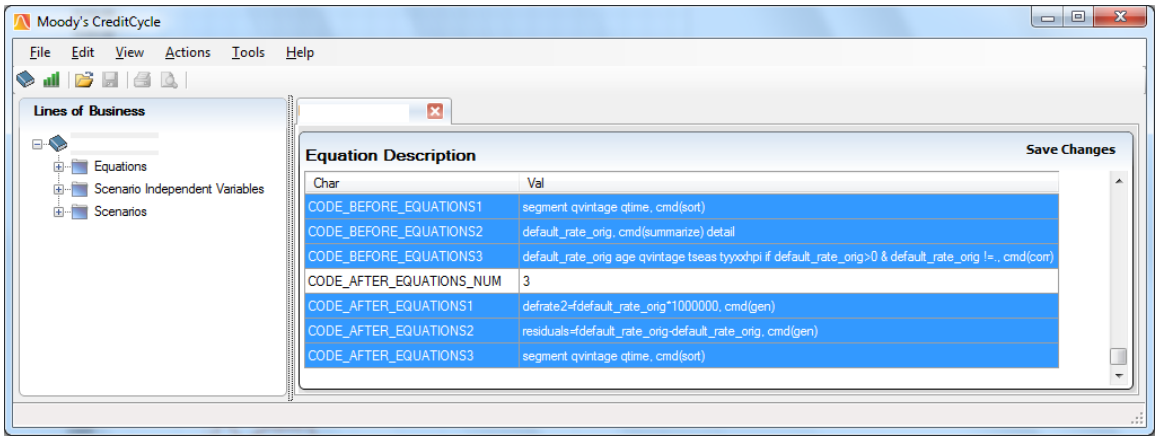


Reverse Stress Testing Techniques into Moody's CreditCycle

Moody's CreditCycle model characteristics table (modelling code) allows users to modify the following:

(1) The variables to model,
(2) Transformation of the variables to model,
(3) Variables to include in equations for modelling (right-hand side drivers),
(4) Estimation technique to apply to each model,
(5) Estimation technique options,
(6) Constraints to apply to model estimation,
(7) Cubic spline knots for modelling the age component,
(8) Length of validation window for out-of-sample testing,
(9) Open code functionality.

The open code functionalityof our platform –point (9) above- gives users the option to write a range of commands whilst running the model estimations. This functionality is located on the model characteristics table, where there are three lines for open code prior to model estimation and three lines post-model estimation.



The format of the code to type in the CODE_BEFORE_EQUATIONS and CODE_AFTER_EQUATIONS lines is as follows:

variables constraints , cmd(*command*)

where command is the specific command requested (from list below), the variables are any specified variables within the model (note the first one specified in regression commands is the independent variable) and constraints are any requested constraints for the command (i.e. if segment==100) – this can be left blank if no constraints required.

The commands available for populating the CODE_BEFORE_EQUATIONS and CODE_AFTER_EQUATIONS lines are:

- **gen** – generate a new variable (age24=1 if age==24, cmd(gen))
- **egen** – generate a variable from multiple variables (vt_var=group(qvintage qtime), cmd(egen))
- **collapse** – make a dataset of summary statistics (default_rate_orig , cmd(collapse))
- **replace** – replace contents of existing variable (y5=1 if stock_default !=. ,cmd(replace))
- **summarize** – provide summary statistics (default_rate_orig if segment==100 , cmd(summarize))
- **sort** – sort data by given variables (segment qvintage qtime , cmd(sort))
- **cap drop** – drop variables if exist (drop testvar , cmd(cap))
- **tsset** – declare data to be time series data (cross_section_seg_qv qtime, cmd(tsset))
- **pca** – principal component analysis (default_rate_orig age if segment==100, cmd(pca))
- **corr** – correlation analysis (default_rate_orig age if segment==100, cmd(corr))
- **tab** – tabulate data (age if segment==100 , cmd(tab))
- **var** - vector autoregressive models (default_rate_orig if segment==100 & qvintage==150,cmd(var))
- Regression techniques: **regress, ivregress, qreg, logit, probit, var, stepwise, nestreg.**
- **predict** – obtain predictions after estimation (reg_values, cmd(predict))
- **estat** – provides post-estimation statistics (vce , cmd(estat))

