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UNDERSTANDING ASSET CORRELATION DYNAMICS FOR STRESS TESTING

MODELINGMETHODOLOGY

AUTHORS

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ABSTRACT

The Moody's KMV approach to modeling asset correlation in measuring portfolio credit risk is to decompose a borrower's risk into systematic and idiosyncratic components. Pairs of borrowers within a portfolio are correlated through their exposures to systematic factors. Specifically, there are two sets of inputs that determine the pair-wise correlation. The first set of inputs is the proportion of risk that is captured by the systematic factors, or R-squared values. The second set of inputs is the correlations among the respective systematic factors, or systematic factor correlations.

Understanding how the components of asset correlation change through time will allow us to investigate how asset correlation dynamics behave during periods of economic stress. Although the time-varying correlation of equity returns has been extensively researched, we have found few studies on the dynamics of asset correlation over time.

In this paper, we explore how both R-squared values and systematic factor correlations change through time. We show that R-squared values are more volatile than the systematic factor correlations. We also study the relationship between changes in R-squared and changes in factor variance, as well as the relationship between changes in factor correlation and changes in factor variance.

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1 OVERVIEW

The Moody's KMV approach to modeling asset correlation in measuring portfolio credit risk is to decompose a borrower's risk into systematic and idiosyncratic components.[1] Pairs of borrowers within a portfolio are correlated through their exposures to systematic factors. Specifically, there are two sets of inputs that determine the pair-wise correlation.

- The proportion of risk that is captured by the systematic factors, or R-squared values.
- Correlations among the respective systematic factors or systematic factor correlations.

Understanding how the components of asset correlation change through time will allow us to investigate how asset correlation dynamics behave during periods of economic stress. For example, does empirical evidence show that asset correlation tends to increase when the systematic factor variance is high? If such relationships exist, it would be beneficial to utilize them for stress testing purposes.

The objective of this study is to understand the asset correlation dynamics over time from the perspective of stress testing. Although the time-varying correlation of equity returns has been extensively researched, we have found few studies on the dynamics of asset correlation. We will explore how both R-squared values and systematic factor correlations change through time, from which we can infer the asset correlation dynamics. We will also study the relationships between changes in R-squared and factor correlations and changes in factor variance. In addition, we will show that R-squared values are more volatile than factor correlations, and that R-squared changes have a larger impact on asset correlation.

In this study, we estimate a 1-year R-squared using weekly returns for all firms in the Moody's KMV global public firm database. We estimate the 1-year R-squared because it is more dynamic in nature (by construction) than the 3-year R-squared Moody's KMV recommends to its clients. Moody's KMV still finds that the 3-year R-squared is better at predicting future correlations; however, because the purpose of this study is for stress testing, we choose to investigate the more dynamic 1-year R-squared. Using the 1-year R-squared, we construct the time series of R-squared percentiles by geographical region and financial or industrial classification. For North American companies, we perform firm-level univariate regressions to assess the relationships between changes in R-squared (or factor correlations) and changes in the variance of systematic factors.

Similar to the R-squared analysis, we utilize a systematic factor correlation estimate from a shorter time window (1-year or 3-year) rather than a long term factor correlation estimate (e.g., 10+ years). Because the 1-year or 3-year factor correlation estimate is more volatile than the long term correlation estimate, it is more appropriate to use when we explore the dynamics of factor correlations and its relationship with changes in factor variance. It is worth noting that Moody's KMV still recommends estimating the systematic factor correlation using a long window (i.e., 10+ years) when predicting future correlations because of the mean reverting behavior of correlations, as well as the precision of the correlation estimate.

This paper is organized in the following way.

- Section 2 briefly describes the Moody's KMV Global Correlation Model™ (GCorr) correlation structure.
- Section 3 discusses the properties of the data underlying this study, including the asset return series, R-squared, and composite factors (defined in Section 2).
- Section 4 illustrates the relative stability of composite factor correlations over the last decade.
- Section 5 presents the R-squared dynamics graphically for North American financial and industrial firms from 1990Q1 through 2008Q4.

[1] Asset correlation is the correlation between the changes in credit quality measures for any two borrowers. Note that the borrowers can be from any asset class (commercial real estate, retail, corporate, etc.).

- Section 6 provides firm-level regression analyses on the contemporaneous relationship between changes in R-squared (or factor correlation) and changes in factor variance. In addition, we provide an example comparing the impacts on asset correlation of R-squared changes versus factor correlation changes.
- Section 7 concludes the paper with highlights of major findings.

2 GCORR CORPORATE CORRLEATION STRUCTURE

The GCorr Corporate factor model provides pair-wise asset correlations for roughly 34,000 firms which are currently publicly traded. As depicted in Figure 1, in the GCorr Corporate model framework, corporate risk is decomposed into systematic and idiosyncratic portions.[2] Furthermore, the systematic risk factor is a weighted sum of country and industry factors to which the firm has exposure. In this paper, the term *composite factor* is used to denote the systematic risk factor. Formally, the composite factor of firm k is expressed as:

$$\phi_k = \sum_{C=1}^{49} w_{k,C} r_C + \sum_{I=1}^{61} w_{k,I} r_I \tag{1}$$

Where r_C and r_I denote a country risk factor and an industry risk factor, respectively, and $w_{k,C}$ and $w_{k,I}$ denote the respective weights for firm k.

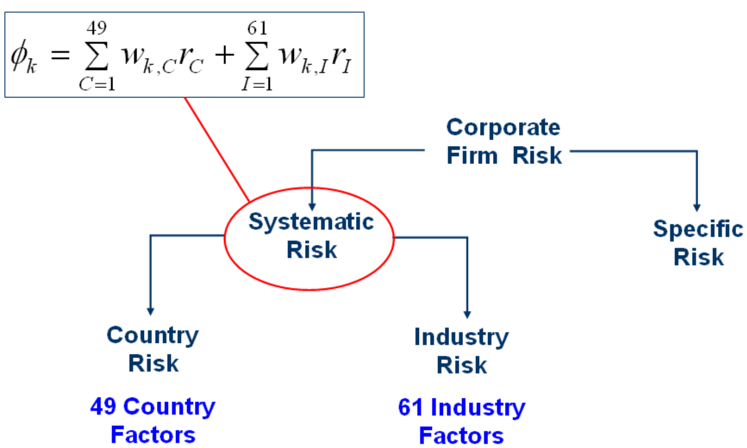

FIGURE 1 GCorr Correlation Structure

Meanwhile, the asset return of firm k is linearly related to the return on the composite factor and the relationship can be determined by the following regression:

$$r_k = \alpha_k + \beta_k \phi_k + \varepsilon_k \tag{2}$$

The R-squared from the firm level regression indicates the percentage of the firm's total risk, which is captured by systematic factors:

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[2] For a complete description of the GCorr model, see Hu, Z., A. Kaplin, A. Levy, Q. Meng, N.Patel, Y. Tsaig, Y. Wang, T. Yahalom, and F. Zhu. "Modeling Credit Portfolios."

$$RSQ_k = \frac{\beta_k^2 \sigma_{\phi_k}^2}{\sigma_{r_k}^2} \quad (3)$$

In this framework, the correlation between any two firms is given by:

$$corr(r_i, r_k) = \frac{\text{cov}(\beta_i \phi_i + \varepsilon_i, \beta_k \phi_k + \varepsilon_k)}{\sigma_{r_i} \sigma_{r_k}} = \frac{\beta_i \beta_k \sigma_{\phi_i} \sigma_{\phi_k} corr(\phi_i, \phi_k)}{\sigma_{r_i} \sigma_{r_k}} = \sqrt{RSQ_i}\sqrt{RSQ_k}\, corr(\phi_i, \phi_k) \quad (4)$$

Therefore, the asset correlation between two firms can be specified by two sets of parameters: the R-squared value of each of the two firms, and the correlation among the composite factors. It is also worth noting that both R-squared and the composite factor correlations can be written as a function of the composite factor variance and other variables. In Section 6, we run the firm level univariate regressions of changes in R-squared or factor correlations onto changes in composite factor variance. We examine the magnitude of the betas from the regressions to help determine the change in R-squared or factor correlations given a change in composite factor variance.

The representation of asset correlation as the product of square roots of R-squared values and composite factor correlations has the advantage that we can analyze the effect of each. We take this approach below; we analyze R-squared and factor correlation separately, and compare their respective impacts on correlation.

3 DATA

This section summarizes the raw data and several key variables that underlie this research. Section 3.1 gives an overview of the raw dataset with emphasis on weekly firm-level asset return series. Moreover, we discuss how we control our sample, as well as the classification methods. Section 3.2 outlines a 1-year moving window procedure to estimate the firm-level R-squared and construct its time series. In Section 3.3, we utilize the same 1-year moving window procedure to obtain the time series of composite factor correlations and variances.

3.1 Firm-level Asset Return Series

Our raw data consists of weekly asset return series and weekly composite factor series from the Moody's KMV production database. It covers more than 50,000 public firms in 49 countries and 61 industries from January 1990 through December 2008. The weekly asset returns are derived from equity returns and liability structure information using an option-theoretic framework. The equity returns are calculated from Wednesday to Wednesday, and are adjusted for corporate actions such as dividend payouts, splits, and mergers.

Although over 50,000 unique firms appear in this dataset, the number of firms varies over time due to bankruptcy, merger/acquisition, new firm entry, etc. The potential population shift in the dataset can make it difficult to interpret the empirical results. Therefore, we decide to control the population by restricting the dataset to firms which existed throughout the entire period of January 1990 through December 2008. In addition, due to possible industry effects, we classify firms as financials or industrials and look at each group separately. This study focuses on companies in North America, including United States and Canada. Approximately 26% of the firms in the database are from this region. The controlled North American population has 173 financial firms and 1,045 industrial firms.

