5 FEBRUARY 2010

MOODY'S ANALYTICS

MODELING METHODOLOGY
FROM MOODY'S KMV

Implications of PD-LGD Correlation in a Portfolio Setting

Authors

Qiang Meng
Amnon Levy
Andrew Kaplin
Yashan Wang
Zhenya (Ella) Hu

Contact Us

Americas
+1-212-553-5160
clientservices@moodys.com

Europe
+44.20.7772.5454
clientservices.emea@moodys.com

Asia (Excluding Japan)
+85 2 2916 1121
clientservices.asia@moodys.com

Japan
+81 3 5408 4100
clientservices.japan@moodys.com

Abstract

This paper discusses the implications of the Moody's Analytics PD-LGD correlation model on portfolio analysis. We provide numerical results to illustrate the impacts of PD-LGD correlation on risk and return measures of credit portfolios.

Under the PD-LGD correlation model framework, recovery is correlated with the firm's underlying asset process via both systematic factors and idiosyncratic shocks. PD-LGD correlation introduces additional variability into instrument value and portfolio value distributions. At the instrument level, value distribution becomes more dispersed under the PD-LGD correlation model, since a good credit state is not only associated with a low default probability, but also with a high expected recovery amount. The opposite is true for a bad credit state.

At the portfolio level, the values of defaulted instruments are correlated with systematic factors. An implication is that during an economic downturn, not only will the number of defaults be higher, defaulted instruments tend to realize a lower recovery amount as well. As a consequence, portfolio value distribution will have a heavier tail, resulting in higher risk measures (e.g., Unexpected Loss and capital) when accounting for PD-LGD correlation. Spreads will also widen to compensate for the increased risk. As a real-world example, PD-LGD correlation increases the capitalization rate of the International Association of Credit Portfolio Managers (IACPM) portfolio from 5.24% to 7.23%, a relative increase of 37.8%.

For comparison purposes, this paper also provides results from a stressed LGD model and illustrates that the downturn LGD recommended in Basel II may not be conservative enough to compute the capital amount associated with 10bp target probability.

Table of Contents

1 Overview

Overwhelming evidence shows that recovery in a default event is closely related to macroeconomic conditions.[1] Generally speaking, recovery is pro-cyclical: during a recessionary period, recovery tends to be lower than during an expansionary period. According to a Moody's Special Comment and shown in Figure 1, during 1983–2001, the regression R-squared of the average recovery rate of corporate bonds against the value-weighted average annual default rate is above 50%. This positive correlation between recovery and general economic conditions can be rationalized using a demand and supply argument—when a firm is in financial distress and needs to sell assets, its industry peers are likely to experience problems as well. This implies that more distressed assets are available in the market, but fewer firms are able or willing to buy those assets. This supply increase and demand decrease drives the price of the distressed assets below their value in best use.

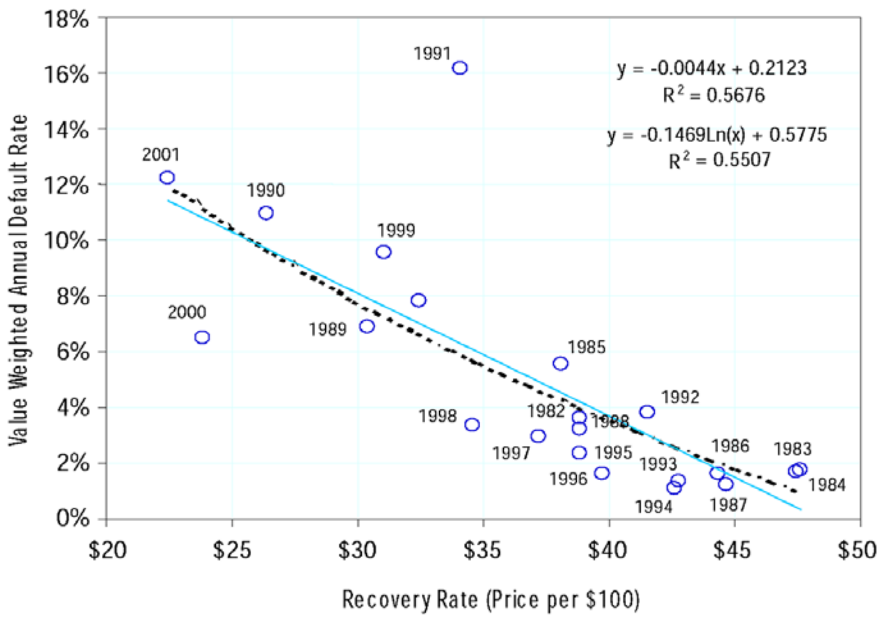

Figure 1 Annual speculative-grade default rate and recovery rate

Source: Default & Recovery Rates of Corporate Bond Issuers, Moody's Special Comment, February 2003.

The recent financial crisis provides additional convincing evidence of the existence and impact of PD-LGD correlation (PLC). According to Keisman and Marshella (2009), during the first half of 2009, the default rate on high-yield bonds reached a record-high level, and recovery rates fell to their lowest level in history. Ignoring the correlation between PD and LGD underestimates the risk in a credit portfolio. Solutions to this issue typically fall into one of two categories:

1. Stressed LGD models, which replace regular LGD by a "stressed" LGD to reflect the adverse economic condition.
2. PD-LGD correlation models, which expand the existing correlation structure to include PD-LGD correlation.

The first approach is relatively inexpensive and straightforward. It only changes the value of input parameters and requires no modification to the model. Examples of stressed LGD models include: the downturn LGD model, recommended by the Basel Committee for regulatory capital calculation; and the stressed LGD model, included in

[1] Altman (2006) provides a review of literature and empirical evidence.

Moody's Analytics LossCalc™. The stressed LGD is usually estimated using default data covering at least one complete economic cycle. For instance, the stressed LGD in LossCalc reflects the worst macroeconomic situation during approximately the past 25 years.

In contrast to stressed LGD models, PD-LGD correlation models directly modify the correlation structure and will account for the possible benefits from diversification in collateral. Moreover, PD-LGD correlation models differentiate an extreme event from a relatively moderate event. In the context of economic capital computation, different target probabilities will be associated with different levels of LGD values. Major challenges associated with a PD-LGD correlation model include estimating the correlation between PD and LGD as well as the correlation between LGD and systematic factors, and conducting proper valuation and portfolio analysis when PD-LGD correlation is accounted for.

