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PARSIMONY IN PRACTICE:
AN EDF-BASED MODEL OF CREDIT SPREADS

MODELING**METHODOLOGY**

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ABSTRACT

In recent years, the Moody's KMV Expected Default Frequency™ (EDF) credit measure has become a standard measure of corporate credit risk among traders and managers of credit risk. Beyond predicting defaults, one other important application of any quantitative credit risk measure is to value credit risky claims such as corporate bonds, loans and credit derivatives.

The goal of this paper is to provide evidence on the valuation performance of an EDF-based valuation model on a comprehensive sample of corporate bond data. This paper serves to document some of the valuation related research done at Moody's KMV and builds on previously published work. We apply our valuation method to a large sample of bond spreads and find that, unlike many other versions of structural models, ours performs quite well. Our model consistently explains more than 70% of the cross-sectional variation in bond spreads.

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1 OVERVIEW

In recent years, the Moody's KMV Expected Default Frequency™ (EDF) credit measure has become a standard measure of corporate credit risk among traders and managers of credit risk. The EDF credit measure is calculated using a structural framework conceptually similar to the Black-Scholes-Merton (BSM) framework (see Black and Scholes (1973) and Merton (1974)) for modeling corporate liabilities. Moody's KMV implements the Vasicek-Kealhofer (VK) version of this model (see Kealhofer (2003a) for details) to calculate the EDF credit measure. Various studies have demonstrated that the EDF credit measure is a powerful predictor of corporate defaults (see Kealhofer and Kurbat (2002) and Miller (1998) for details). Beyond predicting defaults, one other important application of any quantitative credit risk measure is to value credit risky claims such as corporate bonds, loans and credit derivatives. Many academics and practitioners believe that default risk measures based on Merton's Structural approach do not perform well in the valuation of credit risky claims.[1] In fact, poor valuation performance of most structural models is widely considered to be a central weakness of this approach.

The goal of this paper is to provide evidence on the valuation performance of an EDF-based valuation model on a comprehensive sample of corporate bond data. This paper serves to document some of the valuation related research done at Moody's KMV and builds on the work published in Bohn (2000) and Kealhofer (2003b). We have enhanced the basic model discussed in these papers and offer new evidence on the performance of the model.

In contrast with conclusions published in much of the academic literature, our results are quite encouraging. We show that the Moody's KMV implementation of the BSM-style- structural model produces spreads that explain more than 70% of the cross-sectional variation in the secondary market spreads on corporate bonds (note that the sample is filtered for bonds with questionable prices and includes bonds issued only by those companies that have publicly traded equity). Considering the poor quality of corporate bond data, compared to say, equity market data and also that corporate bonds are very complex instruments with many features (e.g., covenants, parent-subsidiary effect, etc.) remaining unmodeled, this degree of fit seems quite satisfactory. This goodness-of-fit result is quite robust in the sense that it holds consistently across a daily sample for 5 years. Moreover, the result holds in different industry sectors and across different credit ratings. What is remarkable about these findings is that this type of fit is achieved through a parsimonious model with just three unknown parameters that are calibrated to a broad cross-section of bonds. This result is in contrast with the reduced form approach that typically calibrates issue specific parameters for fitting the models. Thus, in many typical implementations of reduced-form models, the number of parameters increases with the number of firms in the cross-section and good data availability for each issue becomes a crucial condition for model calibration.[2] Our model puts far lower demands on the spread data and thus has distinct advantages in most practical situations of interest.

We examine the pricing errors in some detail. We find that a trading strategy based on pricing errors generates risk-adjusted excess returns. We take this as evidence that at least a part of the pricing errors likely result from the inefficiencies in the bond market relative to equity markets and/or stale bond data.

2 VALUATION FRAMEWORK: RISK-NEUTRAL VALUATION

Our valuation framework is designed to compute the values of credit risky claims like bonds, loans, and credit derivatives. At a conceptual level, we follow the risk-neutral valuation methodology that is grounded in the No-Arbitrage principle. More specifically, we compute a transformation that converts our default probabilities under the physical measure (the EDF credit measures) to default probabilities under the risk-neutral measure (the Quasi Default Frequencies or QDFs). The main parameter in this transformation is the *market price of risk* (denoted by λ_m in this paper). This parameter essentially captures corporate debt investors' attitude toward risk. Alternatively, λ_m can be interpreted as the market's Sharpe ratio or expected excess return demanded by investors per unit of risk. This attitude toward risk for credit market investors is best reflected in the prices (or spreads) of credit risky claims. Consequently, we

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[1] Kao (2000) summarizes this view, "Empirically, most criticisms of Merton's basic model center on its difficulty in explaining the observed term premium of the corporate bond yield curve…Jones, Mason and Rosenfeld (1984) showed that the basic model is not successful in pricing investment-grade corporate bonds, even for those issuers with a simple capital structure."

[2] See, for example, Duffee (1999) for a detailed empirical implementation of a reduced form model. He calibrates the model to bonds issued by 161 firms that are active in the corporate bond market.

use these data to calibrate Market Sharpe Ratio. Credit spread data are also used to calibrate other model parameters like the firm size premium and risk-neutral sector Loss Given Default (LGD). The details are given in the following paragraphs.

Under the risk neutral valuation principle, the model spread (EDF Implied Spread or EIS) on a defaultable zero-coupon bond can be characterized as,[3]

$$EIS_T = -\frac{1}{T}\ln(1 - CQDF_T * LGD) \qquad (1)$$

where T is the tenor of the bond, *CQDF* is the cumulative default probability under the risk neutral measure and *LGD* is the loss given default.

Thus, if we were to value a bond using the above model, we begin with the known tenor T. We will need an estimate of *LGD*, for which multiple possibilities exist. We could potentially obtain it from statistical analysis of historical LGD data or (as we will show later) use bond spreads to calibrate this quantity as a parameter. For now, we will assume that this number is known and for convenience set its value to 0.55 for senior unsecured bonds.[4] This number is based on the historical LGD experience for senior unsecured bonds.[5] To the extent this estimate is inaccurate, our estimates of other model parameters will also deviate from their true economic values.

For calibration of the *CQDFs*, we start with the EDF credit measures, our default probabilities under the physical measure. When asset returns are assumed to follow a Geometric Brownian motion process, one can show that *CQDF* can be obtained from *CEDF* through the following transformation,[6]

$$CQDF_{iT} = N\left[N^{-1}\left(CEDF_{iT}\right) + \frac{\mu_i - r}{\sigma_i}\sqrt{T}\right] \qquad (2)$$

In our current valuation framework, we rewrite this relationship by imposing the Capital Asset Pricing Model (CAPM) on asset returns. CAPM says,

$$\begin{aligned}
\left(\mu_i - r\right) &= \beta_{im}\left(\mu_m - r\right) \\
&= \frac{\rho_{im}\sigma_i\sigma_m}{\sigma_m^2}\left(\mu_m - r\right) \\
\frac{\left(\mu_i - r\right)}{\sigma_i} &= \rho_{im}\frac{\left(\mu_m - r\right)}{\sigma_m} \\
\lambda_i &= \rho_{im}\lambda_m \\
&\Rightarrow \\
CQDF_{iT} &= N\left[N^{-1}\left(CEDF_{iT}\right) + \rho_{im}\lambda_m\sqrt{T}\right]
\end{aligned} \qquad (3)$$

[3] Some simplifying assumptions are used in deriving this model.

[4] We use only the senior unsecured bonds in most of the model calibration and testing. These bonds constitute a majority of the corporate bond universe.

[5] See, for example, Altman and Kishore (1996).

[6] See Appendix A for a derivation.

Thus,

$$EIS_T = -\frac{1}{T}\ln(1 - N\left[N^{-1}\left(CEDF_{iT}\right) + \rho_{im}\lambda_m\sqrt{T}\right] \times LGD) \qquad (4)$$

Because *CEDF* is known and *LGD* values can be set based on historical data, ρ_{im} and λ_m are the two main unknown parameters. ρ_{im} is the correlation coefficient of individual asset returns with the market returns and represents a firm-specific parameter. Market Sharpe Ratio is constant across the whole cross-section of assets. Conceptually, both parameters can vary over time.

Our estimates of ρ_{im} are based on the Moody's KMV Global Correlation Model. That leaves λ_m as the main unknown parameter in the valuation model in equation (4) above. However, before we turn to the details of parameter estimation, we need to consider possible systematic misspecification in this model.

As we will show later, one major systematic bias that shows up in the bond spreads is the "size effect." We find that the spreads on bonds issued by smaller firms are systematically higher than those on bonds of larger firms, after controlling for various known spread drivers such as EDF credit measure, agency rating, seniority, tenor and industry. We take such findings as clear evidence of a size effect in bond spreads, which is not too surprising given the long history of a known size effect in the stock market. We capture the size effect by (a) calibrating the model in equation 4 to large firms only and (b) including a Size Premium term to explain the spreads for small firms. Thus, the general model with size premium becomes,

$$S_T = \beta_z f\left(z_i\right) - \frac{1}{T}\ln(1 - N\left[N^{-1}\left(CEDF_{iT}\right) + \rho_{im}\lambda_m\sqrt{T}\right] \times LGD) \qquad (5)$$

where z is the firm size, *f(z)* is a size-function whose form is calibrated from the bond data, β_z is the size-premium parameter, and $\beta_z f\left(z_i\right)$ is the size premium.

3 COMPUTING CREDIT SPREADS FROM BOND PRICES

Before we can take the above model to the bond data, we have to deal with several issues related to bond spreads. First, we need to calculate the bond spreads from the observed bond prices. We need to choose an appropriate default-free reference curve for this purpose. Authors of academic papers typically choose a U.S. Treasury curve for this purpose. Market practitioners, on the other hand, mostly seem to choose a LIBOR-based curve. Recent evidence indicates LIBOR-based curves are better suited for this purpose. We use a curve that is a minor variant of the LIBOR curve in the sense that it makes a small downward adjustment to the LIBOR rate to account for the small amount of credit risk reflected in LIBOR rates. We call this reference curve the *Zero-EDF curve*.[7]

A second issue is that many bonds have embedded call options and this option effect convolutes the calculation of a credit spread. We remove the effect of options by calculating the *option adjusted spread (OAS)* over the default-free curve. We rely on the industry standard methodology for calculating an OAS to deal with this issue.

Finally, most of the bonds are coupon bonds rather than zero-coupon bonds. Since we already produce the full term structure of default probabilities in our EDF model, conceptually there are no difficulties in modeling the coupon bonds themselves. However, modeling the coupon bonds directly can present some estimation difficulties given the noisiness of the data. Fortunately, our research shows that a simplified calculation implemented by collapsing the multiple cash flows of a coupon bond at a single point in time produces reasonable results. Essentially, we approximate the coupon bond

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[7] See Bohn (2000) for more details on this procedure.

with a zero-coupon bond. The tenor for this equivalent zero-coupon bond is equal to the duration of the coupon bond.[8] This approximation works fairly accurately in most situations of interest and considerably simplifies the estimation and intuitive explanation of results (See Appendix B for some evidence on the rationale and accuracy of this approximation).

4 WHY USE BOND MARKET DATA FOR CALIBRATION?

Current calibration of the model is based on corporate bond data. Historically, the corporate bond market has been the main venue for trading corporate credit risk. In recent years, the Credit Default Swap (CDS) and Secondary Loan markets have become more active. We chose the bond market rather than the CDS or loan market for the current calibration based on the following considerations.

a. It has the widest coverage of names (among the credit markets), thus allowing us to get a more representative calibration of market-wide parameters like the Market Sharpe Ratio and the size premiums.

b. It has the longest history of data available, thus allowing us to study the behavior of the model over a long time period and make refinements.

c. Features of bond instruments are fairly well understood in the industry and many established methodologies (like the OAS calculation) are available.

However, leveraging market information from alternative markets such as those for CDS and loans is an active area of research at Moody's KMV.

5 BOND DATA SOURCES AND DATA FILTERING

Our primary source of corporate bond data is the EJV database from Reuters. It is a comprehensive database covering about 80,000 U.S. corporate bond issues.[9] The data include comprehensive terms and conditions and daily pricing. Since most of the bonds do not trade on a daily basis, the quality of price data varies widely. At Moody's KMV, we have designed an elaborate set of filters and tests to remove the uninformative prices. The filters remove outliers, stale prices, data entry errors, bonds with unusual features, asset-backed securities, and many other types of noise-inducing data points. On a daily basis, we filter out about 80 to 90% of the raw EJV prices before we calibrate our model.