Note that the sample period (January 1990 through December 2008) includes three business-cycle contractions, as defined by the National Bureau of Economic Research (NBER), as well as the blowup of Long Term Capital Management (LTCM) in 1998.

3.2 Firm-level R-squared

The firm-level R-squared is the coefficient of determination in the regression of the (log) asset return on the (log) composite factor return. To explore the time-varying dynamics of R-squared, we follow a moving window approach to estimate the empirical R-squared each quarter. We estimate the 1-year R-squared quarterly using a 1-year moving window, and calculate the first R-squared estimate using the 1-year weekly asset return and composite factor series between January 1990 and December 1990. Next, we move the window forward by one quarter, and we estimate the R-squared for the period April 1990 through March 1991. We obtain a time series of 1-year R-squared in the end for each firm, with 73 observations in this time series.

We use the same 1-year moving window procedure to construct the time series of composite factor variance and composite factor correlation (Section 3.3). Therefore, we estimate both factor variance and factor correlation in the same moving windows as firm-level R-squared. There are 73 observations in each time series.

From the time series of 1-year R-squared, we derive the time series of quarterly changes in R-squared (Section 5.2) which has 72 observations. Likewise, the time series of quarterly changes in composite factor variance (or correlation) can be produced. The resulting time series of changes in factor variance is linked to those of changes in R-squared (or factor correlation).

Similarly, we can estimate the 3-year R-squared quarterly using a 3-year moving window. Figure 2 and Figure 3 illustrate three percentiles of the 1-year and 3-year R-squared between January 1992 and December 2007 for one sample, respectively.[3] Not surprisingly, the time series of 3-year R-squared is much smoother than the 1-year R-squared. Although Moody's KMV finds that the 3-year R-squared is better than the 1-year R-squared at predicting future asset correlations, we utilize the 1-year R-squared in this study. The more dynamic 1-year R-squared is more suitable for investigating contemporaneous movements of R-squared. It is worth pointing out that the 3-year R-squared is better at predicting future correlation due to the mean reverting behavior we see in correlations. For example, Figure 10 and Figure 11 show that R-squared values appear to follow a cyclical pattern. Similarly we see a cyclical or mean reverting pattern for the composite factor correlation as shown in Figure 6 and Figure 7. Note that there are about 50 weekly observations in a 1-year window, so the 1-year R-squared is reasonably well-measured.

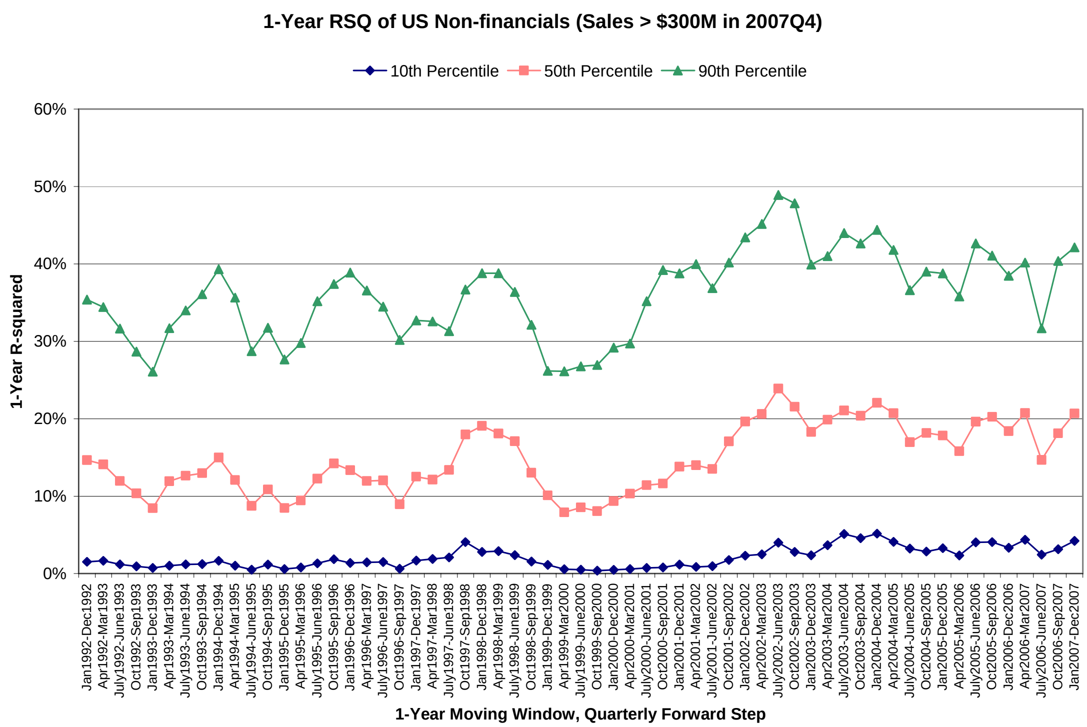

FIGURE 2 Time Series of 10th, 50th, 90th Percentiles of 1-year R-squared

[3] U.S. non-financials with sales greater than $300 million in 2007.

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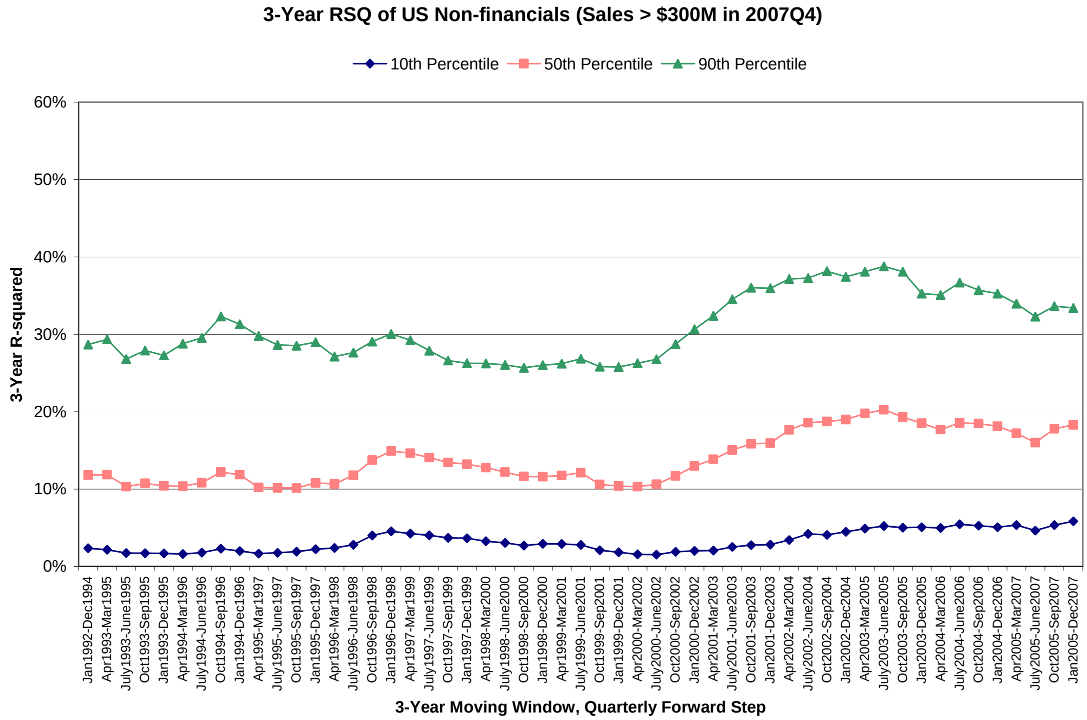

FIGURE 3 Time Series of 10th, 50th, 90th Percentiles of 3-year R-squared

3.3 Composite Factors

Recall that the firm-specific composite factor summarizes all systematic risk factors affecting a firm. It is defined as a weighted sum of country return factors and industry return factors. The weights depend on the firm's exposure to different countries and industries. For example, if firm k has a 100% country exposure to the U.S., its composite factor is the following:

$$r_{\phi_k} = \sum_I w_{k,I} r_I + r_{US} \tag{5}$$

In the remainder of this paper, we investigate the dynamics of three variables: composite factor variance, composite factor correlation, and firm-level R-squared. Ultimately, we are interested in linking the dynamics of the composite factor variance to those of the others. In this analysis, we apply a common moving window methodology to these variables to ensure consistency in exploring dynamics. To obtain time series of weekly composite factor variances (or correlations), we use the same 1-year moving window approach for estimating R-squared (Section 3.2). For instance, we estimate the weekly variance of composite factor quarterly using the 1-year moving windows.

Figure 4 and Figure 5 present five different percentile points of the U.S. composite factor variances through time by the financial and industrial classifications. The narrow band between the 10th and 90th percentiles indicates that there is not much cross-sectional variation in composite factor variance across U.S. industries. This is because empirically the variance of r_{US} is the dominant term in the construction of the U.S. composite factor variance.

As a side note, we find the U.S. composite factor variances are significantly positively correlated with the S&P 500 volatility and VIX.[4] During the recessions, the level of composite factor variance is generally high. All percentiles reach

[4] Chicago Board Options Exchange Volatility Index.

their historical highs in 2008. We also compare the dynamics of composite factor return versus its variance. The empirical results suggest that there is a strong negative contemporaneous relationship between them. Intuitively, higher market volatility is usually accompanied by lower market return, and vice versa. We do not include these results in this paper.

