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MODELING CORRELATION OF STRUCTURED INSTRUMENTS IN A PORTFOLIO SETTING*

MODELINGMETHODOLOGY

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ABSTRACT

Traditional approaches to modeling economic capital, credit-VaR, or structured instruments whose underlying collateral is comprised of structured instruments treat structured instruments as a single-name credit instrument (i.e., a loan-equivalent). While tractable, the loan-equivalent approach requires appropriate parameterization to achieve a reasonable description of the cross correlation between the structured instrument and the rest of the portfolio. This article provides an overview of how one can calibrate loan-equivalent correlation parameters. Results from taking the approach to the data suggest that structured instruments have far higher correlation parameters than single-name instruments.

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TABLE OF CONTENTS

1 INTRODUCTION

Credit events are correlated. Portfolios of corporate or retail loans can exhibit wide swings in loss as economic factors common to the underlying entities drive defaults or deterioration in credit quality. While challenging, the modeling and data issues related to single-name credit instruments are well understood and documented (see [10] and [11]). However, the correlation of structured instruments is not as well understood. The complex payoff structure, along with the correlation within the collateral pool makes it difficult to describe the correlation between a structured instrument and other instruments. In fact, traditional approaches to modeling economic capital, credit-VaR, or structured instruments whose underlying collateral is comprised of structured instruments treat structured instruments as a single-name credit instrument (i.e., a loan-equivalent).[1] While tractable, the loan-equivalent approach requires appropriate parameterization to achieve a reasonable description of the cross correlation between the structured instrument and the rest of the portfolio. This article addresses this challenge by calibrating the loan-equivalent correlation parameters to dynamics observed in a granular model of the structured instrument. The granular model accounts for the terms of the structured instrument and the characteristics of the underlying collateral pool. The loan-equivalent correlation parameters are calibrated to correlations of the structured instrument with other structured instruments, as well as with single-name credit instruments. When using the Basel II recommendations in [2], along with Equifax and Moody's Economy.com data to parameterize the granular model, this study presents striking results suggesting a far higher degree of correlation across structures than observed for the underlying collateral pool, or for other classes of single-name instruments such as corporate exposures.

As indicated above, the analysis begins with a granular model of the structured instrument. The underlying reference entities associated with the collateral pool are used to simulate collateral losses, which are translated to structured instrument loss using its subordination level. For ease of exposition we assume pass-through waterfalls throughout the article, so that the waterfall structure and subordination level is completely determined by the attachment and detachment points. The structured instrument is said to be in distress if it incurred a loss. The simulated structure instruments losses are used to calculate a probability of distress and loss given distress for each of the structured instruments, and a joint probability of distress for a structured instrument and other instruments in the portfolio. The joint probability of distress and the individual probabilities of distress are then used to back out an implied 'asset return' correlation associated with the structures' loan equivalent reference entities (henceforth, *the loan-equivalent correlation*). Now, taking the probability of distress as the loan-equivalent probability of default, the loss given distress as the loan-equivalent loss given default, and the deal maturity as the loan-equivalent maturity, completes the parameterization of the loan-equivalent.

In addition to the benefits associated with using loan-equivalents in a portfolio setting, the simplicity associated with loan-equivalent correlations serves as a useful summary statistic for understanding portfolio-referent risk characteristics. For example, this study finds that the correlation between two structured instruments is much higher than the average correlation between instruments in their collateral pools. This is because the idiosyncratic shocks in a collateral pool offset each other and the systematic portion is left to "run the show." Moreover, loan-equivalent correlations between two subordinated tranches can be substantially lower than their senior counterparts. The difference is driven by the higher correlation exhibited between two credit instruments in the tail region of the loss distribution (the loss region of the collateral pool where a senior tranche enters distress). This is because a senior tranche distress is most likely driven by systematic shock, which makes it more likely that it will be accompanied by a distress in the other senior tranche.

The article also studies the impact of overlap in reference entities associated with different collateral pools on the correlation of Collateralized Debt Obligations (i.e., CDOs). Although it is intuitive that the loan-equivalent correlations increase with the degree of overlap, the effect is found to be stronger for more junior tranches. This follows from the junior tranches being more susceptible to idiosyncratic noise. With higher degree of overlap, the tranches share more of the idiosyncratic shocks.