Here, we note that two sources of bond transaction prices have become available in recent years: (a) TRACE data from NASD, and (b) insurance companies' bond transactions data from Capital Access International (CAI). Since few bonds transact regularly, the prices in these data sources do not have good continuity and the coverage on a given day may or may not be representative of the total bond universe. This could be true even though TRACE now covers bonds that account for about 75% of the trading volume in investment grade corporate bonds. Further, the transaction prices are quite volatile because of liquidity type effects like the bid-ask bounce. Extra volatility occurs in TRACE data because retail and institutional trades are generally executed at different prices but are not reported separately. Thus, as of now, these data sources are not best suited for a direct calibration of our model.[10] Instead, we use transaction information to carefully screen the EJV bond prices.[11]

In addition to filtering out bonds with no trades in recent past, we also eliminate many other categories of bonds that could have unusual spread behavior. These types of bonds include convertibles and putable bonds, bonds with sinking funds, Payment in Kind bonds, bonds issued by Real Estate Investment Trusts (REITs), small issues (less that $10million in amount outstanding), issues maturing in less than 1 year or more than 10 years, and bonds with OAS exceeding 2000 basis points. The exclusions are helpful in reducing the noise in parameter estimates.

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[8] Using either the Option Adjusted Duration or the Macaulay Duration yields similar results.

[9] The EJV database has substantial international coverage, too. We are currently researching the power of the model on international data.

[10] This situation may change as these data sources become more comprehensive over time. Monitoring these sources is an area of research at Moody's KMV.

[11] We reject any bond that had no transaction recorded in our transaction databases in the last quarter. We believe that the EJV prices for such bonds are likely to be noisy.

6 ESTIMATION METHODOLOGY

The estimation methodology is based on equation (5) above. We could use the OAS computed from bond prices as observations on spread (S) and try to estimate simultaneously the three unknowns λ_m, β_z, and *LGD* in equation (5) above. It turns out, however, that the simultaneous estimation is unstable and comes up with estimates that are economically unintuitive and too volatile. Clearly, this indicates an identification problem. We circumvent the problem by adopting a *three-step estimation* approach. This approach brings in extra information on the economic behavior of the parameters and thus solves the identification problem. The first step is estimation of λ_m, where we use only large firms' data so that we can set the β_z to zero. Since λ_m is a single parameter for the whole cross-section, we can use an average value for LGD. We use an LGD estimate of 0.55 based on historical calculations cited earlier. Once we estimate λ_m, we reintroduce small firms' data and estimate the sector size β's. The final step is to use the estimated values of λ_m and β_z and compute the cross-sectional variation in LGDs implied by the predefault bond spreads. The estimates resulting from this three-step procedure are stable across time and economically much more meaningful as we will see below.

7 EMPIRICAL EVIDENCE ON SIZE PREMIUM

Before we discuss the parameter estimation further, we will give a brief empirical characterization of the bond spread data and how the variation in bond spreads relates to the variation in spread drivers like EDF credit measures and firm size. Table 1 shows a few illustrations of the size effect in bond spreads.

TABLE 1 Illustrations of size premium in bond spreads

Company	Total Sales (b)	OAS 6/25/01	OAS 10/25/01
Cooper Tire	$ 1.9	147 bps	243 bps
Safeway	$ 19.5	94 bps	98 bps
Waste Management	$ 6.6	169 bps	184 bps
El Paso Corp.	$ 30.6	156 bps	141 bps
Carnival Corp.	$ 2.8	126 bps	277 bps
Burlington	$ 5.3	89 bps	118 bps

We choose pairs of issuers with the same rating, same EDF value, and same industry sector but different firm size (measured by sales) and compare bond spreads on bonds with similar tenor and seniority. As we can see from the table, bonds of smaller firms tend to have significantly higher spreads than comparable bonds issued by larger firms, after controlling for other determinants of spreads.

Figures 1, 2, and 3 show the evidence of size premium in bond spreads in a more general form. Here, median spreads are shown for different size and EDF quartiles for bonds issued by Industrial, Financial, and Utilities firms. We control for the impact of tenor by dividing the samples by tenor quartiles. We therefore show four graphs, with each limited to a particular tenor quartile, for each sector.

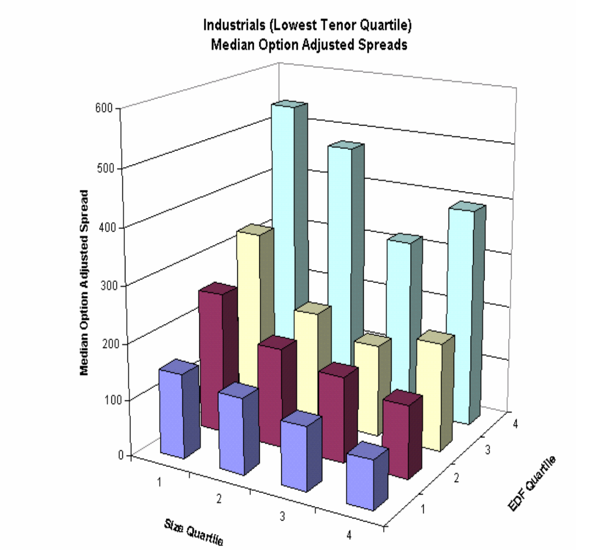


Fig. 1(a)

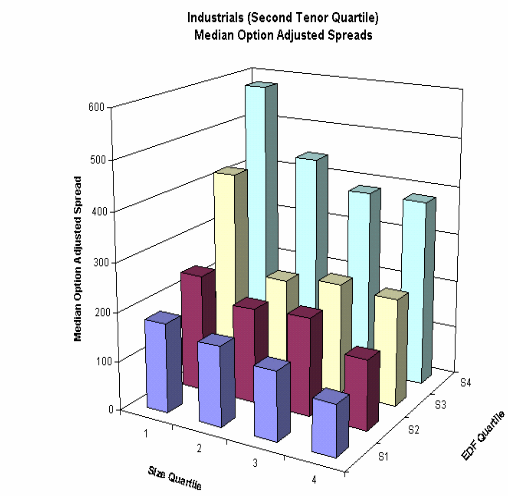


Fig. 1(b)

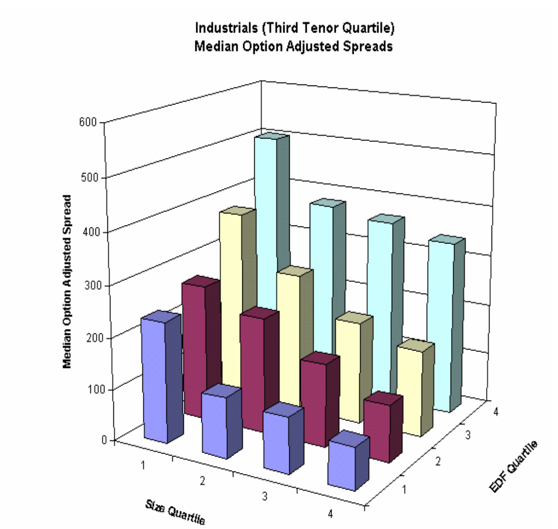


Fig.1(c)

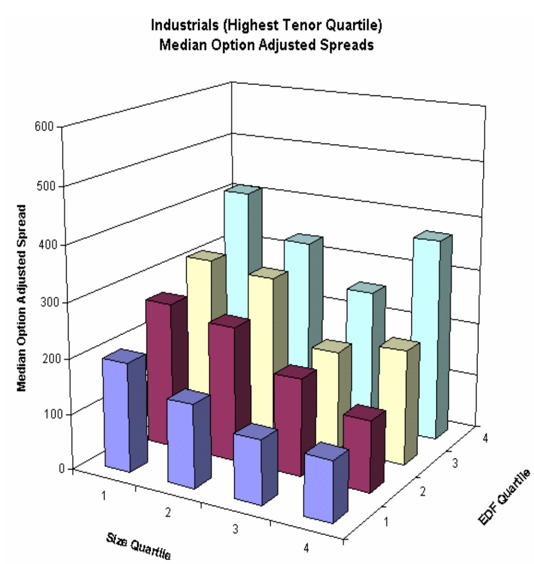


Fig 1(d)

FIGURE 1 Median OAS Across EDF and Size Quartiles For Different Tenor Quartiles (INDUSTRIALS)

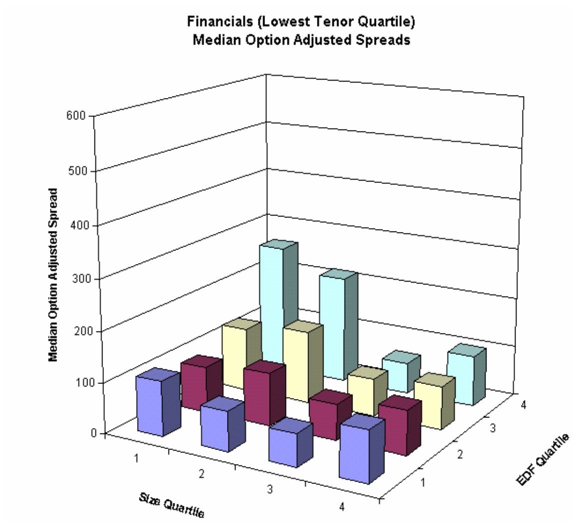
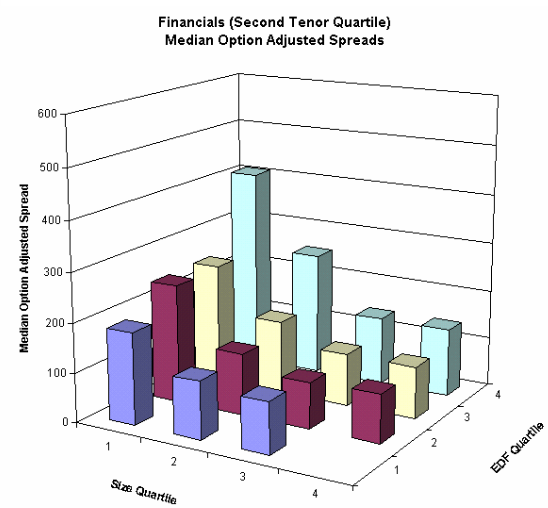


Fig. 2(a)



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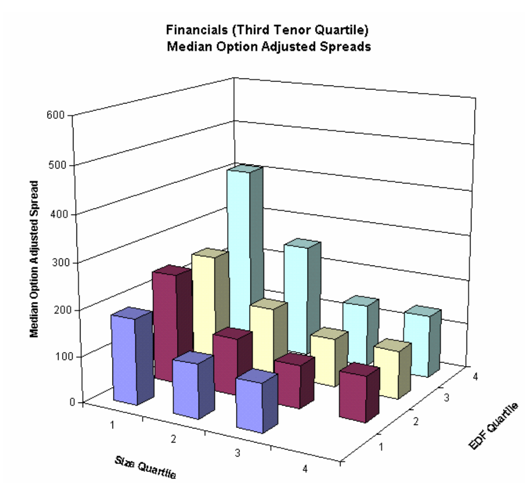

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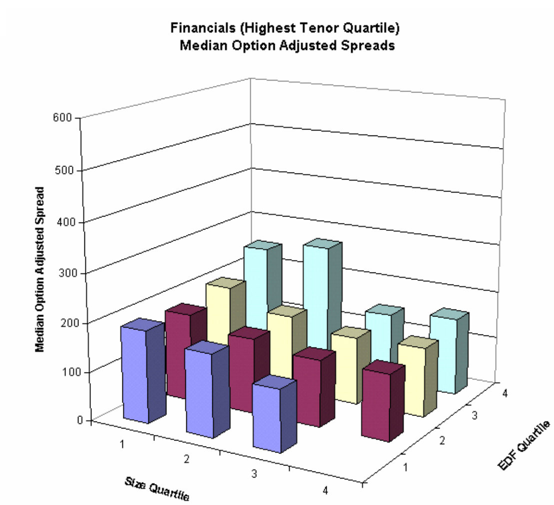


Fig. 2(d)

FIGURE 2 Median OAS Across EDF and Size Quartiles For Different Tenor Quartiles (FINANCIALS)

