An Empirical Assessment of Asset Correlation Models

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1. Overview

Over the last thirty years there has been considerable research on the use of forecasting models to estimate correlation structure of security returns in the areas of asset management and risk management. In the area of credit risk management, portfolio models such as CreditMetrics™, CreditRisk+™, KMV's Portfolio Manager™ and McKinsey's Portfolio View™ have gained various degrees of acceptance and application by financial industry. In general, these models do not measure credit correlations directly because credit events such as defaults and credit migrations are rare. Instead, a common approach to estimate credit correlations, as exemplified by CreditMetrics™ and KMV's Portfolio Manager™, is to infer credit correlations from credit event probabilities and the correlations of the underlying asset values that drive these credit events. In turn, the correlations of the underlying assets can be estimated by a given correlation model, which can be constructed from asset return data. Moreover, the asset return data, in general, is of higher quality and larger quantity than credit event data.

Rigorously validating and monitoring the out-of-sample performance of these correlation models are vital to their acceptance and successful implementation in practice. In this paper, by applying both statistical and economic criteria, we validate the out-of-sample performance of a number of asset correlation models using data on more than 27,000 firms worldwide.[1] Our results show that well-constructed correlation models can produce reasonably accurate estimates of future correlations, thereby leading to more optimal portfolio construction and more accurate measurement of risk contributions of individual assets. The results also show that the widely used one-factor models and Industry Average Model perform worse than suggested in the financial literature. Furthermore, we show that country effects play an important role in determining correlation structure. These results have significantly extended the current understanding of correlation models.

It is well known that historical correlations provide relatively poor estimates of future correlations. This is because the historical correlations contain random noise in addition to useful information. Previous studies, such as Elton and Gruber (1973) and Elton, et al. (1978), showed that various factor and average models outperform historical correlations in forecasting future correlations by filtering out noise from the historical correlations. These studies also showed that single factor models could outperform more complicated multi-factor models because the added factors in the multi-factor models tend to pick up more random noise than information. Only recently, studies such as Eun and Resnick (1992) and Chan, et al. (1999), have been able to demonstrate that some well-constructed multi-factor models can outperform single factor models in minimizing out-of-sample portfolio variances. Interestingly enough, Eun and Resnick (1992) showed a macro factor model failed to outperform a traditional single-factor

[1] The correlation model in CreditMetrics™ provides estimates of ***equity*** correlations instead of asset correlations. In general, equity correlations are poor proxies for asset correlations in credit risk calculations, as demonstrated by Zeng and Zhang (2001).

model, suggesting the difficulties in linking macro factors to asset correlations.[2] It is worth pointing out that all the aforementioned studies have used relatively small datasets, with the largest sample size of 500 firms in Chan, et al. (1999). Besides, most of the studies focused on North American firms and did not extensively examine the performance of correlation models on international firms.

With a dataset of weekly returns of more than 27,000 firms diversely distributed in more than 40 countries and 61 industries, we examine the out-of-sample performance of several widely used correlation models. These models can be categorized into three groups: historical model, average models, and factor models. The historical model assumes that the past realized correlation is the best estimate of the future correlation. Average models assume that all the elements in the correlation matrix for a sample of firms can be approximated by the average correlation of their peer group. Examples of average models include widely used Country and Industry Average Models, whereby the correlation between a pair of firms is measured by the average correlation of their country or industry, respectively. Factor models assume that co-movements among asset returns are driven by a set of common factors. Well-known examples of factor models are single factor models such as Capital Asset Pricing Model (CAPM) and multi-factor models derived from Arbitrage Pricing Theory (APT). In our study, we examine two variants of single factor models and two versions of KMV's Global Correlation Model™, Global Correlation Model Version 1 and Global Correlation Model Version 2 (GCM1 and GCM2 hereafter).

To ensure the robustness of our findings, we construct several sub-samples based on various characteristics of sampled firms and different divisions of in-sample and out-of-sample periods in evaluating the performance of these correlation models. We apply statistical criteria to measure the accuracy of forecasted correlations against realized correlations and economic criteria to assess the impact of the forecasts on portfolio management. Our main findings are:

- The correlation models display meaningfully different forecasting abilities. More complicated models do not necessarily give better out-of-sample performance.

- Well-constructed correlation models such as Global Correlation Model™, produce more accurate estimates of future correlations in terms of smaller differences and higher correlations between the forecasted and realized values. The gain from these more accurate estimates of future correlations translates to the economic gain of more optimized portfolios with significantly smaller realized risk and more accurate measurement of risk contributions of individual assets to their portfolios.

- By all the criteria considered, GCM2 outperforms all the alternative models. GCM1 also performs well, being one of the top three models in all the four sub-samples under the statistical criteria and the second best under the economic criteria. Global Correlation Model™ works best probably because its hybrid one-factor/multi-factor approach

[2] McKinsey's Portfolio View™ uses a framework in which a macro factor model drives the default probability and default correlations.

captures most of the information in the data with its underlying multiple factors while reducing the sampling errors with its single composite factor structure.

- Both of the one-factor models, given their popularity in equity portfolio management and their success in equity correlation modeling in previous studies, perform worse than expected. The widely used Industry Average Model also performs poorly. The Country Average Model stands out as the best of the three average models. These results suggest that country effects play a significant role in correlation structures. A single factor structure or industry factors by themselves are too parsimonious to account for all the common movements of asset values of firms in more than 40 countries and 61 industries worldwide.

The rest of the paper proceeds as follows: Section 2 discusses the correlation models being tested. Section 3 describes the data and the statistical and economic criteria used to assess model performances. Section 4 reports the empirical results from using those criteria on various testing groups from the chronologically selected in-sample and out-of-sample. Section 5 offers some final thoughts and concludes the paper. The results from the randomly selected in-sample and out-of-sample are discussed in the appendix.

2. Alternative Correlation Models

We examine eight models for estimating the correlation structure of asset returns. Broadly speaking, they can be grouped into three categories: Historical Model, Average Models and Factor Models. We briefly discuss the economic intuitions behind them and how they can be used to forecast correlations in the following. For more details, see Das and Ishii (2001a).

Historical Model

The most direct way to estimate future correlations is to calculate all pairwise correlations over a historical period and assume that these historical correlations are the best estimates of their future values.

Average Models

- **Overall Average Model:** This model assumes that all elements of the correlation matrix are equal and any observed deviations from the mean are random, that is, the differences between the observed sample correlations and the average are due to noise.

- **Country Average Model**: This is a finer version of average models. Instead of using the overall average of all pairwise correlations, Country Average Model assumes that correlations for firms within a country and correlations between firms from one country and another are constant. For example, the correlation between a US firm and a Japanese firm can be estimated by the average of pairwise historical correlations between all US firms and all Japanese firms over the estimation period. Similarly, the correlation between a US firm and another US firm can be estimated by the average of pairwise historical correlations among all US firms.

- **Industry Average Model**: It is similar to the Country Average Model. The difference is that it uses the averages of correlations within or across industries. Both the Country Average Model and the Industry Average Model are commonly used by practitioners.[3]

To get a sense of typical country and industry average asset correlations, please refer to Das and Ishii (2001b).

Factor Models

Factor models assume that asset returns are primarily driven by a set of common factors that explain the co-movements of asset returns. Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT) are special cases of factor models. Using these factor models, the variances and covariances (hence correlations) of asset returns can be calculated via their factor variances and covariances. We test four factor models in this study:

- **Large Firm Index Model**: This is a CAPM-type one-factor model. The market factor is the value-weighted average of asset returns of all firms in the KMV database. This factor tends to capture the overall effect of global economy. We call it Large Firm Index Model because the index is dominated by the largest firms.

- **Small Firm Index Model**: In this one-factor model, the factor is the average of asset returns of all firms, weighted by the logarithms of market values of firms. Small firms dominate this index as taking log shrinks the large market values and the majority of the companies in the estimation database are small to mid-size firms. This index tends to capture the overall effect of the global economy as it affects small firms.