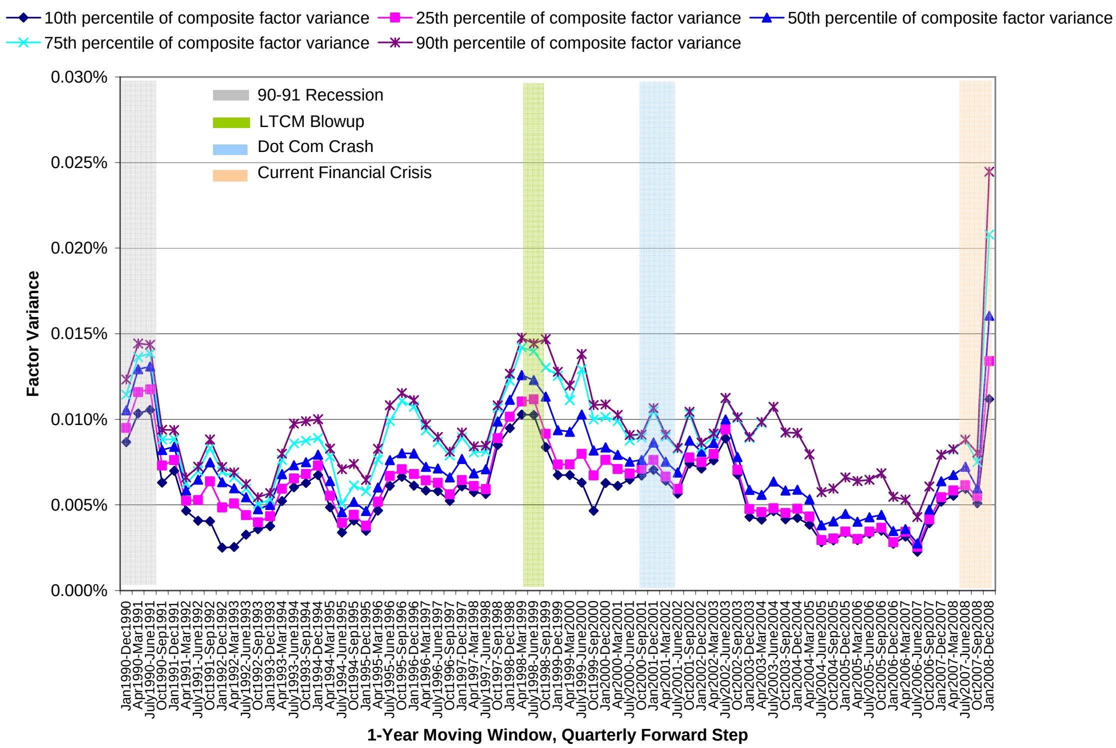

FIGURE 4 Percentiles of Composite Factor Variances over Time: Controlled Sample of U.S. Financials

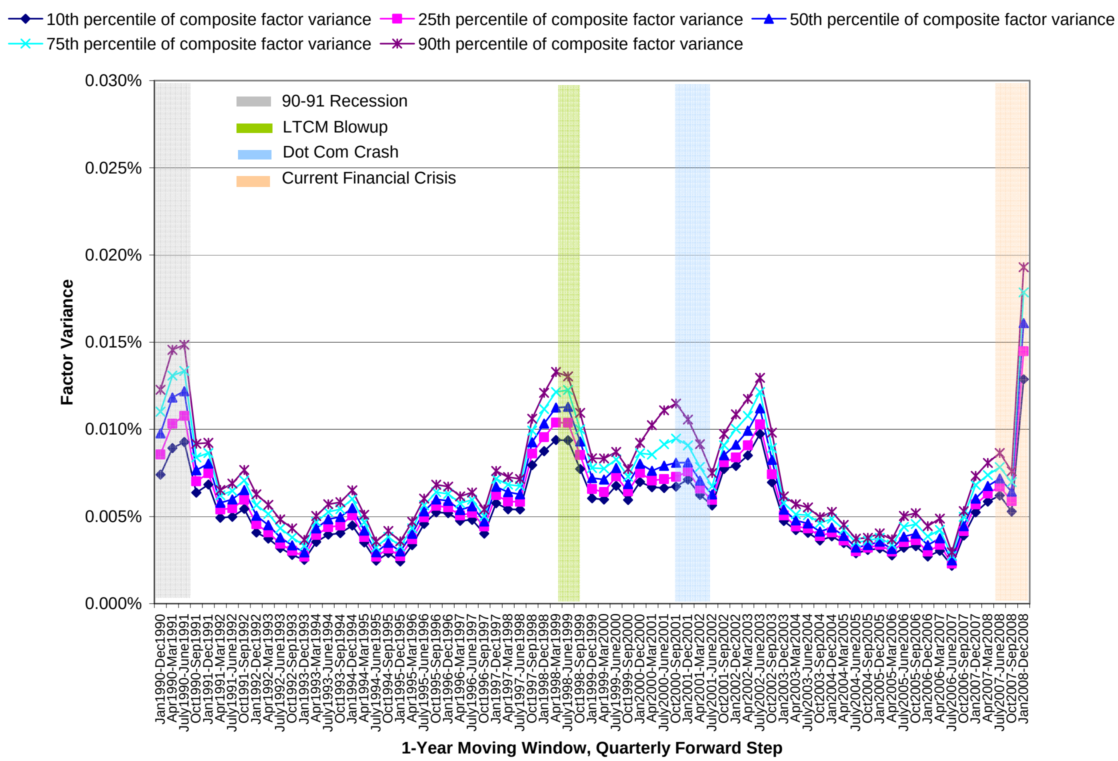

FIGURE 5 Percentiles of Composite Factor Variances over Time: Controlled Sample of U.S. Industrials

4 COMPOSITE FACTOR CORRELATION DYNAMICS

In a similar spirit as the R-squared analysis, we choose to estimate the composite factor correlations using a shorter window so that the dynamics are more pronounced. Since we explore the composite factor correlation dynamics for the purpose of stress testing, we would like to utilize a more volatile time series. In our first set of analysis we will look at how 3-year composite factor correlation estimates deviate from a long run 10-year estimate.[5] When we study the relationship between changes in factor correlation and changes in factor variance in Section 6.2, we use a 1-year factor correlation estimate, consistent with the R-squared analysis of Section 5.

This section presents a preliminary analysis of the composite factor correlation dynamics for the U.S. and the U.K. We construct some composite factors using a 100% weight on one country index and one industry index. For example, the U.S.-Bank index is the sum of the U.S. country index and the Banks and S&Ls index. It can be conceived as the composite factor for a hypothetical firm fully exposed to the U.S. and the Banks and S&Ls industry. Similarly, we can construct the U.S.-Real Estate and U.K.-Automobiles indices, among others.

For a particular pair of composite factors, say U.S.-Bank and U.S.-Real Estate, we can estimate their historical correlation over the 10-year period from 1998Q4 through 2008Q4. Alternatively, we use a 3-year moving window and estimate their historical correlation within each window every year. For those pairs of composite factors we generate, the differences between the 3-year correlation estimates and the 10-year correlation estimate are generally marginal. Figure 6 illustrates this using four industry pairs within the U.S. The largest absolute difference between 3-year correlation estimates and the 10-year correlation estimate is only 5%. As shown in Section 6.3, changes in R-squared are more

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[5] The 3-year composite factor correlation is estimated annually using a 3-year moving window. The first 3-year correlation estimate is calculated using 3 years of weekly composite factor returns between October 1998 to October 2001, or equivalently the beginning of 1998Q4 to the beginning of 2001Q4. Next, the window is moved forward by one year and we estimate the factor correlations for the period October 1999 to October 2002.

dynamic than changes in factor correlation within the U.S. Figure 7 shows the differences for the same four industry pairs in the U.K. The largest absolute difference is 7%.

Figure 8 and Figure 9 present the differences between the 3-year and 10-year estimates for selected industry pairs across the U.S. and the U.K. Figure 8 shows four pairs of different industries, while Figure 9 looks at four same-industry pairs. All absolute differences in the figures are bounded by 10%.

FIGURE 6 Difference between 3-year Correlation Estimates and 10-year Correlation Estimate: Four Industry Pairs in U.S.

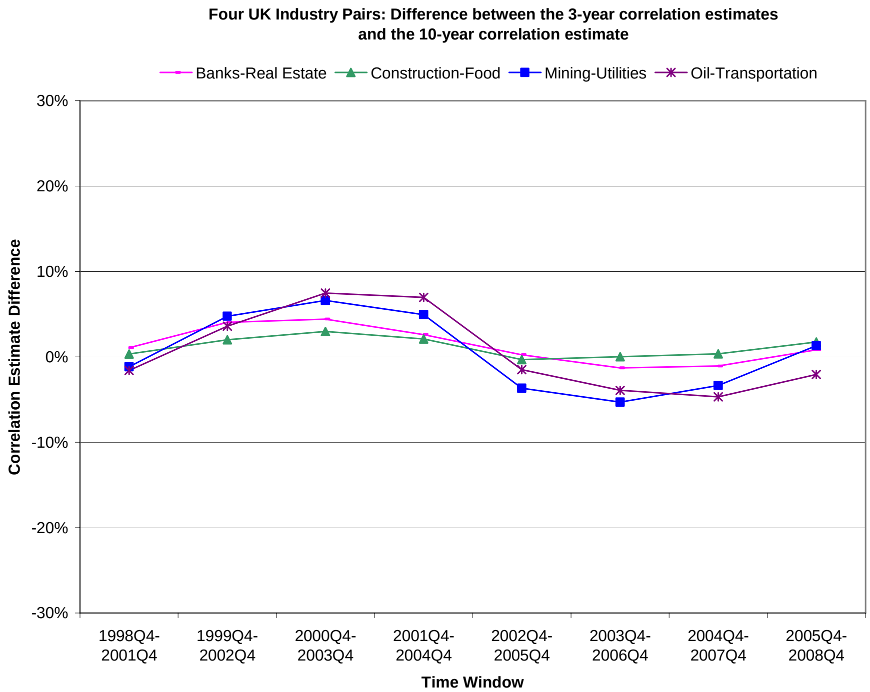


FIGURE 7 Difference between 3-year Correlation Estimates and 10-year Correlation Estimate: Four Industry Pairs in U.K.