The two model categories that account for PD-LGD correlation have been developed for the same purposes and thus have similar impacts. In brief, compared to a model in which systematic risk in recovery is unaccounted for, both PD-LGD correlation and stressed LGD will lead to a decrease in MTM value, and an increase in spreads, Unexpected Loss (UL), and capital. Miu and Ozdemir (2005) analyzed a loan portfolio and showed that the impact of PD-LGD correlation on capital amount is equivalent to that of an increase in expected LGD of about 35% to 41%. This finding suggests that it is possible to use the two model types in a complementary manner. For instance, the Basel Committee (2005) specifies that the downturn LGD "may be derived from forecasts based on stressing appropriate risk drivers." PD-LGD correlation models facilitate the estimate of stressed LGD associated with different stress scenarios and different target probabilities, as shown later in this paper. Moreover, stressed LGD values can be used as a conservative measure of LGD and combined with PD-LGD correlation. This style of parameterization fits into the stress testing practices of many institutions.

RiskFrontier® accounts for PD-LGD correlation in credit portfolio analysis. The model utilizes LossCalc and Moody's Analytics Global Correlation Model™ (GCorr) data to overcome challenges in parameterization. With this functionality, users can construct an integrated correlation structure that models not only correlation between asset returns, but also correlation between LGD and asset return of the borrower, and the correlation between LGD values of different instruments. The model accounts for the impact on portfolio risk associated with the compounding effects of severe losses during periods when defaults are concentrated. In addition, it accounts for the impact of systematic risk in recovery on valuation and returns. The integrated correlation structure provides portfolio managers the ability to minimize recovery risk in their portfolio through diversification.

This paper analyzes the implications of Moody's Analytics PD-LGD correlation model, and provides numerical results to illustrate the impacts of PD-LGD correlation on risk and return measures of credit portfolios. For comparison purposes, it also presents some results from a stressed LGD model. This paper is organized in the following way

- Section 2 provides an introduction to the modeling framework and parameter estimate[2].
- Section 3 analyzes the impact of PD-LGD correlation on valuation and portfolio analysis.
- Section 4 discusses implication of the model with a focus on how risk and return measures behave under different PD-LGD correlation assumptions; Sections 3 and 4 also provide a comparison between the PD-LGD correlation model and the stressed LGD model.
- Section 5 summarizes the major findings and conclusions.

2 Modeling Framework

Recovery uncertainty is usually modeled using a probability distribution. For example, in RiskFrontier, LGD is modeled as a Beta distribution. A Beta distribution is governed by two parameters, LGD and *k*, which determine the mean and variance of recovery, respectively. If PD-LGD correlation is not accounted for, the Beta distribution for an instrument is independent of the associated asset return. Moreover, macroeconomic factors are also irrelevant in terms of recovery amount estimation. In contrast, the Moody's Analytics' PD-LGD correlation model explicitly specifies the relationships between recovery and asset value, and between recovery and macroeconomic factors.

[2] For more details, refer to *Incorporating Systematic Risk in Recovery: Theory and Evidence*, Levy and Hu (2007), and *Modeling Credit Portfolios*, Chapter 14.

In this modeling framework, the returns on the asset process (A_t) and on the collateral value process ($RR_{T,t}$ which is the basis for recovery upon default) are modeled as two correlated Brownian Motion processes:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dr_{A,t},$$
$$\frac{dRR_{T,t}}{RR_{T,t}} = \mu_{RR_T} dt + \sigma_{RR_T} dr_{RR_T,t}. \tag{1}$$

The default event is defined as the situation when the asset value hits a default threshold. The subscript T of the collateral value process denotes default time. As such, there is a collateral value process $RR_{T,t}$ associated with each possible default time T. The drift rates of the return on A_t and $RR_{T,t}$ are μ_A and μ_{RR_T}, and the diffusion rates are σ_A and σ_{RR_T}, respectively. Moreover, the instantaneous correlation between $r_{A,t}$ and $r_{RR_T,t}$ is assumed to be equal to $\rho_{A,RR}$. The collateral value $RR_{T,t}$ is the estimated or appraised value of the pledged collateral associated with a specific default time T.

The random portion, $r_{A,t}$ and $r_{RR_T,t}$, of the two processes are determined by macroeconomic factors (denoted by ϕ_A and ϕ_{RR}) as wells as firm-specific idiosyncratic shocks (denoted by ε_A and ε_{RR}). The model allows ϕ_{RR} to be different from ϕ_A. This is evident in the case of secured debts whose pledged collateral is typically not the total asset of the borrower. Even for unsecured debts, the collateral value may depend mostly on tangible assets, and is thus not influenced by the same macroeconomic factors as the total asset value. Let R_A^2 and R_{RR}^2 represent the R-squared for asset and collateral return processes, then $r_{A,t}$ and $r_{RR_T,t}$ can be written as the following:

$$r_A = R_A \phi_A + \sqrt{1-R_A^2}\cdot \varepsilon_A$$
$$r_{RR} = R_{RR}\cdot \phi_{RR} + \sqrt{1-R_{RR}^2}\cdot\left[\rho_\varepsilon \varepsilon_A + \sqrt{1-\rho_\varepsilon^2}\cdot \varepsilon_{RR}\right] \tag{2}$$

The subscripts t and T have been dropped for simplicity. Notice that the asset idiosyncratic shock ε_A also has an impact on the recovery process. That being said, the asset and collateral value processes are correlated through macroeconomic factors as well as firm-specific shocks. Mathematically, $\rho_{A,RR}$ can be written as the sum of a systematic component and an idiosyncratic component:

$$\rho_{A,RR} = R_A \cdot R_{RR} \cdot \mathrm{Corr}\left(\phi_A, \phi_{RR}\right) + \sqrt{1-R_A^2}\cdot\sqrt{1-R_{RR}^2}\cdot \rho_\varepsilon \tag{3}$$

The modeling framework introduces several new parameters associated with PD-LGD correlation: $\rho_{A,RR}$, R_{RR}, and ρ_ε. As these three parameters satisfy equation (3) only two need to be estimated. The choice of parameters to be estimated depends on the availability of data. Usually it is very difficult to obtain time series of idiosyncratic shocks, and thus difficult to estimate ρ_ε directly. As such, in our model, $\rho_{A,RR}$ and R_{RR} are provided by users and ρ_ε is calculated using equation (3). Notice that the parameter σ_{RR} in equation (1) represents the volatility of the collateral value process and can be inferred from the LGD and k parameters of the user-specified Beta distributions.