- **Global Correlation Model™, Version 1 and Version 2 (GCM1 and GCM2)**: KMV developed its first asset correlation model CORRMOD™ in 1993. The model covered firms in North America. It was extended and updated to cover international firms in 1997 and was renamed the Global Correlation Model (GCM1). The model is a hybrid one-factor and multi-factor model. For details, please refer to Crosbie (1999). It has been used in all versions of KMV Portfolio Manager™ 1x and Gcorr™ since 1997. The second generation of the model (GCM2) was constructed in 2001 with broader coverage of firms and longer data history for the release of Portfolio Manager™ 2.0. However, GCM2 in Portfolio Manager™ 2.0 is based on the dataset of the whole period

[3] We frequently get requests from our clients and prospective clients to provide average country and industry correlations. Industry consultants often use average correlations, industry average correlations in particular, in their calculations.

while GCM2 in this study was estimated, using the same methodology, but on data only up to May 1996 in order to perform true out-of-sample testing.

It is worth pointing out that all of these correlation models can be regarded as special cases of factor models. For example, the Overall Average Model can be considered as a single-factor model with the same systematic risk (R-squared) for all firms. The Country Average Model can be considered as a special case of a multi-factor model where the factors are country factors and the systematic risks to these country factors are the same for all firms in each country. The Industry Average Model can be interpreted in similar fashion. The Historical Model is an extreme case where the number of factors equals the number of firms.

3. Data and Evulation Criteria

3.1. Data

The dataset used in this study is from KMV EDF™ production database and Global Correlation Model™ estimation database. It covers 27,303 firms in more than 40 countries and 61 industries over the period from May 1988 to June 1999. About 40% of the firms are in the U.S. and the rest are international. The weekly asset returns are derived from equity returns and liability structure information by using an option-theoretic approach.

The in-sample period runs from May 1988 to May 1996 with 422 weeks and the out-of-sample period is from June 1996 to June 1999 with 161 weeks. The division is motivated by the fact that the first version of the Global Correlation Model™ was built using the dataset up until May 1996. Sample correlations calculated over the in-sample period are the historical correlations. The Overall Average, Country Average and Industry Average Models are calculated using these historical correlations. The Large Firm Index Model, the Small Firm Index Model, and the two versions of Global Correlation Model™ are estimated using the in-sample data. Sample correlations calculated from the out-of-sample period are treated as realized correlations. The median of the numbers of weekly returns for all firms is 212, but they vary across firms. To ensure the sample correlations are calculated with meaningful sample size, a firm needs to have at least 78 observations (roughly 18 months) each for in-sample and out-sample periods to be included in the testing sample. This reduces the number of firms in the testing sample to 8,957.

Previous studies have shown that sample correlations are highly dependent on firms' size and other characteristics (Chan, et al (1999) and references therein). It is, therefore, not surprising that the performances of correlation models vary with samples under study. We evaluate the accuracy of the forecasted correlations on four different sub-samples.

The first sub-sample consists of 1,000 firms with the smallest numbers of missing weekly returns over both the in-sample and the out-of-sample periods. As is standard in the literature, our testing methodology assumes that the realized *ex-post* correlations are proxies for the true *ex-ante* values of what correlation models strive to forecast. There are two advantages of using a sample with the fewest missing observations over both the estimation and forecast periods. First, the sampling errors of realized correlations in the out-of-sample period are minimized. Second, the forecasting models can be constructed more accurately. Larger and more stable firms tend to have more observations, because either they are traded more frequently, or they have longer history, or, for that matter, they tend to be covered by data vendors. Therefore, the results on this sub-sample can, arguably, be viewed as primarily for firms in those categories.

The second group is a more *representative* sub-sample that consists of 1,000 randomly selected firms. This is not a truly random sample from the KMV database as we have already trimmed out many firms with too few valid weekly asset returns. Because the sub-sample also excludes a large percentage of smaller firms that do not have much debt, we think that the results of this sub-sample offer a general sense on how well the correlation models perform for typical firms that financial institutions lend to.

We also construct two sub-samples of 1,000 firms each based on asset return volatilities. We calculate sample volatilities of asset returns from the in-sample period for each firm. The *stable* sub-sample is selected randomly from the sample of firms with higher than the median in-sample volatility and the *volatile* sub-sample is selected from the sample of firms with lower than the median in-sample volatility.

To ensure that the test results do not depend on the division of in-sample and out-of-sample periods, we repeat our evaluation exercises on a different division of in-sample and out-of-sample periods. Specifically, we randomly select half of the weeks as the estimation period and the other half as the forecast period. Note that this division is possible since the time series property of asset returns is not used in the estimation of the correlation models or the calculation of the realized correlations.

3.2. Evaluation criteria

We use both statistical and economic criteria in evaluating the performances of the correlation models. Statistical criteria take account of the sampling variation of the forecasted and realized values in examining the accuracy of the forecasts. Specifically, we examine the mean absolute error, the mean squared error, and the regression performance of the forecasts. Economic criteria focus on the economic significance of the forecasts. We examine the predictive abilities of the correlation models to construct optimal portfolios and to measure risk contributions of individual assets to their portfolios.

We apply several widely used statistical criteria. First, we look at the distribution of the absolute forecast errors, which are defined as the absolute differences between the realized and the forecasted values. The Mean Absolute Forecast Error (MAFE), as well as percentiles, is examined. We also examine the Root of Mean Squared Error (RMSE). Mathematically, given

n pairs of matching actual and forecasted correlations (A_i, F_i), MAFE and RMSE are computed as follows:

$$MAFE = \sum_{i=1}^{n} | A_i - F_i | / n \text{ and } RMSE = \sqrt{\sum_{i=1}^{n} (A_i - F_i)^2 / n} .$$

While forecast errors indicate the magnitude of deviations of forecasts from realizations, we also investigate how much information forecasts contain about realizations by regressing the realized values on the forecasted values and examining the correlation, slope coefficient and intercept from the regression. "Noise" in forecasts pulls down the correlation coefficient and attenuates the slope coefficient while biasing the intercept coefficient. On the other hand, if the forecasted correlations are accurate predictions of the realized correlations, the slope coefficient and correlation coefficient should be close to one, and the intercept coefficient should be close to zero.

Ultimately, the most relevant question to a practitioner is how well a correlation model helps in managing his or her portfolios. To answer this question, we carry out two portfolio management exercises that mimic the applications of the modern portfolio theory. The first one is to form the global minimum variance portfolio using each correlation model, then compare it to both passively managed portfolios and the *ex-post* (true) optimal portfolio. The best model should produce the lowest portfolio variance out-of-sample. The second exercise is to evaluate the abilities of the correlation models to determine risk contributions of individual assets to their portfolio. To control for sampling variation, for each portfolio exercise, we randomly select 100 firms from the sample population and repeat the exercises 100 times.

To form the global minimum variance portfolio, the correlation matrix is estimated by each of the eight correlation models. This information together with *ex-post* variances is then used to allocate assets to minimize the portfolio variance. We choose to examine the minimum variance portfolio with the use of realized variances in order to focus on the importance of correlation estimation without the influence of expected returns and variances. The passively managed portfolios are constructed as either value weighted or equally weighted.

In practical portfolio management, short sales are difficult, or impossible in some circumstances. Furthermore, a well-known caveat to apply mean-variance analysis is that the optimized portfolio is sensitive to the input values (in the context of global minimum variance portfolio, that means correlation and variance forecasts, but not expected returns). A small change of input values could have a drastic impact on the composition of an optimized portfolio. To mitigate the effect of forecasting errors and to mimic the actual practice as closely as possible, we constrain the weights of individual assets to be nonnegative and less than or equal to 2%. The *best* model should produce the lowest portfolio variance, which in turn, should be the closest to the realized variance of the *ex-post* minimal variance portfolio.

To achieve a more optimal portfolio, portfolio managers need to swap lower *Sharpe* ratio assets with the higher ones. The Sharpe ratio is calculated as the ratio of return to risk contribution.[4] Risk contribution, defined as an asset's covariance with the rest of the portfolio, measures how much risk the asset contributes to the variance of the whole portfolio. Accurate measurement of risk contributions of individual assets and business units to the whole portfolio and business is essential to capital allocation and performance measurement. Thus we also evaluate the abilities of the correlation models to determine risk contributions of individual assets to their portfolio. Equally weighted portfolios are used in the exercise and the Pearson correlation and the Spearman correlation coefficients between forecasts and realizations are examined. More accurate models produce higher correlation coefficients between forecasted and realized risk contributions.