Recent literature on structured instruments correlation primarily addresses correlations between the reference entities in the collateral pool and their effect on tranche pricing. For example, [1] examines the impact of an increase in correlation among underlying assets on the value of a CDO equity tranche, and shows that CDO equity can be short on correlation.

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[1] Exceptions include Moody's KMV RiskFrontier® which models the terms of the subordinated note along with the correlation structure of the underlying collateral pool as it relates to the other instruments in the portfolio. Please see [6] for additional details.

The Normal copula model, studied in detail in [5], has become the industry standard for pricing structured instruments. It often results in a market price-implied correlation smile. Several papers address this phenomenon, for example [7] explains it using Variance gamma distributions. This article, however, is focused on the correlation structure between a structured instrument and other instruments in the portfolio. This structured instrument correlation is a function of the correlation of the underlying collateral instruments among themselves and with other instruments in the portfolio, as well as of the deal's waterfall structure and the instrument's subordination level. In [3], Coval, Jurek, and Stafford show that similar to economic catastrophe bonds, structure instruments are highly correlated with the market, but offer far less compensation than economic catastrophe bonds. In [4] they suggest that this lower compensation is due to the fact that rating agencies base their ratings for structured instruments on the distress probability and expected loss, ignoring the high correlation with the market.

The remainder of the article is organized in the following way:

- Section 1 analyzes the correlation of two Asset Backed Securities (i.e., ABSs) with varying characteristics.
- Section 2 analyzes the correlation between Collateralized Debt Obligations and other credit instruments. The section considers the impact of overlapping names in two CDO collateral pools, as well as the correlation between a CDO and a loan to a reference entity in the collateral pool. This section also analyzes the correlation between tranches of the same deal.

2 MODELING THE CORRELATION OF TWO ASSET-BACKED SECURITIES

This section analyzes the correlation structure between two ABS subordinated notes. As discussed in the introduction, parameters of the loan-equivalent ABS are calibrated and the correlation properties of the loan-equivalent reference entity's asset return are analyzed. The section starts with analysis of the ABS using a structural approach; the underlying reference entities associated with the collateral pool are modeled and related to losses on different subordinated classes. Also within the structural framework, the correlations between different ABSs are used to calibrate the loan-equivalent reference entity correlations. The section then analyzes the correlation structure of the loan-equivalent reference entity.

Consider an ABS with a homogeneous collateral pool (for example, California auto-loans with similar credit risk criteria). Because the collateral pool is homogeneous, the systematic portion of reference entity *i*'s asset return process (r_i) can be modeled using a single-factor (Z) with the same factor loading (R) as follows:

$$r_i = R \cdot Z + \sqrt{1-R^2} \cdot \varepsilon_i .$$

Henceforth, we use the square form of the factor loading, R^2, because of its interpretation as the proportion of asset return variation explained by the common factor Z. The single factor Z and the idiosyncratic portion ε_i are standard Brownian motion processes, and therefore so is the asset return.

A default occurs when the asset return process drops below a default threshold DT:

$PD_i = P\{\text{reference entity } i \text{ defaults}\} = P(r_i < DT),$ where $DT = N^{-1}\left(PD_i\right)$ is the default threshold and $N^1(.)$ denotes the inverse cumulative function for a standard Normal distribution. Values for parameterization of the instruments in the collateral pool are publicly available.[2]

In analyzing the characteristics of ABSs, simplifying assumptions were made. In particular, a single period is assumed, and the analysis abstracted from subtleties such as reinvestment, collateralization or wrapping which insures the senior notes. The collateral pools were composed of instruments with LGD=70%, and reference entities with *R2*=10%. Each deal consists of 7 ABSs at different subordination levels. The maturity of each deal was set at 1 year. In order to obtain the ABSs probabilities of distress and losses given distress, as well as to account for collateral pool correlations properly, all of the ABS deals were simulated 100,000 times in Moody's KMV's RiskFrontier. See [6] for an overview of the methodology used in RiskFrontier®.

[2] For example, *PD* and *LGD* estimates are available through Moody's Economy.com, and *R2* estimates are available through Moody's KMV.

