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# THE RELATIONSHIP BETWEEN AVERAGE ASSET CORRELATION AND DEFAULT PROBABILITY

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## ABSTRACT

Asset correlation and default probability are critical drivers in modeling portfolio credit risk. It is generally assumed, as in the Basel II Accord, that average asset correlation decreases with default probability. We examine the empirical validity of this assumption in this paper. Overall, we find little empirical support for this decreasing relationship in the data for corporate, commercial real estate (CRE), and retail exposures. For corporate exposures, there is no strong decreasing relationship between average asset correlation and default probability when firm size is properly accounted for. For CRE and retail exposures, the empirical evidence suggests that the relationship is more likely to be an increasing one.

We also provide economic arguments against the assumption that average asset correlation decreases with default probability. For corporate exposures, defaulting firms do not necessarily experience increases in their firm-specific risk if there is a systematic negative shock that causes widespread defaults. For retail exposures, sub-prime borrowers are more sensitive to general economic conditions and thus experience higher asset correlations than prime borrowers. Our analyses suggest that it is imprudent to assume a decreasing relationship between average asset correlation and default probability in measuring portfolio credit risk. In light of these economic arguments and empirical evidence, we encourage the Basel Committee to revisit the use of this relationship in bank capital requirement.

**ACKNOWLEDGEMENTS**

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# 1 OVERVIEW

Asset correlation and probability of default (PD) are critical drivers in modeling portfolio credit risk. The most common approach to modeling default correlation in portfolio credit risk calculation is to combine default probabilities with asset correlations. Basically, two borrowers will default in the same period if both of their asset values are insufficient to pay their obligations. Asset correlation helps define the joint behavior of the asset values of the two borrowers. This idea has become the basis for many portfolio credit risk models, for example, the multi-factor model framework in the Moody's KMV (MKMV) Portfolio Manager™ and RiskFrontier™, and the so-called Asymptotic Single-Risk Factor (ASRF) model supporting the Basel II IRB credit risk capital charge. In the context of the ASRF model, the single systematic risk factor may be interpreted as reflecting the state of the economy, and all borrowers are linked to one other by this single risk factor. The asset correlations determine how the asset value of one borrower depends on the asset value of another borrower. With this single factor set-up, the asset correlations can be considered as the dependence of the asset value of a borrower on the general state of the economy.

It is generally assumed, as in the Basel II Accord, that average asset correlation decreases with an increase in default probability. This relationship suggests that, on average, risky borrowers would have smaller asset correlations. Within the ASRF, the decreasing relationship has the effect of dampening the capital charge curve as a function of default probability. This decreasing relationship was first introduced by Basel in 2001.[1] It is generally recognized that this decreasing relationship is justified by the desire by regulators to reduce pro-cyclical effects of the new accord. Since then, some institutions have also utilized the Basel II average asset correlation function for other portfolio credit risk calculations, such as measuring required economic capital. Despite its importance, there have been few studies on the empirical relationship between asset correlation and default probability. The most cited study is Lopez (2004), who studied the empirical relationship between average asset correlation, firm probability of default, and firm size. The study found that average asset correlation is a decreasing function of PD and an increasing function of firm size. Furthermore, the study argued that variables other than PD, for example, firm size, need to be considered in determining average asset correlation in the ASRF regulatory framework. The increasing relationship between average asset correlation and PD is also found in German corporates by Düllmann and Scheule (2003), but they did not find unambiguous relationship between asset correlation and PD. Using SME data, Dietsch and Petey (2004) found that the relationship is actually positive in the German population and U-shaped in France.

Unlike the previous studies that focus only on corporate data, this paper examines the validity of this assumption for corporate, commercial real estate (CRE), and retail portfolios. Using various sources of data for these asset classes, we investigate the empirical relationship between average asset correlation and default probability. Overall, we find little empirical support for this decreasing relationship in the data for corporate, CRE, and retail exposures. For corporate exposures, there is no strong decreasing relationship between average asset correlation and default probability when firm size is properly accounted. For CRE and retail exposures, the empirical evidence suggests that the relationship is more likely to be an increasing one. We also provide economic arguments and empirical evidence against the assumption that average asset correlation decreases with default probability. For corporate exposures, defaulting firms do not necessarily experience increases in their firm-specific risk if there is a systematic negative shock that causes widespread defaults. For retail exposures, sub-prime borrowers are more sensitive to the general economic conditions and thus, have higher asset correlations than prime borrowers. Our analyses suggest that it is imprudent to assume a decreasing relationship between average asset correlation and default probability in measuring portfolio credit risk. In light of these economic arguments and empirical evidence, we encourage the Basel Committee to revisit the use of this relationship in determining bank capital requirement.

The rest of this paper proceeds as follows:


- "Basel II Average Asset Correlation" on page 6 describes the Basel II average correlation functions.
- "Corporate" on page 7 reports results for corporate exposures.
- "Commercial Real Estate" on page 16 reports results for CRE exposures.
- "Retail" on page 18 reports results for retail exposures.


[1] Basel Committee on Banking Supervision, 2001c. "Potential Modification to the Committee's Proposals," Press release dated November 5, 2001.

- "Impact of Correlation Misspecification on Regulatory Capital" on page 20 shows the impact of underestimation of asset correlation on capital charge.
- "Conclusion" on page 21 concludes our findings.

# 2 BASEL II AVERAGE ASSET CORRELATION

In the ASRF model of Basel II, the asset value of a borrower is driven by a factor model:

$$r_i = \sqrt{R_i}\phi + \sqrt{1-R_i}\varepsilon_i \tag{1}$$

where $r_i$ is the asset return[2] of borrower $i$, $\phi$ is the systematic factor representing the *state of economy*, $R_i$ is the R-squared, the percentage of systematic risk, and $\varepsilon_i$ is the idiosyncratic factor of borrower $i$. Two borrowers are correlated with one another because they are both exposed to the systematic factor (with potentially varying degree). Mathematically, the correlation of borrower $i$ with borrower $j$ is given by:

$$corr(r_i, r_j) = \sqrt{R_i}\sqrt{R_j} \tag{2}$$

According to the Advanced Internal Ratings Based Approach (A-IRB) of the Basel II, the asset correlation parameter $R$ is a decreasing function of PD:

$$R = a \times \frac{1-\exp(-c \times PD)}{1-\exp(-c)} + b \times \left(1 - \frac{1-\exp(-c \times PD)}{1-\exp(-c)}\right) \tag{3}$$

Parameters $a$, $b$, and $c$ depend on borrower type. For corporate borrowers and the so-called low asset correlation CRE exposures, a=0.12, b=0.24, and c=50.[3] For the high asset correlation CRE exposures, a=0.12, b=0.30, and c=50. For retail borrowers, Basel recommends asset correlation of 15% for residential mortgage and asset correlation of 4% for revolving retail lines of credit. Otherwise, a=0.03, b=0.16, and c=35 for other retail exposures.