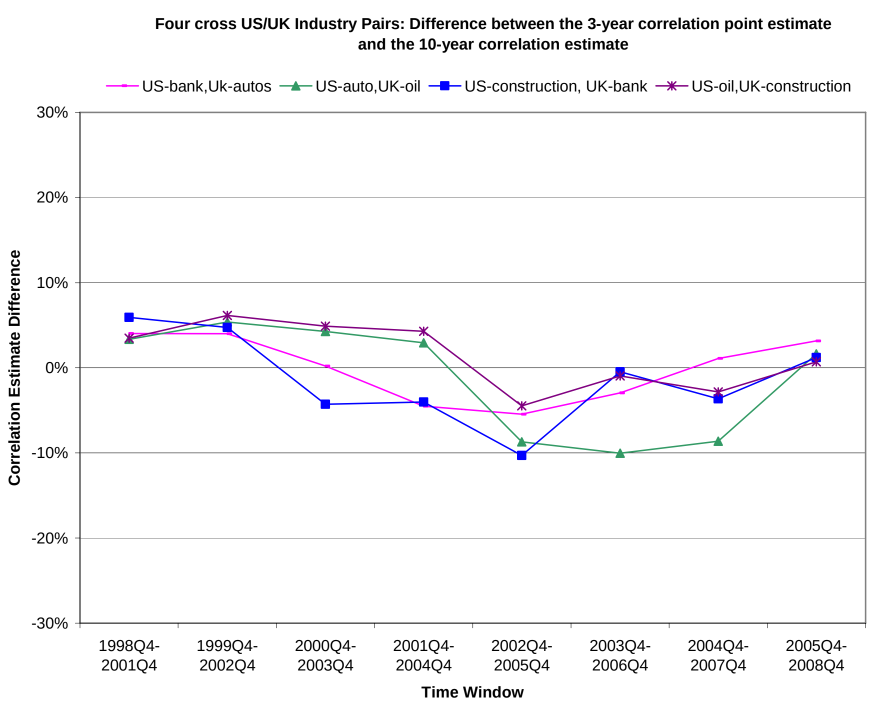


FIGURE 8 Difference between 3-year Correlation Estimates and 10-year Correlation Estimate: Four Industry Pairs cross U.S. and U.K.

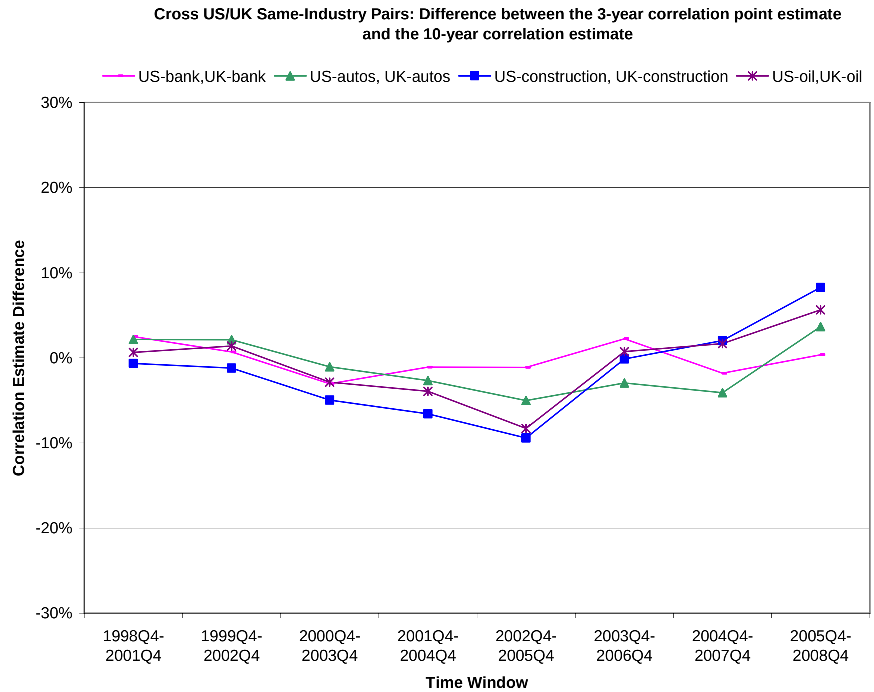


FIGURE 9 Difference between 3-year Correlation Estimates and 10-year Correlation Estimate: Four Same-industry Pairs cross U.S. and U.K.

5 R-SQUARED DYNAMICS

This section presents the R-squared dynamics for North American companies by financial or industrial classification. The most recent three NBER contraction periods and the 1998 LTCM blowup are highlighted in each figure. As mentioned in Section 3.1, the population is controlled so that the same set of firms is present throughout the time sample. A potential issue with the uncontrolled population is that it includes firms that have exited early or entered over time. For the objective of investigating the time-varying R-squared, the distribution of R-squared of uncontrolled samples certainly exhibits some bias. In light of potential interest in the R-squared dynamics of uncontrolled samples, Appendix A presents the R-squared dynamics for all firms in North America and Europe. The discussions in Sections 5.1 and 5.2 focus on the controlled samples only.

5.1 R-squared Level

After restricting the dataset to firms which existed through the sample period, there are 173 financial and 1,045 industrial firms in North America. Time series of the R-squared percentiles for the controlled samples of North American financial and industrial firms are displayed in Figure 10 and Figure 11, respectively. First, it appears that percentiles of R-squared are closely correlated within either financial or industrial group. This suggests that the R-squared of all firms in either group are affected by some common macro factor(s). Second, the median R-squared level of industrial firms is lower than that of financial firms. Third, the median R-squared in either financial or industrial group appears higher than its counterpart in Appendix A. This may be attributable to the fact that firms existing throughout the period are larger in size. Large firms tend to have higher R-squared than small firms.

The general dynamics of the R-squared for North America firms is very intriguing. There is a deep decline coming out of the 1990–1991 recession. After the Dot Com crash in 2001, the median R-squared seems to have increased. In the first two NBER contraction periods and the LTCM crisis, the realized R-squared values for both financials and industrials increase going into these periods, then decline. During the Dot Com crash in 2001, the realized R-squared values quickly reverse the downward momentum and increase abruptly before the end of recession. In the financial crisis starting in 2007Q4, the realized R-squared values for both financials and industrials initially drop and then increase significantly. Note that all R-squared percentiles for industrials hit their respective historical highs in 2008. We also study the R-squared dynamics exhibited by European firms. Appendix B presents the R-squared percentiles for the controlled European firms.

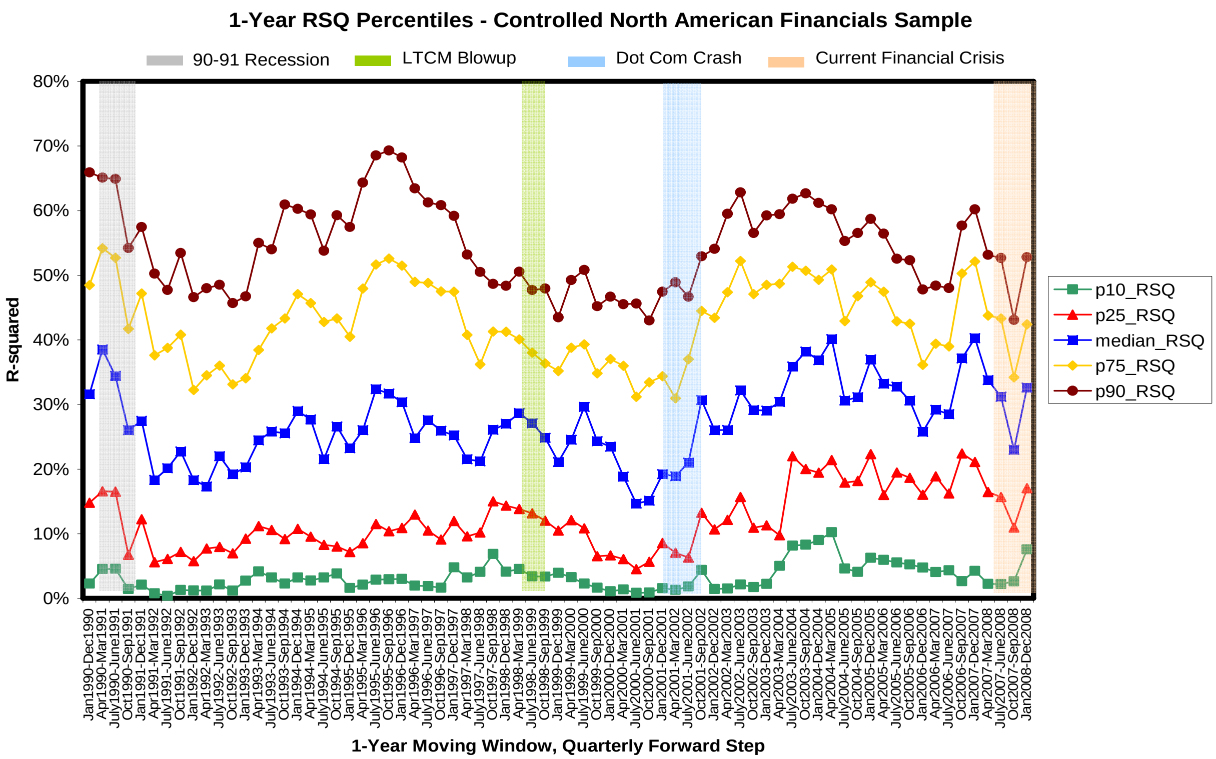


FIGURE 10 Time Series of R-squared Percentiles: Controlled North American Financials