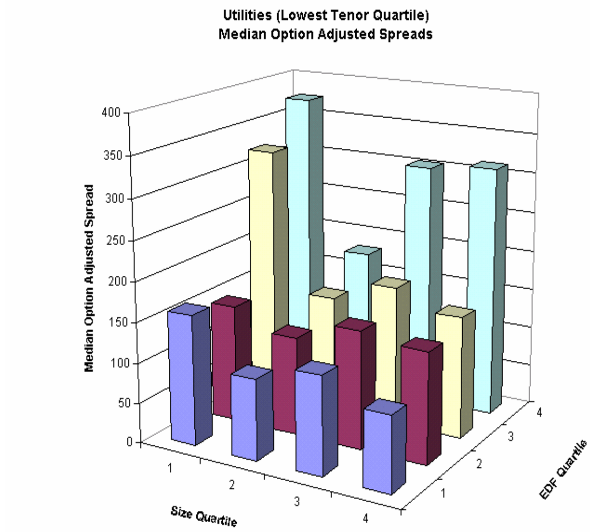


Fig. 3(a)

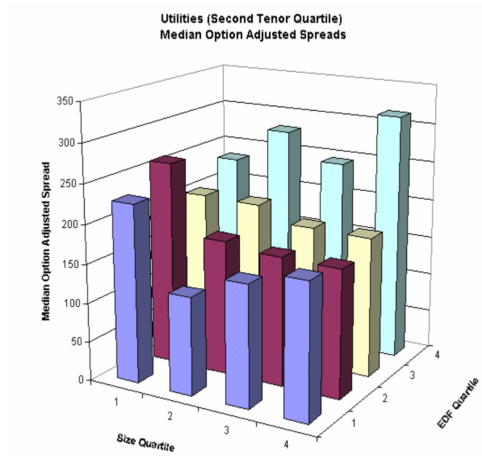
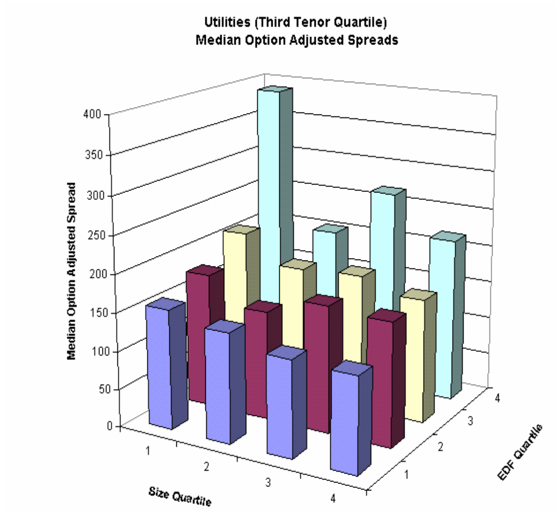


Fig. 3(b)



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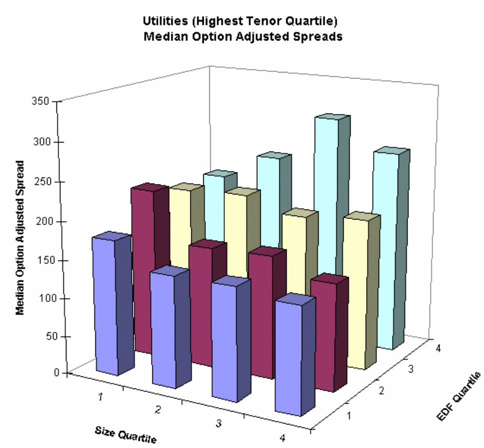

Fig. 3(d)

FIGURE 3 Median OAS Across EDF and Size Quartiles For Different Tenor Quartiles (UTILITIES)

Three patterns are observed clearly from these graphs:

- Spreads increase with EDF values. This result is intuitive and consistent with our valuation model. It also indicates consistency between debt and equity markets, i.e., EDF values go up when equity prices decrease (all else equal) making the debt riskier and resulting in lower bond prices (or increased bond spreads).
- Overall, controlling for EDF values, spreads decrease with size, i.e., bond issues of larger firms tend to offer lower spreads even though they have the same default probability as measured by EDF credit measures. This effect is not seen clearly at high EDF levels; we lose clarity because the median EDF levels for the samples are around 50 basis points and the variation of EDF levels above the median is too high to be controlled effectively.
- The general levels of spreads are different across the three sectors. For example, the spreads for utilities and financial firms seem lower than for industrial firms. Further, the impact of size seems to be different across the sectors, being the least prominent for utilities.[12] This is presented in Figures 4(a) and 4(b) for two typical tenor and EDF quartiles. In these pictures, it is apparent that for similar EDF values and tenors, Industrial firms bear a higher premium compared to Utilities and Financial firms. Secondly, this premium decreases more steeply for Industrial firms. Hence, we break up our sample across the three sectors when estimating the size-premiums.

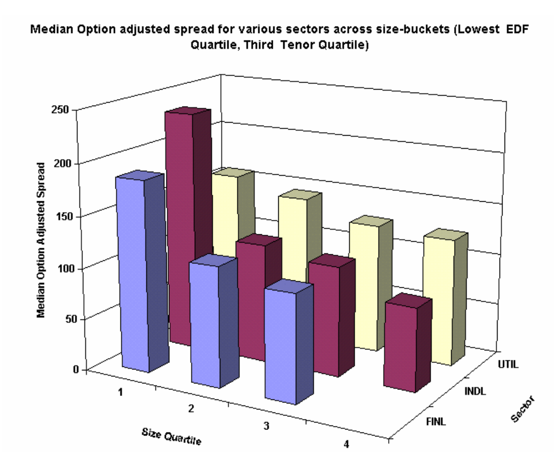

Fig. 4(a)

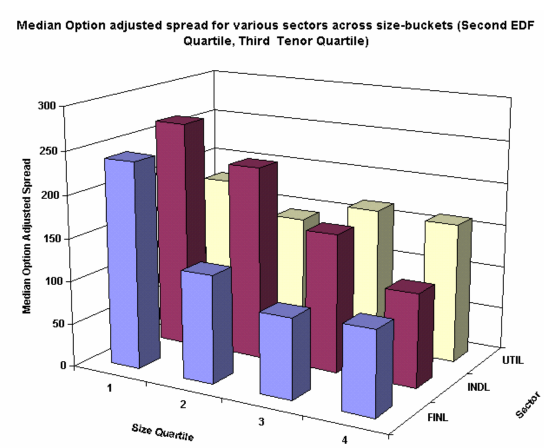

Fig. 4(b)

FIGURE 4 Median Option-Adjusted Spread across Sector and Size Quartiles

Figure 5, below, shows the time-series of daily lambda estimates for the five-year period 1999-2003. A consistently positive Market Sharpe Ratio shows that investors in the corporate bond market are indeed risk averse. Therefore the conversion of physical default probabilities to risk neutral probabilities is important for accurate pricing of bonds. We note that the Market Sharpe Ratio estimates are fairly stable over the short run but may have some long-run variation. Conceptually, since Market Sharpe Ratio represents market participants' risk tolerance, it is unlikely to vary much on a day-to-day basis. Considering that the estimates shown are based on cross-sectional data, they show considerable stability in the short run. The magnitude of Market Sharpe Ratio estimates hovers around 0.5, a value that is commonly understood to be close to the lambda for the U.S. equity market as well. This consistency with the equity market estimates is quite comforting.

[12] In fact, one can argue, based on these graphs, that the size premium is weak or even nonexistent for Utilities. The functional form we choose to model the size premium would accommodate such a possibility if that is what the data show. We do not pre-specify any size premium coefficient, rather we let that be determined by the data.

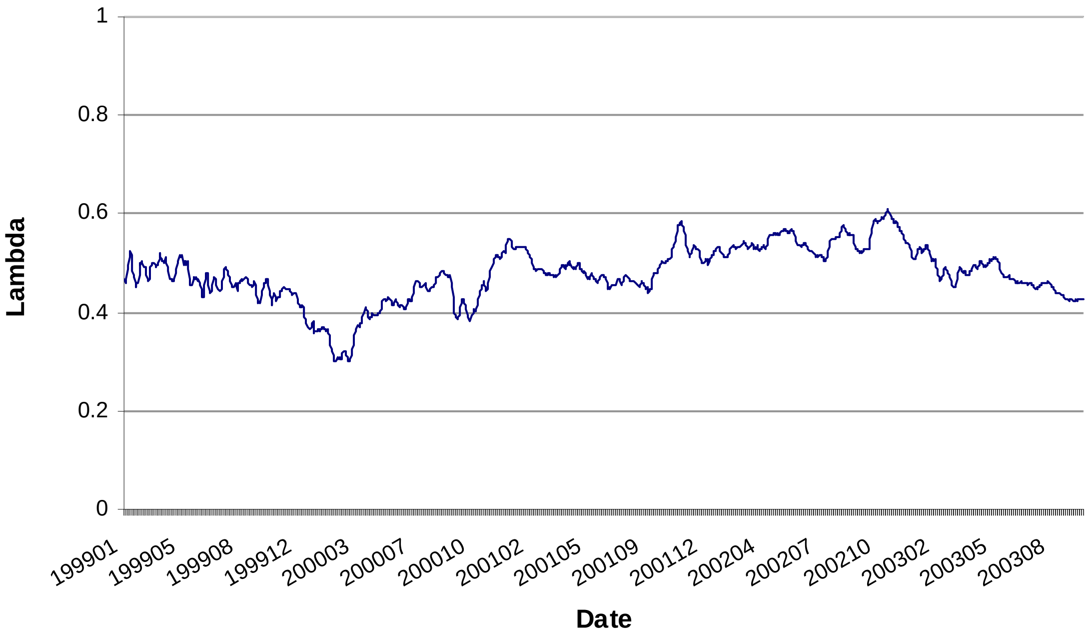

FIGURE 5 Market Sharpe Ratio Estimates 1999-2003

Figure 6, below, shows the estimates of size premium for bonds issued by firms with a firm size approximating $400 million (ln(size) = 6). This firm size is typical of the smallest firms with public bonds outstanding in the United States and thus the premiums shown in the figure represent the upper limit of the size premium.[13] Several points are worth noticing. First, the magnitude of the size premium slowly varies over time and the variation seems to relate to the economic cycle (though our five year sample is not long enough to make that claim more rigorously). Size premiums are the biggest at the trough of the cycle that is consistent with the general notion that small firms are the worst affected in recessions. Second, the magnitude of the size premiums can be large, suggesting that (a) the size effect is a systematic feature of bond spread data, and (b) the size premium captures not only the illiquidity due to trading frictions but also the price effects of informational frictions and rewards for any unmodeled sources of risk. Finally, size premiums vary across the three broad sectors, in general being smallest for Utilities and largest for Financials. This is intuitive because utilities tend to have less business risk and even the smallest ones will likely suffer less in recessions compared to the smallest firms in the Financial and Industrial sectors.

[13] In rare cases, firms smaller than $400 million do issue corporate bonds and the size premium may be higher in those few cases.

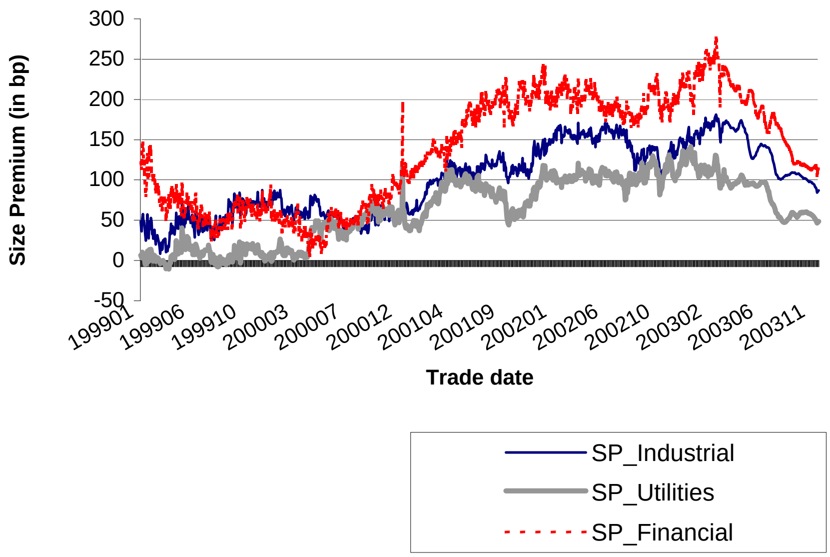

FIGURE 6 Size Premium Estimates for Small Firms (firm size of $400 million)

Next, we briefly discuss the LGD estimates. We initially divide the sample of bonds into 13 industry sectors and 3 seniority categories (Senior Unsecured, Non Senior and Subordinated) producing 39 LGD buckets. The understanding behind such a matrix is that seniority and industry dimensions are likely to be important in capturing the cross-sectional variation in LGD estimates. Since we do not have sufficient data in the Subordinated category in each industry sector, we estimate Subordinated LGDs across only 3 broad industry sectors (Industrials, Financials and Utilities) rather than 13 industry sectors, thus reducing the number of LGD buckets to 29. We use the estimated values of the Market Sharpe Ratio and size premium as inputs to estimate LGD as an unknown parameter.