Figure 1 plots the Basel II recommended asset correlation functions for different types of borrowers. Asset correlation ranges from 12% to 24% for corporate borrowers[4] and it decreases as PD increases. A similar pattern is assumed for CRE and retail exposures. Asset correlation ranges from 12% to 24% for low asset correlation CRE portfolios and 12% to 30% for high asset correlation CRE portfolios. Asset correlation for retail exposures ranges from 3% to 16%. All asset correlation functions imply a decreasing relationship between asset correlation and PD.

[2] Here the "asset return" can be broadly interpreted as the variable that drives the credit quality change of the borrower.

[3] There is a downward adjustment applied to European middle market firms with annual sales between €5 million and €50 million.

[4] This does not include the small firm adjustment. For small firms the asset correlation ranges between 8% and 20%.

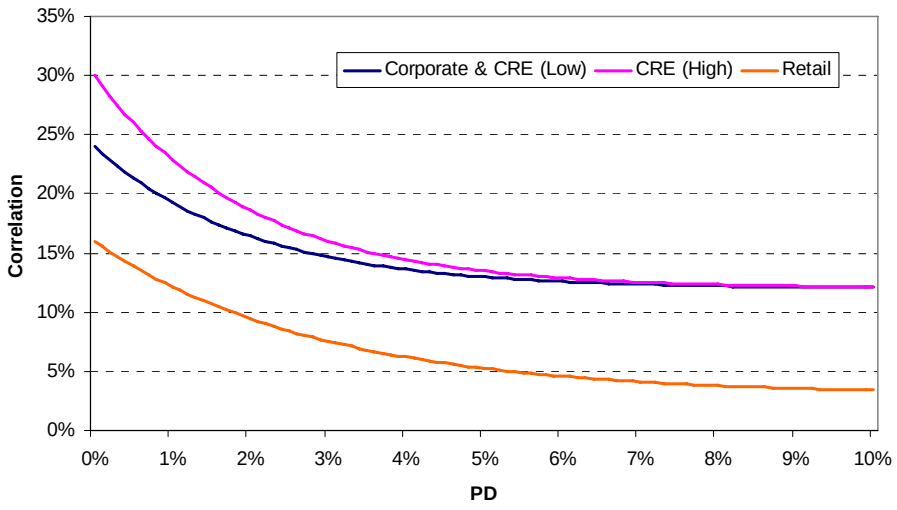


FIGURE 1 Asset Correlation Parameters in Basel II IRB Framework

Although the decreasing relationship in (3) was first specified for regulatory capital calculation, it has since gathered general acceptance in the banking industry for other credit risk calculation, for example, determining required economical capital. Given the critical role that asset correlation plays in determining portfolio credit risk, it is important to understand the dynamic relationship between average asset correlation and default probability. Our paper now focuses on this relationship.

# 3 CORPORATE

This section presents our results on the relationship between asset correlation and PD for corporate exposures. We discuss results from both the asset return data and the realized default experience. We also examine the behavior of firms as they approach default.

## 3.1 Evidence from Asset Return Data

We first investigate the empirical relationship between asset correlation and PD using Moody's KMV's asset return and Expected Default Frequency™ (EDF) data. Moody's KMV uses an option theoretic approach to calculate EDF credit measures. This approach measures probability of default for more than 30,000 public traded companies worldwide. During EDF calculation, asset returns can be obtained from equity returns and financial statements. The sample data of asset returns of these publicly traded companies serves as the development dataset of Moody's KMV's Global Correlation Model (GCorr). The R-squared values from the GCorr possess a similar economic interpretation as the R's from Equation (1), as both represent the percentage of systematic risk and determine the asset correlation. In the subsequent analysis of this section, we use the R-squared values from the GCorr as a measure of asset correlations.

Figures 2 and 3 show the relationship between PD and R-squared for U.S. industrial and financial firms, respectively. We divide all firms into five different PD groups based on EDF quintiles. Median EDF for each PD group is shown along the horizontal axis.

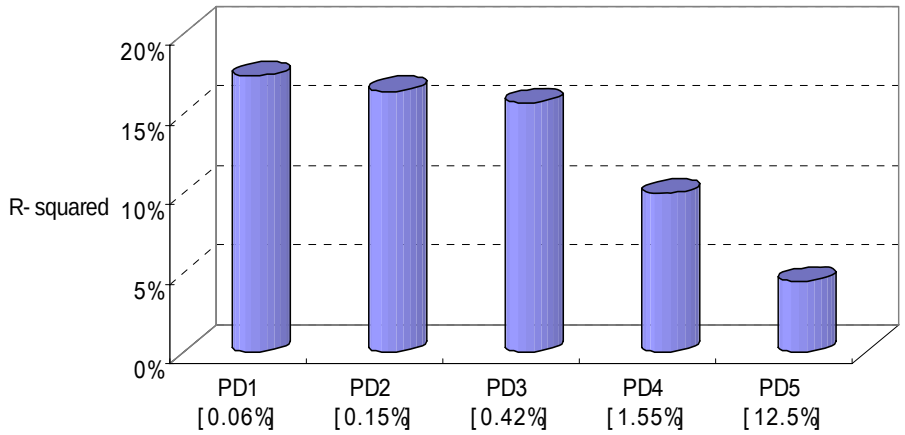


FIGURE 2 Median R-squared values for different PD groups – Industrial Firms

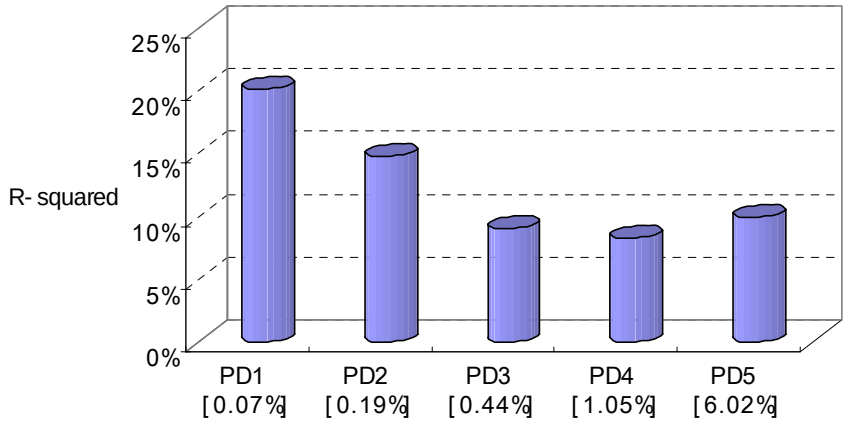


FIGURE 3 Median R-squared values for different PD groups – Financial Firms

Figures 2 and 3 suggest asset correlation decreases with PD, as recommended by Basel II. However, our findings show that this relationship is mainly driven by firm size. Large firms tend correlate more with the economy and, hence, have higher R-squared values. They also tend to have smaller PD values because they typically experience lower asset volatility. The primary reason we observe a decreasing relationship between PD and R-squared is that larger firms tend to have higher R-squared values and lower PD values, while smaller firms have lower R-squared values and higher PD values.