Figure 1 demonstrates how the ABS one-year distress probability changes with the subordination level for collateral pools differing in their PD values (ranging from as low as 10 basis points to as high as 22 percent). This PD range covers a variety of instruments – from prime loans to sub-prime mortgages that have reference entities with annual default probabilities in the double digit range. As expected, the ABS probability of distress is decreasing in the subordination level. Moreover, it appears the distress probability curve starts concave before it switches and becomes convex. This convexity switch occurs at the maximum point of the bell-shaped collateral loss distribution function. This maximum point is at a higher subordination level, the higher the collateral PD.

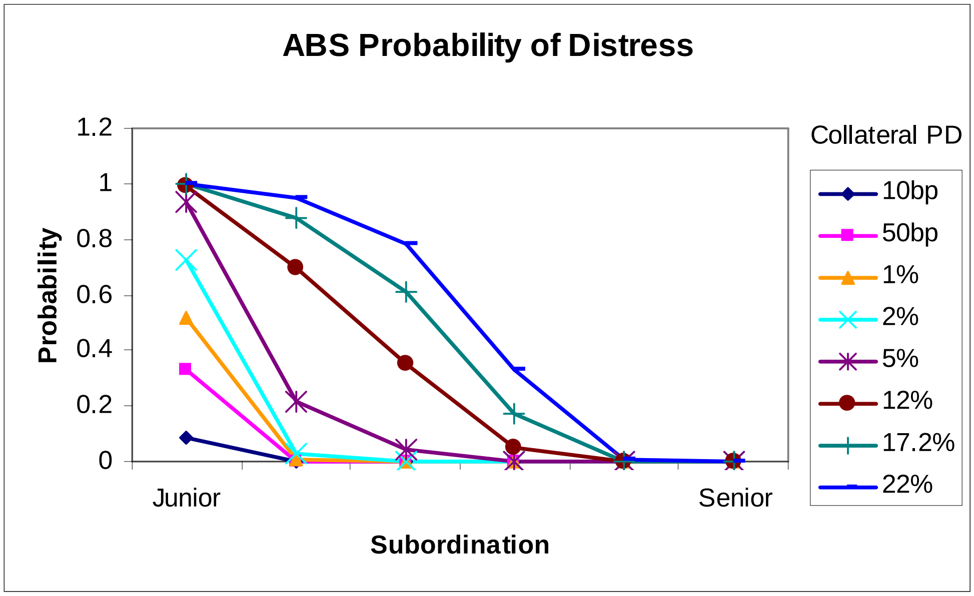

FIGURE 1 ABS probability of distress changing with subordination for collateral parameters: R^2=10%, LGD=70%, and collateral PD ranging from 10 basis points to 22 percent

To gain a better understanding of the correlation structure of these ABSs, the loan-equivalent reference entity's R^2 is computed for each deal. To this end, the simulation holds two similar copies of each ABS. The collateral pools for the two copies have statistically similar instruments (but not the same instruments, i.e., different idiosyncratic shocks). The empirical Joint Probability of Distress (JPD_{sim}) of the similar ABS pairs is calculated from the simulation results. Using a Normal Copula we choose the loan-equivalent reference entity's 'asset return' correlation ρ_{12}^{A} that results in the empirical joint probability of distress. That is,

$$JPD(PD_1, PD_2, \rho_{12}^{A}) = JPD_{sim},$$

where $JPD(PD_1, PD_2, \rho_{12}^{A})$ is the joint one-year probability of default of the two ABSs in a Normal Copula with correlation ρ_{12}^{A} and one-year default probabilities PD_1, PD_2. To be clear, the 'asset return' correlation ρ_{12}^{A} is different from the ABSs' distress correlation ρ_{12}^{D}. The two are, of course, related as exhibited in the following equation:

$$\rho_{12}^{D} = \frac{JPD(PD_1, PD_2, \rho_{12}^{A}) - PD_1 \cdot PD_2}{\sqrt{PD_1 \cdot (1-PD_1) \cdot PD_2 \cdot (1-PD_2)}}.$$

The ABS 'asset return' correlations are generally much higher than the underlying collateral. In our example, the asset return correlation for any pair of auto-loans, one from each ABS, is the product of their factor loadings (or the R^2), $\sqrt{10\%} \cdot \sqrt{10\%} = 10\%$. However, the 'asset return' correlation for the ABS loan-equivalents themselves starts with 29% for the junior note and low collateral PD, and rapidly increases with subordination (Figure 2). Therefore, the average collateral correlation is a highly biased estimate for the loan-equivalent correlation. This disparity in ABS correlations and their collateral correlations can be attributed to the fact that most of the idiosyncratic noise in the collateral pools washes out for large pools due to the law of large numbers. The idiosyncratic shocks in the collateral pool offset each other, resulting in a huge increase to the systematic factor's explaining power.