As an illustration, Figure 7 shows our LGD estimates for the Telecom sector for the year 2002. LGD estimates for this sector were the highest across all sectors in the year 2002 and they tend to peak in the third quarter of 2002 before coming down. This pattern is broadly consistent with the fact that 2002 was the worst year for the entire telecom industry. Since the entire industry was in trouble, liquidation value of any telecom assets would be poor and thus the bond market expected a large loss given default. Practitioners confirm that the sector started recovering toward the end of 2002 as reflected in the estimates here.

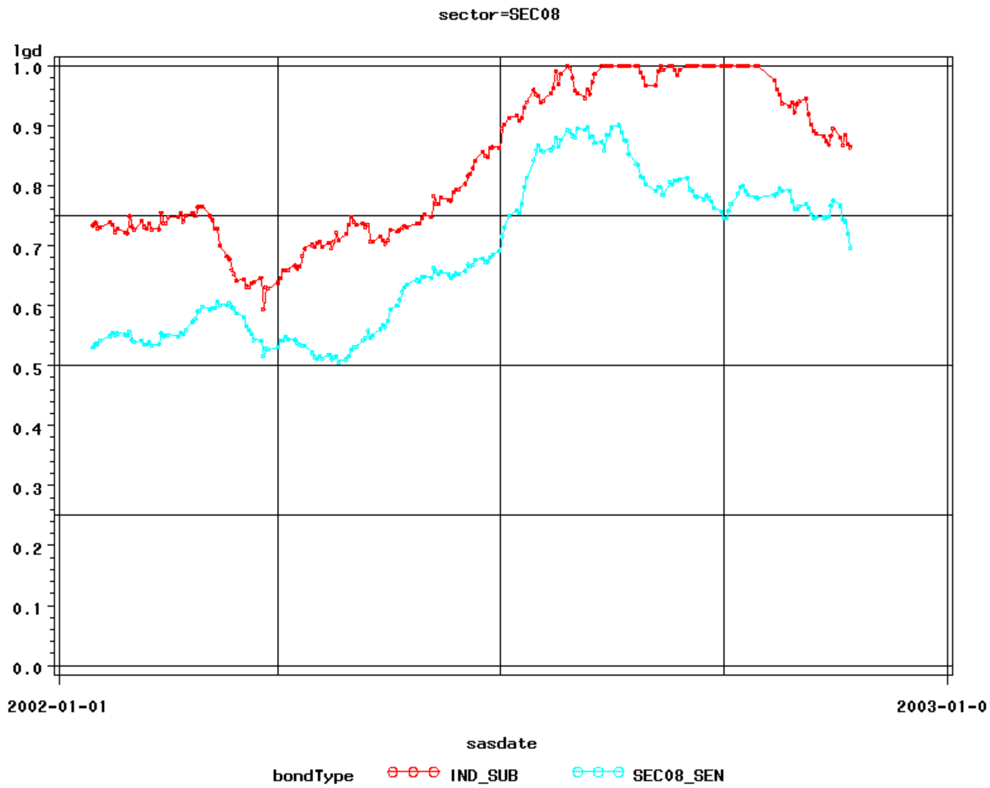

FIGURE 7 LGD Estimates (Senior and Subordinated Bonds) for the Telecom Sector

8 MODEL PERFORMANCE

How to measure model performance? One of the strengths of a valuation model is in its ability to explain the cross-sectional variation of spreads. To demonstrate this, we studied the correlation between EDF implied spreads and Option adjusted Spreads from 1999-2003. As shown in Figure 8, our model is able to explain as much as 70% of the variance fairly consistently (with the exception being the beginning of 2000). In most cases, this correlation is fairly close to 80%.

We next examine if the model output has any systematic bias. The existence of a bias implies that the model is mis-specified along a particular dimension. Figure 9 shows the time series of median, 25th percentile and 75th percentile of spread errors. Errors are defined as the difference (OAS-EIS). For the overall sample, the median error stays between 0 to 20 basis points, a range that is within the transaction cost bounds for corporate bonds. 25th percentile is uniformly less than 50 basis points and so is the 75th percentile except for a brief period when it went as high as 150 basis points. Except for this one period of larger errors, we consider this performance quite satisfactory. This is because corporate bonds are very complex instruments with features like call options, liquidity effects, default risk, recovery risk, and interest rate risk. Further, the bond market and bond data have problems like low liquidity, little transparency and stale data. Our model, with just three cross-sectional parameters calibrated to bond data (Market Sharpe Ratio, Size Premium Beta and LGD) is a very parsimonious representation of these complex instruments and their markets. No issue specific parameters are fitted here. The errors look reasonable in view of the parsimony offered by the model.

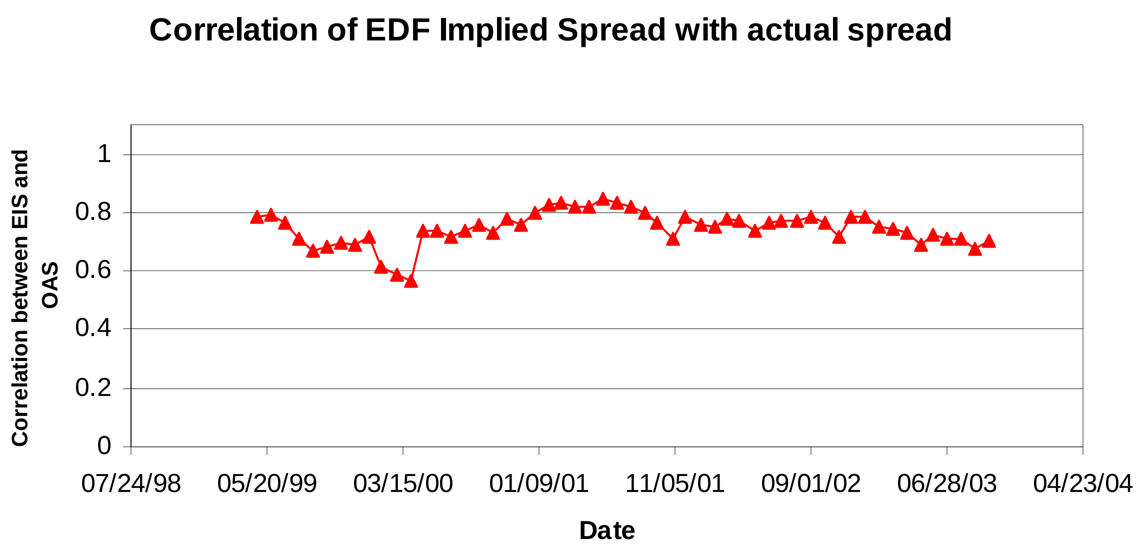

FIGURE 8 Correlation between EIS and OAS (1999-2003)

Since no firm specific parameters are calibrated, our model puts far fewer demands on the bond data than a reduced form model that estimates firm-specific parameters from the bond data. All that we need is a reasonably sized cross-section of bond spreads, with adequate representation of various industry and seniority sectors. Even if an individual firm's data has many gaps, we can still produce reasonable estimates of EIS. This property of the model is very attractive for credit markets like bonds and CDS where continuous trading of particular names is the exception rather than the rule. One direct consequence of this property is that our model can generate model spreads for those several thousand names for which we have EDF credit measures, firm size, and firm ρ_{im} data, but for which reliable bond spread data are scarce. In contrast, a reduced-form model calibrated to historical bond spreads of the same firm would be much more limited in its coverage.[14]

Figure 10(a) through (c) show the median and 25th and 75th percentiles of errors broken down by broad industry sectors. Median errors are closer to zero and percentile bands are tighter for Financials and Utilities compared to Industrials. This reflects the fact that Utilities and Financials are much smaller and much more homogeneous groups compared to Industrials and thus less subject to un-modeled influences in the bond data.

The EDF implied spreads are driven by EDF credit measures, tenors and size. So we also studied the median errors across these three measures for each sector. In general, there was no prominent bias for either measure suggesting that this framework is unbiased and fairly robust to different segments of data.

[14] See, for example, Duffee (1999) who limits his estimation of a reduced form model to a sample of only 161 firms with good, continuous bond data.

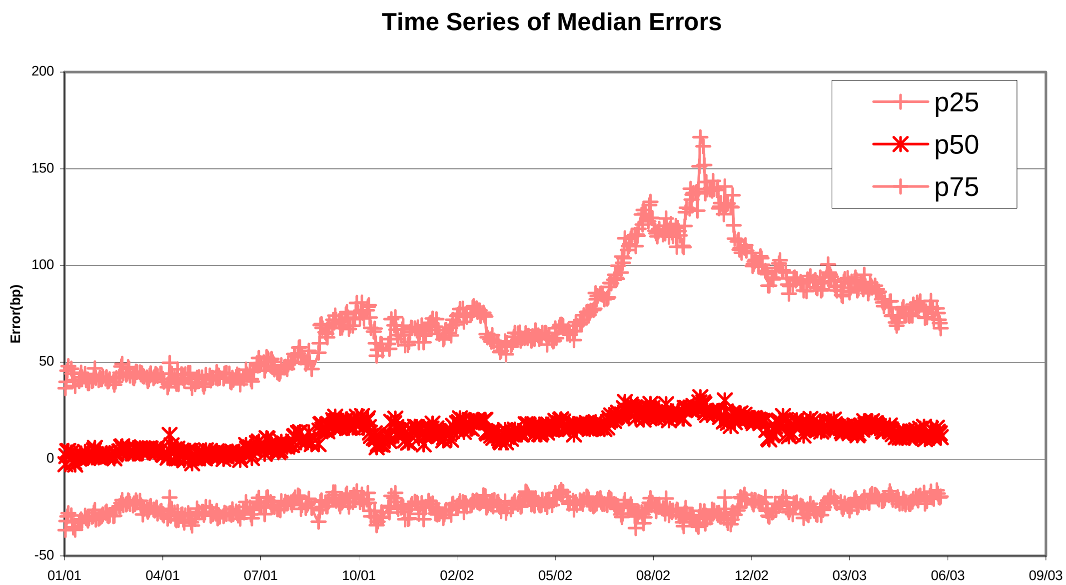

FIGURE 9. Time series of Median Errors (OAS-EIS), 2001-2003

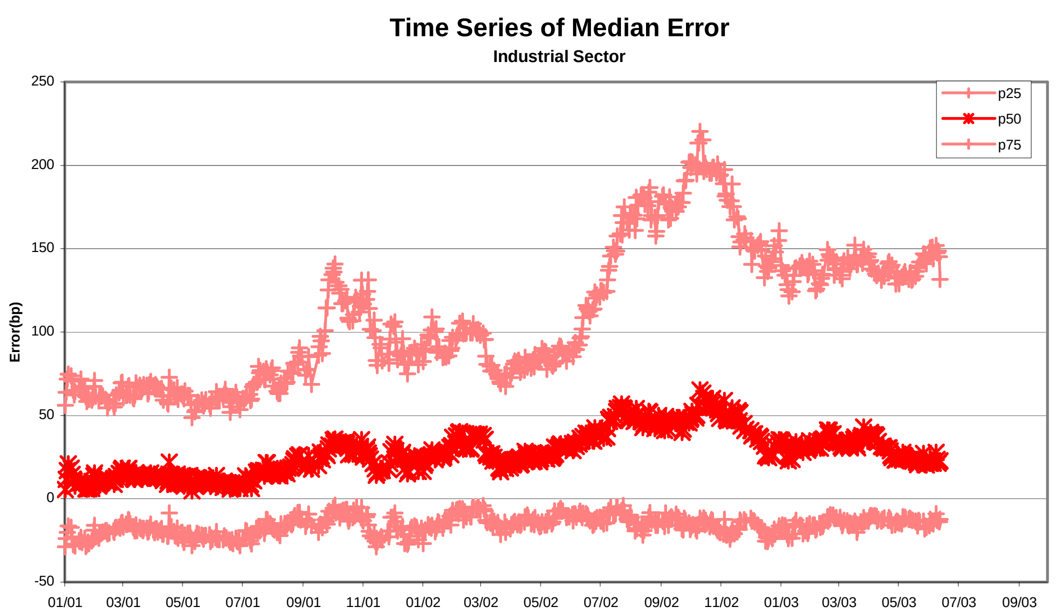

FIGURE 10(a)Time series of Median Errors for Industrial firms (OAS-EIS), 2001-2003

