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INCORPORATING SYSTEMATIC RISK IN RECOVERY: THEORY AND EVIDENCE

MODELING**METHODOLOGY**

ABSTRACT

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This paper proposes a theoretical framework to account for systematic risk in recovery and to address the correlation between the firm's underlying asset process and recovery. Under the proposed framework, the expected value in default under the risk neutral measure can be expressed as a linear function of the expected value under the physical measure. This allows for a simple mapping between expected recovery observed in the data and a measure that can be applied when using risk neutral valuation methods. When calibrating the model to parameters observed in the data, the risk neutral adjustment results in spreads that are 14% higher for a typical bond, and over 30% higher in some cases. When validating against market data, the evidence suggests that market spreads reflect systematic risk in recovery. We found that approximately 80% of our sample was estimated with a lower absolute error when using the risk neutral adjustment to compute model implied spreads.

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1 INTRODUCTION

Recovery in the event of default is closely tied to macroeconomic conditions and a firm's credit quality (see, for example, Gupton and Stein (2005)). As shown in Figure 1, recovery is generally pro-cyclical. During expansion, recovery tends to be higher than during market downturns. Similarly, there is a positive correlation between a firm's credit quality and recovery even after conditioning on macroeconomic effects; expected recovery is higher for high credit-quality firms, and lower for low credit-quality firms. This paper develops a theoretical framework to account for these dynamics. Moreover, it provides empirical evidence for how these dynamics impact bond prices.

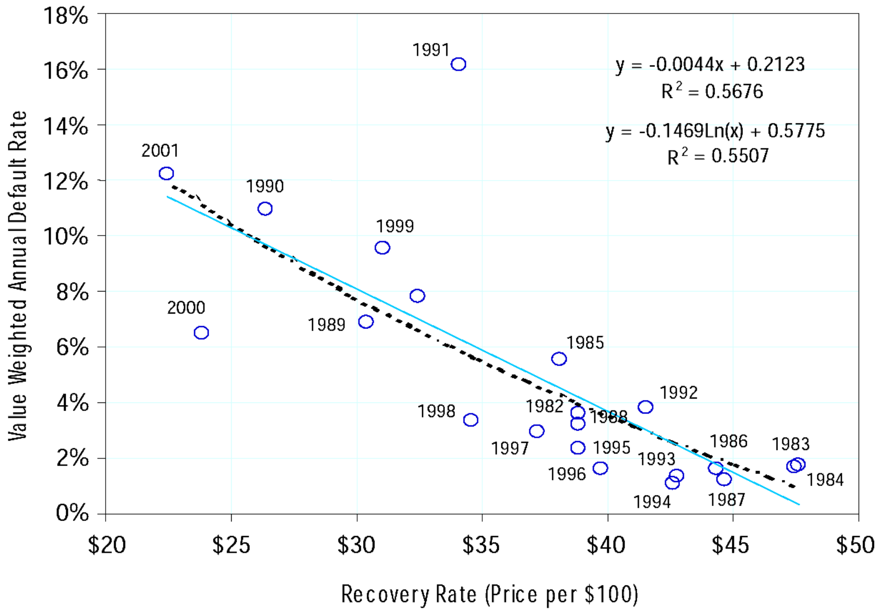

FIGURE 1 Speculative-grade default rates and recovery rates are inversely correlated
Source: "Default & Recovery Rates of Corporate Bond Issuers," Moody's Special Comment, February 2003

The role of systematic risk in recovery becomes evident when pricing bonds because assets that bear systematic risk are typically associated with a risk adjustment. Motivated by the existing evidence and economic theory, we propose a framework within the context of a structural model where recovery is driven by systematic/industry factors common to a firm's asset value and an idiosyncratic factor. This model is compatible with a multi-factor model and similar to the one introduced in Frye (2000a) who analyzes the impact of correlation between recovery and credit on credit portfolio risk. Under the theoretical framework, we show that the relationship between expected recovery under the physical measure and the expected recovery under the risk-neutral measure is linear. This defines a simple relationship between expected recovery, which is typically observed in the data, and expected risk-neutral recoveries that can be used in traditional risk-neutral valuation techniques.