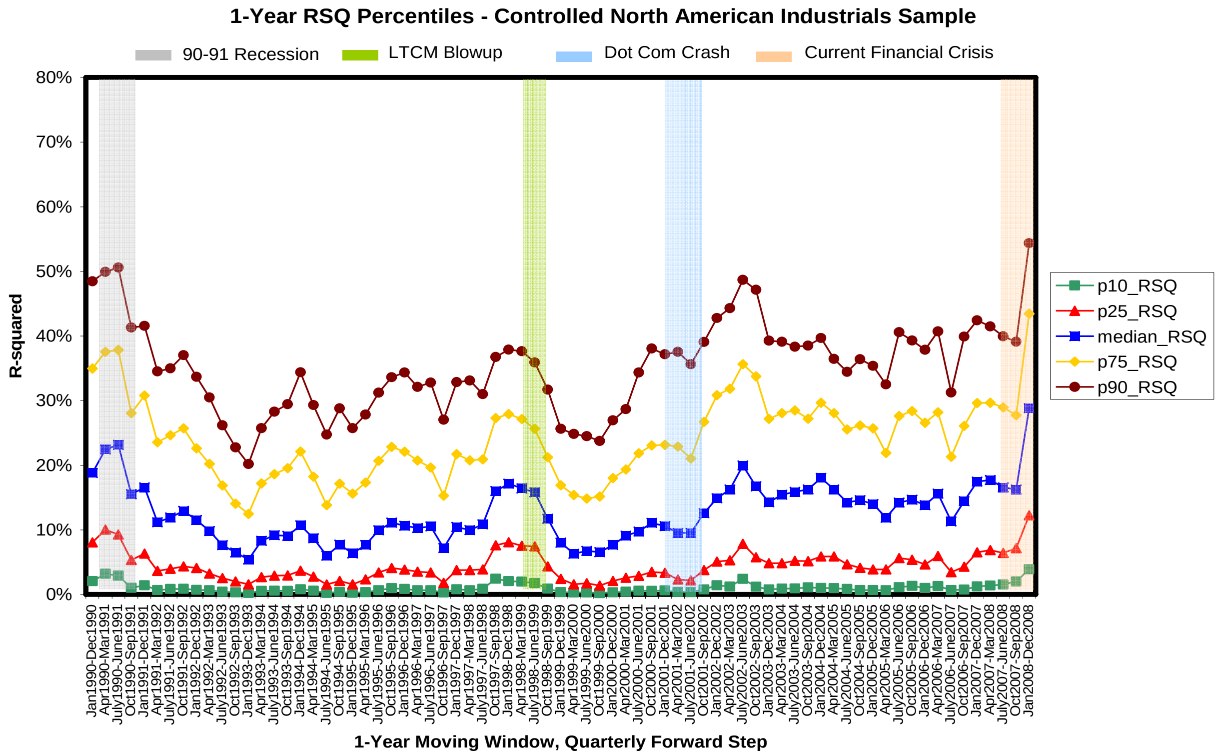


FIGURE 11 Time Series of R-squared Percentiles: Controlled North American Industrials

5.2 R-squared Change

We are also interested in the distribution of the firm-level R-squared change over time. The quarterly change in R-squared at time t is defined to be the raw change (not percent change) in R-squared level from time t to $t-1$. Therefore, time series of quarterly changes in R-squared for every firm can be derived from the time series of R-squared level. Figure 12 and Figure 13 display the percentiles of R-squared changes for the controlled North American financial and industrial firms, respectively. Note that the median line represents the 50th percentile of all firm-level R-squared changes over the past quarter, not the quarterly change in the median R-squared levels. The same applies to all percentiles.

It appears that the median quarterly changes in R-squared for financials are more erratic than industrials. For both financial and industrial samples, the median R-squared changes seem to have a mean-reverting behavior around the zero horizontal line. We observe similar patterns for the controlled European firms.

In the financial crisis of 2008, the 50th to 90th percentiles of changes in R-squared for both financial and industrial firms are at their highest levels on record. For example, the 90th percentile of changes in R-squared for industrial firms is approximately 27%. This value represents the approximate magnitude of maximum changes in R-squared to date, and can be used as an input for stress testing purposes.

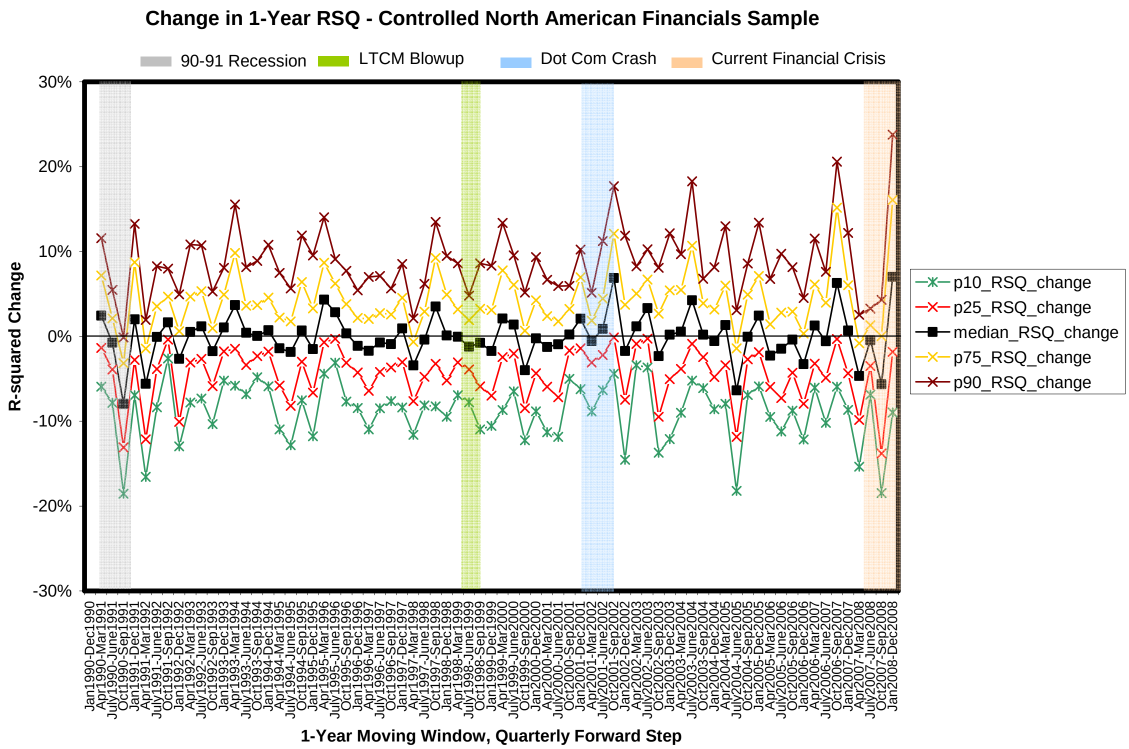


FIGURE 12 Percentiles of R-squared Changes: Controlled North American Financials

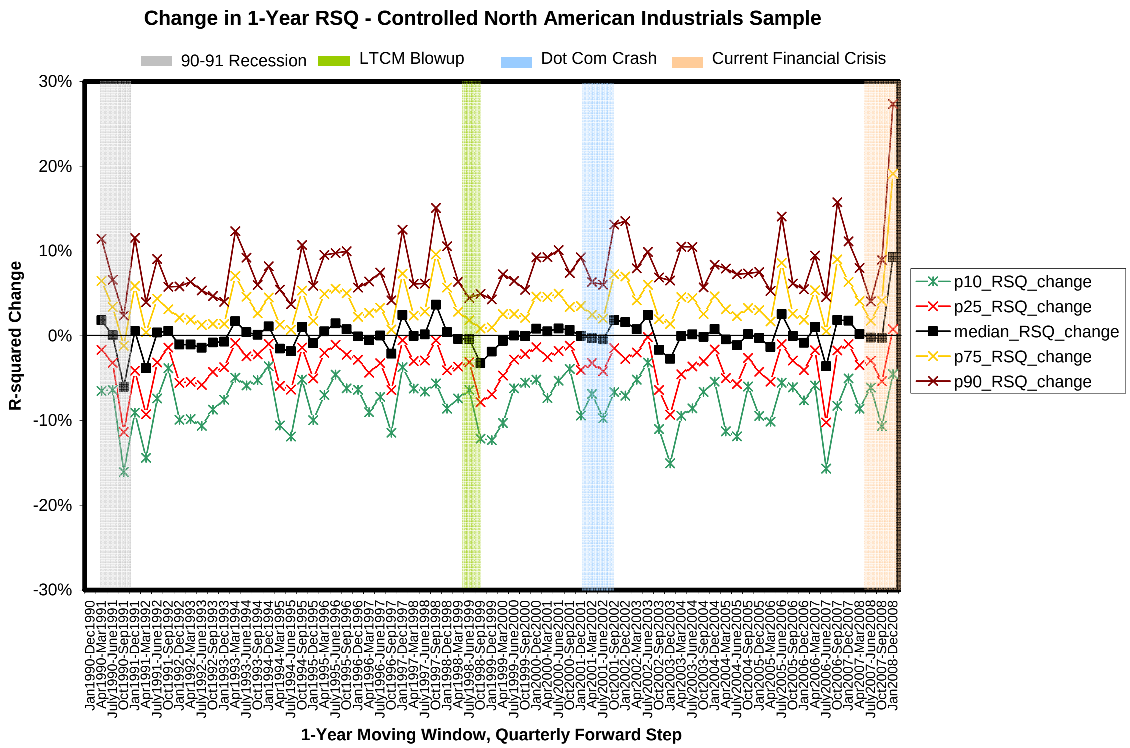


FIGURE 13 Percentiles of R-squared Changes: Controlled North American Industrials

6 CONTEMPORANEOUS RELATIONSHPS

Section 5 presents the R-squared dynamics for North America at an aggregated level. The remainder of this paper focuses on how asset correlation dynamics is related to the composite factor variance, a proxy to market variance. Because the construction of asset correlation involves two components, R-squared and the composite factor correlation, we perform regression analyses that relate changes in each component to changes in composite factor variance. To be consistent with the R-squared analysis in Section 5, we use the 1-year moving window procedure to construct the time series of quarterly changes in factor variance and factor correlation. We find two significant contemporaneous relationships. Both the change in R-squared and the change in factor correlation are positively related to the contemporaneous change in factor variance. Sections 6.1 and 6.2 provide details on the time series regression studies. In Section 6.3 we dissect the asset correlation dynamics and illustrate the relative importance of dynamics of R-squared and composite factor correlation in determining asset correlation.