To test this hypothesis, we need to investigate the relationship conditional on firm size. To do this, we divide the total sample into five different groups based on size quintiles. Size is measured by sales for industrial firms and by book value of assets for financial firms. We further divide each group into five different PD groups based on EDF quintiles. Figures 4 and 5 show the median R-squared for each size-PD group for industrial firms and financial firms, respectively. The results indicate that median R-squared generally increases with the size for both industrials and financials. However, R-squared does not decrease uniformly with PD. Visually, PD and R-squared do not show a strong decreasing relationship, while PD and size show a strong increasing relationship.

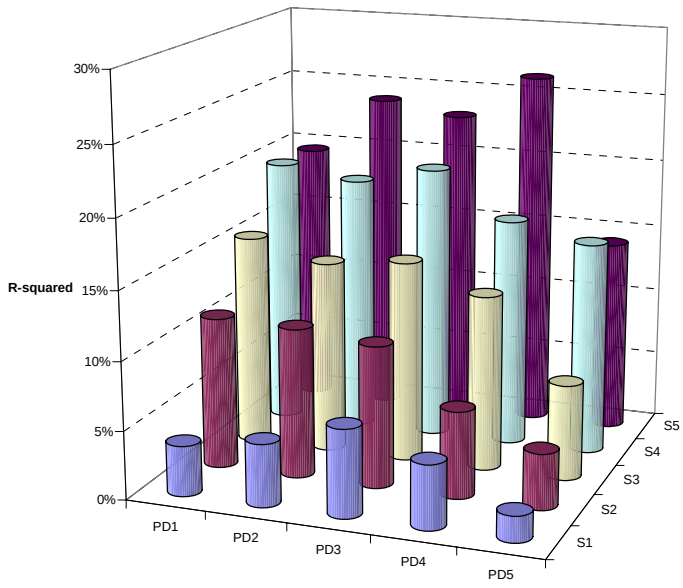


FIGURE 4 Median R-squared for each size-PD Group – Industrial Firms

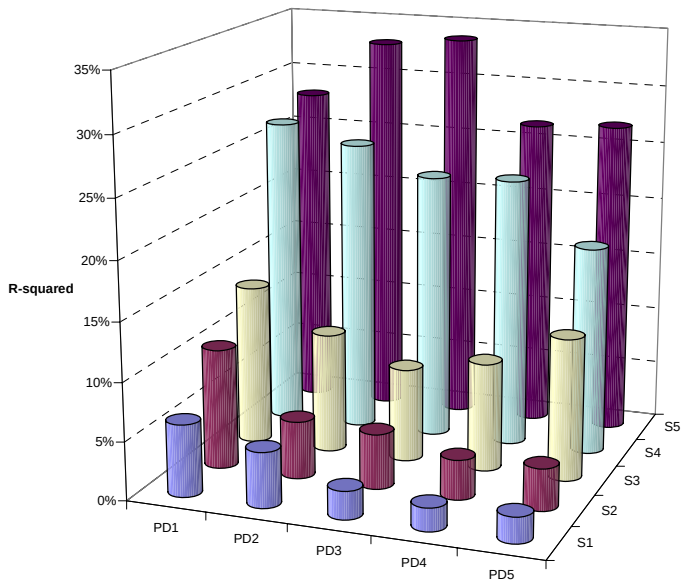


FIGURE 5 Median R-squared for each size-PD Group – Financial Firms

To further illustrate the point that size plays an important role in explaining the variation in R-squared while PD does relatively little, we run following regression models:

$$Rsquared_i = \alpha + \beta \cdot \log(Size_i) + \varepsilon_i \tag{4}$$

$$\varepsilon_i = \alpha + \beta \cdot PD_i \tag{5}$$

where $\varepsilon_i$ is the residual obtained from the first regression in equation (4). We first regress the firm's R-squared values on the log of their sizes. Table 1 summarizes the regression result. The coefficient for size is positive and significant, which indicates that firms' R-squared values increase with size. R-squared from the regression is 45.03%, which indicates size itself explains about 45% of the total variation in R-squared. Next, we regress residuals from the first regression on PD. By construction, these residuals are independent of size. Thus, we can explore the relationship between PD and R-squared while controlling for firm size. Regression result is summarized in Table 2. The coefficient for PD is positive and significant, which indicates R-squared increases as PD increases. The sign of the coefficient does not imply a negative relationship between PD and R-squared. In addition, R-squared from the second regression is 0.0032, implying the additional portion in total variation in R-squared explained by PD is very minimal. Taken together, these regression results suggest that size is the main driver in explaining the variation in R-squared.

Table 1 Regression Result - R-squared values on log(Size)

| | Coefficient | Standard Error | t Statistic | R-squared |
|---|---|---|---|---|
| Intercept | 0.0749 | 0.0019 | 38.5918 | 0.4503 |
| log(Size) | 0.0193 | 0.0003 | 57.9019 | |

Table 2 Regression Result – Residuals on PD

| | Coefficient | Standard Error | t Statistic | R-squared |
|---|---|---|---|---|
| Intercept | -0.0014 | 0.0010 | -1.3188 | 0.0032 |
| PD | 0.0505 | 0.0139 | 3.6273 | |

To further test the relationship in (3) while controlling for firm size, we also estimate the following regression:

$$Rsquared_i = \alpha + \beta \cdot \log(Size_i) + \gamma \cdot \omega_i + \varepsilon_i \tag{6}$$

where $\omega_i = 0.12 \cdot \frac{1-\exp(-50 \times PD_i)}{1-\exp(-50)} + 0.24 \cdot \left(1 - \frac{1-\exp(-50 \times PD_i}{1-\exp(-50)}\right)$

Note that $\omega_i$ is the recommended R-squared, and it is a decreasing function of PD. If there is a significant relationship between R-squared and PD after controlling for firm size, we expect $\gamma$ to be significant. However, as summarized in table 3, $\gamma$ turns out to be insignificant at the 95% level. Furthermore, additional R-squared obtained by including $\omega_i$ in the model is 0.0004 (0.4507–0.4503), which is minimal. The result confirms that, while size provides distinct information explaining the variation in R-squared, PD provides little additional information. In other words, after controlling for the firm's size, the asset return data does not support the decreasing relationship in (3).