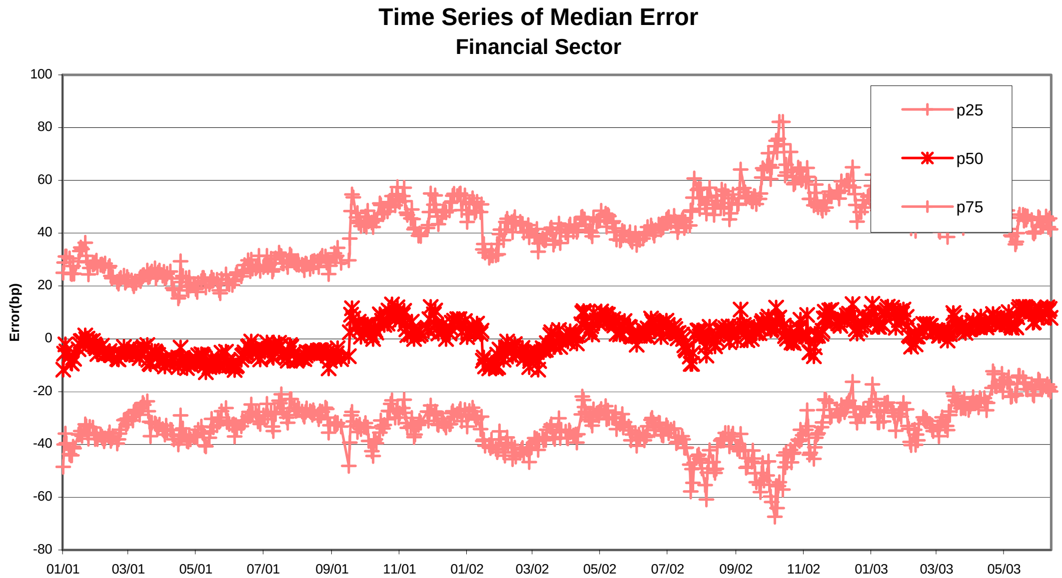

FIGURE 10(b)Time series of Median Errors for Financial firms (OAS-EIS), 2001-2003

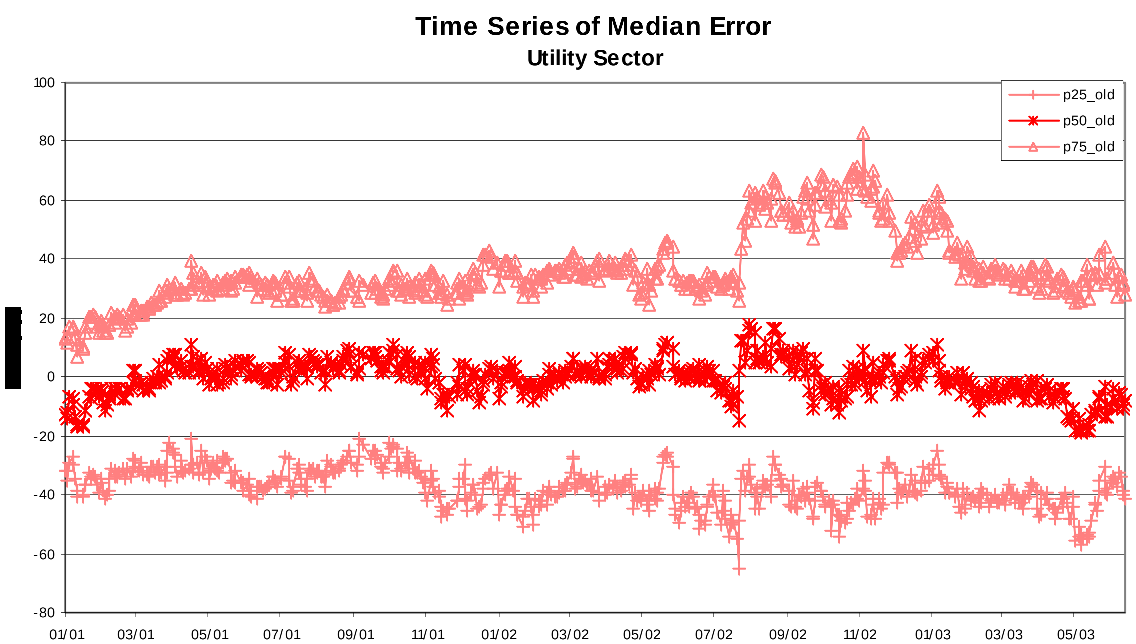

FIGURE 10(c)Time series of Median Errors for Financial firms (OAS-EIS), 2001-2003

9 EXAMPLES OF MODEL FIT

In this section, we illustrate the fit of the EIS model for a few well-known firms. This discussion is based on Figures 11(a) through 11(h). These graphs also highlight some of the patterns we observe and some of the data issues we face even after using our filters. Following are some of the features of the data and the markets that adversely affect the fit of the model:

a. Real market data from the bond market for an issuer is missing for certain periods, and the prices we receive from the vendor are only indicative and may reflect stale prices. Since equity markets are far more active, EDF credit measures and hence EIS are unlikely to have the same problem in most cases. This difference in relative activity of the two markets will drive a wedge between the OAS and the EIS and make the model fit worse. Some good examples of this effect are, AGL Corporation in Figure 11(b) and IBM in Figure 11(f). The OAS suddenly jumps and then stays flat, while the EIS exhibits more continuous movement. The model seems to be effective here. Given that the OAS oscillates around the EIS levels, the model can be judged to perform well in this example

b. In some cases, missing bond data are not filled in by indicative data. In these cases we find breaks in the time-series plots; it is not clear what to compare the EIS against. This phenomenon is most likely an effect of sparse trading in the underlying bond issue. General Electric in Figure 11(d) demonstrates this phenomenon.

c. A third pattern that impacts our fit is when equity markets lead debt markets and therefore EIS leads OAS, until OAS catches up with EIS and they begin moving together. This behavior could result from the bond market acting less efficiently than the equity market in incorporating the price impacting information. While the model indicates a trading opportunity in these cases, it also decreases the fit between OAS and EIS levels, incorrectly suggesting a diminished performance of the model. This is observed in portions of Raytheon (Figure 11(e)) and Merrill Lynch (Figure 11(g)). In the next section, we consider this hypothesis in more detail by examining if the model error is at least partially caused by the relative inefficiency of the bond market.

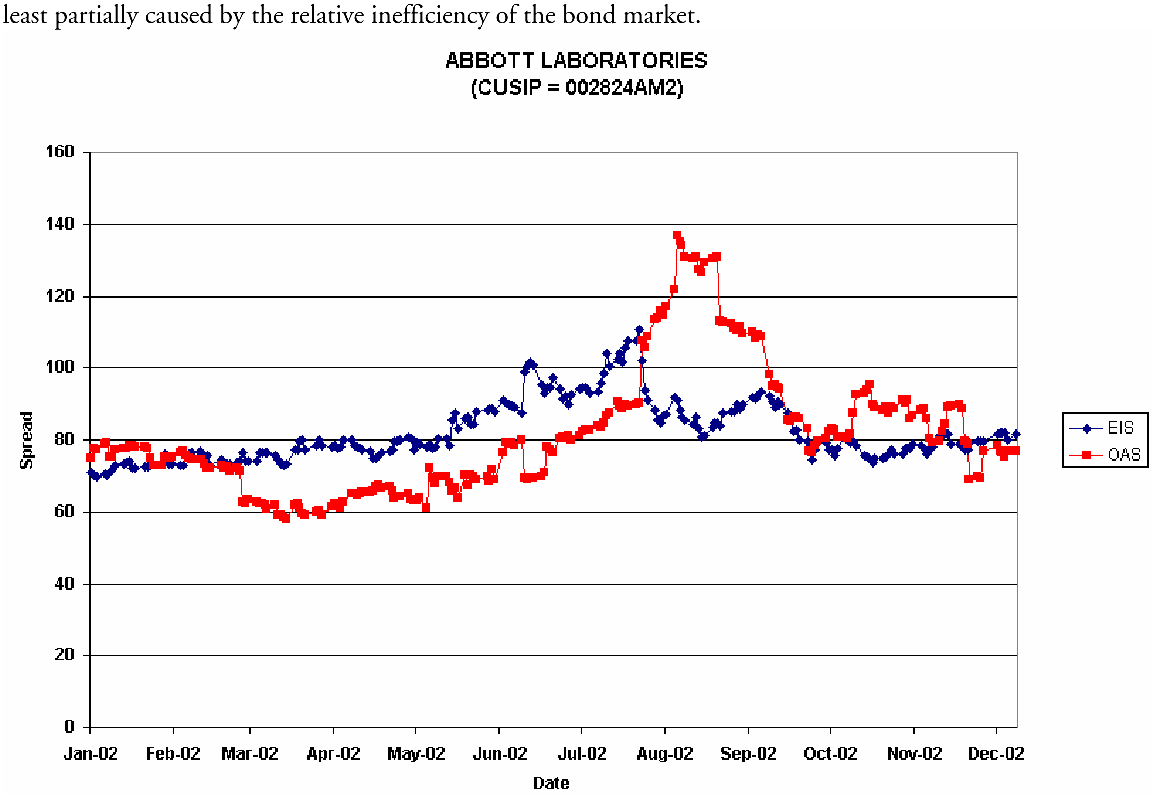

FIGURE 11 (a)

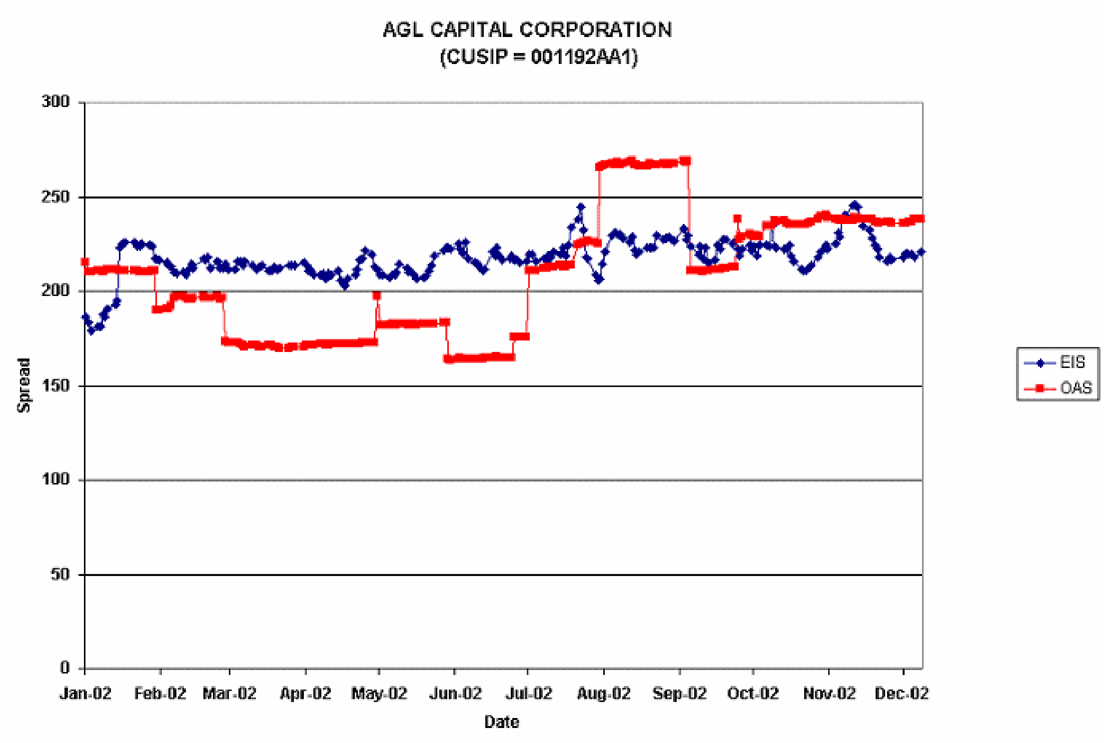

FIGURE 11 (b)