6.1 Relationship of R-squared and Factor Variance

For North American firms, two approaches are used to assess the contemporaneous relationship between changes in R-squared and changes in composite factor variance. First, correlations between the two time series are computed at the firm level. Second, we employ univariate regression to measure the relationship and provide significance statistics.

Figure 14 presents the major percentiles of correlation between changes in R-squared and changes in composite factor variance, organized by financial or industrial classification. The signs of correlation are almost always positive regardless of the financial and industrial classification. The same characteristics are also observed for European and Japanese companies.

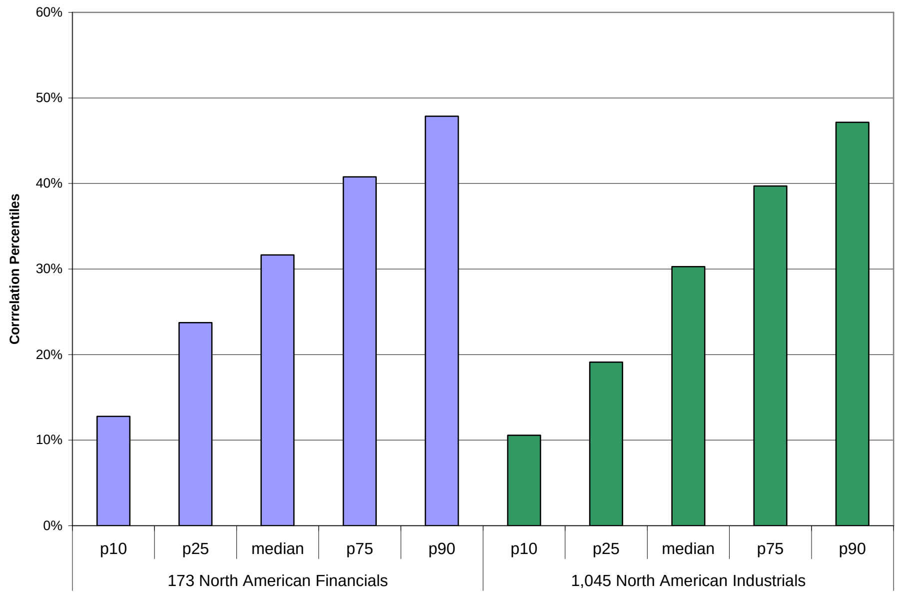


FIGURE 14 Percentiles of Correlation between Changes in R-squared and Changes in Factor Variance

Next, we use regressions to formally quantify the significance of relationship between changes in R-squared and changes in composite factor variance. A time series regression is performed for every North American firm in the controlled sample over the period of 1990Q1 to 2008Q3.[6] The regression results are summarized in Table 1 by financial or industrial classification. Table 1 displays the 10th, 50th, and 90th percentiles of beta estimates, the percentage of firms with beta estimates significant at 5%, and median R-squared from the regressions.[7]

TABLE 1 Relationship between Changes in R-squared and Changes in Factor Variance: North American Financials and Industrials

Sample Population	Dependent Variable	Independent Variable	Beta Percentiles			Pct. (Sig Level < 5%	Median R-squared
			p10	p50	p90		
173 North American financials	RSQ change	Change in Composite Factor Variance	598	1857	3546	76%	10%
1045 North American industrials	RSQ change	Change in Composite Factor Variance	410	1521	3087	66%	9%

[6] The quantitative analyses in this paper are based on the data from 1990Q1 to 2008Q3. At the end of this research, we were able to obtain the 2008Q4 data and updated dynamics plots (e.g., Figure 10) by extending the time series by one more quarter.

[7] For regressions that exhibit serial correlation due to overlapping time windows, parameter significance is determined through the use of Newey-West standard errors.

6.2 Relationship of Factor Correlation and Factor Variance

Recall that the pair-wise asset correlation is determined not only by the respective R-squared, but also by the correlation among composite factors. In the same spirit as Section 6.1, this section addresses the contemporaneous relationship between changes in composite factor correlation and changes in composite factor variance.

We look at 61 weekly U.S.-industry composite factors from 1992Q2 to 2008Q3. They are constructed using a 100% weight on the U.S. country index and one of the 61 distinct industry indices. Table 2 presents results from regressing changes in each of 1,830 pair-wise correlations of the U.S.-industry indices on the median changes in their variances.[8] The beta estimates are highly significant. This suggests that, for the U.S. there is a strong positive relationship between changes in factor correlation and contemporaneous changes in factor variance.

TABLE 2 Relationship between Changes in Factor Correlation and Changes in Factor Variance: 61 U.S. Industry Indices

Dependent Variable	Independent Variable	Beta Percentiles			Pct. (Sig Level < 5%	Median R-squared
		p10	p50	p90		
Changes in each of 1830 pair-wise correlations: 61 U.S. industry indices.	Median changes in factor variance: 61 U.S. industry indices.	761	1210	1855	99.9%	22.8%

6.3 Decomposing Asset Correlation Dynamics

Based on the significant contemporaneous relationships found in Sections 6.1 and 6.2 , this section explores the relative importance of changes in R-squared (or composite factor correlation) on changes in asset return correlation. As explained in Section 2, the correlation between two firms, *a* and *b*, can be described as:

$$corr(r_a, r_b) = \sqrt{RSQ_a}\sqrt{RSQ_b}\,corr(\phi_a, \phi_b) \qquad [6]$$

We proceed by analyzing how changes in composite factor variances impact R-squared versus the composite factor correlations. Table 3 summarizes the standard deviations of quarterly changes in three variables: the weekly factor variance, the factor correlation, and R-squared. For North American firms, Table 4 reviews the beta percentiles from regressions of changes in R-squared (or composite factor correlation) on contemporaneous changes in composite factor variance. By combining the relevant parameters (marked by asterisks) in Table 3 and Table 4, we can compute how a change in composite factor variance impacts R-squared and composite factor correlation, and, consequently, the resulting impact on asset correlation.

TABLE 3 Standard Deviations of Quarterly Changes in Factor Variance, Factor Correlation and R-squared

Variable	Range of Standard Deviations	Median Standard Deviation
Change in factor variance: 61 U.S. industry indices	[0.009%, 0.0016%]	0.0011%*
Change in each pair-wise factor correlation: 61 U.S. industry indices	[0.86%, 5.96%]	2.62%
Change in RSQ: North American financials	[3.36%, 10.71%]	8.17%
Change in RSQ: North American industrials	[2.51%, 11.10%]	7.01%

[8] As shown in Figure 4 and Figure 5, there is not much cross-sectional variation across the U.S. industries of the composite factor variance.

TABLE 4 Review of Percentiles of Beta Estimates

Dependent Variable	Independent Variable	p10 Beta	p50 Beta	p90 Beta
Change in RSQ: North American financials	Change in factor variance	598	1857*	3546
Change in RSQ: North American industrials	Change in factor variance	410	1520*	3087
Change in pair-wise correlation of U.S. industry factors	Change in median factor variance	761	1210*	1855

We present an example below using the median standard deviation of factor variance changes as well as the median beta estimates. Given a 3 standard deviation change in composite factor variance (i.e., 0.0033%), the resulting quarterly raw changes in R-squared and the composite factor correlation are similar in magnitude:

- Change in R-squared (financials) = (1857)(.0033%) ~ 6%
- Change in R-squared (industrials) = (1520)(.0033%) ~ 5%
- Change in factor correlation = (1210)(.0033%) ~ 4%.

As a side note, caution needs to be taken when interpreting these values. For instance, the 6% quarterly change in R-squared is the expected change in response to a 3 standard deviation change in composite factor variance. It does not mean that 6% represents the 3 standard deviation change in R-squared for financials. In fact, we know from Table 3 that the 3 standard deviation change in R-squared is approximately 24% for a typical financial firm.

Although the expected quarterly changes in R-squared and factor correlation are close given the 3 standard deviation change in factor variance, the percentage of change in R-squared is much more pronounced because its level is generally lower than the factor correlation. Hence, changes in R-squared are more dynamic than changes in factor correlation. We use an example to demonstrate that the more volatile R-squared values indeed have a larger impact on asset correlation dynamics.

Suppose also that that two industrial firms, a and b, face typical parameters:

$$RSQ_a = RSQ_b = 20\%$$

$$corr(\phi_a, \phi_b) = 70\%$$

Therefore, their unadulterated asset correlation is:

$$corr(r_a, r_b) = (20\%)(70\%) = 14\%$$

In this example, the impact on asset correlation of allowing the composite factor variance to only impact R-squared is:

$$\Delta corr(r_a, r_b) = (25\%)(70\%) - (20\%)(70\%) = 17.5\% - 14\% = 3.5\%$$

Meanwhile, the impact on asset correlation of allowing the composite factor variance to only impact factor correlation is:

$$\Delta corr(r_a, r_b) = (20\%)(74\%) - (20\%)(70\%) = 14.8\% - 14\% = 0.8\%$$

The impact on asset correlation of allowing the composite factor variance to impact both R-squared and factor correlation is:

$$\Delta corr(r_a, r_b) = (25\%)(74\%) - (20\%)(70\%) = 18.5\% - 14\% = 4.5\%$$

In short, within the context of factor variance dynamics, the exercise demonstrates that dynamics in R-squared have much more of an impact on asset correlation than dynamics in factor correlations.