Table 3 Regression Result – R-squared values on log(Size) and $\omega_i$

| | Coefficient | Standard Error | t Statistic | R-squared |
|---|---|---|---|---|
| Intercept | 0.0670 | 0.0051 | 13.2242 | 0.4507 |
| Log(Size) | 0.0190 | 0.0004 | 47.7381 | |
| $\omega_i$ | 0.0467 | 0.0279 | 1.6773 | |

## 3.2 The Change in Asset Correlation as a Firm Approaches Default

A major argument for the decreasing relationship between PD and R-squared is that, as a firm approaches default, its firm-specific risk increases, and its systematic risk (hence R-squared) decreases. On the other hand, one could argue this relationship is not always true. When a significant negative shock triggers widespread defaults in a specific sector, the percentages of systematic risk of those firms could increase as the result of the large negative shock. This section examines these arguments empirically.

To study the change in asset correlation as firms approach default, we estimate the time series of R-squared of defaulted firms from 24 months prior to default to immediately before default. For this analysis, our default data spans from 1994 to 2006, including defaults for U.S. non-financial firms. Figure 6 shows the total number of defaults by year from 1994 to 2006. 2001 is of specific interest, as this year experienced the largest number of defaults in the sample period, a result of the large, negative shock to the overall economy.

Figure 7 shows the time series dynamics of the 25$^{th}$, median, and 75$^{th}$ percentile of R-squared values of all defaulted firms as they approach default. The median R-squared decreased slightly as firms approached default, but the decrease was not very steep. Figure 8 presents the results for all firms that defaulted in 2001, during a significant negative economic shock. In this case, the median R-squared increased slightly at the time of default for firms that defaulted in 2001.

Interestingly, when we focus on a specific sector, for example, telecom, we observe increases in R-squared as defaulted firms in the sector approach default. The telecom industry experienced widespread defaults in 2001 due to the large negative shock to the entire sector. In this case, R-squared actually increases as PD increases (Figure 9), which contradicts the decreasing relationship between PD and R-squared.

To summarize the results from Figures 7 through 9, we do not find strong, empirical support for the argument that firms' systematic risk decreases as they approach default. On the contrary, they could actually increase during a sector-wide negative shock.

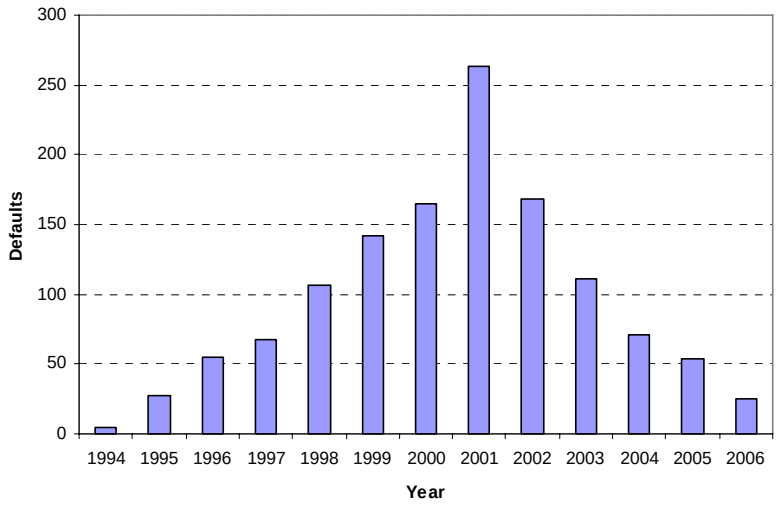


FIGURE 6 Annual Number of Defaults

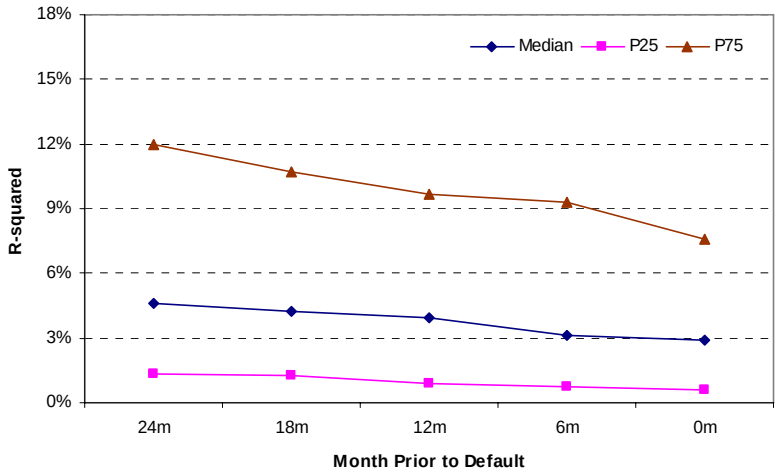


FIGURE 7 R-squared Dynamics – All Firms Defaulted 1994 to 2006 (1,261 firms)

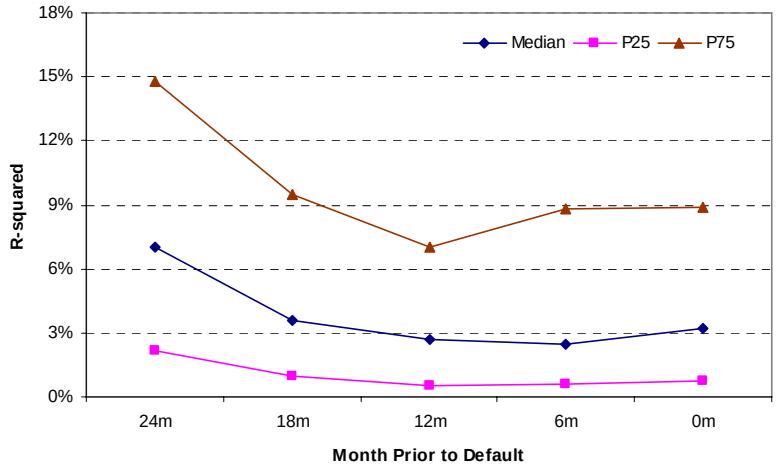


FIGURE 8 R-squared Dynamics – All Firms Defaulted in 2001 (263 firms)