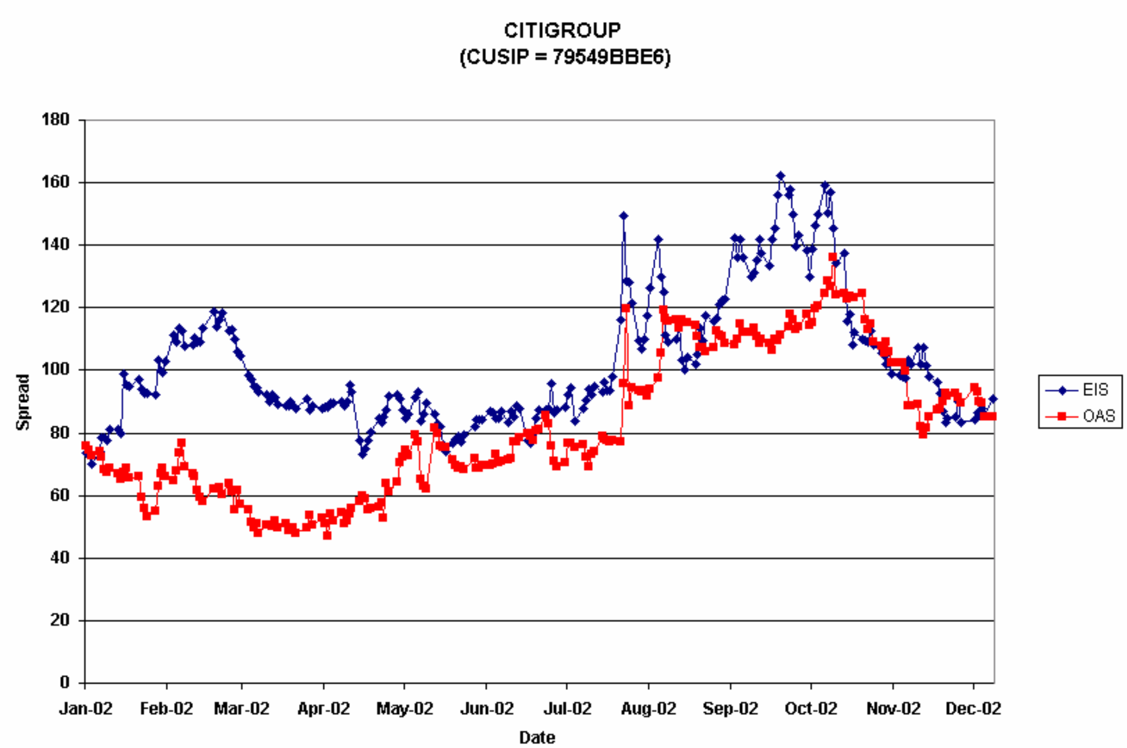

FIGURE 11 (c)

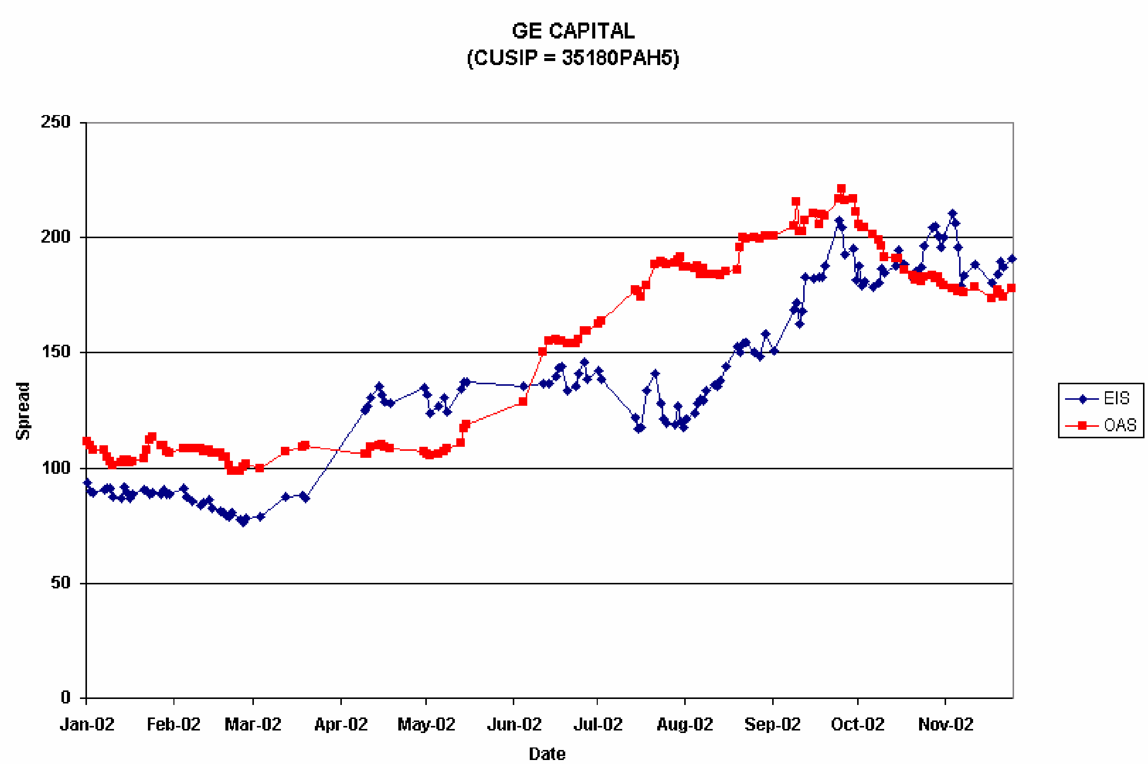

FIGURE 11 (d)

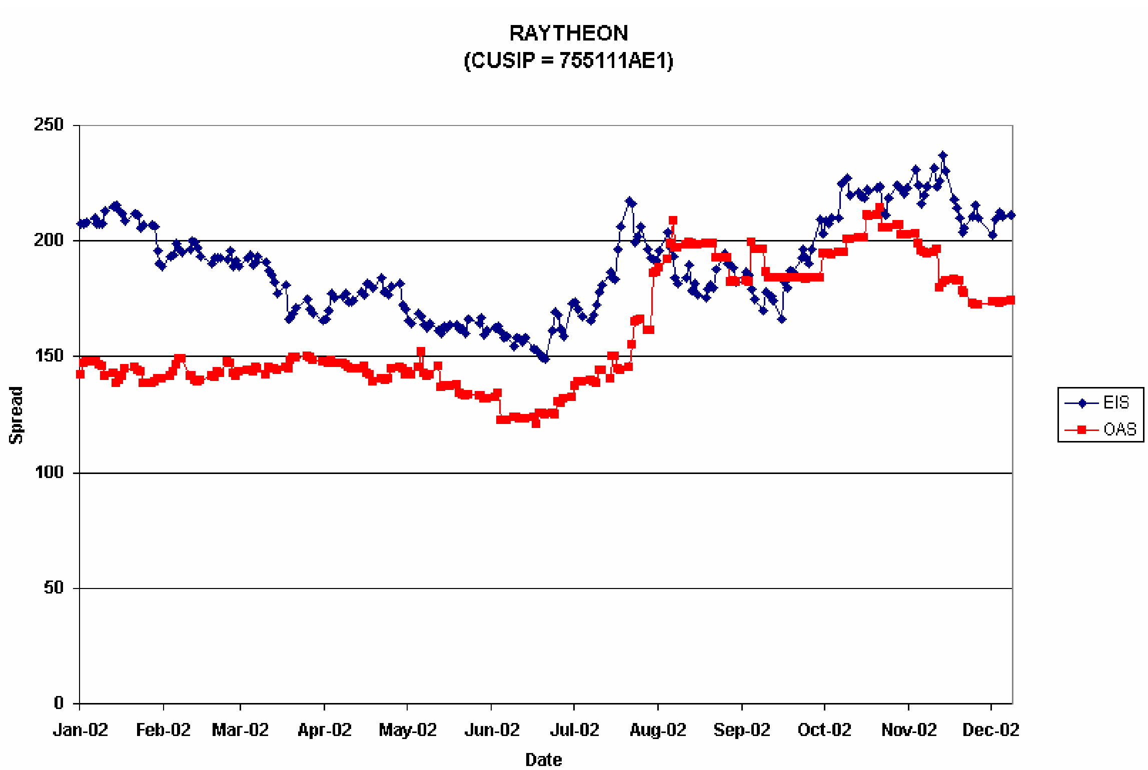

FIGURE 11 (e)

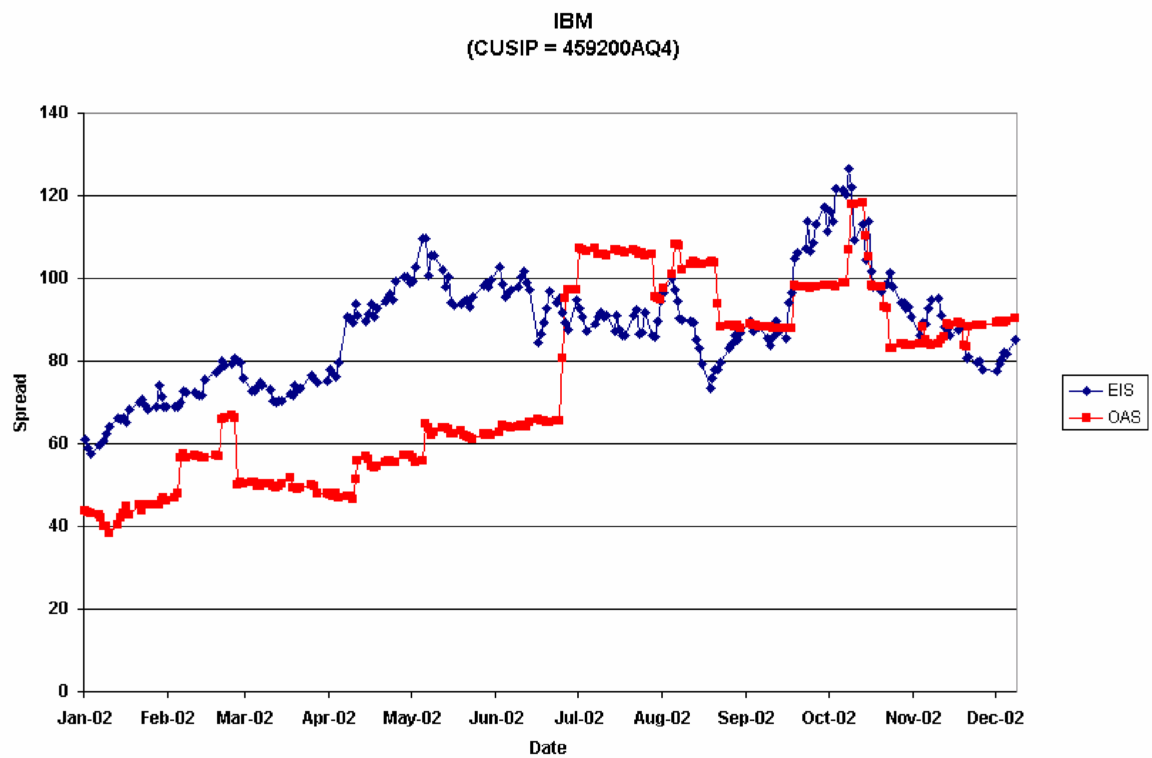

FIGURE 11 (f)