In thinking about why a relationship exists between general economic conditions and recovery, Schleifer and Vishny (1992) argue that when a firm in financial distress needs to sell assets, its industry peers are likely to be experiencing problems themselves, leading to asset sales at prices below value in best use. Such illiquidity makes assets cheap in bad times, and causes ex ante variation in debt capacity across industries and over the business cycle. Acharya, Bharath, and Srinivasan (2005) find supporting evidence that creditors recover less if the industry in distress has illiquid surviving firms, if the industry is more levered, and if it has assets that are industry-specific, or in other words, assets that are not easily redeployed to other industries. Pulvino (1998) provides empirical evidence of this effect in the airline industry.

Similarly, Gupton and Stein (2005) document a positive correlation between a firm's credit quality and recovery; expected recovery is higher for high credit-quality firms, and lower for low credit-quality firms. They find this positive relation prevalent even after conditioning on macroeconomic effects. Intuitively, one would expect this relationship, since both recovery and credit quality should be influenced by the value of the firm's assets even after conditioning on the state of the macroeconomic environment.

In this paper, model parameters are estimated using expected recovery given by MKMV's LossCalc™. This dataset is unique in that it provides a time series of firm-level expected recoveries. Previous studies involving the time series behavior of recovery (see, for example, Fry (2000b), Hu and Perraudin (2002), Altman et al. (2003), or Düllmann and Trapp (2004)) typically analyze the properties of recovery in aggregate. It is not surprising that most of these studies focus on aggregated data since recovery information at the firm level is only observed at default, which means there will typically be no time series for any single firm. The benefit of using the LossCalc™ dataset is that it allows for estimation of firm-specific recovery parameters. With this dataset, we are therefore able to analyze the cross-sectional impact of systematic risk in recovery on bond prices. One exception is Carey and Gordy (2004) who provide an interesting empirical analysis of firm-level ultimate recoveries. Since the typical firm in their sample remains in bankruptcy for 1.2 years, ultimate recoveries differ from LossCalc which provides expected price of recovery at default.

It is worth relating the methodology developed in this paper with traditional reduced-form models (e.g., Duffie and Singleton (1999)). Reduced-form models exogenously specify an arbitrage-free evolution for the spread between default-free and credit-risky bonds and allow for estimation of expected loss; the multiplicative product of the risk-neutral recovery rate times the default probabilities. These models, however, do not typically allow for estimation of the recovery rate (risk-neutral or physical). Jarrow (2001) and Madan, Güntay, and Unal (2003) are exceptions. Jarrow (2001) develops a methodology for recovery estimation by using information from the equity market along with the reduced form framework. Meanwhile, Madan, Güntay, and Unal (2003) develop a model that allows them to estimate the risk-neutral density of recovery rates by using information from bonds with issuers that have available prices for both junior and senior issues.

The model's predictions related to the impact of accounting for systematic risk in recovery are explored through a calibration exercise. Using parameters for a bond with average characteristics, the risk-neutral adjustment implies modeled bond spreads that are 14% higher for a bond with average characteristics; the model-implied spread increases from approximately 64 bps to 73 bps. Moreover, we find that for a reasonable range of parameters, bond spreads will increase by 5% to over 30% as a result of incorporating systematic risk in recovery.

As predicted by the model, we find a positive relation between the option adjusted spreads (OAS) observed in the data and risk-neutral recovery adjustment. In fact, over 80% of the sample exhibits a reduction in root-square error. Additionally, empirical evidence supports the model predictions after conditioning on model spreads that exclude risk-neutral recovery adjustment.

The rest of this paper is organized as follows. Section 2 introduces the modeling framework and presents the main theoretical results. Section 3 describes how the variables necessary to estimate the model are obtained. Section 4 describes parameter estimates. Section 5 describes the calibration exercise and model validation. Section 6 presents the conclusions that can be drawn from the model analysis.