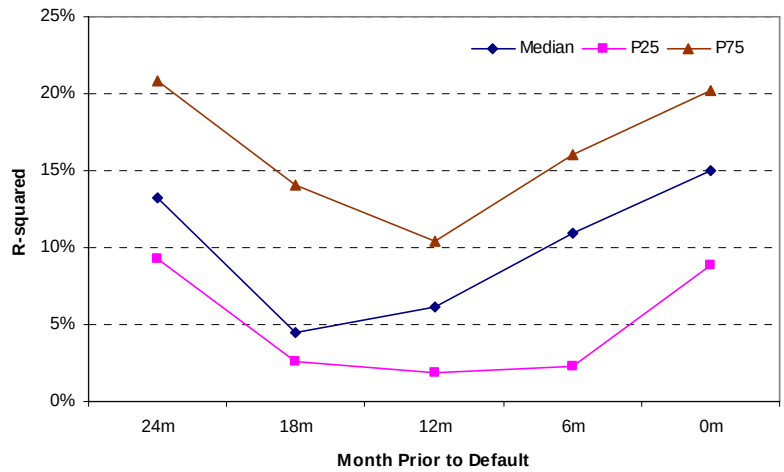


FIGURE 9 R-squared Dynamics – All Firms Defaulted in Telecom Industry 2001 (26 firms)

## 3.3 Evidence from the Default Data

In the previous sections, we investigated the relationship between asset correlation and PD using asset returns data. In this section, we explore the relationship using realized defaults data. We estimate the quarterly realized default rate for each size PD group (as defined in section 3.1) from April 1980 through September 2008. We then estimate the default-implied asset correlation from the realized default rate series. Default-implied asset correlation can be estimated using following equations:[5]

$$JDP_{ij} = N_2(N^{-1}(PD_i), N^{-1}(PD_j), R_{ij}) \tag{7}$$

$$JDP_{ij} = \sum_t w_t \frac{D_t^i}{N_t^i} \frac{D_t^j}{N_t^j} \tag{8}$$

$PD_i$ and $PD_j$ are the mean realized default rates for groups i and j, respectively. $JDP_{ij}$ is the joint default probability between groups *i* and *j*, and it is estimated using the realized default rates as in equation (8). $w_t$ is the weight applied to each quarter. We use equal weights in our study. Default implied asset correlation is then backed out from the equation (7).

The default-implied asset correlation for each size-PD group for industrial firms[6] is presented in Figure 10. Even after controlling for size, we do observe a negative relationship between PD and asset correlation. However, as we shall see subsequently, this decreasing relationship is, to a certain degree, introduced by the biases associated with estimating default-implied asset correlation for groups with a low number of defaults, either as the result of low PD or a low number of firms within the group.

[5] For more details on the estimation methodology, please see *Asset Correlation, Realized Default Correlation, and Portfolio Credit Risk.*
[6] For financial firms, there were no defaulted firms for many size-PD groups, which made investigating the relationship between PD and asset correlation difficult.

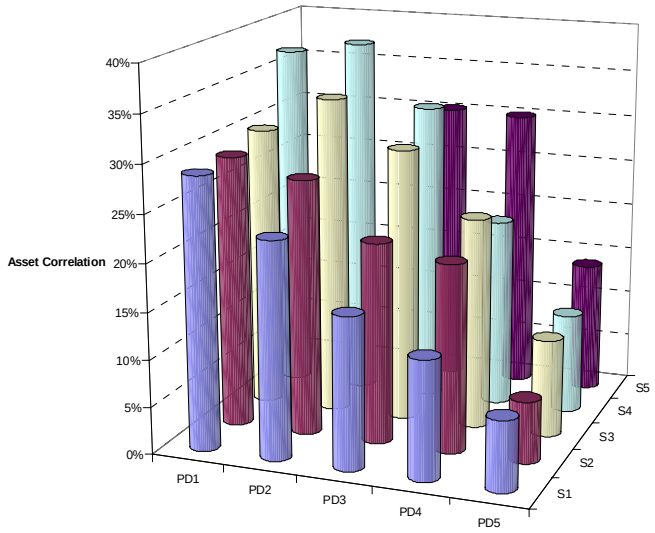


FIGURE 10 Default-Implied Asset Correlation for Each Size PD Group – Industrial Firms

To understand the magnitude of the biases associated with estimating default-implied asset correlation, we conducted the following simulation exercise: [7]

1. Generate correlated asset returns using the single-factor model framework in equation (1). An asset correlation parameter is assumed to impose a correlation structure on the simulated asset returns. For each point of time (total number of time periods = T), generate N asset returns (the number of firms = N). Thus, we generate T x N asset returns.
2. Count defaults – Whenever an asset return falls below the default threshold, count defaults. PD parameter is used to determine default threshold.
3. Get default rate series – Using the number of defaults counted for each period and the number of firms (N), obtain default rate series.
4. Estimate default-implied asset correlation – Using the default rate series, estimate default-implied asset correlation.
5. Repeat steps 1 through 4 10,000 times to form a distribution of default-implied asset correlations.

The simulation exercise suggests that when PD is low, asset correlation is overestimated. Figures 11 through 13 show the median default-implied asset correlation estimates along the different levels of PD. The total number of periods is assumed to be 114, the actual number of quarters between April 1980 and September 2008, (28.5 years). The total number of firms is assumed to be 165, the average number of firms in each group. As shown in Figures 11 through 13, the simulated default-implied asset correlation shows a decreasing relationship with PD, even though the true R-squared stays constant. Default-implied asset correlation is overestimated in the low PD range, and the size of bias decreases as the level of PD increases. The absolute difference between true and estimated value can be as large as 33% in the case where PD is 1bp and the true R-squared is 1%. The bias associated with the estimates is partly, if not entirely, responsible for the observed decreasing relationship between PD and default-implied asset correlations.

[7] For the in-depth analysis of how each parameter affects default correlation estimates, please see *Asset Correlation, Realized Default Correlation, and Portfolio Credit Risk.*

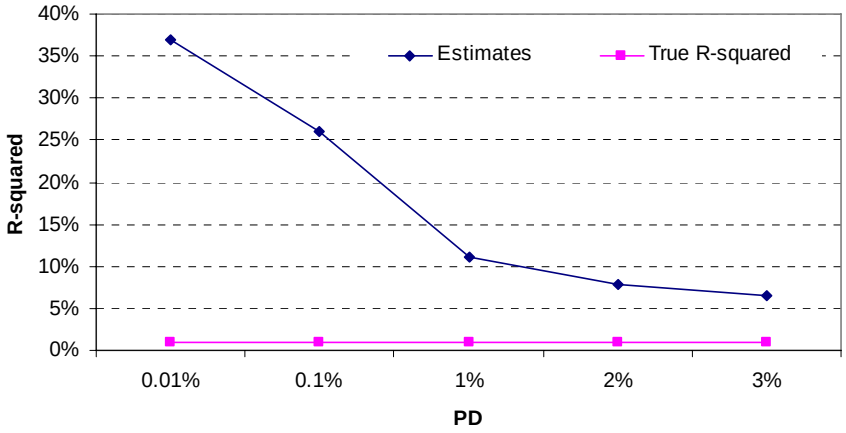


FIGURE 11 Simulated Default-Implied Asset Correlation (True R-squared = 1%)

