USING ASSET VALUES AND ASSET RETURNS FOR ESTIMATING CORRELATIONS

MODELING METHODOLOGY

ABSTRACT

In the Moody’s KMV Vasicek-Kealhofer (VK) model, asset values and asset returns are calculated separately. Moody’s KMV GCorr™ uses weekly asset returns directly from the VK model to calculate asset correlations. As an alternative, asset returns estimated from monthly asset values from Credit Monitor® can be used to estimate asset correlations. This study shows that the asset returns backed out from asset values are vulnerable to capital structure changes and other corporate activities, especially for financial firms. The frequent capital structure changes in financial firms make their correlations from asset values much smaller than the correlations from VK asset returns. Moreover, it is demonstrated that confidence intervals for correlation estimates from three years of monthly returns are much wider than correlation estimates from three years of weekly VK asset returns.

AUTHORS

Fanlin Zhu
Brian Dvorak
Amnon Levy
Jing Zhang
# TABLE OF CONTENTS

1  INTRODUCTION ........................................................................................................ 5

2  ASSET VALUE AND ASSET RETURN IN THE VASICEK-KEALHOFER MODEL ........ 5

3  CASE STUDY ............................................................................................................. 6
   3.1  Ford Motor ......................................................................................................... 6
   3.2  General Motors .................................................................................................. 7
   3.3  Correlation Between Ford and General Motors ............................................... 9

4  CROSS-SECTIONAL STUDY ............................................................................... 11

5  CONFIDENCE INTERVAL OF SAMPLE CORRELATION ...................................... 12

6  CONCLUSION ....................................................................................................... 12

APPENDIX A .................................................................................................................. 13
INTRODUCTION

In the Moody’s KMV Vasicek-Kealhofer (VK) model, asset values and asset returns are calculated separately. Moody’s KMV Global Correlation Model (GCorr) uses weekly asset returns directly from the VK model to compute asset correlations. As an alternative, one can potentially back-out a measure of asset returns from monthly asset values in Credit Monitor and use these monthly asset returns to calculate asset correlations. This paper provides theoretical arguments and empirical evidence that prove how using VK asset returns are a better choice for measuring asset correlations than using asset returns from monthly asset values. The remainder of this document is organized into the following sections:

- Section 2 provides an overview of how asset values and asset returns relate to one another.
- Section 3 provides a case study that clearly demonstrates the quantitative differences between using asset returns directly from the VK model and asset returns estimated from changes in asset value when computing correlations.
- Section 4 analyses a large sample of firms to better understand the cross-sectional differences between the two correlation estimates.
- Section 5 compares the statistical properties of the two correlation estimates.
- Section 6 concludes the paper.

2 ASSET VALUE AND ASSET RETURN IN THE VASICEK-KEALHOFER MODEL

The VK model provides a structural framework for analyzing default probabilities known as Moody’s KMV Expected Default Frequency™ (EDF) credit measures. In the process of estimating EDF values, the model provides measures of asset values and asset returns. Conceptually, the relationship between asset values and asset returns is similar to that of equity values (measured as market capitalization) and equity returns. Equity returns are not typically computed as the change in market capitalization. Instead, returns are computed as the change in value coming from capital gains and dividends. In other words, the return on holding a share of equity is coming from the change in price (accounting for splits) and cash dividends. Meanwhile the change in market capitalization of equity could come from stock issuances, repurchases, and cash dividends that have more to do with changes in capital structure than changes in the business. As a result, measuring equity returns from changes in market capitalization may be substantively different from equity returns measured using price changes and dividends.