2 MODELING FRAMEWORK

Risk-neutral valuation techniques are typically employed when valuing credit-risky instruments. Although the analysis typically accounts for systematic risk in the default probability, the analysis does not commonly account for systematic risk in recovery. Standard parameterizations use the mean recovery observed in the data, not the expected recovery under the risk-neutral measure. Moreover, the methodologies assume that the recovery amount is uncorrelated with the underlying asset process. One can formulate the time zero value of a zero coupon bond with a promised payment of a unit at time M.[1]

[1] Throughout the discussion, we assume interest rates are non-stochastic.

$$V_0 = E_0^P\left[\int_{T=0}^{M} e^{-rT} CF_T \partial T\right] = \int_{T=0}^{M} E_0^P[RR_T \mid Default_T]e^{-rT}\partial CDP_T + (1-CDP_{0,M})e^{-rM}$$
$$= E_0^Q\left[\int_{T=0}^{M} e^{-r_f T} CF_T \partial T\right] = \int_{T=0}^{M} E_T^Q[RR_T \mid Default_T]e^{-r_f T}\partial CQDP_T + (1-CQDP_{0,M})e^{-r_f M} \quad (1)$$

Here, P represents the physical measure, Q represents the risk-neutral measure, r represents the risk adjusted rate of return, r_f represents the risk free rate, CF_τ represents the uncertain cash flow at time T, CDP_T represents the cumulative expected default probability from time 0 to time T, $CQDP_T$ represents the cumulative expected default probability under the risk-neutral measure from time 0 to time T, and RR_T represents the recovery rate associated with a time T default.

We can relate Equation (1) with the typical methodology discussed above by setting $E_0^P\left[RR_T \mid Default_T\right] = E_0^Q\left[RR_T \mid Default_T\right]$. This is consistent with the assumption that conditional on default, the recovery process is uncorrelated with systematic risk or the underlying asset process. However, evidence discussed in the introduction and presented later in this paper suggests that recovery does contain systematic risk.

To account for the documented correlations discussed in the introduction, the firm's asset (A_t) and recovery ($RR_{T,t}$) processes are explicitly modeled. The default is modeled as the time point T where the firm's asset value reaches the default barrier (DP_T). Thus, the default barrier and the asset value parameters define the probability of default for various horizons ($A_T = DP_T$). Meanwhile, recovery in the event of a time T default is modeled as the collateral-value process associated with the credit instrument. Although time T recovery is only observed if default occurs at time T, the recovery process (i.e., the collateral value) exists regardless of a default event, and prior to time T (i.e., time t). Thus, $RR_{T,t}$ represents the time t value of the unconditional recovery process associated with a time T default event. This is different from the conditional recovery process which defines the recovery value in the default state (i.e., conditional on the asset process hitting the default barrier). It is worth pointing out that for $\tau \neq T$, $RR_{T,t}$ need not be equal to $RR_{\tau,t}$.[2] More formally, the joint (unconditional) stochastic processes are defined as follows:

$$\frac{dRR_{T,t}}{RR_{T,t}} = r_{RR_T}\,dt + \sigma_{r_{RR_T}}\,dB_{RR_T,t},$$
$$and \quad (2)$$
$$\frac{dA_t}{A_t} = r_A dt + \sigma_A dB_{A,t}.$$

Here r_{RR_T} and r_A represent the respective drifts for $RR_{T,t}$ and the underlying assets under the physical measure. $\sigma_{r_{RR_T}}$ and σ_{r_A} represent their respective diffusions. Moreover, the instantaneous correlation between B_{RR_T} and B_A is assumed to be equal to $\rho_{r_{RR_T}}$.

Using this basic structure, Theorems 1 through 3 develop a relationship between recovery value under the physical and risk-neutral measure for a default at time T; the term inside the integral on the right hand side of the relationship depicted in Equation (1). As such, we will drop the T subscript so that RR_t and RR_T represent $RR_{T,t}$ and $RR_{T,T}$ respectively.

[2] Expected recovery for a car loan, for example, typically decreases over time as the car's value (i.e., the collateral) depreciates over time.

Theorem 1: The expected recovery value at time *T*, under the physical measure, is equal to:

$$E_0^P[RR_T \mid A_T = DP_T] = RR_0 \left(\frac{DP_T}{A_0}\right)^{\rho_{r_{RR}}\sigma_{RR}/\sigma_A} e^{(r_{RR}T-\sigma_{r_{RR}}^2 T/2)-\rho(r_A T-\sigma_A^2 T/2)\sigma_{r_{RR}}/\sigma_A+\sigma_{r_{RR}}^2 T(1-\rho_{r_{RR}}^2)/2} \quad (3)$$

Proof: See Appendix A for details.