Using LossCalc and GCorr data, Moody's Analytics provides estimates for $\rho_{A,RR}$ and R_{RR}. The recovery data from LossCalc includes 1,424 defaulted public and private firms. Firm size (measured as total sales in annual report prior to default) ranges from zero to $US 48 billion. The recovery model in LossCalc is based on 3,026 global observations of market prices for loans, bonds, and preferred stock one month after default, from 1981–2004. Estimated parameters are available for firms in North America, the UK, Europe, Asia, and Latin America. If a direct estimate is not available, an alternative estimation approach is provided, based on the firm's size and country/industry weights.

The modeling framework discussed in this section provides portfolio managers with the ability to account for PD-LGD correlation when valuing credit instruments and analyzing credit portfolios. Within the context of RiskFrontier methodology, Section 3 discusses how to conduct valuation and simulation when LGD is correlated with PD and macroeconomic factors.

3 Valuation and Simulation with PD-LGD Correlation

Credit risky instruments can be valued using risk-neutral valuation techniques. Typically, the systematic risk associated with default probability is accounted for using a risk adjustment to instrument value. However, before the PD-LGD models were developed, risk adjustment was not usually associated with recovery. This methodology resulted from the assumption that there is no systematic risk in recovery. In a stressed LGD model, the MTM value is reduced due to the increase in LGD. Nevertheless, there is no guarantee that the amount of decrease is in-line with the systematic risk in recovery. As such, it may not be appropriate to use stressed LGD models for valuation purposes. In contrast, the Moody's Analytics PD-LGD correlation model provides a granular framework that computes value and risk in an internally consistent manner. Specifically, in the modeling framework described in Section 2, a discount factor is introduced to compensate for the systematic risk in recovery. Levy and Hu (2006) showed that the expected recovery under risk-neutral and physical measure has the following relationship:

$$E_0^Q[RR_T \mid A_T = DP_T] = E_0^P[RR_T \mid A_T = DP_T] \cdot \alpha_{0,T} \tag{4}$$

where the factor $\alpha_{0,T}$ associated with recovery risk is given by the following equation:

$$\alpha_{0,T} = e^{(-R_{RR}\lambda_{RR} + \rho_{r_{RR}} R_A \lambda_A)\sigma_{RR_T} T} \tag{5}$$

Typically $\alpha_{0,T}$ is no larger than one, thus equation (4) is analogous to the transform of PD from physical measure to risk-neutral measure.

The MTM value of a credit instrument will decrease due to the additional discount of recovery. More generally, the expected value at any time after the analysis date, and in particular the expected value at horizon, will also drop. The expected spread will become wider to compensate for the systematic risk in recovery.

The positive correlation between PD and LGD will also change the shape of value distribution at horizon. In particular, PD-LGD correlation increases the variation of value given non-default. When PD and LGD are assumed to be independent, the expected recovery amounts from a future default event are identical regardless of the credit quality of the instrument at horizon. If we assume that PD and LGD are positively correlated, this correlation will drive expected recovery up for good credit states and down for bad credit states. This pattern, within the context of the lattice model, is illustrated in Figure 2. In the upper panel, the same expected recovery value is assigned to both the good state (node A) and the bad credit state (node B). In the lower panel, the recovery value associated with state A is higher than that associated with state B, resulting in an increase of value for node A and decrease of value for node B. The standard deviation of non-default value distribution thus becomes higher.