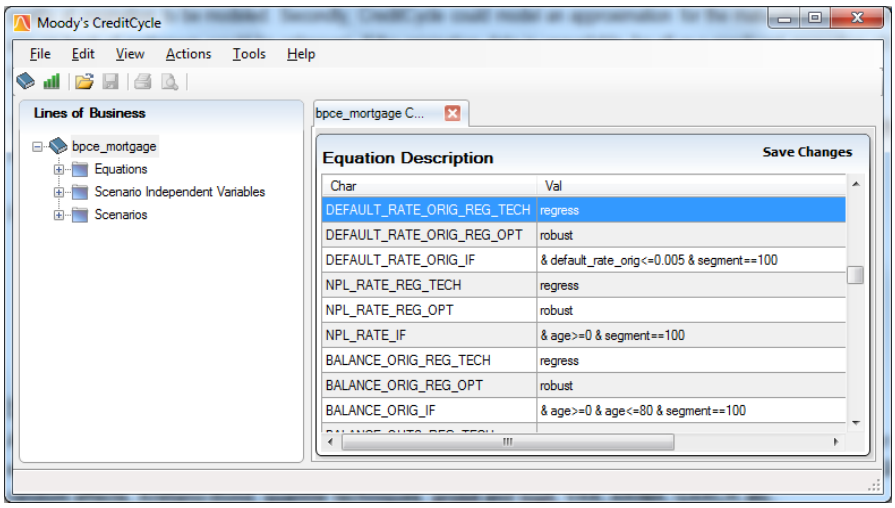
Step 1: Factor Analysis

The **pca** command helps the modeller achieve the reduction in the scenario and parameter spaces. This tool will help the modellers overcome the type-1 multiplicity challenge in reverse stress testing.

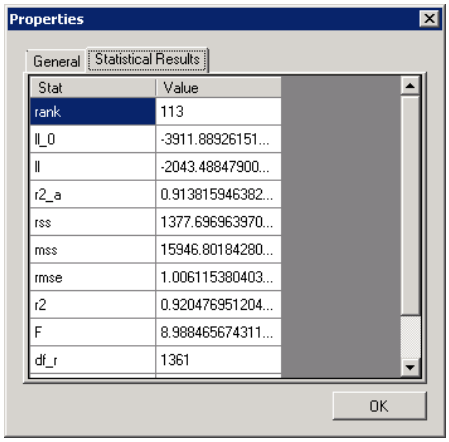
Step 2: From factors to scenarios: Model development/estimations, simulations and scenario analysis

The user can create models **to link the factors to risk parameters** in order to fulfill the RST goals.

After estimating a model in Moodys' CreditCycle, the analyst can also run MonteCarlo or bootstrapping simulations. This can be done for time-series models (e.g., after the **var** command) or for other standard regression techniques (**regress, ivregress, qreg, logit, probit, etc.**). The model characteristics table in CreditCycle allows users to select the model estimation technique (see below example). All fields ending in REG_TECH relate to the regression technique applied to that specific equation.
Regression techniques available within CreditCycle include standard estimation techniques, discrete choice models, time series models and panel data techniques.



The table below shows the model statistics associated with the linear regression performed. These statistics can be viewed with all regression techniques.



Similarly model statistics and coefficients can be viewed on the Last Run file that logs model estimations.

Source	SS	df	MS
Model	15973.963	126	126.777484
Residual	1350.5358	1349	1.00113848
Total	17324.4988	1475	11.7454229

```
Number of obs =    1476
F(126,  1349) =  126.63
Prob > F      =  0.0000
R-squared     =  0.9220
Adj R-squared =  0.9148
Root MSE      =  1.0006
```

Examples of Estimation Techniques:

Quantile Regression

qreg - The **qreg** command fits quantile (including median) regression models, also known as least-absolute-value models (LAV or MAD) and minimum L1-norm models. There is an option associated with quantile regression for modifying the asymmetry parameter. This is a parameter in quantile regression that weights the possibly distinct costs of under-prediction and over-prediction. With this parameter set to 0.5, positive and negative errors are weighted equally. When the asymmetry parameter is 0.5 our best predictor is the median; it does not give as much weight to outliers. When the asymmetry parameter is 0.7, the loss is asymmetric; large positive errors are more heavily penalized then negative errors.

XTREG Regression (FE, RE & MLE Option)

The xtreg command is a regression technique that allows FE, RE & mle options. The mle option maximises the log-likelihood function. The FE option allows the regression to use a fixed effects estimator, whilst the RE option allows the regression to use a random effects estimator.