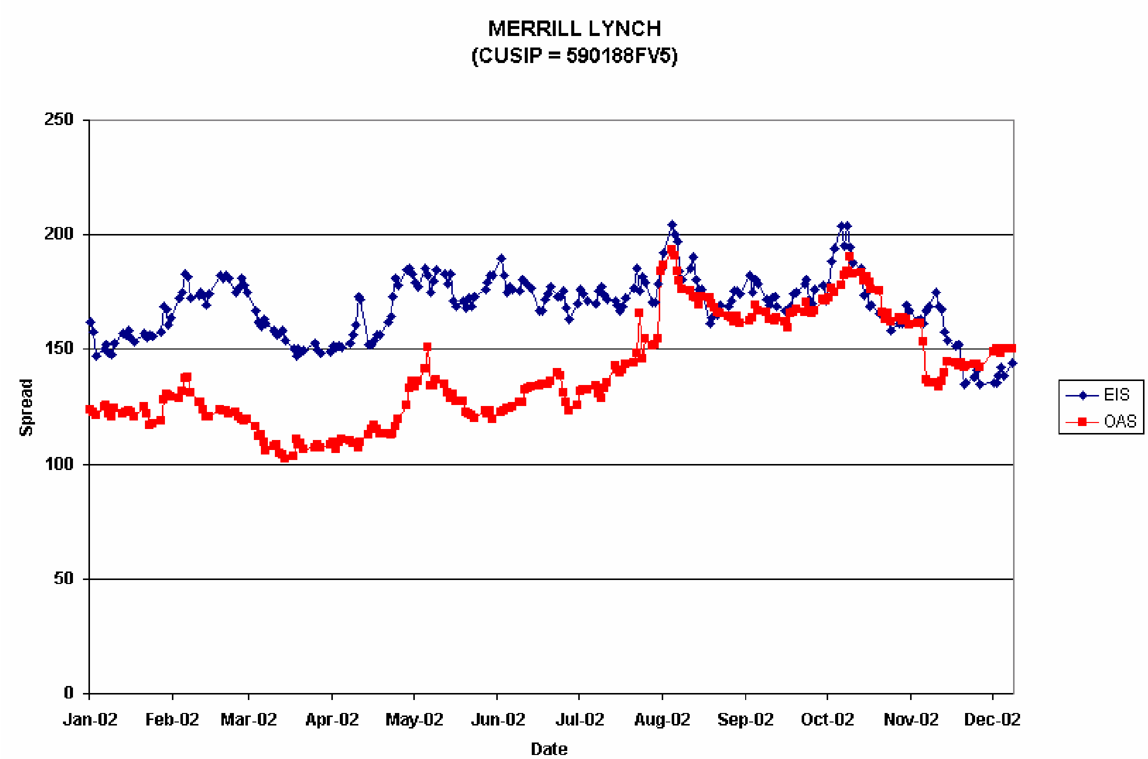

FIGURE 11 (g)

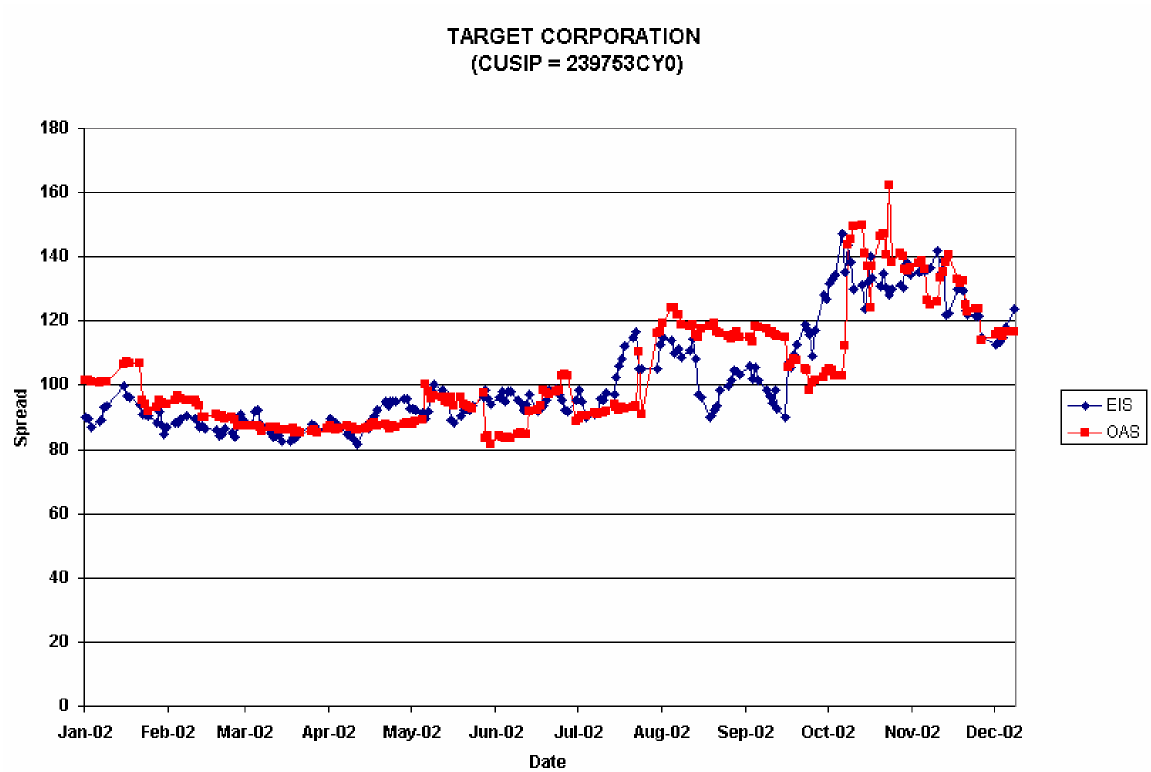

FIGURE 11 (h)

10 TRADING STRATEGY ANALYSIS

There are cases in our sample where the differences between OAS and EIS are large. We conjecture that this may, at least partially, be due to bond market inefficiencies relative to the equity markets. These inefficiencies may stem from differences of information reflected in the claims on the same firm in the two markets, or reduced liquidity in debt markets.

To test this hypothesis, we compare the performance of portfolios of "underpriced" bonds and "overpriced" bonds. We sort our sample in ascending order of the pricing errors measured as (OAS-EIS) and create two bond portfolios titled P(Low) and P(High). P(Low) contains bonds whose errors are smaller than the 25th percentile of all errors and P(High) contains bonds whose errors are bigger than the-75th percentile of all errors. If the EDF credit measure (or the equity market) is the right benchmark for fair pricing and it is the bond market that is relatively inefficient, then P(Low) should be the portfolio of over-priced bonds, and P(High) should be the portfolio of under-priced bonds.[15] If P(High) is a portfolio of underpriced bonds, the underpricing should be corrected over time and this portfolio should yield an excess return compared to a benchmark. The reverse should be true of the P(Low) portfolio. We could obviate the need for a benchmark by comparing the relative performance of just these two portfolios. If, on the other hand, we thought that the bond market were efficient and any wedge between the two markets reflected the inefficiency of equity markets, we could not assign the labels "underpriced" and "overpriced" to P(High) and P(Low) respectively. In that case, both the portfolios would be fairly priced and should yield similar return performance in future. Of course, any analysis of return performance assumes that the two portfolios have a similar risk profile.

Figure 12, below, shows that the portfolio of underpriced bonds outperforms the portfolio of overpriced bonds. The figure displays excess returns over and above 7 Year U.S. Treasury rates. The 4-year aggregate excess return for the

[15] The portfolios are rebalanced on a monthly basis. Our results are substantially unchanged even when we change the rebalancing horizon to six months.

portfolio of underpriced bonds is about 23% (about 6% annualized), while for over-priced portfolio, it is about 0% (about 0% annualized). This indicates annual return differential of 6% annually in their performance. These return numbers confirm that the portfolio we labeled as underpriced is indeed underpriced and the portfolio we labeled as overpriced is indeed overpriced, confirming that the large pricing errors are, at least partially, caused by relative inefficiency of the bond market.[16]

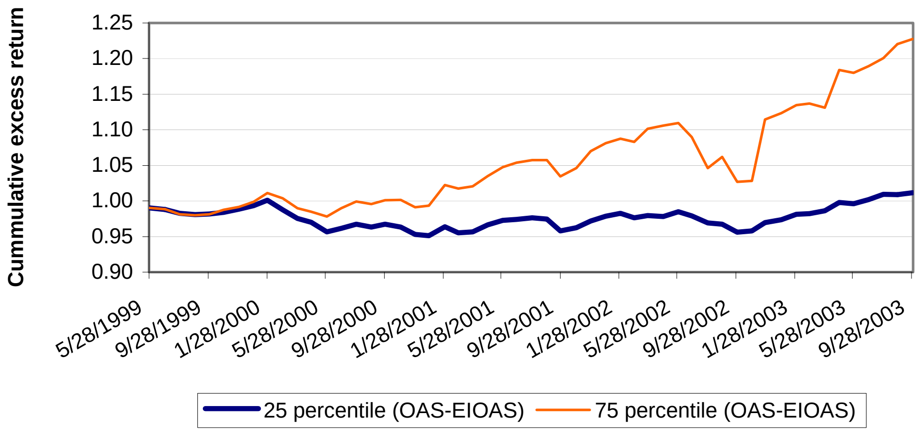

FIGURE 12 Cumulative Excess Return of Portfolio

One possible explanation behind the differences in performances of these portfolios could be the differences in their inherent risks. We test for this explanation by comparing the two major types of risks to which the portfolios are subject: default risk and interest rate risk. We use the median (and value weighted) EDF value as a measure of the portfolio default risk and median (and value weighted) duration as the measure of portfolio interest rate risk. We also compare the asset volatilities as measures of underlying total asset value risk.

Figures 13(a) and (b) demonstrate that these risk-measures are comparable. In fact, the underpriced portfolio seems to carry slightly lower default risk. We also compared the median behavior of duration and asset and equity volatilities for these portfolios and we do not observe any differences in either. These results demonstrate that the differential return performance does not stem from differences in risk profiles of the two portfolios, rather the differential returns point to the underpriced nature of one portfolio relative to the other. Thus, this result supports our hypothesis that the model errors can at least partially be explained by the lagging nature of bond market prices relative to equity prices. While the information may be embedded in equity prices, transforming these prices into EDF credit measures by determining default point and firm asset volatility is critical to identifying market inefficiencies.

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[16] The performance can also be affected by fluctuations in the interest rates. During the sample period (1999-2003), interest rates have generally been on a decline leading to an increase in bond prices. However, to the extent that the median Macaulay durations are similar across the two portfolios (as shown in Figure 13(b)), we have reason to believe that the relative performances of the two portfolios are driven by their mispricings.