4. Empirical Results

In this section, we report the out-of-sample performance of the eight correlation models in the four sub-samples and the portfolio optimization exercises. After presenting all the detailed evidence from these tests, we synthesize these findings. Readers can choose to skip the details of the empirical evidence and proceed to the main points in Section 4.3.

4.1. Correlation forecast accuracy

Sub-sample of firms with the fewest missing weekly returns

Table 1 presents the results for this sub-sample. Panels A and B provide descriptive statistics of the firms for the in-sample period of 1988 to 1996 and the out-of-sample period of 1996 to 1999. 'Asset volatility' is the sample standard deviation of asset returns. 'Weeks' is the number of weekly returns a firm has in the data. 'Pairwise Weeks' is the number of pairwise weekly asset returns that are used to calculate the correlation of two firms. As mentioned before, firms in this sample tend to be large and less volatile. The mean asset volatilities for this sub-sample (17.76% and 18.62% for the in-sample and out-of-sample, respectively) are lower than those for the representative sub-sample (24.48% and 24.35%, see Table 2).

Panel C lists summary statistics of the forecasted correlations by each model. The standard deviation of forecasted correlations for the Historical Model (12.40%) is the largest, suggesting that straightforward extrapolation from the past may be overly accommodative of the noise in the data. The three average models stand at the other end of the spectrum with the smallest standard deviations, while the factor models register in the middle. This is not surprising since the factor models tend to smooth out the correlations and yield less extreme forecasts. The standard deviations of forecasted correlations by GCM1 and GCM2 are higher than those by the

[4] The risk contributions calculated here are based on asset return correlations and volatilities. They are different from those calculated by Portfolio Manager™, which are based on exposure value correlations and unexpected losses.

two one-factor models, reflecting the fact that the multiple factors in GCM1 and GCM2 give a richer correlation structure than a single factor structure.

Panel D presents the performance results for all the correlation models. The first two columns report the mean and the 95th percentile of absolute forecast errors. The third column is the Root Mean Squared Error. GCM2 produces the lowest values in all three measures. The Historical Model is the second best, followed by GCM1. These three models distance themselves from the rest of the group while the Small Firm Index Model produces the largest forecast errors.

The last three columns of Panel D provide further information on forecast performance. Specifically, we regress the realized values on the forecasted values and examine the correlation coefficient, the slope coefficient, and the intercept from the regression. While the mean absolute error and mean squared error criteria penalize a model for making over- or under-estimates, the correlation coefficient focuses more on whether the forecasts tend to move in the same direction as the realized values. By this criterion, GCM2 leads the group with the highest correlation coefficient (0.6729) followed by the Historical Model and GCM1. The Country Average Model turns in as the fourth best with a correlation coefficient of 0.6210, which is only slightly smaller than those from the three best models and much higher than those from the rest three.

The slope coefficients and intercept coefficients from the regressions of realizations on forecasts are reported in the last two columns of Panel D. GCM2 has the largest slope coefficient (0.8231) and the smallest intercept coefficient (0.0122) for the models with a slope coefficient smaller than one. The Country Average Model and Industry Average Model have slope coefficients greater than one. While it is generally preferred that a correlation model produces the smallest absolute forecast error and mean square error, and the highest correlation coefficient, it is more difficult to interpret the slope and intercept coefficients. As it is assumed that forecasted correlations are unbiased estimators of underlying 'true' values of correlations which realized correlations purport to proxy (with or without noise), the slope and intercept coefficients indicate how much information forecasted correlations contain about the 'true' ones. Deviations of the slope and intercept from one and zero respectively reflect how noisy the forecasted values are. Under this assumption, the slope coefficient should be smaller than one. Therefore, when observed slope coefficients are greater than one as in the cases for the Country Average Model and Industry Average Model, it implies that the assumption is violated and one needs to be cautious in interpreting the slope and intercept coefficients. Nevertheless, significant departures of the slope coefficient from one and (or) intercept from zero signal large amount of noise, or bias, or both, of forecasts by a correlation model.

The above results show that GCM2 is the best model for this sub-sample among all correlation models examined. The Historical Model and GCM1 are the second and third best, respectively, though the intercept coefficient for GCM1 is smaller than that of the Historical Model. It is somewhat surprising to see both the one-factor models perform poorly, given their popularity and success in equity management. A plausible explanation is that a single factor structure is too parsimonious to capture all the common movements for firms in our database, which are scattered in more than 40 countries and 61 industries. It is worth pointing out that the majority of the previous studies that show good performance of single-factor models are based on US data. In contrast to single factor models, the Country Average Model performs fairly well.

This seems to suggest that country effect is important in characterizing the correlation structure of international firms. This is consistent with the findings in Eun and Resnick (1984).

Representative Sub-sample

Table 2 lists the testing results for the *representative* sub-sample, which consists of randomly selected 1,000 firms. As we discussed earlier, the average asset volatility for firms in this sub-sample is higher than that for the sub-sample of firms with the fewest missing weekly returns. The numbers of non-missing weekly returns and pairwise weekly returns are generally smaller.

Firms in this sub-sample tend to be smaller than those in the previous one. As smaller firms tend to have larger idiosyncratic risk, it is not surprising to see smaller sample correlations in this sub-sample. The average sample correlation for the in-sample period is 8.51% for this sub-sample, in contrast to 18.46% for the sub-sample of firms with the fewest missing weekly returns. Accordingly, all the models produce lower forecasts, as reported in Panel C of Table 2.

Comparing the performance results with those on Table 1, we note that all models except the Large Firm Index Model have lower correlation coefficients than those for the last sub-sample. Despite the fact that average sample correlations are much higher in last sub-sample than this one, statistics about absolute forecast errors are generally comparable for these two sub-samples. It seems that all the correlation models perform better in the sub-sample of firms with fewest missing weekly returns than in the current sub-sample, presumably because the longer asset return history reveals more information about the underlying correlation structure.

Among all correlation models, GCM2 gives the best performance in terms of absolute forecast error statistics, correlation coefficient and intercept coefficient, while the Country Average Model gives the highest slope coefficient. A significant difference between the results of this sub-sample and those of last sub-sample is that the Country Average Model replaces the Historical Model as one of the top three models. Again this reinforces the importance of the country effect in the correlation structure of international firms.

Stable Sub-sample

Table 3 presents the results for the *Stable* sub-sample, which consists of firms with lower than average asset volatility. As reported in Panels A and B, the mean asset volatilities for the in-sample period and out-of-sample period (14.03% and 15.20% respectively) are smaller than those from the last two sub-samples. The numbers of valid weekly asset returns and pairwise weekly returns are slightly larger than those in the representative sub-sample but smaller than those in the sub-sample of firms with the fewest missing returns. The realized correlations in this sub-sample also lie between those from the previous two sub-samples. All these behaviors are consistent with the fact that asset volatility is negatively correlated with firm size, which in turn, is positively correlated with the number of valid weekly returns in the sample.

Among all correlation models, GCM2 continues to produce the lowest forecast errors, the highest correlation coefficient, and the lowest intercept coefficient. GCM1 comes in as the second best. Among the remaining six models, while three average models have low values of statistics about absolute forecast errors, the Country Average Model also gives the highest slope coefficient, the lowest intercept coefficient, and comparable correlation coefficient (0.4349) to that of the Historical Model (0.4397).

Volatile Sub-sample

This sub-sample is constructed by selecting firms with higher than average asset volatility. Understandably, firms in this sub-sample tend to be small in size, have more missing weekly asset returns, and have smaller realized correlations. The smaller numbers of weekly returns in this sub-sample make it more difficult to estimate and evaluate correlation models.

Table 4 presents the results for this sub-sample. All correlation models perform worse in this sub-sample than in any other sub-sample. For example, the highest correlation coefficient achieved by GCM2 (0.4531) is lower than those in other sub-samples. The lack of data reduces the accuracy of the forecasts and increases sampling errors of the realized correlations as the evaluation benchmark.

Among all correlation models, the overall performance results rank GCM2 as the best performing model in this sub-sample, followed by the Country Average Model and GCM1 as the second and third best.