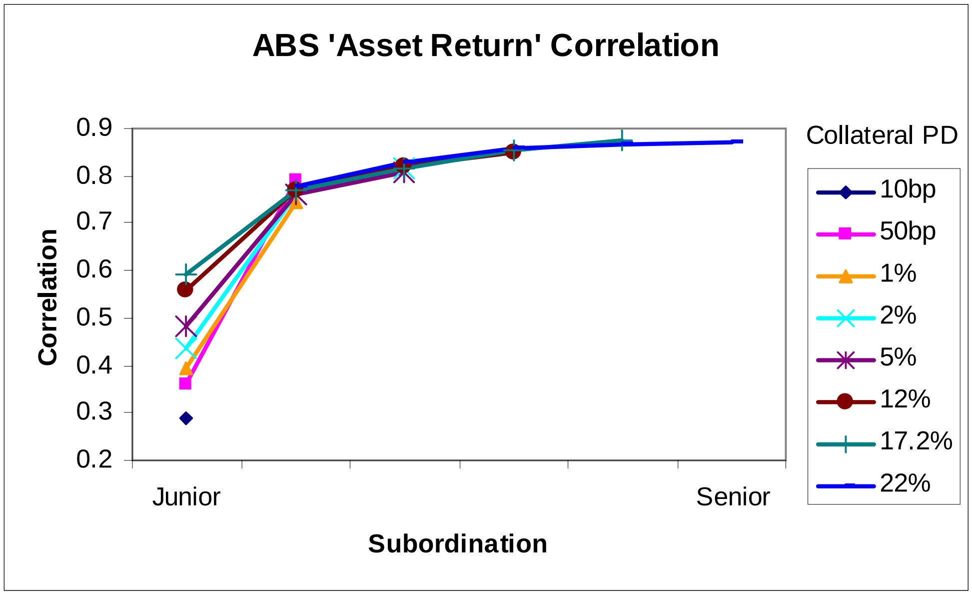

FIGURE 2 ABS loan-equivalent reference entity's 'asset return' correlation for collateral parameters: R2=10%, LGD=70%, and collateral PD ranging from 10 basis points to 22 percent[3]

The systematic shocks dominance effect referenced in the previous paragraph is emphasized further by the homogeneity of the collateral pools. The results in Figure 1 and Figure 2 are based on collateral pools with, for example, homogeneous auto-loans (all in the same state - California). Therefore, the initial correlation between pairs of auto-loans was high to begin with. To examine the significance of pool diversification, consider an example with 50 state factors (*Z1, …, Z50*), where the auto-loans are equally distributed between the states. Each auto-loan *i* has an associated state factor *k(i),* such that $r_i = R \cdot Z_{k(i)} + \sqrt{1-R^2} \cdot \varepsilon_i$. Based on quarterly delinquency rates data from Equifax and Moody's Economy.com, we set the correlation between any two state factors to be 65% ($\rho(Z_k, Z_l) = 0.65$ for any $1 \le k < l \le 50$). Figure 3 compares the ABS 'asset return' correlations between the diversified pool (50 factors) and the homogeneous pool (common factor) for pools with collateral PD=2% and collateral R^2=10% (corresponding with the Basel II parameter recommendations in [2])

[3] Some points in Figure 2 for senior notes with low collateral PD are missing because these ABSs exhibited too few distresses in the 100,000 simulation runs to calculate a meaningful correlation.