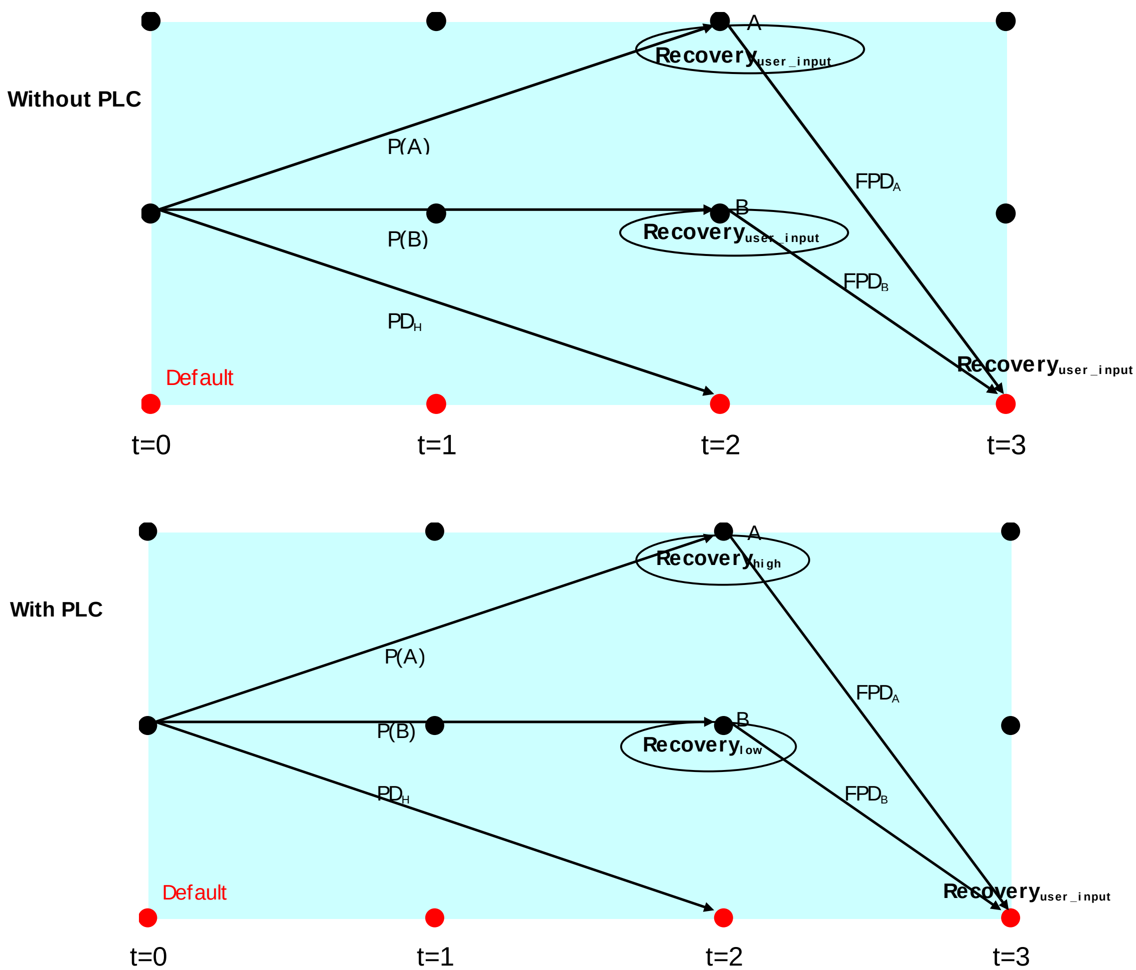

Figure 2 Impacts of PLC on Lattice Valuation

To better understand valuation with PD-LGD correlation, consider the following example:

- A zero-coupon bond matures at time t=3.
- Horizon of analysis, t=2.
- Assume that recovery at t=3 follows a beta distribution with mean recovery=0.5 and k parameter=4.
- The risk-free rate of return is assumed to be 0 for simplicity, market Sharpe ratio is set to be 0.4, for both asset return and recovery.
- The forward PD (under physical measure) associated with nodes A and B are FPD_A and FPD_B (the values are not needed for the calculation). The forward risk-neutral PD values are assumed to be $FQPD_A$=1% and $FQPD_B$=5%, respectively.
- The physical PD and transition probabilities to horizon are set to be PD_H=5%, $P(A)$=65%, and $P(B)$=30%, respectively. The corresponding probabilities under risk-neutral measure are QD_H=7.43%, $Q(A)$=57.35%, and $Q(B)$=35.22%, respectively.[3]

[3] For the conversion formula between physical and risk-neutral probabilities, see *Modeling Credit Portfolios*, Chapter 4.

- Furthermore, suppose the instrument has typical correlation parameters, e.g., $\rho_{A,RR}$=0.4, $R_{RR}^2 = 0.3$, and $R_A^2 = 0.25$.

It can be calculated that $\alpha_{2,3} = 0.9308$, Recovery_{low} = 0.4622, and Recovery_{high} = 0.6161.[4] Notice that the risk neutral probability-weighted average of Recovery_{low} and Recovery_{high} equals the user-input expected recovery:

$$\frac{Q(A)\cdot FQPD_A \cdot \text{Recovery}_{high} + Q(B)\cdot FQPD_B \cdot \text{Recovery}_{low}}{Q(A)\cdot FQPD_A + Q(B)\cdot FQPD_B}$$

$$= \frac{57.35\% \cdot 1\% \cdot 0.6127 + 35.22\% \cdot 5\% \cdot 0.4622}{57.35\% \cdot 1\% + 35.22\% \cdot 5\%} = 0.5 \qquad (6)$$

The value associated with each credit state can be calculated as shown in Table 1.

Table 1 Value associated with each credit state

Credit state	Transition probability	Value (without PLC)	Value (with PLC)
A	0.3	99.5	99.57
B	0.65	97.5	97.15
Default	0.05	50	50

Using the results in the table, after PD-LGD correlation is accounted for it can be further calculated that:

- The expected value at horizon $E[V_H]$ drops from 95.725 to 95.519.
- The standard deviation of value, given non-default, rises from 0.93 to 1.12.
- Transition probabilities and forward default probabilities remained unchanged.

Figure 3 shows the impact of PD-LGD correlation on the distribution of value, given non-default.

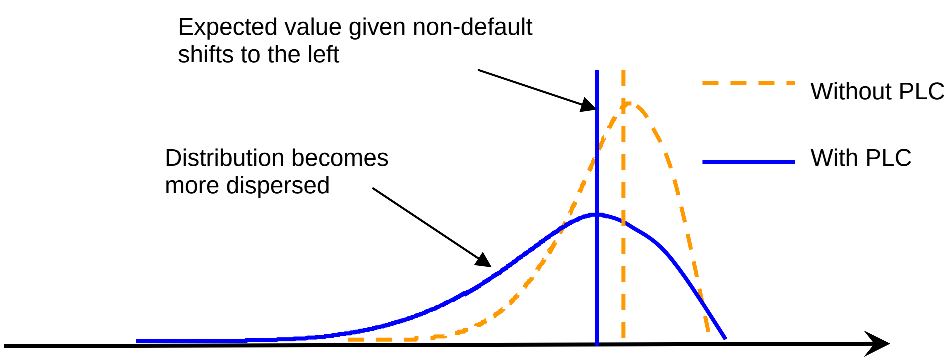

Figure 3 Impacts of PLC on non-defaulted value distribution

[4] For details regarding the actual calculation of $\alpha_{0,H}$, Recovery_{low}, and Recovery_{high}, refer to *Modeling Credit Portfolios*, Chapter 14.

PD-LGD correlation not only impacts the behavior of each individual instrument, more importantly, it impacts the joint behavior of multiple instruments, especially when they are in default. In the context of Monte Carlo simulation in RiskFrontier, when PD-LGD correlation is not accounted for in the model, the LGD of a defaulted instrument will be drawn from the user-specified Beta distribution regardless of factor realization in that trial. Furthermore, if multiple instruments are in default, the LGD values of different instruments are independent of one another.[5] Under the PLC model, the random LGD in each trial is drawn from a particular conditional distribution that is determined by the realization of systematic factors and idiosyncratic shocks of asset return, shown in equation (2).

In general, recovery tends to be higher when systematic factors realize larger (i.e., less negative) values. This positive dependence of recovery on systematic factors has two consequences:

- LGD values of the defaulted instruments are positively correlated, since they are all positively correlated with systematic factors.
- LGD values tend to be high when the number of defaults is large, which usually indicates an economic recession.