To obtain the expected recovery value under the risk neutral measure, we assume complete markets so that r_{RR} and r_A are replaced with r_f:

Theorem 2: The expected recovery value at time *T*, under the risk neutral measure, is equal to:

$$E_0^Q[RR_T \mid A_T = DP] = RR_0 \left(\frac{DP}{A_0}\right)^{\rho_{r_{RR}}\sigma_{r_{RR}}/\sigma_A} e^{(r_f T-\sigma_{r_{RR}}^2 T/2)-\rho(r_f T-\sigma_A^2 T/2)\sigma_{r_{RR}}/\sigma_A+\sigma_{r_{RR}}^2 T(1-\rho_{r_{RR}}^2)/2} \quad (4)$$

Therefore, the relationship between $E_0^P[RR_T \mid A_T = DP]$ and $E_0^P[RR_T \mid A_T = DP]$ can be described as follows:

Theorem 3: The expected recovery value at time *0*, under the risk neutral measure, is equal to:

$$E_0^Q[RR_T \mid A_T = DP] = E_0^P[RR_T \mid A_T = DP] e^{(r_f T-r_{RR}T)-\rho_{r_{RR}}(r_f T-r_A T)\sigma_{r_{RR}}/\sigma_A} \quad (5)$$

One can simplify this further. Let λ_{RR} and λ_A represent the market price of risk (i.e., Sharpe Ratio) for the recovery and asset market. Let $R_{r_{RR}}$ and R_A represent the correlation between the recovery rate process and market (index) return (ϕ), and the asset process and the market (index) return respectively. Using the following two resulting equalities, $\frac{r_{RR}T - r_f T}{\sigma_{r_{RR}}\sqrt{T}} = R_{r_{RR}}\lambda_{RR}\sqrt{T}$ and $\frac{r_A T - r_f T}{\sigma_A\sqrt{T}} = R_A\lambda_A\sqrt{T}$, the relationship in Equation (5) can be written as:

$$E_Q[RR_T \mid A_T = DP] = E_P[RR_T \mid A_T = DP] e^{(-R_{r_{RR}}\lambda_{RR}+\rho_{r_{RR}}R_A\lambda_A)\sigma_{r_{RR}}T} \quad (6)$$

The relationship in Equation (6) provides a parsimonious mapping between recovery under the physical measure and recovery under the risk-neutral measure and is worth discussing in greater detail. Focusing on the first term in the exponent, the expected recovery is driven lower under the risk-neutral measure when recovery is positively correlated with the market. The effect of the second term may seem less intuitive; a positive correlation with the asset process increases expected recovery under the risk-neutral measure. To understand this dynamic, consider an extreme case where $\lambda_{RR} = \lambda_A$, $R_{r_{RR}} = 0$, $\rho_{r_{RR}} > 0$ and $R_A > 0$. In this case, $R_{r_{RR}} > \rho_{r_{RR}} R_A$ so that the risk premium associated with the *conditional* recovery process is negative despite the fact that the *unconditional* recovery process has a risk premium of zero ($R_{r_{RR}} = 0$). To understand this dynamic, first note that the default event pins down the asset return. Next, note that the asset return is equal to the sum of its idiosyncratic and systematic components. Thus a high idiosyncratic asset shock must be associated with a low systematic shock conditional on the default event ($A_T = DP$). More formally, a negative correlation is created between the systematic risk and the idiosyncratic asset processes conditional on the default event. Since the idiosyncratic recovery and idiosyncratic asset process are positively correlated, a negative correlation results between the systematic risk and the conditional recovery process. This effect drives the risk premium down, and may even drive it negative, as in the case of this example. It is worth pointing out that the empirical methodology employed below precludes cases where $R_{r_{RR}} \leq \rho_{r_{RR}} R_A$. This implies that the recovery under the physical measure is always at least as great as recovery under the risk-neutral measure when the asset and recovery markets face the same market price of risk.

A few features of the model are worth discussing. First, it is assumed that recovery follows a log-normal process. This violates a range for the recovery process of [0, 1] that is generally accepted as reasonable.[3] An alternative and perhaps more theoretically appealing approach allows for mean reversion, or bounds on the recovery process. Although theoretically tractable, these processes are notoriously difficult to estimate. Instead, we argue, and find empirical evidence supporting our theory, that the above model provides sufficient flexibility to provide an approximation.