7 CONCLUSION

This empirical study explores the dynamics of the two components of asset correlation: Composite factor correlation and R-squared. We find that the R-squared values appear to be more dynamic than factor correlations. We also regress changes in R-squared and factor correlations onto contemporaneous changes in composite factor variance, and find a positive relationship among these sets of variables. Using the results of the regressions, we use an example to illustrate that, given an increase in factor variance, both R-squared values and factor correlations increase. This also shows that the increase in R-squared contributes more to the overall increase in asset correlation for two typical firms in our samples. This demonstrates the need to incorporate stressed R-squared values in any comprehensive stress testing solution.

APPENDIX A R-SQUARED DYNAMICS (UNCONTROLLED POPULATION)

Figure 15 and Figure 16 illustrate the distribution of R-squared over time for all North American financials and industrials. The median R-squared for industrial firms is around 10%, while the median R-squared for financials is consistently above 10% and can reach 20%. The 90th percentile R-squared for industrial firms is mostly below 40%, about the same magnitude as the 75th percentile of financials.

Figure 17 and Figure 18 illustrate the distribution of R-squared over time for all financials and industrials in Europe.[9] Similar to patterns with North America, there is a strong co-movement across percentiles. On average, financials have higher R-squared than industrials. However, these R-squared time series look drastically different than those of North America. By our definition, the Europe region incorporates many more countries than North America. These countries are a mix of industrialized and emerging economies.

[9] The countries in our Europe sample include: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Spain, Sweden, Switzerland, Turkey, and United Kingdom.