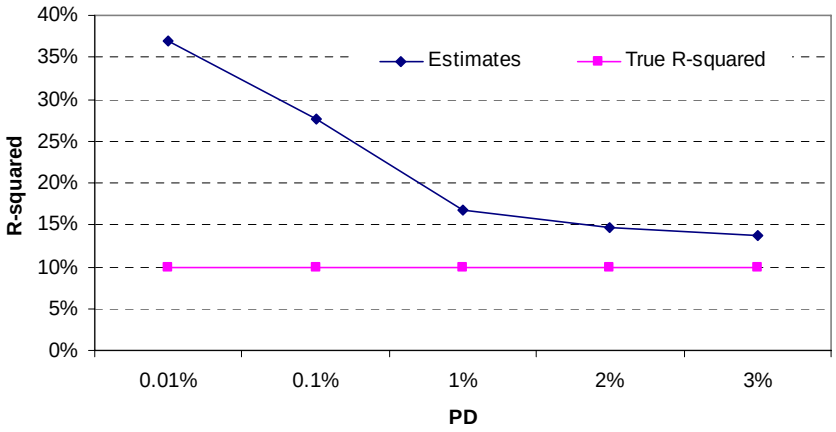


FIGURE 12 Simulated Default-Implied Asset Correlation (True R-squared = 10%)

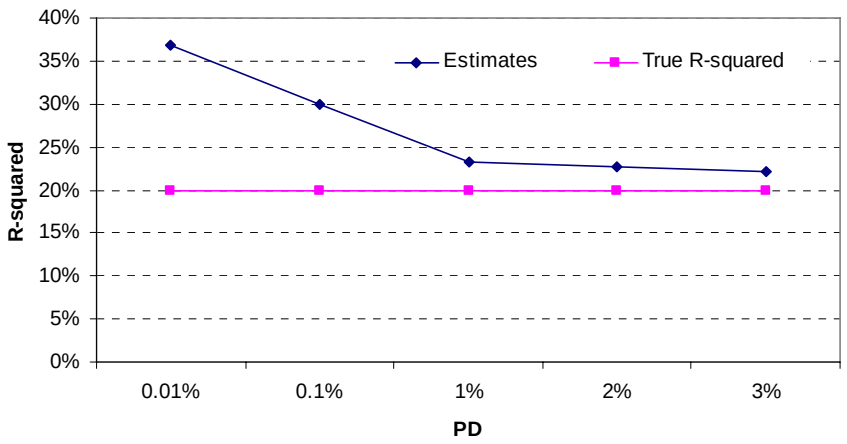


FIGURE 13 Simulated Default-Implied Asset Correlation (True R-squared =20%)

# 4 COMMERCIAL REAL ESTATE

This section describes the relationship between asset correlation and PD among CRE borrowers. Recall that Basel's recommendation for asset correlation for CRE exposures is a decreasing function of PD and ranges from 12% to 24% for low asset correlation CRE portfolios and from 12% to 30% for high asset correlation CRE portfolios. In fact, the function form for low asset correlation CRE exposures is the same for corporate exposures. To test the validity of Basel's recommendation for CRE exposures, we borrow the results from a Moody's KMV study on the asset correlations of US CRE asset correlations. [8] Using American Council of Life Insurers (ACLI) delinquency rates of CRE loans in different regions, we can examine the relationship of delinquency rates implied R-squared with PD values for different property types. Results are plotted in Figures 14 through 18.

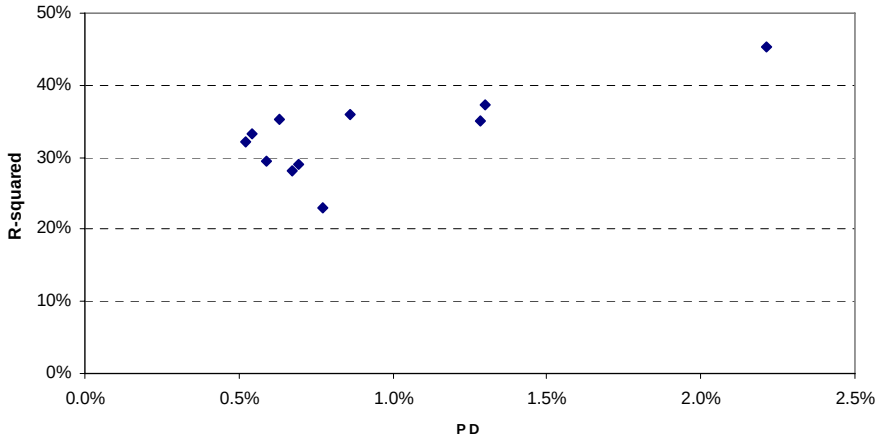


FIGURE 14 GCorr CRE R-Squared Estimates: Multi-Family Housing

[8] Please see *Modeling Asset Correlations for Commercial Real Estate Exposures in Credit Portfolios*.