These two effects jointly lead to a fatter tail of the value distribution. In the numerical example provided in Section 4, we compare the portfolio value distributions produced by the regular LGD model and the PD-LGD correlation model. The dependency of recovery on macroeconomic condition is illustrated in Figure 4.

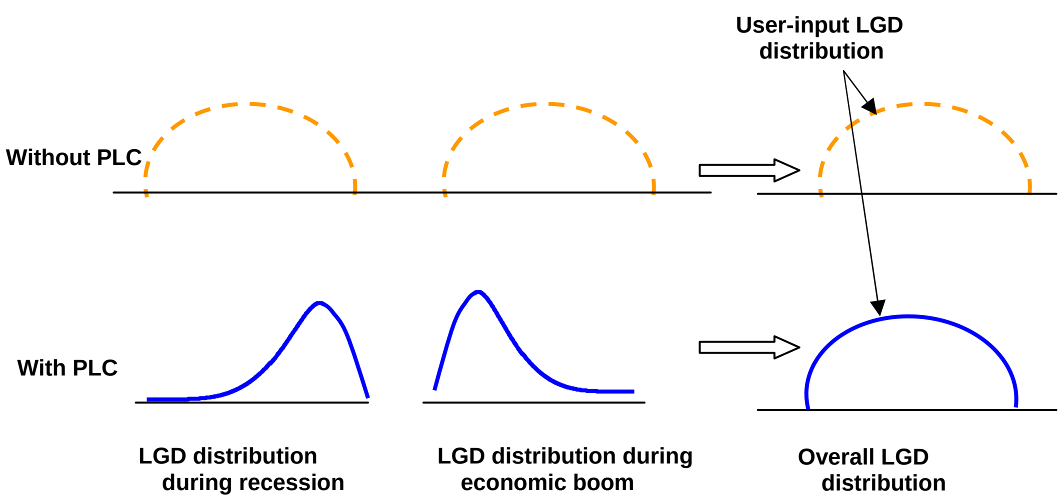

Figure 4 Impacts of PLC on defaulted value distribution

To understand how PD and LGD are correlated in the modeling framework, we explicitly calculated the value of PD and LGD associated with different factor realizations for a homogeneous portfolio consisting of infinitely many instruments in a single-factor model. Under this setting, the effects of idiosyncratic shocks will offset each other, and the number of defaults and average recovery purely depend upon the common factor. Figure 5 shows how the number of defaults changes as the common factor realizes different values. Figure 5 also illustrates the behavior of average LGD (of all defaulted instruments) as a function of the common factor under three different model assumptions: the regular LGD without accounting for PD-LGD correlation, the PD-LGD correlation model, and the stressed LGD model. The regular LGD and stressed LGD used in this exercise are obtained from LossCalc for senior unsecured debts issued by a U.S. firm with PD=1%. The PD-LGD correlation parameters $\rho_{A,RR}$ and recovery R-squared are set to be 0.4 and 0.34, respectively.

[5] In RiskFrontier, users are allowed to assign the same LGD draw ID to different instruments in which case these instruments will have correlated recoveries.

The graph shows that under the PD-LGD correlation model, average LGD and PD are positively correlated, as they both increase when the common factor becomes more negative. Under the other two models, average LGD is constant, with stressed LGD higher than regular LGD, indicating that stressed LGD is more conservative than regular LGD, but neither one reacts to the change in economic condition.

We estimate the PD-LGD correlation parameters and the LossCalc stressed LGD using the same data set—the time series of LGD since 1981—so it is possible to relate the results from the two models. Specifically, the LossCalc stressed LGD is associated with the worst scenario since 1981(i.e., a 1-in-25 year downturn), therefore, it should be comparable to the conditional LGD associated with target probability 4% (or -1.75 in terms of factor realization) from the PD-LGD model. Our example confirms this statement: LossCalc stressed LGD is 0.579 and conditional LGD associated with a -1.75 factor realization is 0.5882.

This exercise suggests that the PD-LGD correlation model can be considered a generalized stressed LGD model; the PD-LGD correlation model describes the behavior of LGD in any scenario. On the other hand, one may back out the PD-LGD correlation parameters from a collection of stressed LGD values associated with various downturn scenarios.

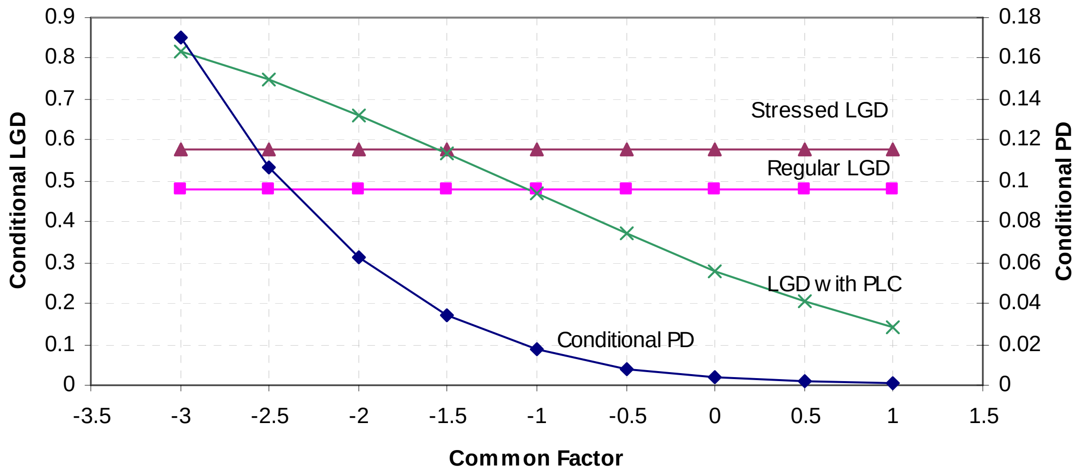

Figure 5 Impacts of PLC on Lattice Valuation

Table 2 summarizes the impacts of the PD-LGD model on MTM value and the distribution of non-default and default value.