Using changes in asset value to compute asset returns suffers from similar problems as using changes in equity value to compute equity returns. As such, in the VK model, asset returns are modeled separately from asset values. Also, returns on equity and debt are combined to arrive at the return on assets. Although capital structure events (e.g., dividend payments, new debt issuances, retirement of debt, new equity issuance, or equity buybacks) impact returns, they do so in an indirect way. For example, a leveraging event impacts asset returns, but through making the default event more likely and equity more risky. However, an equity repurchase or debt issue (both leveraging events) will not directly drive positive or negative asset returns through an increase or decrease in the cash outflow or infusion. In fact, the impact of capital structure changes typically have a minimal impact on asset returns; similar to the impact dividends or equity issues/repurchases have on equity returns. An additional dynamic that the model takes into consideration is the impact of changes in interest rates on the value of debt, which also helps drive asset returns. To summarize, changes in capital structure (e.g., an equity or debt issue) can result in a large jump in asset value. However, asset returns are typically much more stable and represent changes in a firm’s business.

As a side note, it is interesting that in Merton’s original structural credit risk model, equity is modeled as a call option on a firm’s underlying assets. Asset value and volatility are backed out from equity value and volatility. Asset returns are not explicitly calculated. In that model, asset returns can be computed using asset values at the beginning and end of a certain period. The computation of returns is simple within this framework because it is assumed that there are no changes to capital structure. As we see below, this is a strong and unrealistic assumption. Moreover, this assumption can be particularly distortive within the context of modeling returns for the purpose of estimating correlations.

Finally, it is worth pointing out that in the VK model, asset values and asset returns also serve different purposes. Asset values is used to compute Distance-to-Default for a point in time measure. A change in a firm’s capital structure anytime will be reflected in its asset value and default point, which in turn, reflects the credit worthiness after the capital structure
change. On the other hand, asset returns are used to compute asset volatility, which mainly depend on firm’s underlying business, instead of its capital structure.

3  CASE STUDY

This section provides a case study that demonstrates the impact associated with the distortions in measuring returns and asset correlations as discussed in Section 2. In particular, the section provides a quantitative sense for the drivers that result in a difference between using asset returns directly from the VK model and asset returns estimated from changes in asset value. This is done by analyzing returns and correlations for Ford Motor and General Motors. The first two subsections analyze the time series patterns for the two companies separately. The third subsection analyzes differences in correlations estimated using the two return measure.

3.1  Ford Motor Company

Figure 1 displays the weekly asset returns (Aret hereafter) from July 2004 to June 2007. It also displays the weekly asset returns backed out from weekly asset values (Aret_Val hereafter). The figure illustrates how the co-movement of the two return series is typically in sync. However, Aret_Val exhibits periodic spikes that do not correspond with the spikes in Aret. The first -2% spike in Aret_Val occurred during the week of August 4, 2004. The second -2% spike occurred during the week of November 3, 2004. The time interval between these two spikes is exactly three months. The time interval between the third and fourth spike, as well as the fourth and fifth spike are also three months. Those spikes are caused by changes in the quarterly balance sheets.

In Figure 2, we added the weekly equity returns. We can also see that the spikes in Aret_Val do not correspond to the spikes in equity returns. The annualized volatilities for the Aret, Aret_Val, and equity returns are 5%, 7%, and 32% respectively. The correlation between Aret and equity returns is 90%, and the correlation between Aret_Val and equity returns is 47%.
3.2 General Motors Corporation

Figure 3 displays the Aret and Aret_Val for General Motors Corporation. The most striking observation is the substantial negative spike of Aret_Val the beginning of April 2007. As we look through the news, in early April 2006, General Motors Corp. announced that it agreed to sell a 51% controlling interest in its finance arm, General Motors Acceptance Corp. (GMAC), to a consortium of investors. In late November 2006, the deal was closed. Meanwhile, the balance sheet information was updated the week of April 4, 2007. This is an example of how corporate actions can affect Aret_Val, but not Aret.
In Figure 4, weekly equity returns are added. The pattern is similar to that of Ford Motor Company. The annualized volatilities for the Aret, Aret_Val, and equity returns are 5%, 31%, and 42% respectively. The correlation between Aret and equity returns is 87%, and the correlation between Aret_Val and equity returns is 11%.
3.3 Correlation Between Ford and General Motors

Figure 5 illustrates that the co-movement of Aret returns for Ford Motor Company and General Motors Corp. is typically in sync, and it is not surprising that their correlation is 66%. Meanwhile, Figure 6 illustrates that Aret_Val returns for Ford Motor Company and General Motors Corp. are not in sync, especially at the spike on April 4, 2007. The correlation between the two companies is -57% and is mostly coming from this one observation.