Table 1

Performance of correlation forecasting models
for the subsample of firms with the fewest missing weekly returns

Panel A: Descriptive statistics of the subsample in 1986-96

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.1776	0.0738	0.0214	0.0520	0.3089	0.4841
Weeks	401.48	6.18	386	392	412	419
Pairwise Weeks	382.75	8.26	352	369	397	416

Panel B: Descriptive statistics of the subsample in 1996-99

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.1862	0.0834	0.0227	0.0582	0.3337	0.5171
Weeks	153.51	3.83	132	147	159	161
Pairwise Weeks	147.64	4.66	112	139	154	161

Panel C: Descriptive statistics of forecasted correlations

Model	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
GCM2	0.1998	0.1063	0.0000	0.0824	0.4331	0.6500
GCM1	0.2081	0.1210	-0.0455	0.0669	0.4322	0.6500
Historical	0.1846	0.1240	-0.2195	0.0294	0.4352	0.8693
Large firm index	0.1343	0.0748	0.0017	0.0414	0.2791	0.6205
Small firm index	0.0789	0.0777	0.0000	0.0007	0.2259	0.3880
Overall average	0.1742	0.0000	0.1742	0.1742	0.1742	0.1742
Country average	0.0961	0.0733	0.0053	0.0511	0.2753	0.3241
Industry average	0.0798	0.0193	0.0313	0.0501	0.1051	0.2430

Panel D: Forecast performance

	Absolute forecast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0796	0.1954	0.1007	0.6729	0.8231	0.0122
GCM1	0.0858	0.2254	0.1108	0.6440	0.6921	0.0325
Historical	0.0813	0.2057	0.1034	0.6715	0.7041	0.0465
Large firm index	0.1030	0.3675	0.1508	0.0789	0.1371	0.1581
Small firm index	0.1198	0.4242	0.1736	0.1154	0.1931	0.1613
Overall average	0.1008	0.2689	0.1300	0.0000	0.0000	0.1765
Country average	0.0991	0.2638	0.1301	0.6210	1.1012	0.0708
Industry average	0.1152	0.3569	0.1607	0.1636	1.1003	0.0887

* 95-th percentile. **Root Mean Squared Error.

Table 2

Performance of correlation forecasting models
for the subsample of representative firms

Panel A: Descriptive statistics of the subsample in 1986-96

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.2448	0.1316	0.0140	0.0461	0.4986	0.6647
Weeks	278.39	103.50	79	100	402	418
Pairwise Weeks	193.65	92.83	10	73	360	411

Panel B: Descriptive statistics of the subsample in 1996-99

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.2435	0.1316	0.0161	0.0527	0.5020	0.7556
Weeks	140.24	15.60	78	104	157	160
Pairwise Weeks	122.93	18.74	78	87	147	159

Panel C: Descriptive statistics of forecasted correlations

Model	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
GCM2	0.1162	0.0784	0.0000	0.0361	0.2991	0.6115
GCM1	0.1183	0.0853	-0.0631	0.0313	0.3013	0.6500
Historical	0.0851	0.1138	-0.6117	-0.0852	0.2886	0.8996
Large firm index	0.0613	0.0525	0.0000	0.0047	0.1636	0.5851
Small firm index	0.0447	0.0448	0.0000	0.0012	0.1360	0.3394
Overall average	0.0750	0.0000	0.0750	0.0750	0.0750	0.0750
Country average	0.0795	0.0577	-0.0957	0.0319	0.2753	0.4431
Industry average	0.0758	0.0226	0.0313	0.0476	0.1051	0.2430

Panel D: Forecast performance

Model	Absolute forecast error			Regression coefficients		
	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0671	0.1747	0.0867	0.4877	0.5672	0.0540
GCM1	0.0710	0.1894	0.0930	0.4458	0.4765	0.0634
Historical	0.0957	0.2465	0.1228	0.3563	0.2854	0.0955
Large firm index	0.0831	0.2336	0.1141	0.1540	0.2677	0.1033
Small firm index	0.0892	0.2505	0.1231	0.0986	0.2007	0.1108
Overall average	0.0718	0.2152	0.1016	0.0000	0.0000	0.1198
Country average	0.0688	0.1932	0.0919	0.4584	0.7247	0.0622
Industry average	0.0724	0.2110	0.1007	0.1509	0.6093	0.0736

* 95-th percentile. **Root Mean Squared Error.

Table 3

Performance of correlation forecasting models for the subsample of stable firms

Panel A: Descriptive statistics of the subsample in 1986-96

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.1403	0.0549	0.0160	0.0340	0.2136	0.2220
Weeks	300.80	102.47	78	94	405	418
Pairwise Weeks	221.54	100.57	10	76	372	412

Panel B: Descriptive statistics of the subsample in 1996-99

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.1520	0.0741	0.0166	0.0383	0.2811	0.5961
Weeks	141.73	15.63	78	106	157	160
Pairwise Weeks	125.88	18.89	78	88	149	159

Panel C: Descriptive statistics of forecasted correlations

Model	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
GCM2	0.1448	0.0884	0.0010	0.0521	0.3480	0.6148
GCM1	0.1409	0.0994	-0.0533	0.0350	0.3589	0.6459
Historical	0.1216	0.1210	-0.7123	-0.0525	0.3480	0.9913
Large firm index	0.0908	0.0657	0.0000	0.0098	0.2194	0.5102
Small firm index	0.0544	0.0517	0.0000	0.0010	0.1573	0.3327
Overall average	0.1089	0.0000	0.1089	0.1089	0.1089	0.1089
Country average	0.0869	0.0625	0.0007	0.0411	0.2753	0.4431
Industry average	0.0873	0.0307	0.0313	0.0570	0.1408	0.2430

Panel D: Forecast performance

	Absolute forecast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0719	0.1842	0.0924	0.5357	0.6115	0.0459
GCM1	0.0771	0.2071	0.1012	0.4916	0.4991	0.0641
Historical	0.0931	0.2388	0.1194	0.4397	0.3667	0.0898
Large firm index	0.0842	0.2427	0.1161	0.2192	0.3366	0.1038
Small firm index	0.0957	0.2779	0.1325	0.1616	0.3151	0.1173
Overall average	0.0764	0.2191	0.1041	0.0000	0.0000	0.1344
Country average	0.0776	0.2175	0.1042	0.4349	0.7024	0.0734
Industry average	0.0791	0.2332	0.1098	0.2063	0.6791	0.0751

* 95-th percentile. **Root Mean Squared Error.

Table 4

Performance of correlation forecasting models
for the subsample of volatile firms

Panel A: Descriptive statistics of the subsample in 1986-96

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.3457	0.0991	0.2222	0.2309	0.5307	0.6670
Weeks	254.04	98.85	78	99	397	417
Pairwise Weeks	166.46	79.57	10	67	323	410

Panel B: Descriptive statistics of the subsample in 1996-99

Variable	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
Asset Volatility	0.3359	0.1151	0.0724	0.1708	0.5344	0.7380
Weeks	137.38	15.90	78	104	155	161
Pairwise Weeks	117.51	18.36	78	84	144	161

Panel C: Descriptive statistics of forecasted correlations

Model	Mean	Standard deviation	Minimum	5-th percentile	95-th percentile	Maximum
GCM2	0.0946	0.0618	0.0000	0.0314	0.2141	0.5619
GCM1	0.1000	0.0679	-0.0510	0.0284	0.2400	0.5706
Historical	0.0649	0.1096	-0.5902	-0.1011	0.2538	0.8630
Large firm index	0.0380	0.0304	0.0000	0.0032	0.0972	0.2828
Small firm index	0.0363	0.0360	0.0000	0.0009	0.1087	0.2775
Overall average	0.0568	0.0000	0.0568	0.0568	0.0568	0.0568
Country average	0.0765	0.0525	-0.0957	0.0239	0.2753	0.4431
Industry average	0.0672	0.0147	0.0313	0.0428	0.0949	0.1868

Panel D: Forecast performance

	Absolute forecast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0650	0.1736	0.0848	0.4531	0.6454	0.0571
GCM1	0.0689	0.1827	0.0903	0.3786	0.4904	0.0690
Historical	0.1015	0.2598	0.1298	0.2977	0.2388	0.1026
Large firm index	0.0894	0.2421	0.1207	0.0943	0.2730	0.1077
Small firm index	0.0910	0.2442	0.1230	0.0910	0.2222	0.1100
Overall average	0.0765	0.2218	0.1072	0.0000	0.0000	0.1181
Country average	0.0680	0.1880	0.0900	0.4465	0.7483	0.0609
Industry average	0.0732	0.2113	0.1016	0.0844	0.5053	0.0841

* 95-th percentile. **Root Mean Squared Error.