Discrete Choice Models (PROBIT & LOGIT Commands)

Discrete choice models are appropriate where there is dichotomy in the required dependent variable. These models can be utilised if loan-specific data is provided (i.e. modelling a default indicator) or alternatively a vintage level indicator (i.e. default rate > 3%).
probit - The **probit** command fits a maximum-likelihood probit model (binary regression).
logit - The **logit** command fits a maximum-likelihood logit model (logistic regression).

Nested Regression (The NESTREG Command)

NESTREG – fits nested models by sequentially adding blocks of variables and reports comparison tests between the nested models.

Vector Autoregression (The VAR Command)

The VAR command fits a multivariate time-series regression of each dependent variable on lags of itself and on lags of all other dependent variables. VAR also fits a variant of vector autoregressive (VAR) models known as the VARX model, which also includes exogenous variables.

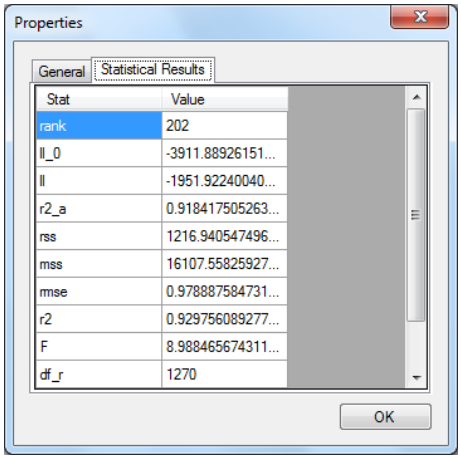
ARIMA Modelling (The ARIMA Command)

The ARIMA command fits univariate models with time-dependent disturbances. ARIMA fits a model of dependent variable on independent variables where the disturbances are allowed to follow a linear autoregressive moving-average (ARMA) specification. The dependent and independent variables may be differenced or seasonally differenced to any degree. When independent variables are included in the specification, such models are often called ARMAX models; and when independent variables are not specified, they reduce to Box-Jenkins autoregressive integrated moving-average (ARIMA) models in the dependent variable. Multiplicative seasonal ARIMA and ARMAX models can also be fit.

Arellano-Bond Estimation (The XTABOND Command)

Linear dynamic panel-data models include p lags of the dependent variable as covariates and contain unobserved panel-level effects, fixed or random. By construction, the unobserved panel-level effects are correlated with the lagged dependent variables, making standard estimators inconsistent. Arellano and Bond (1991) derived a consistent generalized method of moments (GMM) estimator for the parameters of this model; XTABOND implements this estimator. This estimator is designed for datasets with many panels and few periods, and it requires that there be no autocorrelation in the idiosyncratic errors. For a related estimator that uses additional moment conditions, but still requires no autocorrelation in the idiosyncratic errors, use XTDPDSYS. For estimators that allow for some autocorrelation in the idiosyncratic errors, at the cost of a more complicated syntax, use XTDPD.

The properties window (below) for each modeled equation in CreditCycle shows a summary of statistics regarding model performance. Alternatively for checking the in-sample fit, the residuals can be plotted to ensure there is random performance across time / age / vintage. A series for the residuals can be coded by writing **residuals=fname-name, cmd(gen)** using the open code functionality available on the model characteristics table (where name is the name of the field being modeled).



Stat	Value
rank	202
ll_0	-3911.88926151...
ll	-1951.92240040...
r2_a	0.918417505263...
rss	1216.940547496...
mss	16107.55825927...
rmse	0.978887584731...
r2	0.929756089277...
F	8.988465674311...
df_r	1270

The in-sample fit can also be observed by graphing the spliced and modeled series together. The below example shows the actuals (spliced series) against model (fitted series) for three vintages across age.