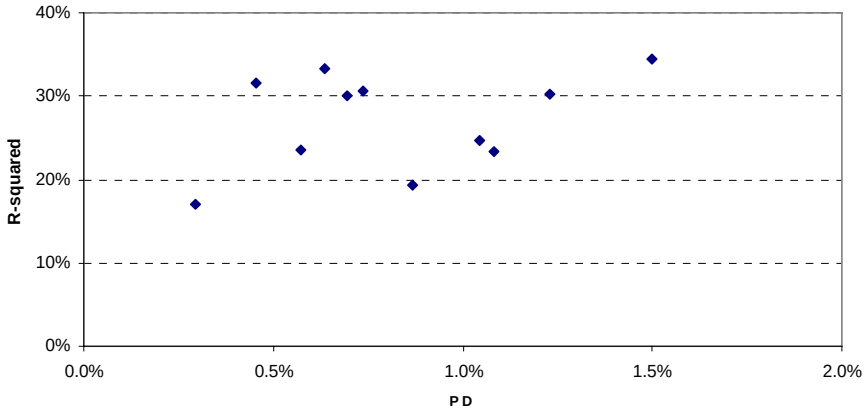


FIGURE 15 GCorr CRE R-Squared Estimates: Retail

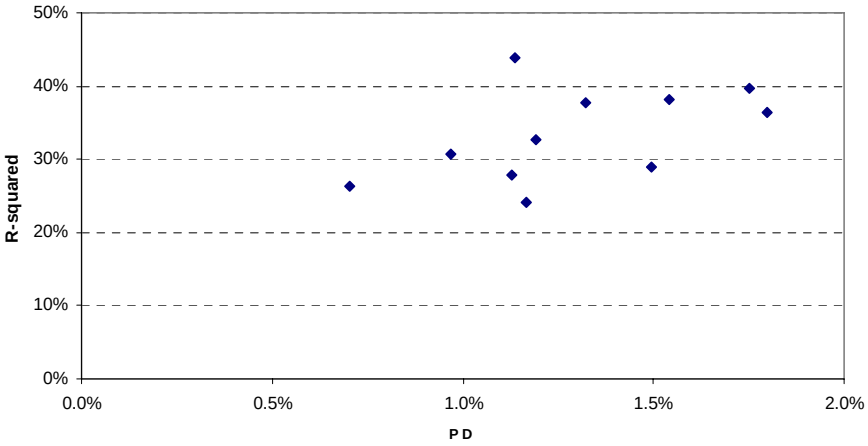


FIGURE 16 GCorr CRE R-Squared Estimates: Office

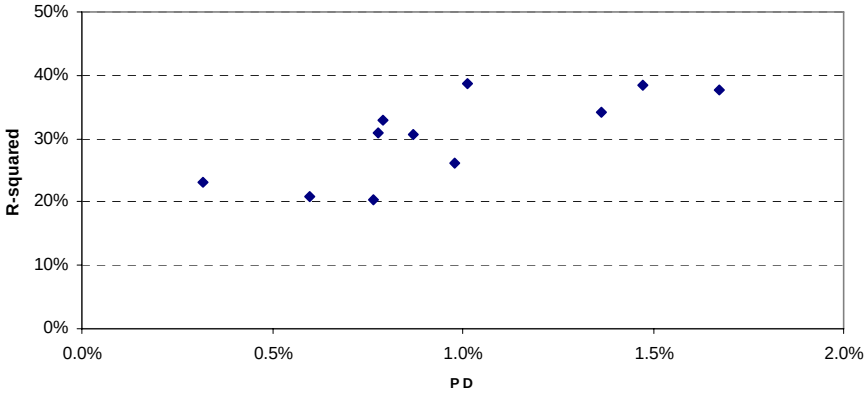


FIGURE 17 GCorr CRE R-Squared Estimates: Industrial

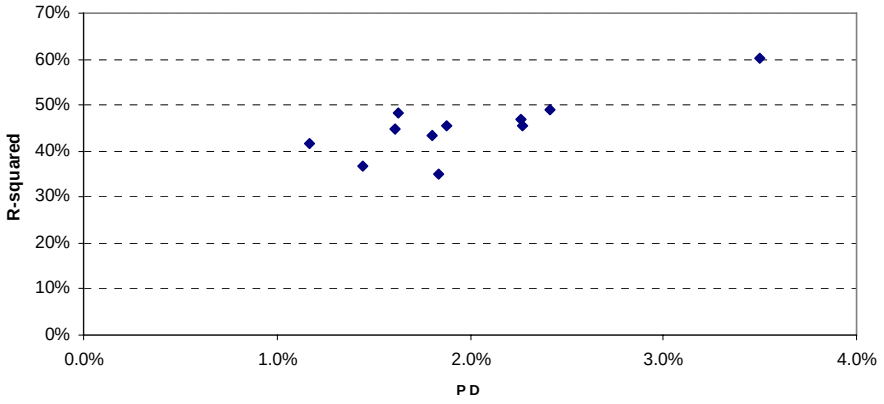


FIGURE 18 GCorr CRE R-Squared Estimates: Hotels

Clearly, the evidence in Figures 14 through 18 does not support a decreasing relationship between PD and asset correlation for CRE exposures. In fact, these estimates show a positive relationship between PD and R-squared, although it is not very strong. Our results suggest that we need to be more careful when exploring the relationship between PD and asset correlation for different asset classes. Assuming a similar pattern obtained from public firm data can potentially underestimate the risk for CRE portfolios in high PD range.

# 5 RETAIL

In this section, we study the relationship between average asset correlation and PD for retail exposures. Recall, the Basel average asset correlation for retail exposures ranges from 3% to 16%, as a decreasing function of PD. This function suggests that a retail borrower with a lower credit score has lower asset correlation than a borrower with a higher credit score. We are not aware of any justifications for this relationship for retail exposures.

There are a number of economic arguments supporting sub-prime borrowers as more sensitive to general economic conditions than prime borrowers and, hence, having a higher percentage of systematic risk (i.e., higher R-squared). First, the ratio of financial obligations over disposable income tends to be higher for sub-prime borrowers. As we see in Figure 19, between 2005 and 2007, financial obligations are less than 15% of disposable income for American prime borrowers and more than 35% for sub-prime borrowers. This data implies that sub-prime borrowers are more sensitive to economic changes.

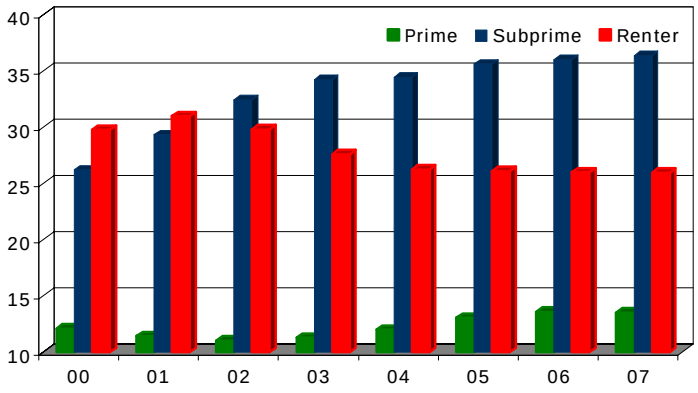


FIGURE 19 Financial Obligation Ratios, Percentage of Disposable Income, Source: Moody's Economy.com

Second, sub-prime borrowers spend a higher percentage of their income on basic necessities such as utilities and commuting. Figure 20 shows that for the lowest 20% of consumers, most likely sub-prime borrowers, the expense for fuel oil, electricity, natural gas, and gas is more than 10% of their spending. These borrowers have fewer cushions against an economic downturn.

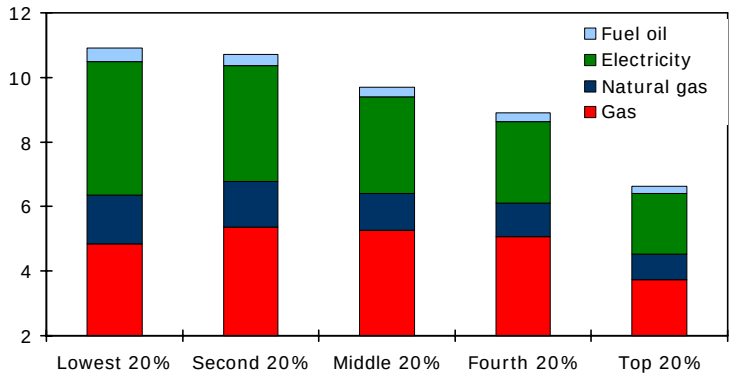


FIGURE 20 Expense Percentage of Consumer Spending, Source: Moody's Economy.com

The above economic arguments suggest that retail borrowers with higher PD values (e.g., sub-prime borrowers) tend to have higher asset correlations. To test this thesis empirically, we estimated R-squared values of various retail loans for 250 Metropolitan Statistical Areas (MSAs) in the U.S. using delinquency rates for various retail loans provided by Creditforecast.com.[9] Figure 21 shows the median of these R-squared values and suggests that median asset correlations are higher for sub-prime borrowers than for prime borrowers.