Table 2 Impacts of the PD-LGD Model

	Without PLC	With PLC
MTM value	Recovery has no systematic risk and the recovery amount is discounted at a risk-free rate.	Recovery has systematic risk and hence will be discounted more heavily, or equivalently, the expected recovery amount under risk neutral measure will be lower than that under physical measure (equation (4)). As a result, MTM will be lower when PLC is taken into consideration.
Non-default value distribution at horizon	The expected future recovery doesn't depend on the credit state at horizon.	Higher future recovery is expected for better credit state at horizon. Therefore, PLC increases instrument value (due to higher expected future recovery) for good credit states and reduces instrument value (due to lower expected future recovery) for bad credit states, and consequently increases the variation of non-default values. Meanwhile, PLC decreases expected value given non-default by discounting future recovery amount more heavily.
Default value distribution at horizon	Distribution of recovery is independent of macroeconomic condition. Recovery rates of different instruments are independent.	Recovery is likely to be higher during economic boom and lower during recession. As such, when there are many defaults (during recession), the LGD values tend to be higher. Thus, the joint value distribution of defaulted instruments has a fatter tail.

4 Impact on Portfolio Risk and Return

As PD-LGD correlation introduces an additional source of uncertainty to credit portfolios, intuitively it will lead to an increase in portfolio risk. Portfolio return must be higher to compensate for the increased risk. This section provides the results from extensive numerical tests, illustrating the implication of the PD-LGD correlation model on risk and return measures such as MTM value, Expected Spread (ES), UL, and Economic Capital.

4.1 Impact of PLC on a Homogeneous Portfolio

We conduct the first numerical experiment using a homogeneous portfolio consisting of 1,000 term loans. To facilitate comparison between a PD-LGD correlation model and a stressed LGD model, the term loans are parameterized to be in line with those used in the stressed LGD-conditional LGD comparison in Section 3 (see Figure 5). In particular, each loan is assumed to have maturity of three years, PD of 1%, and asset R-squared of 0.25. The regular LGD and stressed LGD are obtained from LossCalc. The two PD-LGD correlation parameters, $\rho_{A,RR}$ and recovery R-squared, are set to be 0.4 and 0.34, respectively.

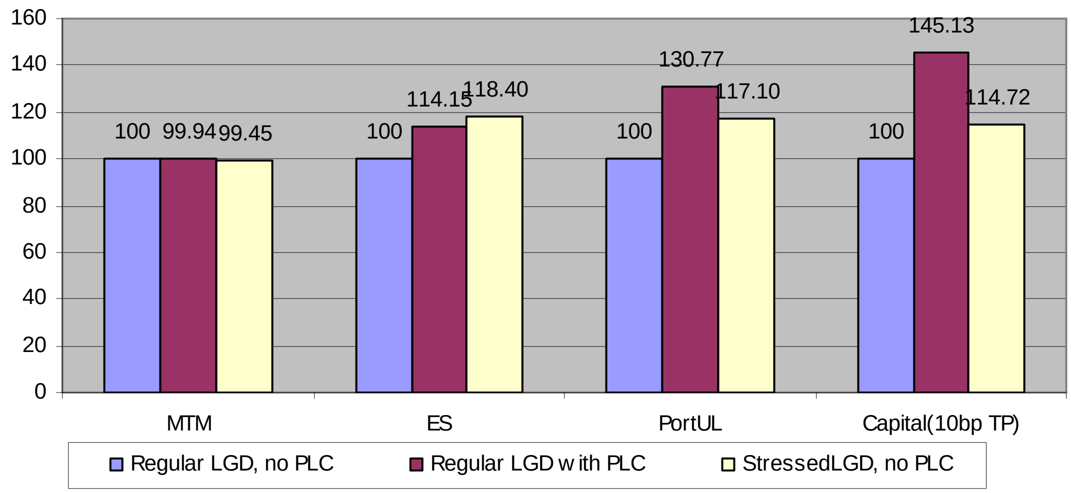

Figure 6 Impacts of PLC and stressed LGD on risk and return measures

Figure 6 shows the change in risk and return measures before and after accounting for PD-LGD correlation. The capital number is associated with a target probability of 10bp. The results are presented as a percentage of values when PD-LGD correlation is not accounted for. As expected, with PD-LGD correlation, MTM value of the portfolio becomes lower, while the ES, UL, and capital amounts are all higher. Furthermore, the relative change in capital (45.13% increase) is the largest among all risk and return measures, indicating that the impact of PD-LGD correlation is more significant in the tail region where the number of defaults is high.

We also present the results from the stressed LGD model. We obtain the value of stressed LGD used in the analysis from LossCalc. In Section 3 we showed that this stressed LGD value is in-line with a target probability of around 4%. As such, the stressed LGD value corresponds to a scenario that is much less severe than the extreme situation associated with capital computation (a 4% event compared to a 0.1% event). As a consequence, the capital amount obtained from the stressed LGD model is much lower than the amount obtained from the PD-LGD correlation model. This finding implies that the downturn LGD recommended in Basel II may not be conservative enough for the purpose of computing the capital amount associated with 10bp target probability, because Basel II downturn LGD is rarely estimated based on data of more than 25 years.[6]

[6] Basel II requires a minimum data observation period that covers at least one complete economic cycle and is no shorter than seven years.

The results presented in this section together with the exercise in Section 3 suggest a possible application of a PD-LGD correlation model in regulatory capital calculation. In particular, one can parameterize downturn LGD values associated with different scenarios or different target probabilities.

4.2 Impact of PLC Under Different Parameterization

The graph in Figure 6 shows a fairly high increase in capital due to PD-LGD correlation. Conversely, in the extreme case where PD is close to zero, LGD (and, hence, PD-LGD correlation) will not be a major factor in portfolio analysis. This suggests that the impact of PD-LGD correlation depends on the characteristics of the portfolio. Besides PD, asset R-squared is another important factor that influences the impact of PD-LGD correlation. High asset R-squared, combined with high recovery R-squared, implies the number of defaults and LGD are more likely to increase at the same time, namely, during economic downturn. Other parameters, such as maturity, mean LGD, and the *k* parameter, can also affect impacts of PD-LGD correlation. Typically, PD-LGD correlation has more significant impacts on a portfolio with higher PD, longer time to maturity, higher R-squared, and higher variance of LGD.