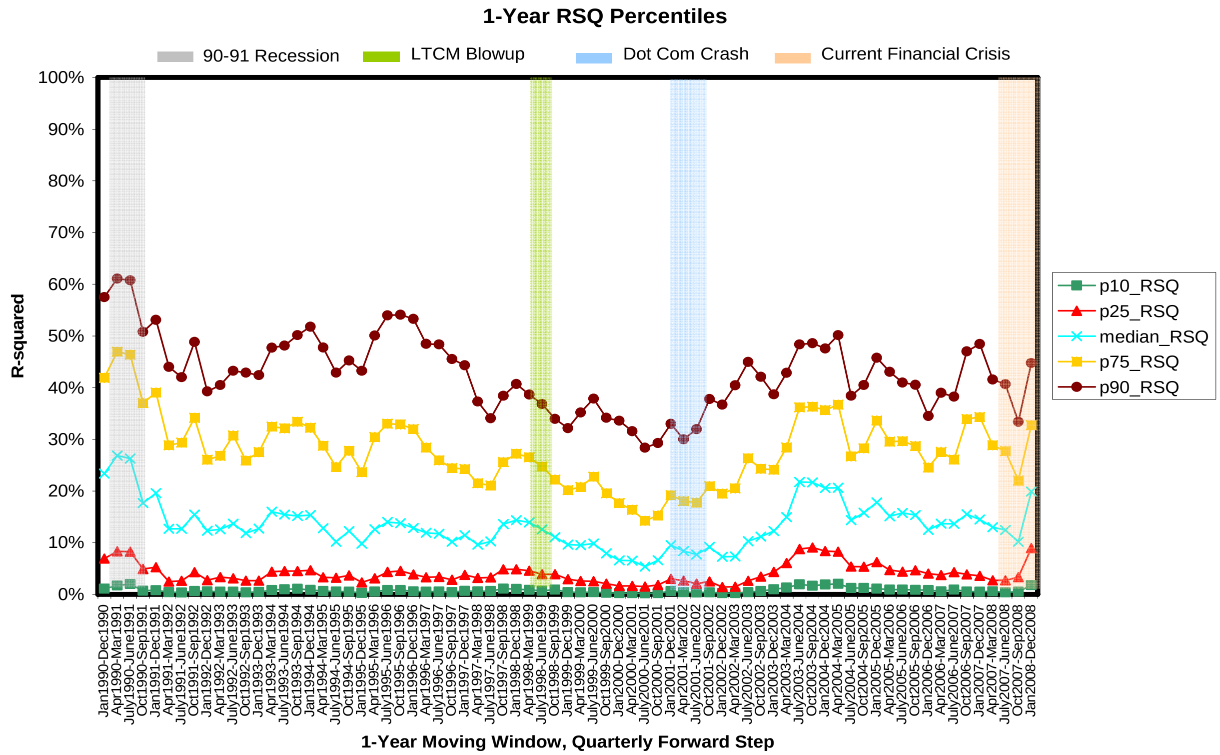


FIGURE 15 R-squared Percentiles for all North American Financials

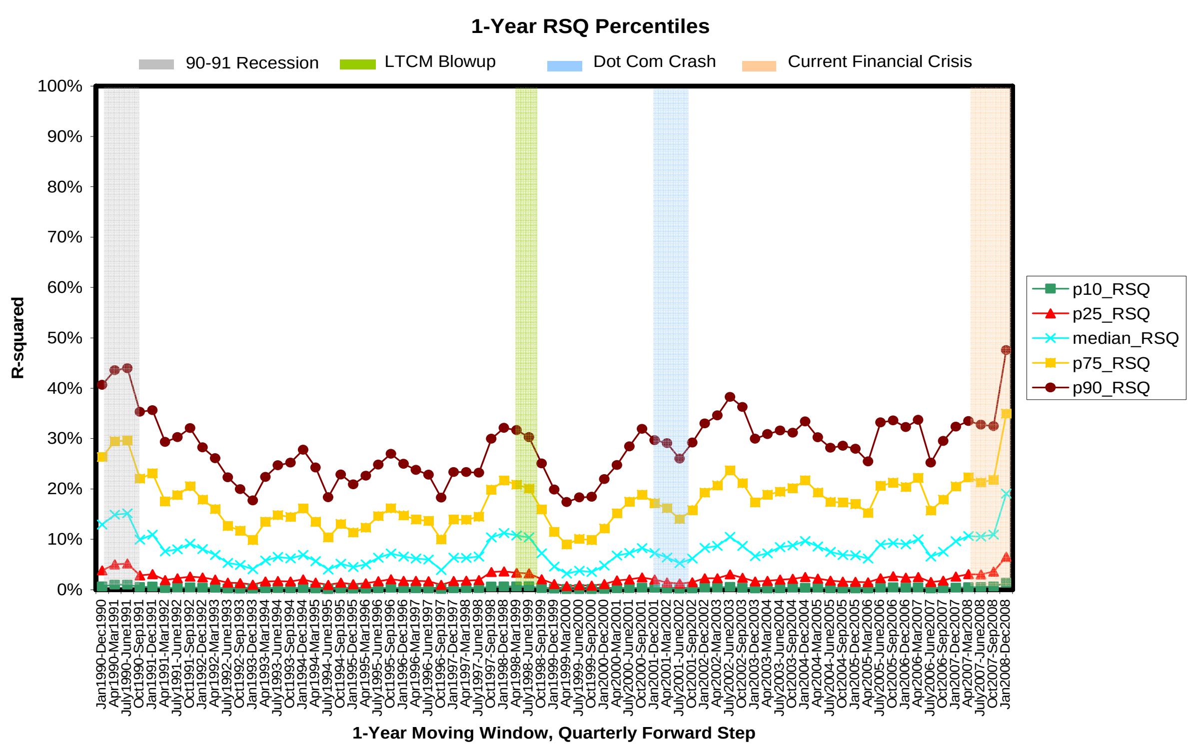


FIGURE 16 R-squared Percentiles for all North American Industrials

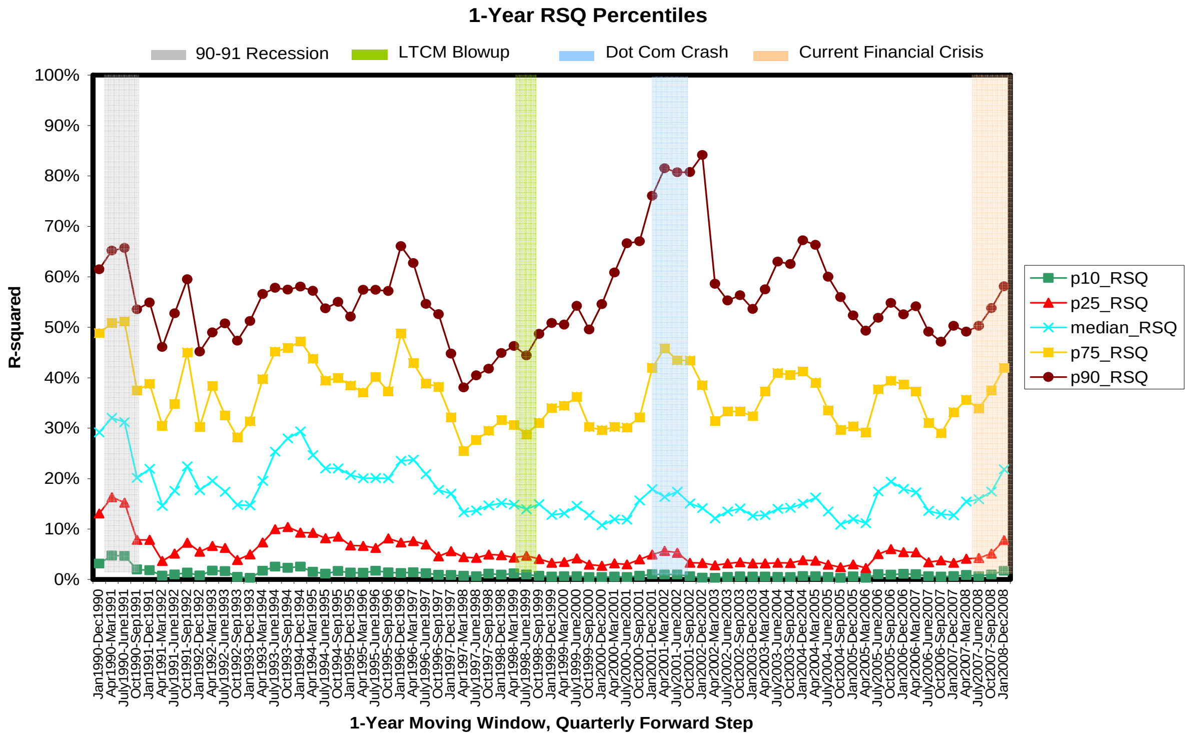


FIGURE 17 R-squared Percentiles for all European Financials

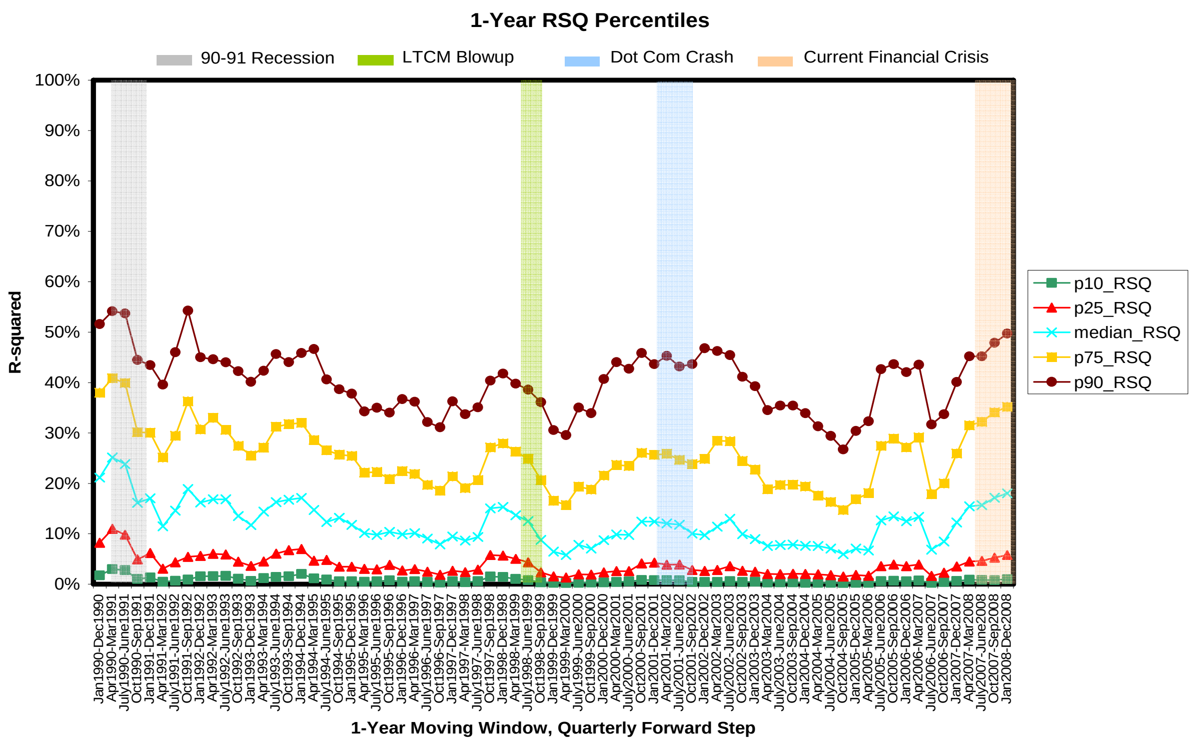
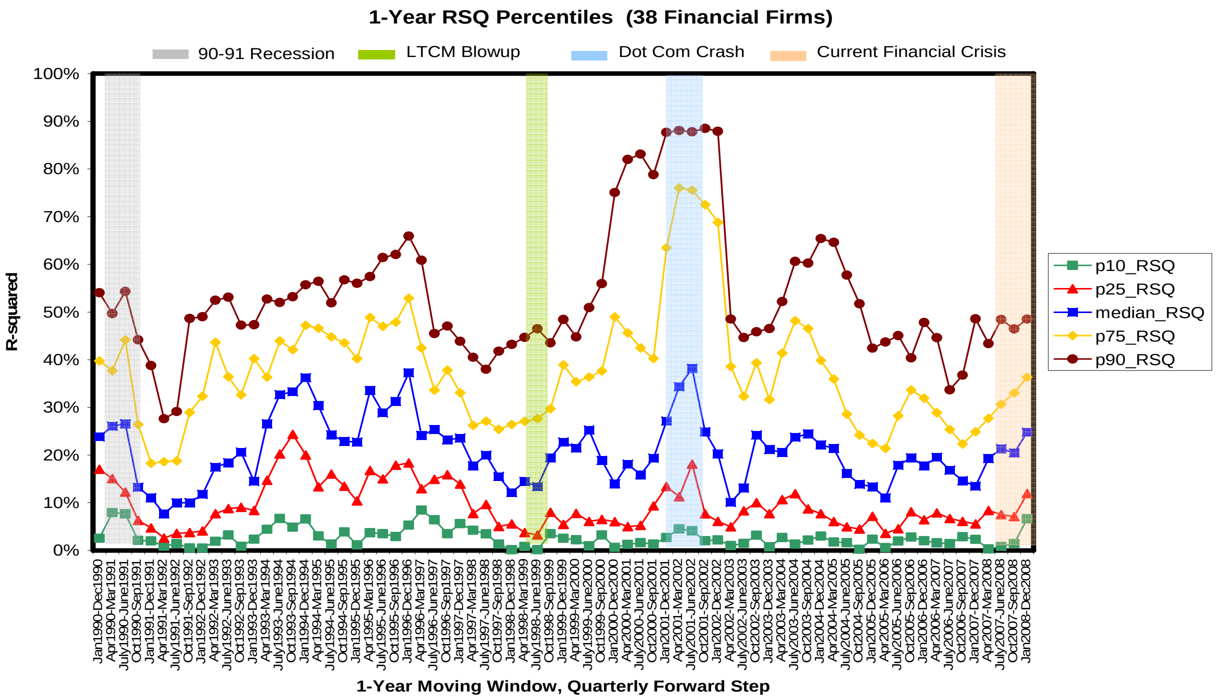


FIGURE 18 R-squared Percentiles for all European Industrials

APPENDIX B R-SQUARED DYNAMICS (CONTROLLED EUROPEAN SAMPLES)



R-squared Percentiles for the Controlled European Financials

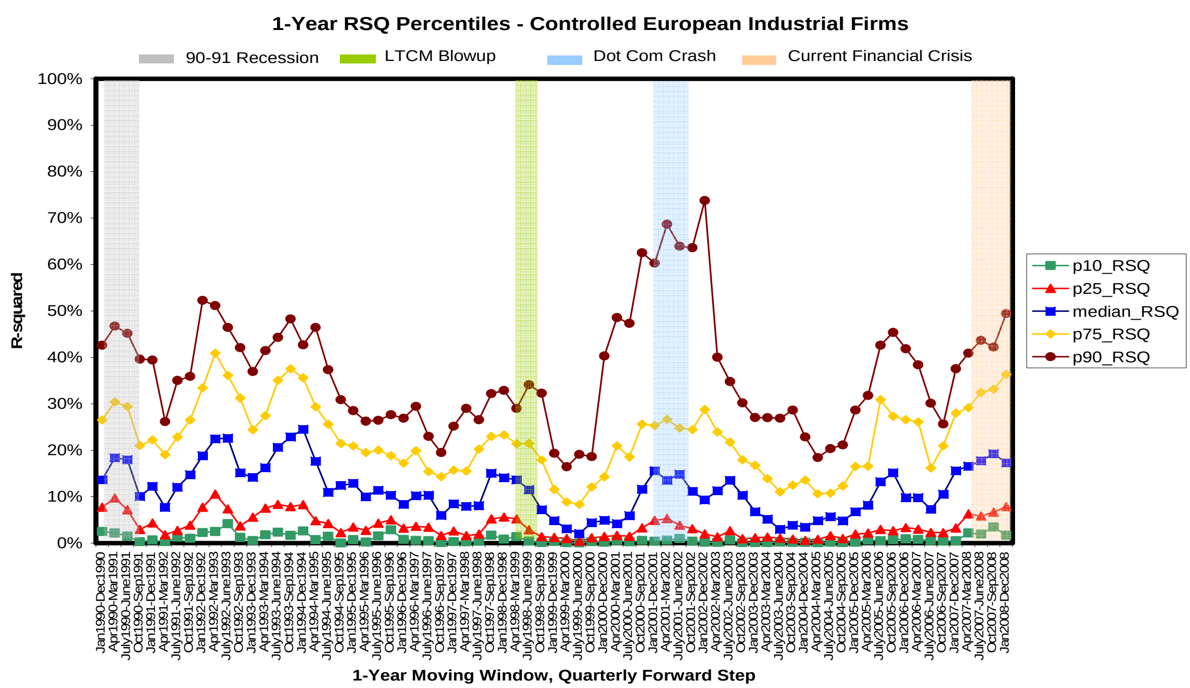


FIGURE 19 R-squared Percentiles for the Controlled European Industrials

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