Figure 1: Fitted values (solid) vs. actual data (dotted)

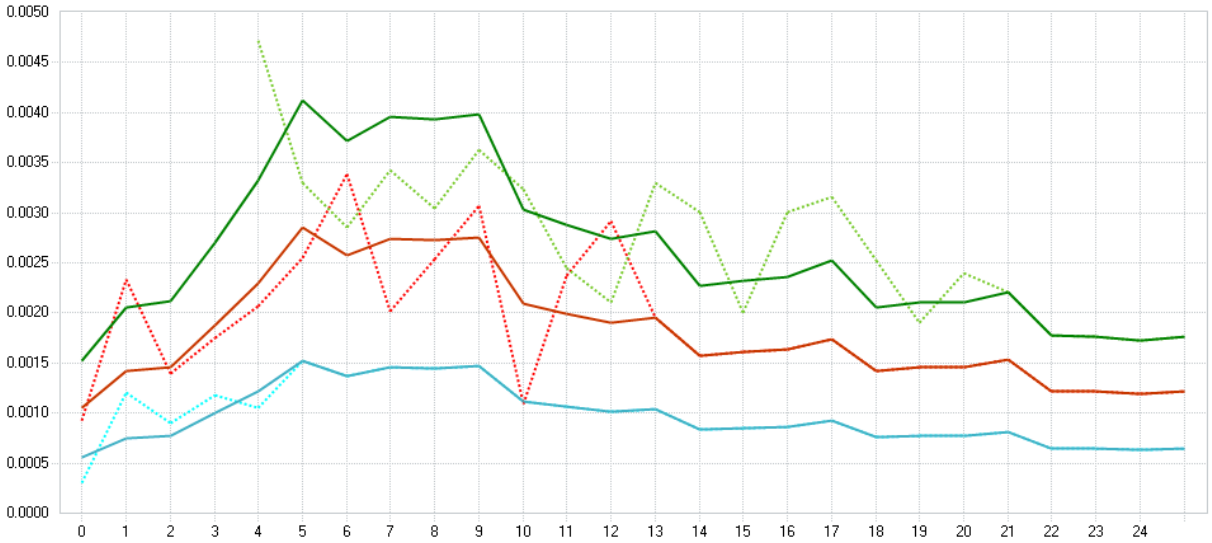


Figure 2: Example of in-sample and residual analysis
UK commercial real estate returns, model fit (left) and residual distribution (right)

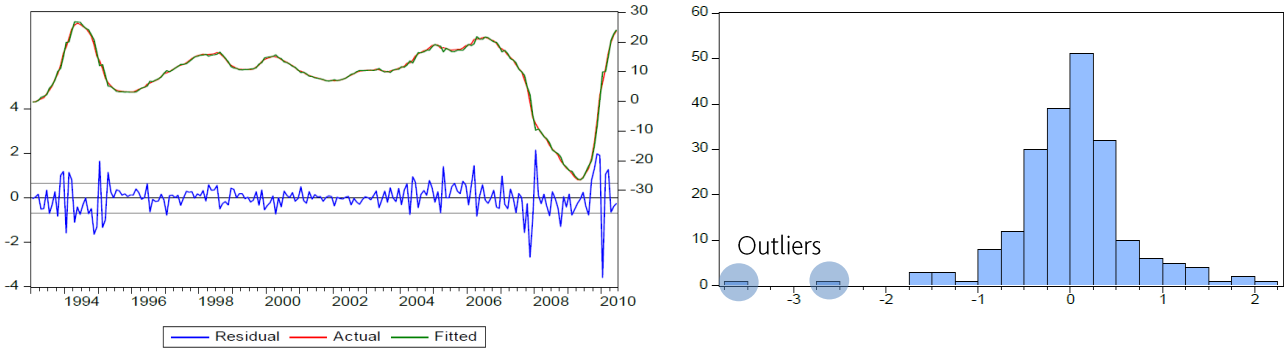
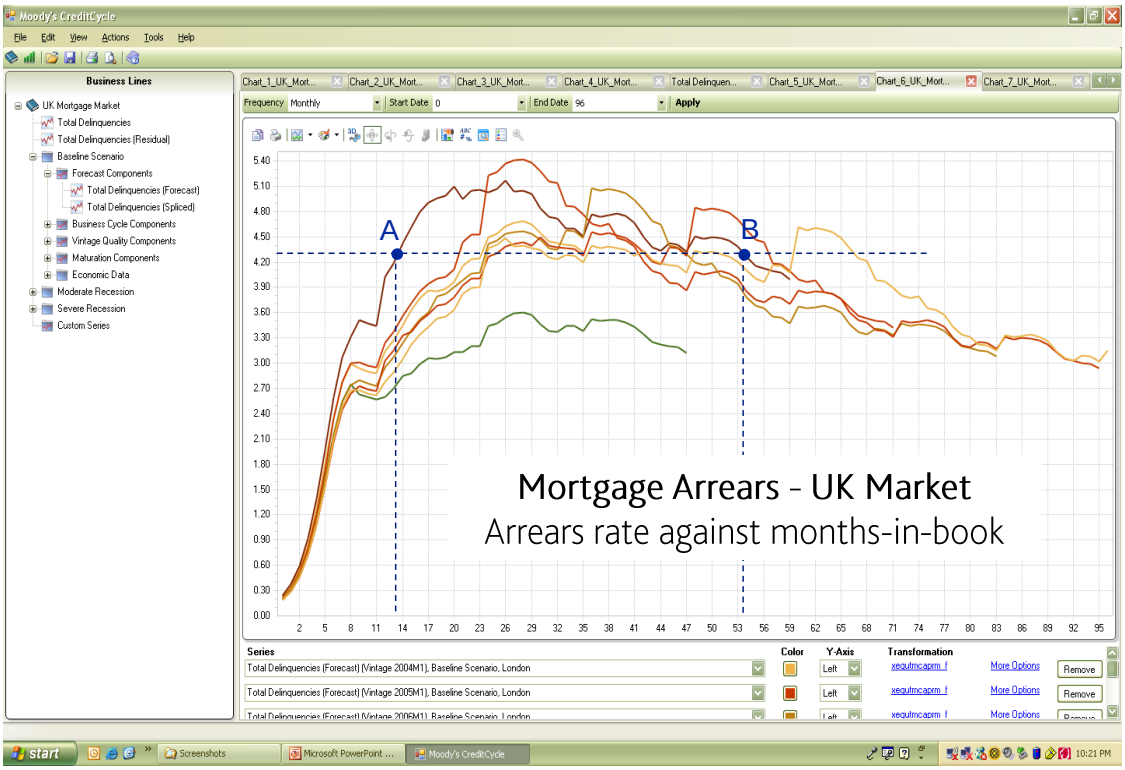