4.2. Portfolio management study

The evidence presented in the last subsection characterizes the performance of correlation models using statistical criteria. In this subsection, we examine the economic significance of these correlation models in managing portfolios. Specifically, we conduct two exercises to examine the abilities of these models to optimize portfolios and to measure risk contributions. First, we form the global minimum variance portfolio using the correlation models and compare them to passively managed portfolios and the *ex-post* optimal portfolios. Equally weighted and value weighted portfolios are used as passively managed portfolios. Second, we calculate the *ex-ante* risk contributions of individual assets to their portfolios using the correlation models and compare them to the realized risk contributions. For each portfolio exercise, we randomly select 100 firms from the sample population and repeat the exercise 100 times.

Table 5 lists the percentage (5% percentile, mean, median and 95% percentile) of volatility reduction from passively managed portfolios through constructing optimal portfolio using each correlation model. When the optimal weights are calculated based on the forecasted correlations by each of the eight models, the *ex-post* volatility of the resulting optimal portfolio is expected to be *lower* than that of a passively managed portfolio, such as an equally-weighted or value-weighted portfolio.[5] It is striking to see that optimal portfolios constructed from any correlation model produce significant reduction of portfolio volatility, with more than 50% volatility reduction on average with each model except the Small Firm Index Model. Needless to say, the relative percentage of volatility reduction depends on the benchmark, which is somewhat arbitrarily chosen here. Nevertheless, these results suggest that actively managing portfolios with a good correlation model does reduce the portfolio risk.

Instead of comparing with passively managed portfolios, it is more meaningful to compare the optimally constructed portfolios to the realized portfolios. This exercise will give us a better sense how close the optimally constructed portfolios are to the 'true' optimal one. The first four columns of Table 6 present a summary report of this exercise. For each of the eight models, when the optimal weights for assets in the portfolio are calculated based on the forecasted correlations, by definition, the *ex-post* volatility of the resulting optimal portfolio will be *higher* than that of the *ex-post* optimal portfolio. The best model should produce an optimized portfolio with its volatility being *closest* to the *ex-post* minimal volatility. Thus we examine the percentage difference between the realized volatility of the optimized portfolio using each correlation model and the *ex-post* minimal volatility. The first three columns in Table 6 give the mean, median and 95th percentile of these percentage differences for the 100 randomly selected portfolios. By these measures, GCM2 leads the group of models with the lowest percentage volatility increase, followed by GCM1 and the Historical Model as the second and third best. The fourth column gives the percentage of times that an alternative model produces an optimized portfolio with lower risk than that by GCM2. This measure provides more information about how well GCM2 performs in comparison with the alternative models. A 50%

[5] Table 5 only lists the results for the case of using a value-weighted portfolio as the benchmark. The results with an equally weighted portfolio are similar.

chance would imply that the two models give comparable performance. Consistent with the results of the mean percentage increases, the chance ('Crossing' in Table 6) of an alternative model produces a less risky portfolio than GCM2 is all less than 30%, with GCM1 (26%) as the most likely followed by the Historical Model (20%) and others.

Figures 1-4 provide more detailed pictures of the performance results of the correlation models in these portfolio optimization exercises. Figure 1 ranks models based on the average percentage increases of portfolio volatilities from the realized minimal volatility by the alternative models. Clearly GCM2 is the best, followed by GCM1, Historical Model, Country Average Model, Industry Average Model, Overall Average Model, Large Firm Index Model, and Small Firm Index Model as the last. Figures 2-4 plot the percentage increases of portfolio volatilities by GCM2 in the global minimum variance exercise against the next three best performing models on individual portfolios. The percentage increases for GCM2 are arranged in an increasing order for the presentational ease. Clearly GCM2 tends to produce optimal portfolios that are less risky and closer to the realized optimal portfolios than the alternatives models.

The last two columns of Table 6 report the means of the Pearson and Spearman correlations between the forecasted and realized risk contributions of assets in 100 equally weighted portfolios consisting of 100 randomly selected firms each. A more accurate correlation model will produce more accurate forecasts of risk contributions hence a larger correlation coefficient between the forecasted and realized risk contributions. Notice that these correlation coefficients are large for all the models, suggesting that the risk contribution calculations are heavily influenced by the volatilities of individual returns for which we used the *ex-post* actual values for all the models here. It also suggests that the effects of sampling errors due to correlation models are partly canceled out in a portfolio. Nevertheless, GCM2 and GCM1 have the first and second highest Pearson and Spearman coefficients, followed by the Overall Average Model. Surprisingly, the Historical Model falls behind all the three average models though only slightly. The two one-factor models perform the worst on these two measures.

Table 5

Percent of volatility reduction relative to value-weighted portfolio

	Percentage decrease of volatility			
Model	P5*	Mean	Median	P95**
GCM2	36.55	56.21	54.92	74.56
GCM1	35.86	55.86	55.52	75.37
Historical	34.46	54.80	53.94	73.89
Large firm index	29.79	51.99	51.75	72.07
Small firm index	24.57	48.62	49.30	70.29
Overall average	34.65	54.73	54.59	74.44
Country average	34.21	54.90	54.89	75.47
Industry average	34.77	54.77	54.94	74.09

*5-th percentile. **95-th percentile.

Table 6

Percent of volatility increase relative to ex-post optimal portfolio

	Percentage increase of volatility				Risk contribution	
Model	Mean	Median	P95*	Crossing**	Pearson	Spearman
GCM2	14.53	13.92	21.16	0.00	0.823	0.811
GCM1	16.28	16.02	24.02	0.26	0.820	0.807
Historical	18.14	17.61	27.89	0.20	0.787	0.778
Large firm index	26.17	24.78	39.44	0.03	0.765	0.733
Small firm index	36.27	34.21	53.19	0.00	0.750	0.672
Overall average	19.74	19.33	28.95	0.09	0.806	0.797
Country average	18.70	18.79	26.73	0.11	0.799	0.789
Industry average	19.51	18.31	28.47	0.06	0.799	0.794

*95-th percentile.
**Percentage of number of portfolio volatilities that are higher than the corresponding ones by GCM2.

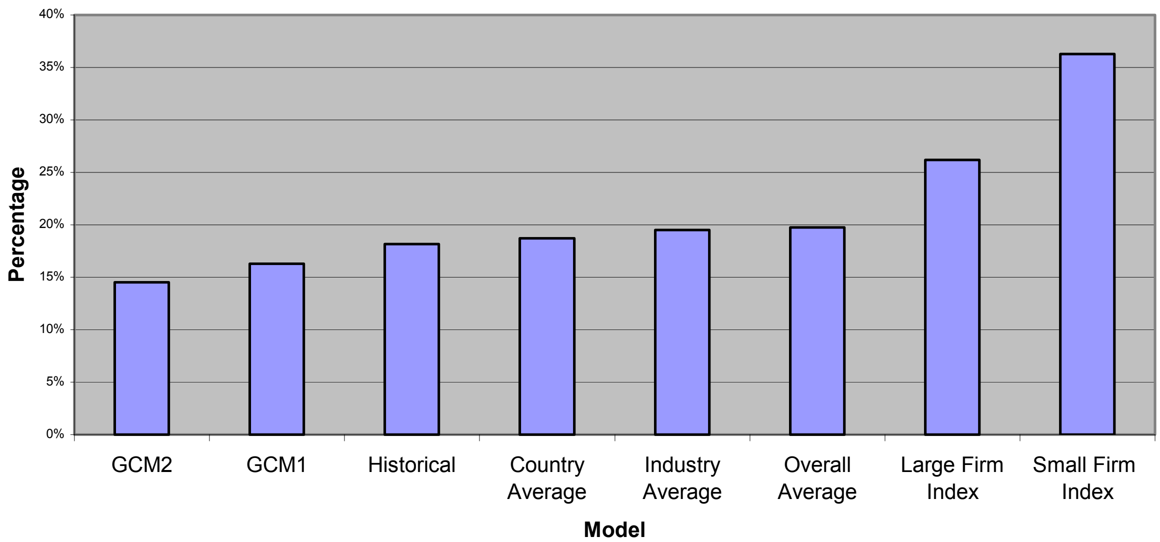

Figure 1: Mean percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios.