Finally, Figure 7 displays weekly equity returns for Ford Motor Company and General Motors Corp. The equity correlation is 62%, and is much closer to Aret weekly asset return correlation (also displayed in the following figure).

FIGURE 5 Weekly Aret Returns for Ford Motor Company and General Motors Corp.
Weekly Aret_Val Returns for Ford and GM
Correlation = -57%

Weekly Equity Returns for Ford and GM
Correlation = 62%

FIGURE 6 Weekly Aret_Val Returns for Ford Motor Company and General Motors Corp.

FIGURE 7 Weekly Equity Returns for Ford Motor Company and General Motors Corp.

To see whether correlation differences are driven by outliers, we filter Aret, Aret_Val, and equity returns. To all return series, we apply the same filtering criteria. Table 1 displays the pairwise correlations using the pre- and post-filtered data. Although pair-wise Aret_Val correlation is now more intuitive than before filtering, the level is still far from Aret or equity correlations.

1 The specific filtering criteria remove observations that are too high or low using measures such as the standard deviation of the series.
TABLE 1  Correlation between Ford Motor Company and General Motors Corp.

<table>
<thead>
<tr>
<th>Returns</th>
<th>Aret Correlation</th>
<th>Aret_Val Correlation</th>
<th>Equity Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>66%</td>
<td>57%</td>
<td>62%</td>
</tr>
<tr>
<td>Filtered</td>
<td>64%</td>
<td>40%</td>
<td>61%</td>
</tr>
</tbody>
</table>

4  CROSS-SECTIONAL STUDY

This section analyzes a large sample of firms to better explain the cross-sectional differences between the two correlation estimates. The starting point is a sample of the 1,000 largest firms in the Moody’s KMV global database. Pairwise asset correlations are estimated using three years of weekly Aret asset returns and weekly Aret_Val returns, respectively. The difference between Aret and Aret_Val correlations is computed for each pair of firms in the sample and presented in Figure 8. From the graph, we see that the correlation difference is skewed on the positive side, which means that on average, the asset correlations (Aret_corr) from Aret returns are larger than the correlations (Aret_Val_corr) from Aret_Val returns. Only about 30% of the sample is within -0.04 and +0.04 (the -0.02 bucket and 0.02 bucket). Approximately 21% of the sample contains the difference between Aret_corr and Vret_corr greater than 16%. This is striking given that the average pairwise correlation is around 16%. It is interesting to note that financial firms were associated with many of these observations. This is not surprising when one considers the observation that financial institutions tend to have relatively complicated capital structures, and are more frequently involved in capital structure events.
5 CONFIDENCE INTERVAL OF SAMPLE CORRELATION

In the above analysis, three years of weekly data was used in estimating both sets of correlations. A subtlety worth pointing out is that Credit Monitor provides monthly asset values only. As such, it is only possible to use monthly data to back out value implied asset returns and asset correlations. Because of this, one would either need a longer time series, or use fewer observations and compromise the accuracy of their correlation estimates. A concern of using longer time series is that a firm’s business is more likely to vary over the sample resulting in a stale measure of correlation. Moreover, additional data requirements result in fewer firms with sufficient data. This section provides a quantitative sense for the benefit of using weekly versus monthly data. This is done by comparing the confidence interval of sample asset correlations with monthly data and weekly data. The technical methodology used in calculating the confidence interval can be found in the Appendix.

Table 2 displays the lower bound, upper bound, and the width of the 90% confidence interval for 3-year weekly and monthly data. Across the most likely asset correlations from 5% to 65%, the widths of the 90% confidence interval using monthly data are more than double that of the weekly data.