Figure 3: Identifying severity of risk parameters
UK Mortgage Market – Arrears Prediction Model in Moody's CreditCycle



Having identified the severity of the shocks through the risk models (credit, market or liquidity) the final step consists of linking those vectors to macro series.

Macroeconomic Data and Scenarios

Macroeconomic data is embedded into the system and can be automatically updated from the platform, linked to Moody's Analytics baseline and alternative scenario databases. We cover all major countries with national and sub-national level data. All scenarios are updated on a monthly basis, allowing the modellers to have access to the most current economic and credit trends available. Moreover, the modellers can bring their own economic series and assumptions and run them through the models. Moody's CreditCycle can forecast the models under multiple economic scenarios simultaneously and scenarios can be duplicated. The platform can host several countries into a single work-file, allowing the user to compare portfolios across geographic regions.

Figure 4 – Economic scenario chart examples, real wage growth, France, history + alternative scenarios

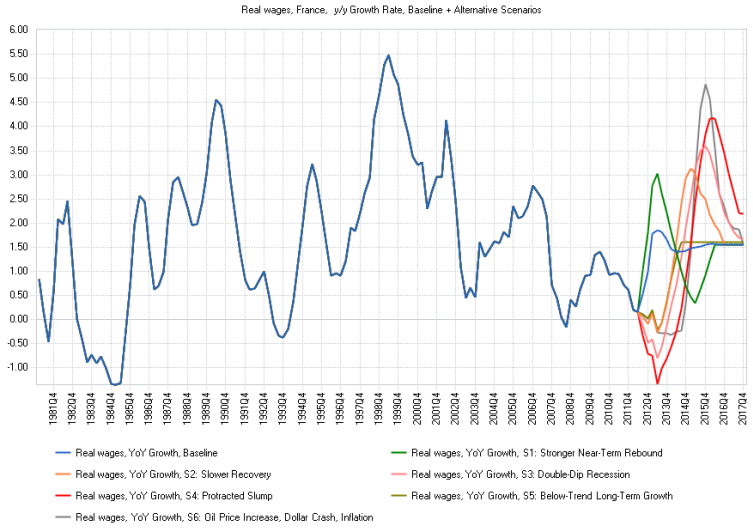
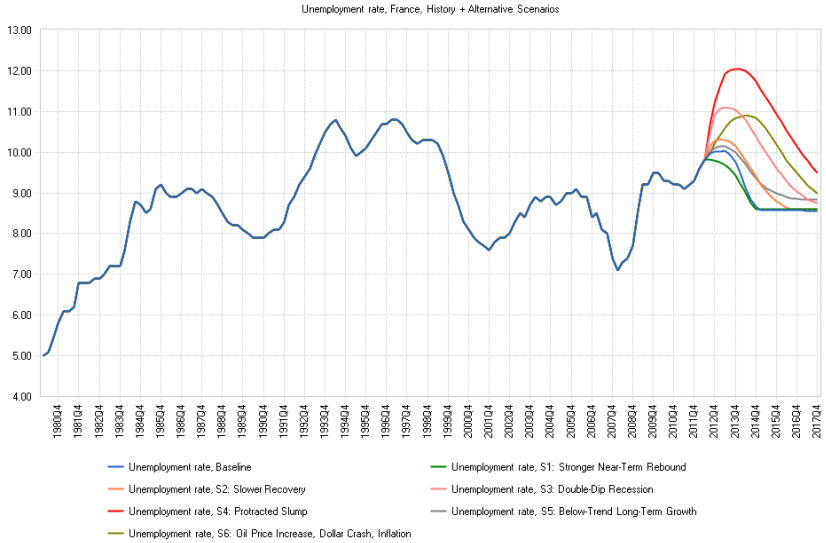
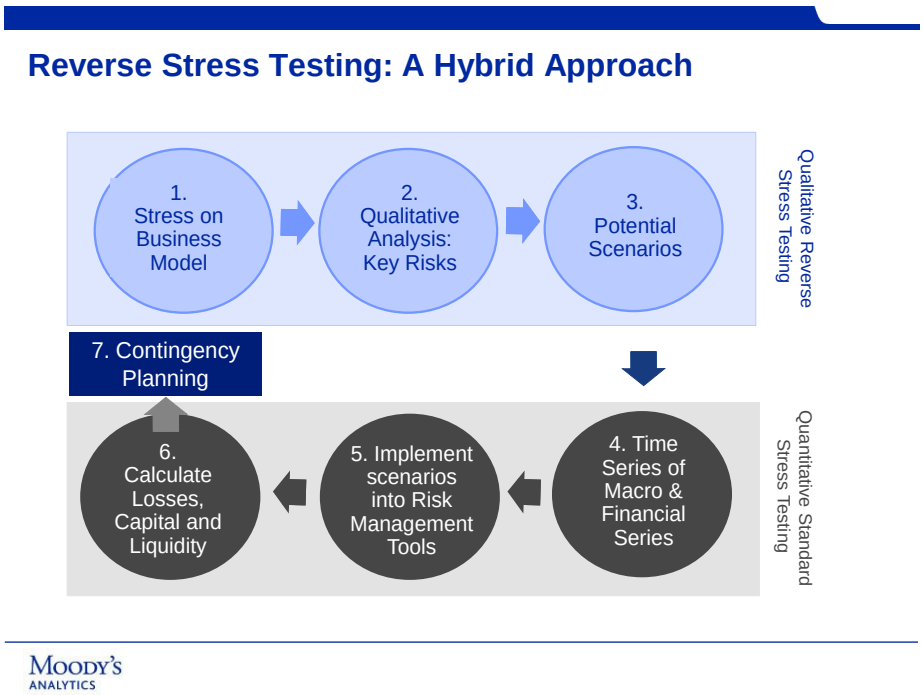


Figure 5 – Economic scenario chart examples, unemployment rate, France, history + alternative scenarios