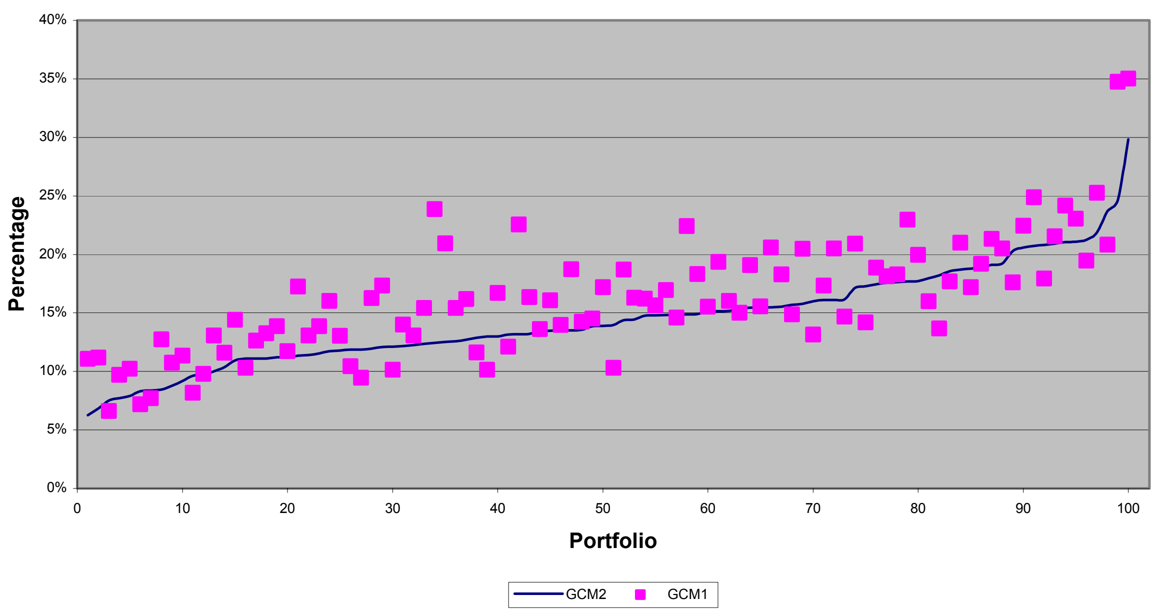

Figure 2: Percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios, GCM2 versus GCM1.

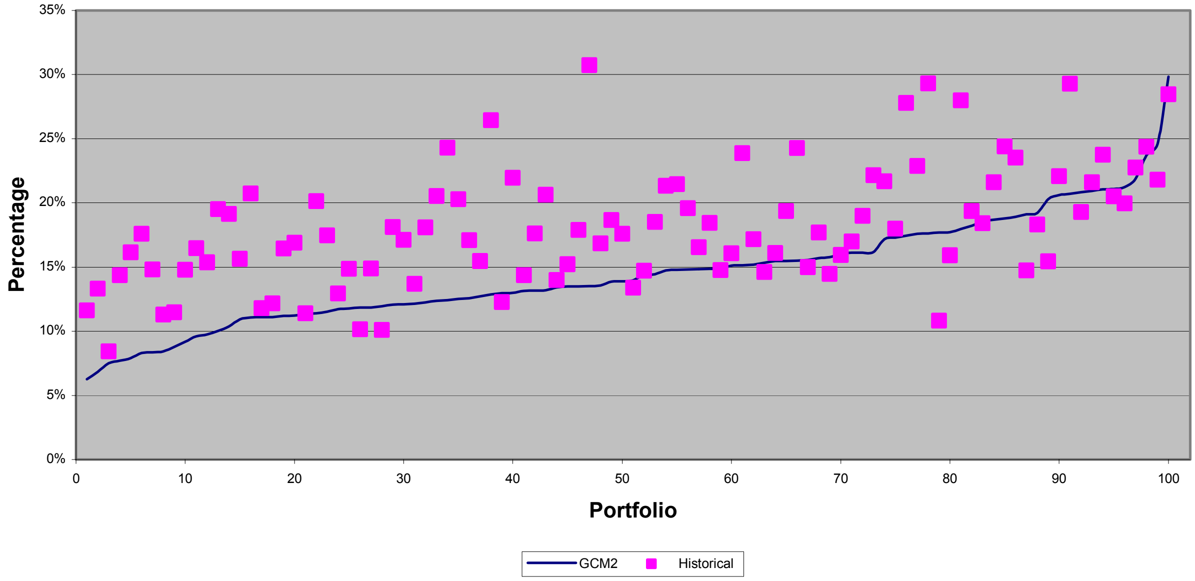

Figure 3: Percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios, GCM2 versus the Historical Model.

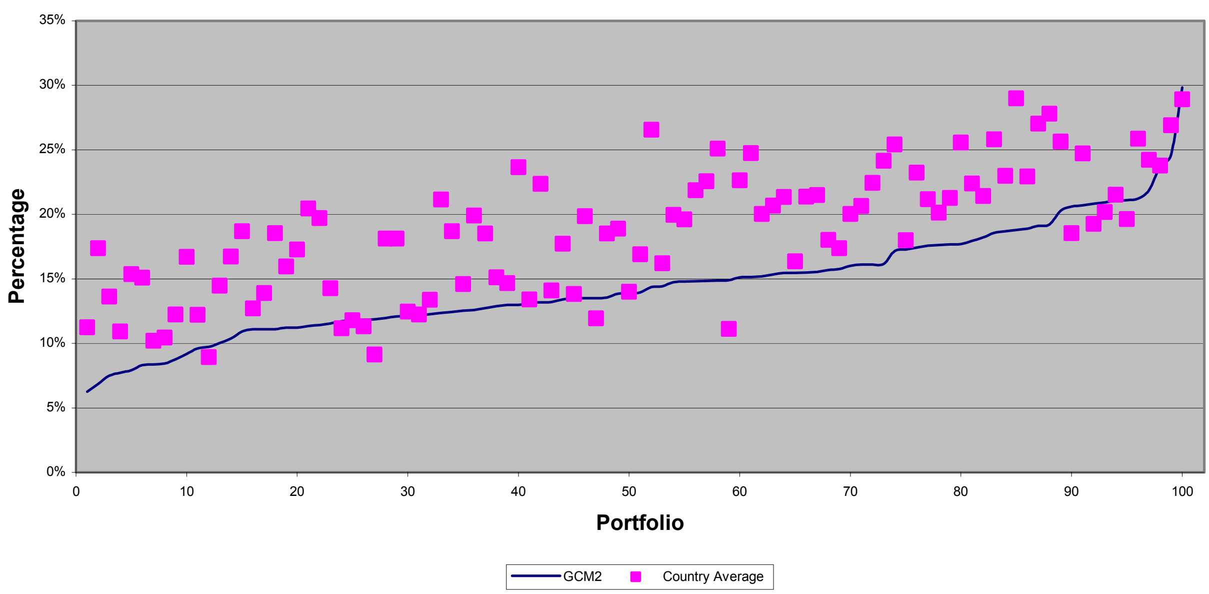

Figure 4: Percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios, GCM2 versus the Country Average Model.

4.3. Summary of findings

The main findings of our study can be summarized as:

1. The correlation forecasting models display meaningfully different predicting abilities. The performance and relative rankings of the models can vary with the underlying samples under study. More complicated models do not necessarily give better out-of-sample performance possibly because added complexity captures more noise than information.

2. All the correlation models perform better for firms with fewer missing weekly returns. It is probably due to the fact that the longer in-sample history allows more accurate estimation for the correlation models, and more out-of-sample realizations reduce the sampling errors of the realized correlations as the proxies for the underlying 'true' correlations.

3. Well-constructed correlation models such as Global Correlation Models produce more accurate estimates of future correlations. These models tend to produce smaller absolute forecast errors and higher correlation coefficients between the *ex-ante* forecasts and the *ex-post* realizations. These more accurate estimates result in more optimal selection of portfolios and better measurement of risk contributions of individual assets to their portfolios.

4. By all the criteria considered, GCM2 outperforms the alternative models. GCM1 also performs well, being one of the top three models in all the four sub-samples under the statistical criteria and the second best under the economic criteria. Global Correlation Model™ works best probably because its hybrid one-factor/multi-factor approach captures most of the information in the data with its underlying multiple factors while reducing the sampling errors to a minimum with its single composite factor.