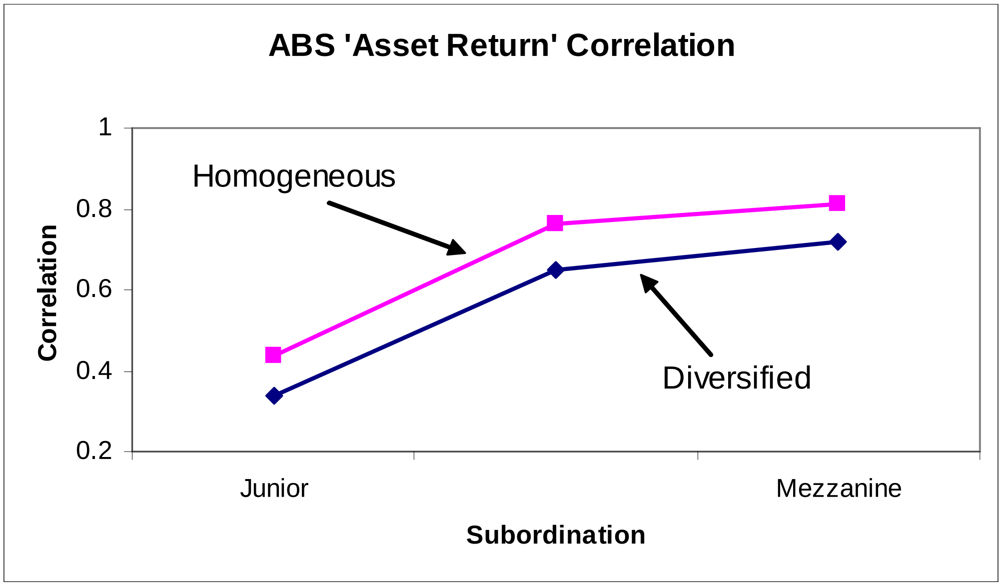

FIGURE 3 ABS loan-equivalent reference entity's 'asset return' correlation – diversified pool (50 factors) vs. homogeneous pool (common factor) with collateral pool parameters: PD=2%, R^2=10%, LGD=70%

Figure 3 demonstrates that the diversified pool is associated with a loan-equivalent reference entity that is less correlated than that implied by the homogeneous pool. This is not surprising given that state factors are not perfectly correlated, allowing for diversification. This exercise is particularly interesting if one is interested in modeling a CDO of ABSs. The exercise provides a sense for the benefits of each ABS in the CDO collateral pool having a different targeted geographic focus, versus each ABS having a relatively similar geographic focus.

3 CDO CORRELATIONS

This section studies Collateral Debt Obligation (i.e., CDO) correlations. Even though CDO deals are similar in spirit to ABS deals, there are several dynamics that impact their correlation structure that are worth exploring in greater detail. First, corporate entities have lower default probabilities and higher R^2 than typical reference entities associated with ABS collateral instruments. Second, it is common to have overlapping names in the collateral pools of different CDO deals. In fact, [9] shows that some names are included in over 50% of deals in Fitch's Synthetic CDO Index (Ford Motor Co – 56.52%, and General Motors Corp – 52.40%). Third, it is common for a credit portfolio to have overlapping names across the single-name portion of the portfolio and the CDO collateral pool. For these reasons we analyze the correlation structure of CDOs in this separate section.

The remainder of the section is divided into two subsections. The first, analyzes the correlation between a CDO and a single-named instrument whose reference entity does not overlap with that of the CDO collateral pool. The second analyzes the correlation between two CDOs with varying degrees of collateral overlap.

3.1 Correlation Between a CDO and a Single-Name Instrument

In [8] Morokoff calculated the loan-equivalent reference entity R^2 for a pass-through CDO tranche from the joint probability of tranche distress and a default by a single name instrument (not in the CDO's collateral pool). He considered a collateral pool of N homogeneous instruments in a single factor environment. Moreover, the single name instrument was assumed similar to the instruments in the homogeneous collateral pool.

The methodology relies on using the independence of defaults conditional on the realization of systematic risk factors to compute the conditional joint default probability, and then takes the expectation over the systematic risk factor:

$$
\begin{aligned}
P(\text{Tranche distress and Single default}) &= E_Z\left(P(\text{Tranche distress and Single default } \mid Z=z)\right) \\
&= E_Z\left(P(\text{Tranche distress } \mid Z=z)\cdot P(\text{Single default } \mid Z=z)\right) \\
&= E_Z\left(P(\#\text{ defaults in pool } \geq \hat{a} \mid Z=z)\cdot p(z)\right) \\
&= E_Z\left(\left(1-P(\#\text{ defaults in pool} < \hat{a} \mid Z=z)\right)\cdot p(z)\right) \\
&= E_Z\left(\left(1-\sum_{\mathrm{k}=0}^{\hat{a}-1} P(\#\text{defaults in pool } = k \mid Z=z)\right)\cdot p(z)\right) \\
&= E_Z\left(\left(1-\sum_{k=0}^{\hat{a}-1}\binom{N}{k} p(z)^k(1-p(z))^{N-k}\right)\cdot p(z)\right) \\
&= p-\sum_{k=0}^{\hat{a}-1}\binom{N}{k} E_z\left(p(z)^{k+1}(1-p(z))^{N-k}\right),
\end{aligned}
$$

where $\hat{a}$ is the required number of defaults in the pool to cause a distress to the tranche, *p(z)* represents the probability of default conditional on the common factor *Z*=*z*:

$$
p(Z)=\mathrm{N}\left(\frac{\mathrm{N}^{-1}(p)-R\cdot Z}{\sqrt{1-R^2}}\right), \text{ and } p=E_Z\left[p(Z)\right].
$$

Similar to our analysis of ABS correlations one can infer an 'asset return' correlation from the default probability, distress probability, and joint default-distress probability (JPD) using a Normal copula:

$$
JPD=N(N^{-1}(\text{tranche Distress Prob}), N^{-1}(p), \rho).
$$

Finally, $R^2_{\text{Loan Equivalant}}$ can be computed as $\frac{\rho^2}{R^2}\cdot$ This follows from the single factor model structure; the squared correlation between two reference entities is equal to the product of their squared correlation with the factor.

The analysis can be extended to allow for the reference entity associated with the single-name instrument to be represented in the collateral pool. In this case, the JPD calculation would require at least $\hat{a}-1$ defaults out of the remaining *N-1* instruments. The analysis is excluded for brevity.

It is worth pointing out that these numerical calculations present an alternative to the simulations used in the ABS section to obtain the correlation results. However, the computation becomes infeasible for large pools, and is therefore more suitable for CDOs. The Normal approximation to the Binomial distribution can be used to alleviate the cumbersome calculations, but for some values of the systematic factors *z* the conditional default probability *p(z)*is too small for the Normal approximation to work.

3.2 The Correlation of CDOs with Overlapping Names

Intuitively, the degree of overlap associated with two CDO tranches increases their correlation; mathematically, the overlapping names share the same idiosyncratic shocks. To better understand the quantitative impact of overlap, simulation methods similar to those used in Section 1 are employed. The relevant difference in this analysis is that a certain percentage of the collateral pool is assumed to overlap when analyzing the correlations across the deals. Figure 4 below presents the tranche loan-equivalent asset return correlations computed using the methodology outlined in Section 1. The analysis was conducted on a high yield deal with typical high yield CDO properties (*PD*=2% and R^2=25%). Along the x-axis, the collateral overlap varies from 0% to 65%. Each curve represents a subordination level for the tranche. It is interesting that the increase of loan-equivalent asset correlation with overlap is more substantial for junior tranches. This follows from the susceptibility of junior tranches to idiosyncratic shocks. After all, it is relatively easy for the junior tranche to be in distress even when the systematic shock is high; all it takes is a single reference entity to realize a particularly low idiosyncratic shock. On the other hand, it is extremely unlikely for senior tranches to be in distress when the systematic shock is high. Therefore, the degree of overlap, which affects idiosyncratic shocks only, has much more effect on junior tranches.