After the identification of the multiple scenarios that could fit the RST starting "outcome", the final stage should consist of a serious analysis regarding the potential "actions" to be taken to prepare for such an event. The diagram below illustrates the whole RST process, concluding on stage 7, "**Contingency Plannning**".



Appendix 1. Multiplicity from a Mathematical Viewpoint

Mathematical Setup

Consider the standard stress-testing process (ST) as a "mapping" from scenarios and risk parameters: $x \in R^N$, into outcomes: $y \in R^M$, such that a set of conditions is satisfied: $\Phi(x, y) = 0$. The mapping $\Phi : X \times Y \to R^M$ represents the system of equations that links scenarios, risk parameters and outcomes. The stress-testing task can be described as follows: For a given stressed scenario and its associated risk parameters, $\tilde{x} \in R^N$, find the corresponding outcomes, $\tilde{y} \in R^M$, such that $\Phi(\tilde{x}, \tilde{y}) = 0$. This standard process is built so that there are as many equations as unknowns: The dimension of y coincides with the dimension of the range of the Φ system. But in practice, the number of scenarios and risk parameters tends to be greater than the number of equations: $N \geq M$. Using the previously defined notation, the reverse stress-testing (RST) process will consist of: (i) starting from the extreme output/outcome, $\hat{y} \in R^M$, (ii) find those potential scenarios and risk parameters, $\hat{x} \in R^N$, (iii) such that $\Phi(\hat{x}, \hat{y}) = 0$.