5. The Historical Model performs satisfactorily for firms with the fewest missing weekly returns when the signal to noise ratio is high. In other situations, the noise in the estimates overwhelms the signal and alternative models work better.

6. On the other hand, average models have the least sampling variabilities. The Country Average Model stands out as the best of the three average models. This suggests that country effects play a significant role in correlation structures. The Country Average Model performs particularly well for volatile firms where the Historical Model does worst. In contrast, the Overall Average Model loses too much information in individual pairwise correlations by assuming all pairwise correlations are constant, and the Industry Average Model registers the worst performance among the three.

7. Both the one-factor models, given their popularity in equity portfolio management and their success in equity correlation modeling in previous studies, perform worse than expected. One plausible explanation is that because the firms in the test samples in our study are more diverse than those in other studies, a single factor is too parsimonious to

capture all the common movements of asset values of firms in more than 40 countries and 61 industries.

5. Concluding Remarks

We evaluated the out-of-sample performances of eight asset correlation models using both statistical and economic criteria. The sample dataset in our study, arguably, is the most comprehensive sample ever assembled, covering more than 27,000 firms worldwide in more than 40 countries and 61 industries. We ensure the robustness of our results by looking at various sub-samples of the data and different division of the in-sample and out-of-sample periods. Our findings add new insights to the existing understanding of forecasting asset correlations as well as confirm some results from previous studies.

Consistent with prior studies, we find that historical correlations often contain as much random noise as useful information and therefore they are poor estimates of future correlations. More often than not, data with sufficient length and quality is not readily available. This is particularly true for small firms and firms with short history. Therefore, various average models and factor models are developed with the hope that they filter out the noise from historical correlations while keeping the useful information. As expected, some of these models often outperform historical correlations. When more factors are introduced, a model usually has greater explaining power of historical correlations. However, it is not guaranteed that a multi-factor model can outperform a more parsimonious one, suggesting that multi-factor models are prone to over-fitting the data.

In contrast to previous studies, we find that single factor models perform worse than expected. On the other hand, the Country Average Model has performed reasonably well. These results suggest that the correlation structures of asset values of international firms are richer than those prescribed by single factor models. This, in turn, suggests that portfolio diversification across countries needs correlation models that captures country effects.

By all the criteria considered, the Version 2 of Global Correlation Model™ outperforms all the alternative models. The earlier version also does reasonably well, especially with the portfolio optimization study, and stands out as the second best forecasting model. It is worth pointing out here that the real tests of the first version have been conducted by our clients at more than fifty financial institutions in using Portfolio Manager™ managing their debt portfolios. The Global Correlation Model™ works best probably because its underlying factor structure captures most of the information while minimizing the noise in the data. While we're fully aware of the limitations of all the correlation forecasting models given their forecasting errors, it's clear that well constructed asset correlation models such as Global Correlation Model™ can be powerful tools for accurate measurement of asset correlations.

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Appendix

In this appendix we report the test results on the correlation models with an estimation period and a forecast period consisting of randomly selected weeks. We believe that repeating our performance evaluation exercises by this random division of the in-sample and the out-of-sample can confirm that the main findings won't change with different sample periods.

Another motivation is to minimize the impact of asset volatilities on correlations. Recent research (e.g., Sheedy (1997)) indicates that realized *ex-post* asset correlations vary over time and variations are driven by the systematic factor volatilities. Although it is a standard practice to use out-of-sample correlations as a benchmark to evaluate forecasting models, changes of systematic factors over the in-sample and out-of-sample periods may adversely affect the interpretation of performance results. One way to mitigate this possibly uneven volatility bias is to randomly select weeks for the in-sample and out-of-sample periods. This approach of random selection may raise the concern that, unlike more typical tests, there are no chronological orders for the dates of the in-sample and out-of-sample periods. The asset returns in the in-sample used to construct correlation forecasting models may contain useful information about those from the out-of-sample. We don't consider this to be a serious problem. As asset returns can be considered approximately a random walk, correlation models don't require any time-series properties of the sample periods. More importantly, even if the random division of in-sample and out-of-sample periods should bias performance testing results, it would probably help the Historical Model most. Since our purpose is to assess the performance of the alternative correlation models relative to the Global Correlation Model™, the random division should not affect our results in a favorable way.

There are 583 weeks of data in the KMV database. We randomly select 292 weeks as the in-sample period and the rest as the out-of-sample period. A minimum of 156 observations each from the estimation and forecast periods are required for any firm in the testing sample. The number of firms satisfying this condition is 8959.

Since GCM1 in the current use was built based on the period from May 1988 to May 1996, we have to exclude it from this test. Otherwise, we follow the exactly same procedures to construct testing samples and apply the same criteria used in the chronologically selected in-sample and out-of sample study.

The performance results are presented in Tables 7 and 8. Since the weeks employed as estimation and forecast periods are randomly selected, asset returns in both periods show much greater similarity than it would be for other divisions of sample periods. Thus, it is not unexpected that the performance results are generally better than those in our earlier tests for all the correlation models.

The general characteristics of the performance results, however, remain similar to what we observed in Section 4. One noticeable difference is that the correlation coefficient between realizations and forecasts for the Historical Model with the sub-sample of stable firms is higher than that for GCM2. However, the overall performance results still clearly show that GCM2 is the best performing model.

Table 7

Performance of correlation forecasting models with randomized division of in-sample and out-of-sample

Panel A: Subsample for firms with the fewest missing weekly returns

	Absolute forescast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0681	0.1695	0.0862	0.7703	0.7777	0.0707
Historical	0.0906	0.2140	0.1118	0.7299	0.7115	0.1067
Large firm index	0.1140	0.3853	0.1586	0.2196	0.3918	0.1602
Small firm index	0.1387	0.4088	0.1797	0.2154	0.4177	0.1709
Overall average	0.0981	0.3101	0.1371	0.0000	0.0000	0.1973
Country average	0.1296	0.2787	0.1532	0.7097	1.3394	0.1036
Industry average	0.1349	0.3643	0.1740	0.1762	1.0689	0.1256

Panel B: Subsample for representative firms

	Absolute forescast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0608	0.1582	0.0785	0.6336	0.6645	0.0529
Historical	0.0950	0.2343	0.1193	0.4656	0.4159	0.1034
Large firm index	0.0940	0.2584	0.1261	0.2148	0.4865	0.1097
Small firm index	0.0991	0.2725	0.1325	0.1220	0.2782	0.1211
Overall average	0.0826	0.2455	0.1146	0.0000	0.0000	0.1327
Country average	0.0792	0.2039	0.1025	0.5307	0.9347	0.0704
Industry average	0.0826	0.2402	0.1134	0.1914	0.8190	0.0796

Panel C: Subsample for stable firms

	Absolute forescast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0815	0.2120	0.1059	0.6803	0.8641	0.0511
Historical	0.0849	0.2117	0.1074	0.7523	0.7113	0.0894
Large firm index	0.1213	0.3380	0.1611	0.4651	0.8324	0.1226
Small firm index	0.1657	0.4156	0.2085	0.3229	0.8716	0.1690
Overall average	0.1089	0.3165	0.1455	0.0000	0.0000	0.2204
Country average	0.1591	0.3884	0.1981	0.4600	1.1855	0.1422
Industry average	0.1171	0.3321	0.1559	0.3447	0.7371	0.1193

Panel D: Subsample for volatile firms

	Absolute forescast error			Regression coefficients		
Model	Mean	P95*	RMSE**	Correlation	Slope	Intercept
GCM2	0.0575	0.1467	0.0739	0.5779	0.6672	0.0458
Historical	0.0942	0.2340	0.1188	0.3696	0.3175	0.0996
Large firm index	0.0909	0.2358	0.1207	0.1575	0.4877	0.1027
Small firm index	0.0870	0.2312	0.1177	0.1368	0.3171	0.1061
Overall average	0.0762	0.2185	0.1058	0.0000	0.0000	0.1188
Country average	0.0685	0.1824	0.0896	0.4968	0.9030	0.0576
Industry average	0.0741	0.2124	0.1027	0.1032	0.6528	0.0795

* 95-th percentile. **Root Mean Squared Error.