Figure 7 illustrates the impacts of PD-LGD correlation with different PD values. The test portfolio used here is similar to the one used in Section 4.1 except that the mean LGD is set to be 0.4 instead of using LossCalc LGD. Similar numerical tests have been conducted for maturity, asset R-squared, and average LGD. The patterns of PLC impacts with respect to these parameters are similar to that shown in Figure 7. As a numerical example, the percentage changes in risk and return measures resulted from taking account of PLC for the portfolio with PD=1% are as follows: PLC leads to a 0.06% decrease in MTM value, a 16.1% increase in ES, a 36.0% increase in portfolio UL, and a 52.6% increase in portfolio capital.

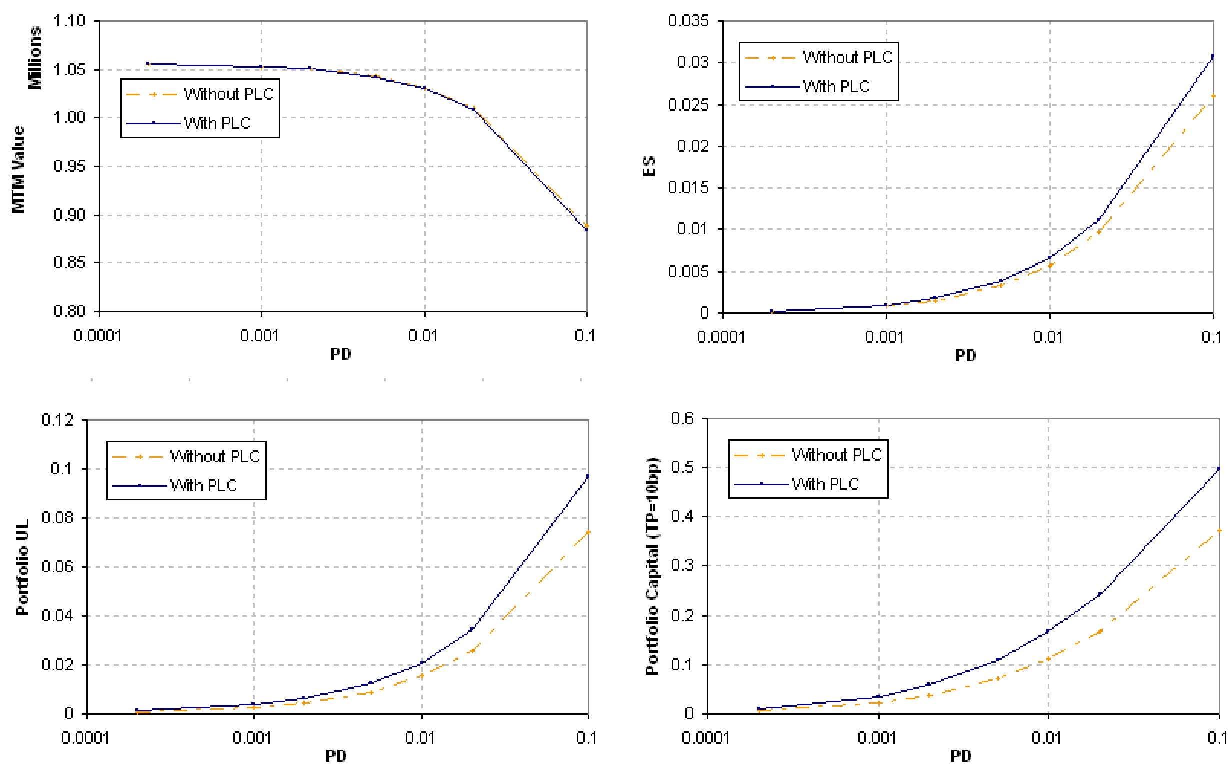

Figure 7 Impacts of PLC under different PD

4.3 Impact of PLC on the IACPM / ISDA Portfolio

To understand better how PD-LGD correlation impacts a portfolio and portfolio-referent risk, we conduct tests on the IACPM / ISDA portfolio, which consists of 6,000 term loans from 3,000 obligors across seven countries and 61 industries. This portfolio has been used for the purpose of comparing different economic capital models. For more details, we refer to Convergence of Credit Capital Models by the International Association of Credit Portfolio Managers (IACPM).

Table 3 shows a comparison of risk and return measures before and after accounting for PD-LGD correlation. In particular, the capitalization rate changes from 5.24% to 7.23%, a relative increase of 37.8%.

Table 3 Comparison of risk and return measures before and after PD-LGD correlation

	With PLC	Without PLC	Diff%
MTM Value	99,821,638,404	99,936,639,631	-0.1%
Expected Spread	773,709,812	645,677,105	19.8%
Unexpected Loss	1,393,631,998	1,169,939,275	19.1%
Capital	7,216,441,832	5,238,672,469	37.8%

As shown in Figure 6, the percentage change in capital is much higher than the change in any other risk and return measure, implying that PD-LGD correlation impacts the tail region more heavily. This finding is evident from a comparison between value distribution before and after PLC is accounted for, using the IACPM / ISDA portfolio, shown in Figure 8. The figure shows that PD-LGD correlation impacts the overall portfolio distribution in a manner similar to the way it impacts the non-default value distribution (see Figure 3). In particular, the value distribution when PLC is accounted for has a lower peak and fatter tail.