A useful definition is that of a "projection function": $\pi_x : X \times Y \to X$ with $\pi_x(x, y) = x; \forall (x, y)$ such that $\Phi(x, y) = 0$. Similarly, $\pi_y(x, y) = y$. Using this concept, the ST process is defined as: (a) given $\tilde{x} \in R^N$, (b) find $\tilde{y} \in R^M$ such that (c) $\tilde{y} \in \pi_y(\pi_x^{-1}(\tilde{x}))$. The RST process is therefore: (i) for an outcome $\hat{y} \in R^M$, (ii) find $\hat{x} \in R^N$ such that (iii) $\hat{x} \in \pi_x(\pi_y^{-1}(\hat{y}))$. As a mapping/correspondence, RST consists of the following composite formula: $\pi_x \circ \pi_y^{-1}$.

In practice, the π_x^{-1} relationship is typically well-behaved: Every value of x is associated with a single outcome y. But π_y^{-1} can consist of multiple values of x. Studying the properties of π_y^{-1} is at the core of the understanding of the RST process.

1.1: Type-1 Multiplicity: Indeterminacy

In most RST frameworks, we are faced with the task of matching a large number of risk and macroeconomic variables with a limited set of assumptions: # variables > # equations, or $N > M$. Indeterminacy takes the form of a **continuum** of solutions (scenarios and risk parameters) whose mathematical properties will help the modeller identify avenues to close the extra degrees of freedom.

In formal terms: if $N > M$, then shape of the solution set $\hat{X} = \{x \in X; x \in \pi_x(\pi_y^{-1}(\hat{y}))\}$ will depend on the mathematical properties of the Φ system (and therefore the π_y^{-1} mapping). Under some "regularity" conditions on Φ (e.g.; sufficient conditions to satisfy the assumptions for the implicit function theorem) the solution set $\hat{X}$ is a smooth-manifold of dimension $N - M$. In other words, there exists a continuum of scenarios and risk parameters (set $\hat{X}$) that are consistent with the original outcome $\hat{y} \in R^M$.

This indeterminacy needs to be dealt with before any further attempt to successfully reverse engineer the process. Solutions will require (a) additional ad hoc assumptions on some parameters (expert judgment, market-wide assumptions or values in line with regulatory guidelines), and/or (b) additional equations based on empirical findings; e.g., $LGD = f(PD)$, and/or (c) reducing the number of risk and macro factors that define a scenario while keeping its shape. The end goal is to close the "degree of indeterminacy" to zero and end up with as many equations as unknowns.

1.2: Type-2 Multiplicity: Inverse Mapping

Even after closing the gap between equations and unknowns ($N - M = 0$), a new challenge emerges when trying to reverse engineer the ST process: The inverse of a function may not behave as a function. Consider a stressed value of the risk factors, x_0, that is mapped to a vector of outcomes, y_0, such that $\Phi(x_0, y_0) = 0$. Mapping y_0 back (i.e., reverse engineering the process) could give us a value of x that is different from x_0: There may exist another vector x_1 that is consistent with the same outcome $\Phi(x_1, y_0) = 0$. Specific characteristics of the ST system Φ will help us ensure that (at least locally) one can "invert" the process and obtain a reliable RST mapping $\pi_x \circ \pi_y^{-1}$. Applications of results such as "Inverse Function" and "Implicit Function" theorems will help us understand the shape of the solution-set $\hat{X} = \{x \in X; x \in \pi_x(\pi_y^{-1}(\hat{y}))\}$. Under some "regularity" conditions on Φ the set $\hat{X}$ is a zero-dimensional smooth-manifold (set of isolated points). Moreover, each point in $\hat{X}$ **locally** maps (a) $X \to Y$ through $\pi_y \circ \pi_x^{-1}$ (stress testing) and (b) $Y \to X$ through $\pi_x \circ \pi_y^{-1}$ (reverse stress testing) in a one-to-one (and unique) smooth fashion (overcoming type-2 multiplicity).

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