Table 8

Percent of volatility increase relative to ex-post optimal portfolio

Model	Mean	Percentage increase of volatility Median	P95*	Crossing**	Risk contribution Pearson	Spearman
GCM2	10.01	9.68	15.34	0.00	0.857	0.851
Historical	12.99	12.46	19.39	0.15	0.813	0.814
Large firm index	23.79	23.72	32.41	0.00	0.714	0.670
Small firm index	29.25	28.99	38.55	0.00	0.676	0.598
Overall average	15.46	15.17	22.23	0.03	0.751	0.763
Country average	13.50	13.01	19.48	0.10	0.784	0.796
Industry average	15.01	14.75	20.87	0.02	0.765	0.772

*95 percentile.
**Percentage of number of portfolio volatilities that are higher than the corresponding ones by GCM2.

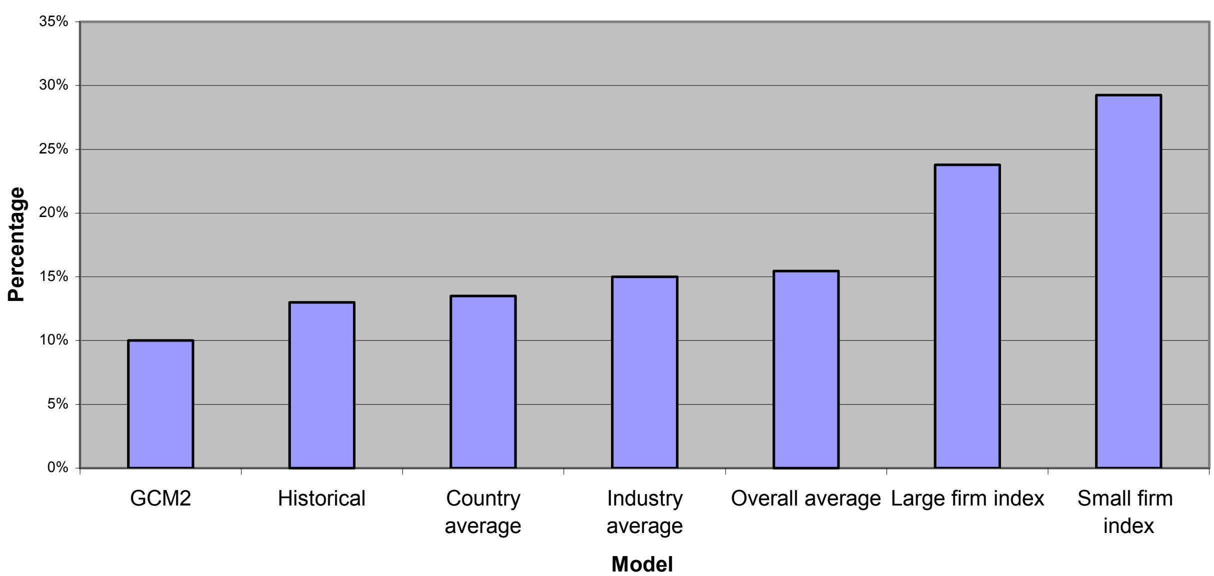

Figure 5: Mean percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios.

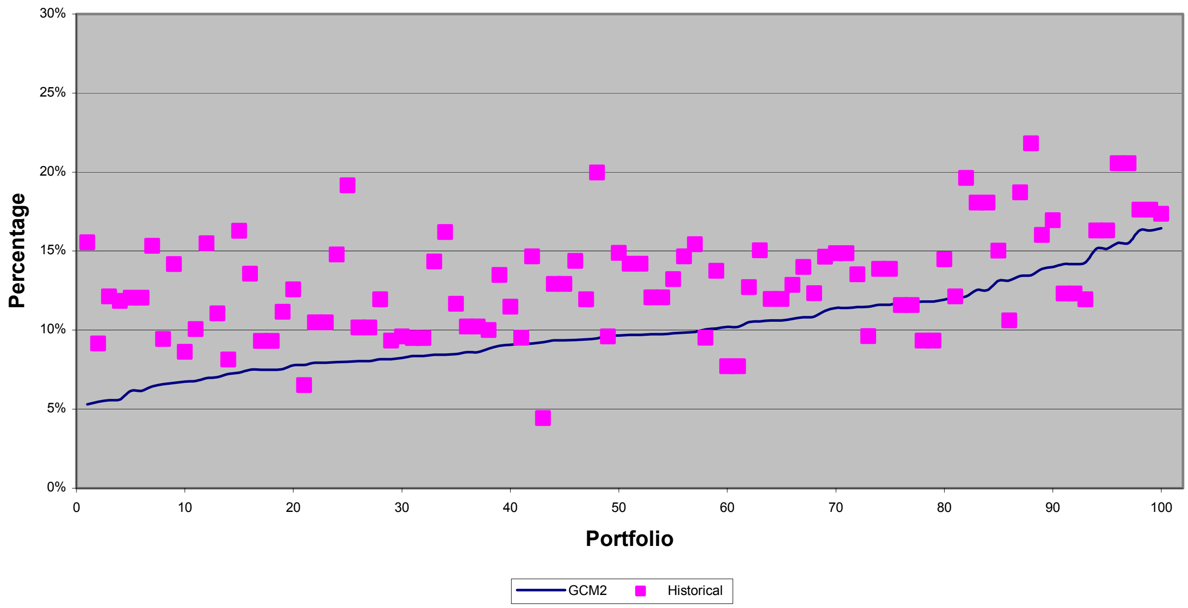

Figure 6: Percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios, GCM2 versus the Historical Model.

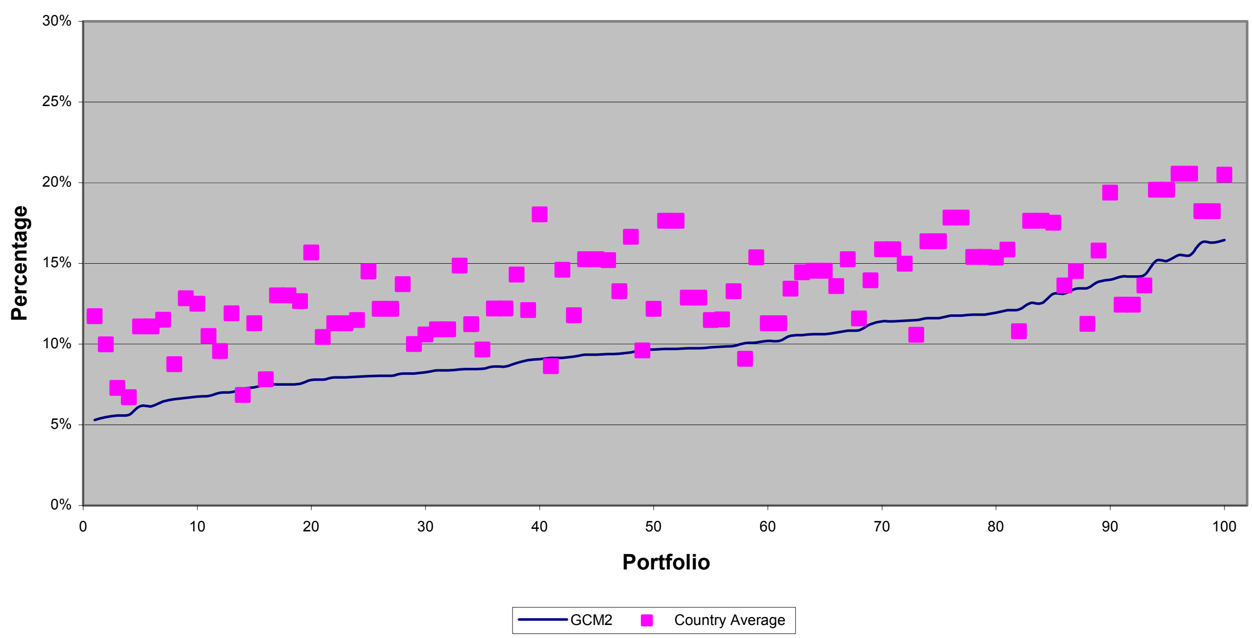

Figure 7: Percent of increase of volatilities of constructed optimal portfolios relative to the realized optimal portfolios, GCM2 versus the Country Average Model.