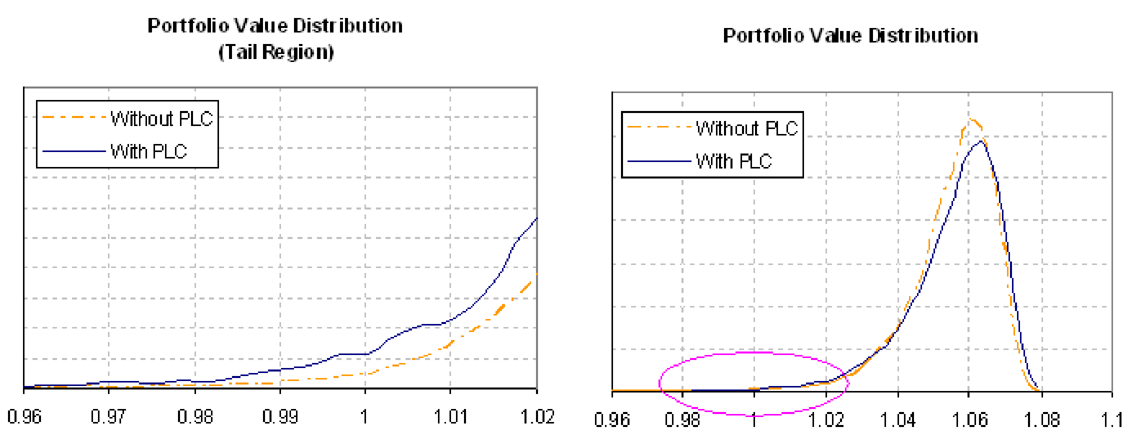

Figure 8 Impacts of PLC on portfolio value distribution

It is worth examining how PD-LGD correlation impacts capital amount. As shown in Figure 5, under the PD-LGD correlation model, higher LGD is associated with more extreme events. Therefore, the impact on capital depends on the target probability for capital calculation. The 37.8% difference in the above table is associated with a target probability of 10bp. The amount of increase will be lower as the target probability becomes larger. This trend can be seen from the cumulative portfolio value distribution.

Figure 9 shows the cumulative distribution curves before and after accounting for PD-LGD correlation. At target probability 5bp, PD-LGD correlation results in a decrease of $2.45 billion (approximately 2.45% of MTM value) while at target probability 50bp, the amount of decrease due to PLC shrinks to $1.3 billion (approximately1.3% of MTM value). This difference results from the much lower conditional LGD (although still higher than regular LGD) associated with 50bp target probability than that associated with 5bp target probability.

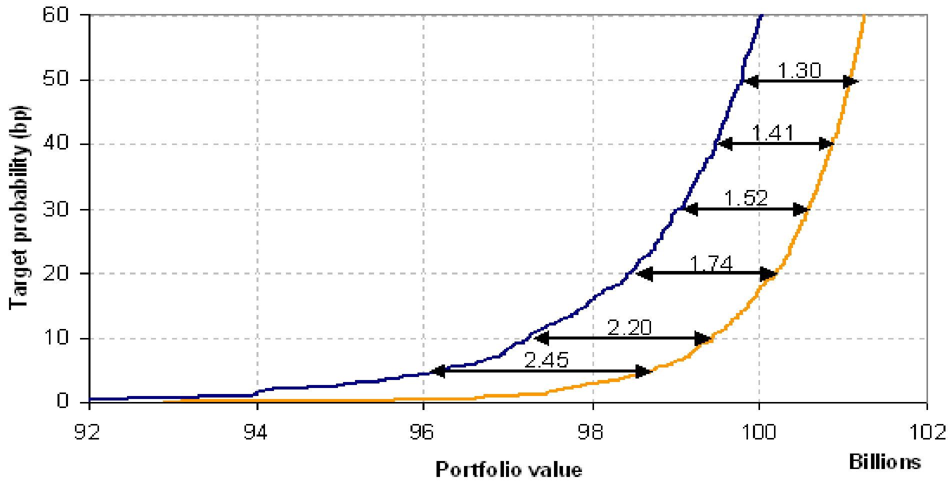

Figure 9 Portfolio value difference vs. target probability

5 Conclusion

The correlation between PD and LGD can be accounted for by using either a more conservative value for LGD (i.e., a downturn or stressed LGD), or by extending the correlation structure and explicitly modeling the PD-LGD correlation. This paper presents results from Moody's Analytics' PD-LGD correlation model. The model provides a granular framework that computes risk and value in a consistent manner. The model utilizes Moody's Analytics LossCalc and GCorr data to overcome key challenges in parameterization. Numerical tests show that portfolio value decreases and risk, such as UL and capital, increases after accounting for PD-LGD correlation. The impacts of PD-LGD correlation depend upon portfolio characteristics such as PD, maturity, and asset R-squared. Furthermore, PD-LGD correlation has a more significant impact in the tail region. For the IACPM / ISDA portfolio, PD-LGD correlation leads to a 37.8% increase in capital.

In this study, we also compare outputs from the PD-LGD correlation model and the LossCalc stressed LGD model. The two models imply very similar stressed LGD values under standard parameterization. The PD-LGD model is a potential application for deriving stressed LGD values associated with different scenarios or different target probabilities.

Our study also shows that the downturn LGD recommended in Basel II may not be conservative enough to compute the capital amount associated with 10bp target probability.

References

Altman, Edward, "Default Recovery Rates and LGD in Credit Risk Modeling and Practice: An Updated Review of the Literature and Empirical Evidence," New York University Salomon Center, Stern School of Business, 2006. http://pages.stern.nyu.edu/~ealtman/UpdatedReviewofLiterature.pdf

Basel Committee on Banking Supervision, "International Convergence of Capital Measurement and Capital Standards: A Revised Framework," 2004. http://www.bis.org/publ/bcbs107.htm.

Basel Committee on Banking Supervision, "Guidance on Paragraph 468 of the Framework Document," 2005. http://www.bis.org/publ/bcbs115.pdf.

Dwyer, Douglas and Irina Korablev, "Moody's KMV LossCalc™ v3.0: Modeling Methodology," 2009.

International Association of Credit Portfolio Managers (IACPM) and International Swaps and Derivatives Association (ISDA), "Convergence of Credit Capital Models," 2006.

Keisman, David and Tom Marshella, "Recoveries on Defaulted Debt in an Era of Black Swans," Moody's Global Corporate Finance, Moody's Investors Service, 2009.

Levy, Amnon and Zhenya Hu, "Incorporating Systematic Risk in Recovery: Theory and Evidence," white paper, Moody's KMV, 2007.

Miu, Peter and Bogie Ozdemir, "Basel Requirement of Downturn LGD: Modeling and Estimating PD & LGD Correlations," Journal of Credit Risk, Vol. 2, No. 2, pp.43-68, 2006.

Moody's KMV Portfolio Research, "Modeling Credit Portfolios: RiskFrontier® Methodology," 2009.

Acknowledgements

The authors wish to thank Douglas Dwyer, Davide Cis, Joel Reneby, and Henry Lam for their comments and suggestions.