As a final note, it is important to point out that correlation estimates using historical data are notoriously noisy. One of the primary motivations behind the GCorr methodology design is to use a classification structure (i.e., industry and country) to infer correlations and an estimation methodology that reduces estimation noise. To this end, GCorr routinely predicts future correlations more accurately than historical correlation. ²

TABLE 2  Confidence Interval of Sample Correlation Using 3-year Data

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L05</td>
<td>U95</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>0.10</td>
<td>-0.03</td>
<td>0.23</td>
</tr>
<tr>
<td>0.15</td>
<td>0.02</td>
<td>0.28</td>
</tr>
<tr>
<td>0.20</td>
<td>0.07</td>
<td>0.32</td>
</tr>
<tr>
<td>0.25</td>
<td>0.12</td>
<td>0.37</td>
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<tr>
<td>0.30</td>
<td>0.17</td>
<td>0.42</td>
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<td>0.35</td>
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</tr>
<tr>
<td>0.50</td>
<td>0.39</td>
<td>0.59</td>
</tr>
<tr>
<td>0.55</td>
<td>0.45</td>
<td>0.64</td>
</tr>
<tr>
<td>0.60</td>
<td>0.51</td>
<td>0.68</td>
</tr>
<tr>
<td>0.65</td>
<td>0.57</td>
<td>0.72</td>
</tr>
</tbody>
</table>

6 CONCLUSION

In this paper, we demonstrated that asset value can be unduly influenced by corporate transactions, especially for financial firms. For example, using the case of Ford Motor Company and General Motors Corp., the impact was particularly striking where corporate activity resulted in an unrealistic asset value based correlation estimate that was negative. When analyzing a broad sample, we find that the difference between asset return-based correlations and asset value-based correlation estimates are typically nontrivial. This is particularly true for financial institutions that tend to have relatively complicated capital structures and are more frequently involved in capital structure events. Finally, this paper presents evidence indicating that using monthly data (as is available in CreditMonitor) as opposed to the weekly data (as is used in estimating GCorr) compromises the accuracy of sample asset correlations.

² For additional details see An Empirical Assessment of Asset Correlation Models, Zeng, Bin and Zhang, Jing.
APPENDIX A

Estimation of a standard error for Pearson’s correlation is not completely trivial; the statistic is not normally distributed. To get around this problem, Pearson’s correlation is converted to Fisher’s $z$, which converges to normal distribution as the sample size increases. The confidence interval is then computed using Fisher’s $z$ under the assumption of normality. The values of Fisher’s $z$ in the confidence interval are then converted back to Pearson’s correlations. Fisher’s $z$ is defined in the following way:

$$ Z = \frac{1}{2} \ln \frac{1 + \rho}{1 - \rho} - N(\mu_z, \sigma_z^2) $$

where

$\rho$ is the sample correlation,

$$ \mu_z = \frac{1}{2} \ln \frac{1 + \rho_0}{1 - \rho_0}, $$

$$ \sigma_z^2 = \frac{1}{n - 3}, $$

$\rho_0$ is the true correlation,

and $n$ is the number of pairs of data.

Next, the two-sided confidence limits for $\zeta$ are computed as

$$ \zeta_l = z_{(1 - \alpha/2)} \sqrt{\frac{1}{n - 3}} $$

$$ \zeta_u = z_{(1 - \alpha/2)} \sqrt{\frac{1}{n - 3}} $$

where $z_{(1 - \alpha/2)}$ is the 100(1-\alpha/2) percentile of a standard normal.

Finally, $\zeta_l$ and $\zeta_u$ are then transformed back to derive the confidence interval for $\rho$:

$$ r_l = \frac{\exp(2\zeta_l) - 1}{\exp(2\zeta_l) + 1} $$

$$ r_u = \frac{\exp(2\zeta_u) - 1}{\exp(2\zeta_u) + 1} $$

[1]

[2]

[3]