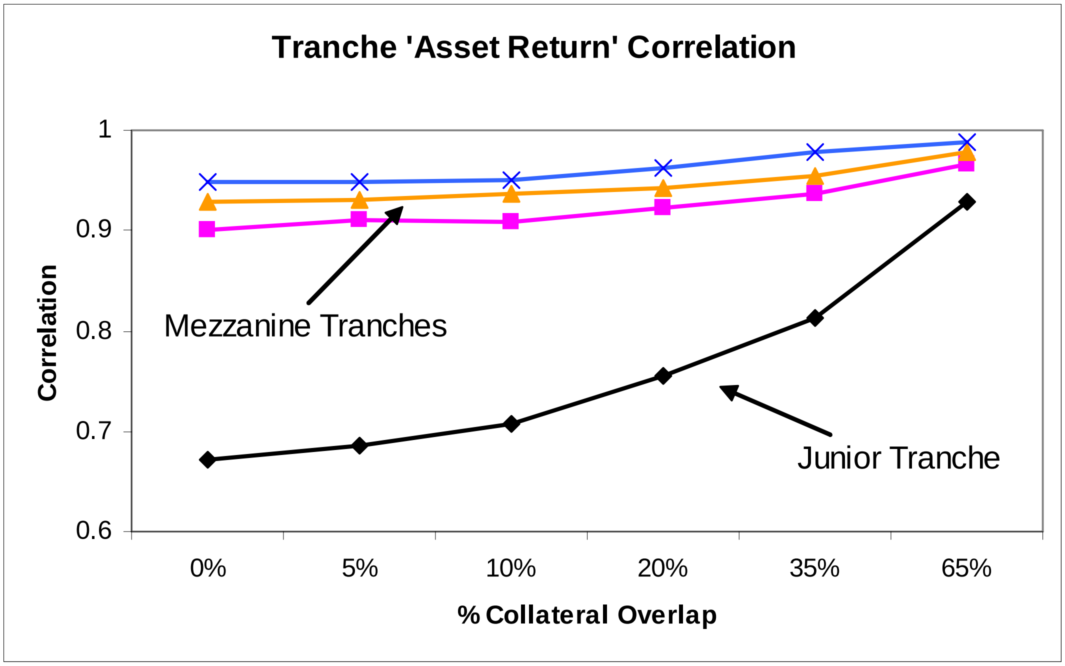

FIGURE 4 The effect of collateral overlap on tranche loan-equivalent reference entity's 'asset return' correlation with collateral pool parameters: PD=2%, R^2=25%, LGD=70%

Tranches of the same CDO deal can be viewed as an extreme case of overlap (100% of the names overlapping). The correlation between two identical tranches in the same deal is trivially 1, and therefore this discussion is focused on modeling different tranches of the same deal. If we were to parameterize a loan-equivalent for each of the tranches using the methodology above, the correlation between tranches would be underestimated.

From a loan-equivalent perspective, this is similar in spirit to having two loans with the same reference entity; the correlation between the two loans cannot be solely determined by the factor loadings since they share idiosyncratic risk. One approach to dealing with this issue is to decompose the idiosyncratic shock so that part of it is shared across the tranches, and the rest remains different for the different tranches. When possible, this shared portion is set to increase the correlation between the tranches in the appropriate amount.

REFERENCES

1. Agca, S. Islam, S. (2007) *Can CDO Equity Be Short on Correlation?* working paper.
2. Basel Committee on Banking Supervision (2005) *An Explanatory Note on the Basel II IRB Risk Weight Functions,* Bank for International Settlements.
3. Coval, J. D, Jurek, J. W, Stafford, E. (2008) *Economic Catastrophe Bonds,* HBS Finance working paper No. 07-102, available at: http://ssrn.com/abstract=995249
4. Coval Coval, J. D, Jurek, J. W, Stafford, E. (2008), *Re-Examining the Role of Rating Agencies: Lessons from Structured Finance,* working paper.
5. Gregory, J, Laurent, J.P (2004), *In the core of correlation,* RISK, October, 87-91.
6. Levy, A. (2008), *An Overview of Modeling Credit Portfolios,* Moody's KMV whitepaper.
7. Moosbrucker, T. (2006), *Explaining the Correlation Smile Using Variance Gamma Distributions*, to appear in The Journal of Fixed Income.
8. Morokoff, W. (2006), Modeling ABS, RMBS, CMBS: *'Loan Equivalent' Approach in Portfolio Manager,* unpublished work done at Moody's KMV.
9. Thompson, A, Rajendra, G. (2006) *Global CDO market 2005 review*, Deutsche Bank whitepaper.
10. Zeng, Bin and Zhang, Jing, *An Empirical Assessment of Asset Correlation Models (2001),* Moody's KMV whitepaper, available at: http://www.moodyskmv.com/research/files/wp/emp_assesment.pdf
11. Zhang, Jing, Zhu, Fanlin, and Lee, Joseph, *Asset Correlation, Realized Default Correlation, and Portfolio Credit Risk (2008),* available at: http://www.moodyskmv.com/research/files/wp/Asset_Correlation_and_Portfolio_Risk.pdf