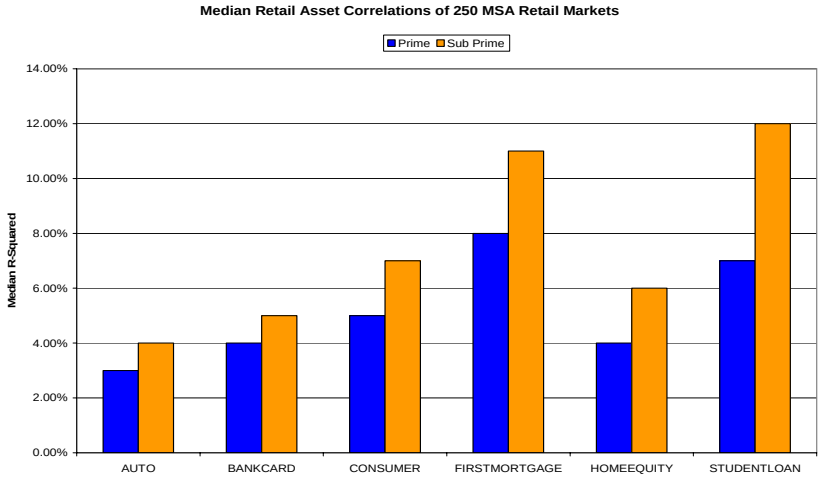


FIGURE 21 GCorr Retail Median R-squared Estimates

In addition to our empirical evidence, Cowan (2004) showed that default correlation increases as the internal credit rating declines, using a large portfolio of sub-prime loans from an anonymous sub-prime lender. In their paper, they present the first formal study of default correlation within a sub-prime mortgage loan portfolio. They analyze six-month default correlation using both actual defaults (foreclosures) and a more broad definition of delinquency. As anticipated,

[9] For more details, please see Modeling Retail Correlations in Credit Portfolios.

the magnitude of default correlation increases as the internally assigned risk grade declines. To facilitate a like-to-li comparison, we calculated implied asset correlation from their reported default correlations and present the results Figure 22. Again, the results do not support a decreasing relationship between average asset correlation and PD.

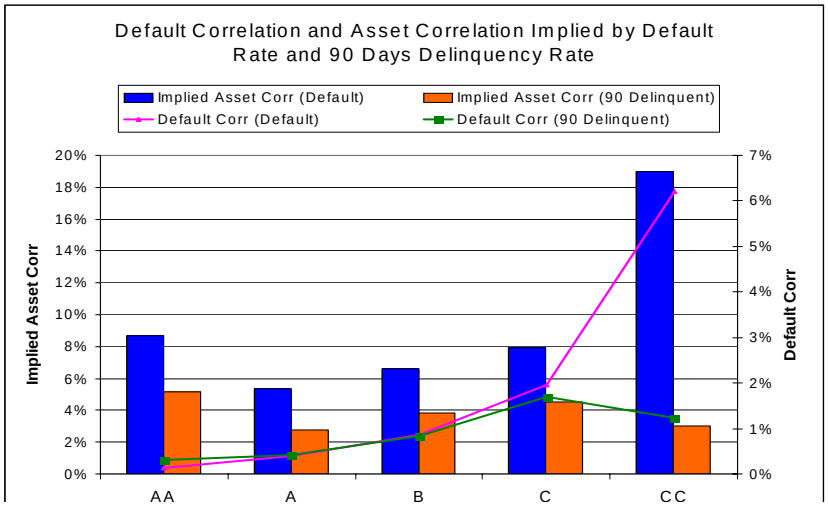


FIGURE 22 Default Correlations and Implied Asset Correlations

# 6 IMPACT OF CORRELATION MISSPECIFICATION ON REGULATORY CAPITAL

In this section, we demonstrate the impact of potential misspecification of average asset correlation on the calculation of regulatory capital. Conceptually, the (unjustified) decreasing relationship specified in (3) would yield (unjustified) lower regulatory capital for borrowers with high PD. To illustrate this, we use the data from our CRE section. Recall, under Basel IRB, the capital charge is given by:

$$capital = LGD \cdot N\left( N^{-1}(PD) \cdot \sqrt{\frac{1}{1-R}} + N^{-1}(0.999) \cdot \sqrt{\frac{R}{1-R}} \right) - PD \cdot LGD \qquad (9)$$

where R is the average asset correlation or R-squared in a single-factor framework. R is obtained from the Basel asset correlation function and actual estimates for CRE. For each property type, we fitted a linear line based on the actual PD and R-square estimates. All five property types imply a positive relationship between R-squared and PD. We then use these linear functions to attain R for each PD level, and calculate capital charges using equation (9). Figure 23 presents the capital charges along with PD based on different correlation estimates, together with the Basel II function.[10] Figure 23 illustrates that the decreasing relationship between PD and R-squared assumed in Basel's asset correlation function can lead to an underestimation of capital charge. This underestimation can be more severe in the high PD range. A similar result is expected for retail exposures as well, as our estimates suggest that there is a positive relationship between PD and R-squared for retail instruments. This simple analysis demonstrates that capital can vary largely with the assumptions made to the asset correlation parameter. Using the parameters from the existing Basel II requirement could lead to significant under-estimation of capital requirements for risky borrowers.

[10] LGD is assumed to be 50%


FIGURE 23 Capital Charge Based on Different Correlation Estimates

# 7 CONCLUSION

Our analyses find little empirical support for the decreasing relationship between asset correlation and default probability for CRE and retail. In addition, we also find that, after controlling for firm size, there is no strong negative relationship between asset correlation and default probability for corporate. The exact relationship between asset correlation and default probability is complicated and may not be consistent across asset classes. For CRE and retail exposures, the empirical evidence suggests that the relationship is more likely to be an increasing one. The stylized decreasing relationship in the IRB approach of the Basel II Accord does not appear to have theoretical nor empirical support and may underestimate portfolio risk for high default probability portfolios. In light of our research, in addition to the previous results by Düllmann and Scheule (2003) and Dietsch and Petey (2004), we encourage the Basel Committee to revisit the use of this relationship in bank capital requirement.

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