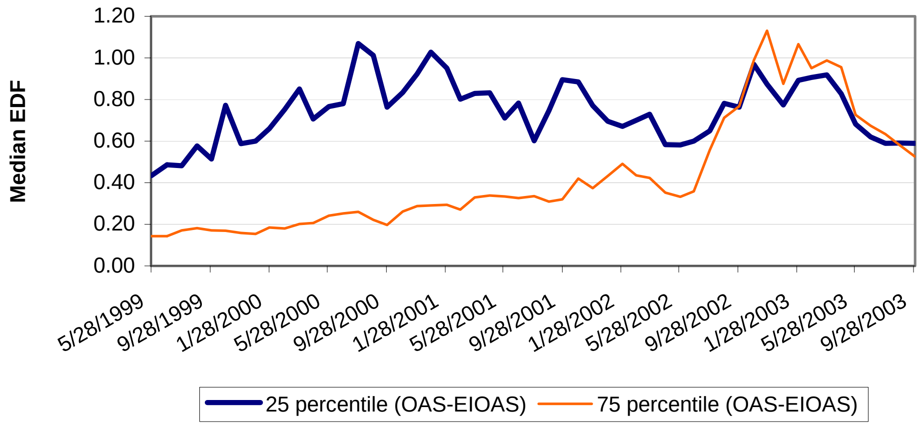

FIGURE 12(a)(i) Median EDF of Portfolio

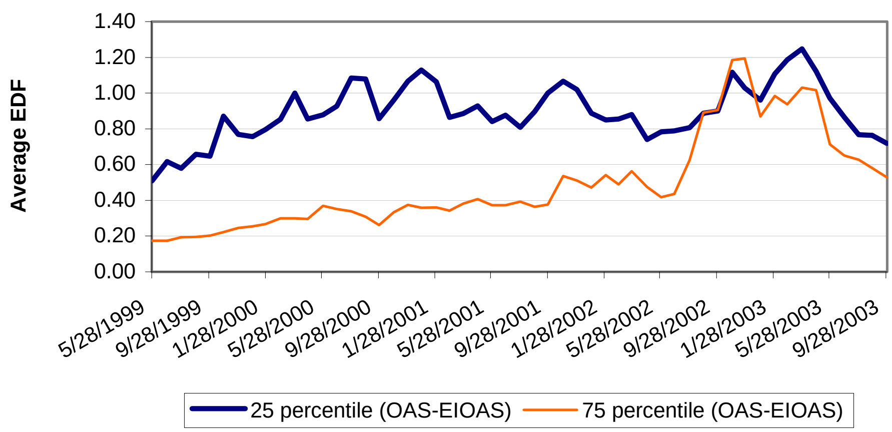

FIGURE 13(b)(ii) Average EDF of Portfolio

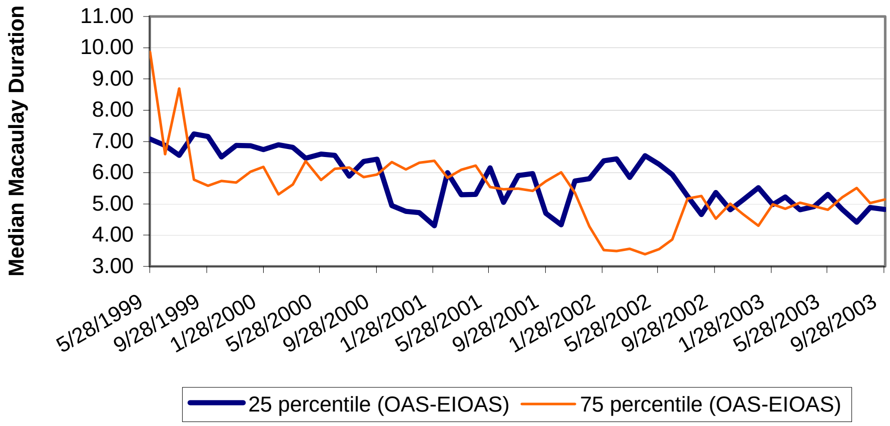

FIGURE 13(c)(i) Median Macaulay Duration of Portfolio

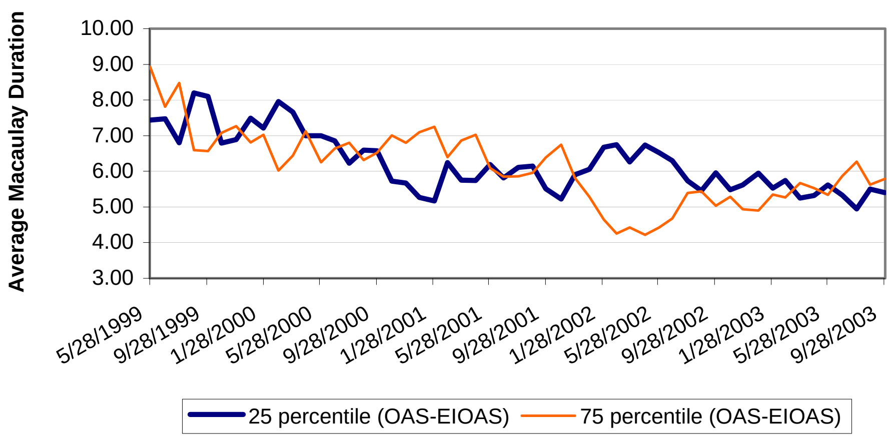

FIGURE 13(c)(ii) Average Macaulay Duration of portfolio

11 CONCLUSIONS

We implement a simple model for the valuation of credit risky claims. The model leverages EDF credit measures derived from the Moody's KMV structural model of default and builds a mechanism to convert them to the risk-neutral measure using an estimated Market Sharpe Ratio. We calibrate the Market Sharpe Ratio using corporate bond market prices. The risk-neutral default probabilities so derived can be used to price various credit risky instruments using standard risk-neutral valuation methodology. We apply our valuation method to a large sample of bond spreads and find that, unlike many other versions of structural models, ours performs quite well. Our model consistently explains more than 70% of the cross-sectional variation in bond spreads. We show that at least part of the remaining unexplained variation comes from the fact that equity markets tend to lead the bond markets in terms of incorporating new information.

APPENDIX A: A DERIVATION OF CEDF TO CQDF TRANSFORMATION

Consistent with our EDF model, we assume that the asset process is given by:

$$\frac{dA}{A} = \mu dt + \sigma dZ \tag{6}$$

Using Ito's Lemma on the above asset process, we get:

$$d\ln A = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dZ \tag{7}$$

This can be integrated to get:

$$\ln\left(\frac{A_t}{A_0}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\varepsilon \tag{8}$$

Assuming that default can happen at time *t* if asset value falls below the default point *DPT*, one can derive the following condition (1) on the random shock term *e*, for default to occur:

$$\begin{aligned} &\text{If } A_t < DPT, \\ &\varepsilon < -\frac{\ln\left(\frac{A_t}{A_0}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}} = N^{-1}(CEDF) \qquad \ldots(1) \end{aligned} \tag{9}$$

One can also look at the above process in a risk-neutral setting in which the asset drift is equal to the riskless rate *r*.

$$\frac{dA}{A} = rdt + \sigma\left(dZ + \frac{\mu - r}{\sigma}dt\right) = rdt + \sigma d\tilde{Z} \tag{10}$$

$d\tilde{Z}$ represents the Brownian motion in a risk-neutral framework and is related to the Brownian motion dZ in the physical framework as:

$$\begin{aligned} d\tilde{Z} &= dZ + \frac{\mu - r}{\sigma}dt \\ \tilde{Z}_t &= \tilde{\varepsilon}\sqrt{t} = \varepsilon\sqrt{t} + \frac{\mu - r}{\sigma}t \\ \Rightarrow \tilde{\varepsilon} &= \varepsilon + \frac{\mu - r}{\sigma}\sqrt{t} \end{aligned} \tag{11}$$

Recalling the relationship (1) above, we get:

$$\tilde{\varepsilon} = N^{-1}(\text{CEDF}) + \frac{\mu - r}{\sigma}\sqrt{t} \qquad (12)$$

We can therefore define the cumulative quasi-EDF as:

$$\text{CQDF} = N(\tilde{\varepsilon}) = N\left[N^{-1}[\text{CEDF}] + \frac{\mu\text{-}r}{\sigma}\sqrt{t}\right] \qquad (13)$$

APPENDIX B: ZERO-COUPON APPROXIMATION

Suppose a bond pays multiple cash flows with *Ct* being the cash flow at time *t*. The risk-neutral default probability is $CQDF_t$ to the cash flow at *t*. Assume the bond matures at *t*= *T*. The discount factor of a cash flow at *t* to as-of date is given by DF_t. The as-of date value of all these cash flows is given by:

$$
\begin{aligned}
V_0 &= \sum_{t=1}^{T}\left[CQDF_t - CQDF_{t-1}\right]\left[\left(1-LGD\right)\sum_{i=t}^{T} C_i.DF_i + \sum_{i=1}^{t-1} C_i.DF_i\right] + \left[1-CQDF_T\right]\sum_{i=1}^{T} C_i.DF_i \qquad (14)\\
&= \left(1-CQDF_T + CQDF_T\right)\sum_{i=1}^{T} C_i.DF_i - LGD\sum_{t=1}^{T}\left[CQDF_t - CQDF_{t-1}\right]\sum_{i=t}^{T} C_i.DF_i\\
&= \left(1-LGD\right)\sum_{i=1}^{T} C_i.DF_i + LGD\left[\sum_{i=1}^{T} C_i.DF_i - \sum_{t=1}^{T}\left(CQDF_t - CQDF_{t-1}\right)\sum_{i=t}^{T} C_i.DF_i\right]\\
&= \left(1-LGD\right)\sum_{i=1}^{T} C_i.DF_i + LGD\sum_{i=1}^{T}\left(1-CQDF_i\right)C_i.DF_i\\
&= \left(1-LGD\right)RFV + LGD.RYV
\end{aligned}
$$

where *RFV* and *RYV* are the risk-free value and the risky values respectively, given by:

$$
\begin{aligned}
RFV &= \sum_{i=1}^{T} C_i.DF_i \qquad (15)\\
RYV &= \sum_{i=1}^{T} \left(1-CQDF_i\right)C_i.DF_i
\end{aligned}
$$

Therefore, a setup like above resolves itself to a two-period model. It can be approximated by a bond with a single cash flow, as long as the effective duration approximates the weighted cash flows, which is indeed true in the case of Macaulay duration for low-EDF bonds.

To verify this, we created a synthetic portfolio of bonds with various LGD, coupon, EDF, and R-squared levels to ensure that we cover the entire spectrum of potential bonds. We calculated the EDF-implied spreads as measured by the spread to maturity of a multi-coupon cash-flow structure. We then compared it to the spread as implied by the corresponding zero-coupon bonds with their maturities being the Macaulay durations of the multi-coupon bonds. The results are displayed in Figure B.1, below.

As discussed above, we do not observe any significant bias in either direction by using this assumption. For majority of the bonds, we found that the spreads were within a few basis points of each other.

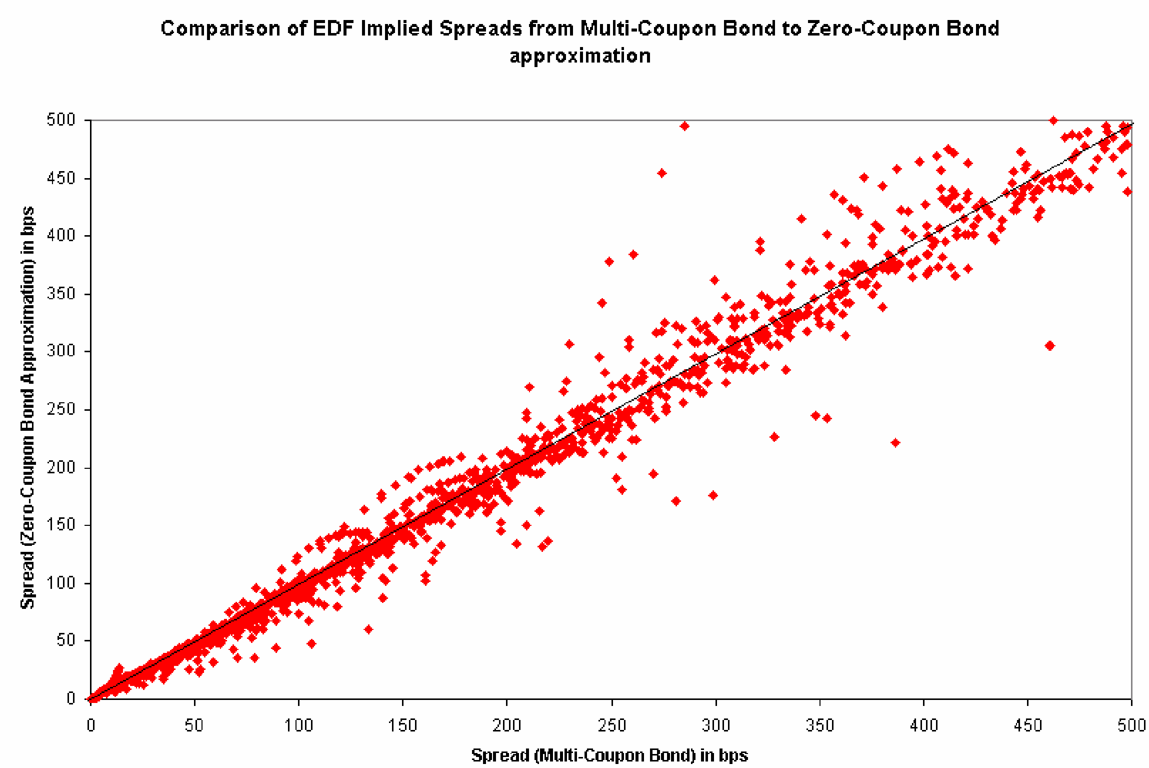
Comparison of EDF Implied Spreads from Multi-Coupon Bond to Zero-Coupon Bond approximation
Spread (Zero-Coupon Bond Approximation) in bps
Spread (Multi-Coupon Bond) in bps
0
50
100
150
200
250
300
350
400
450
500

FIGURE B.1

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