Also, Equation (6) presents a methodology of valuing recovery when the asset process hits the default threshold at time *T*. More generally, the asset process can hit the default threshold at any time before maturity, as represented in Equation (1). To generalize and allow default to occur at any time prior to maturity, the relationship in Equation (6) can be combined with Equation (1) as follows:

$$V_0 = \int_{\tau=0}^{M} E_P[RR_{T,T} \mid A_T = DP_T] e^{(-R_{r_{RR}}\lambda_{RR} + \rho_{r_{RR}} R_A \lambda_A)\sigma_{r_{RR_T}} T} e^{-r_f T} \partial CQDP_T + (1 - CQDP_{0,M}) e^{-r_f M} \qquad (7)$$

It is worth pointing out that we should not expect variance to grow at a constant rate (i.e., $\sigma^2_{r_{RR_\tau}} \neq \sigma^2_{r_{RR}} \tau$). After all, recovery variance is bounded by reasonable bounds on recovery (i.e., [0, 1]) which is inconstant with a constant growth rate for volatility. This issue will be discussed further in the next section on estimating parameters. Similarly, expected recovery should not increase at a constant rate with increasing default time (i.e., $E_Q[RR_{\tau+\Delta t,0}] \neq E_Q[RR_{\tau,0}] e^{r_f \Delta t}$) because, again, we know that recovery faces reasonable bounds (i.e., [0,1]). In short, the relationships described in Equations (2) through (6) should be understood to apply to a recovery process associated with default at a particular time. It is thus reasonable to expect a term structure of variance, { $\sigma^2_{r_{RR},\tau}$ }, and expected recovery, { $RR_{\tau,0}$ }.

Finally, notice that the relationships in Equations (6) and (7) allow for a separate specification of market risk for the recovery process and the asset process. This separation is a result of modeling the underlying recovery and asset processes. It would not be as simple had we directly modeled the recovery process, conditional on default.

3 MEASURING RECOVERY RETURNS AND PARAMETERS

In this section we describe the methodology employed in estimating $R^2_{r_{RR}}$, $\rho_{r_{RR}}$, and $\sigma_{r_{RR}}$ that are used in Equation (8) below.

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[3] In principle, realized recovery can fall outside this range. For example, if cost of recovery is included, recovery can actually be negative.

Ideally, monthly recovery returns (r_{RR_t}) would be computed using changes in the discounted expected value of recovery, $r_{RR_t} = \frac{E_{t+1}[RR_T]/(1+r_{RR})^{T-(t+1)}}{E_t[RR_T]/(1+r_{RR})^{T-t}} = \frac{RR_{T,t+1}}{RR_{T,t}}$. Unfortunately, recoveries are only observed in default. For example, LossCalc provides estimates for the expected recovery conditional on default, $E_t[RR_T \mid A_T = DP]$, and recovery variance conditional on default, $Var_t[RR_T \mid A_T = DP]$. However, we can use the conditional recovery process to determine a conditional recovery return that can be used for the purpose of estimating the necessary parameters:

$$r_{RR|A=DP_{t+1}} = \frac{E_{t+1}[RR_T \mid A_T = DP]/(1+r_{RR|A=DP})^{T-(t+1)}}{E_t[RR_T \mid A_T = DP]/(1+r_{RR|A=DP})^{T-t}} - 1 \tag{8}$$

With $r_{RR|A=DP_t}$ in hand, one can estimate $\sigma_{r_{RR|A=DP}}$, $\rho_{r_{RR|A=DP}} = \frac{Cov(r_{RR|A=DP}, r_A)}{\sigma_{r_{RR|A=DP}}\sigma_{r_A}}$, and $R_{r_{RR|A=DP}} = \frac{Cov(r_{RR|A=DP}, \phi)}{\sigma_{r_{RR|A=DP}}\sigma_{\phi}}$.

Using the mapping described in Theorem 4 below, one can obtain $R_{r_{RR|A=DP}} = \frac{Cov(r_{RR|A=DP}, \phi)}{\sigma_{r_{RR|A=DP}}\sigma_{\phi}}$:

<u>Theorem 4:</u> Three useful properties of the conditional recovery process are:

1. $\sigma^2_{r_{RR}} = \sigma^2_{r_{RR|A=DP}} \Big/ \left(1 - \frac{\rho^2_{r_{RR}} t}{T}\right)$

2. $\rho_{r_{RR}} = (\rho_{r_{RR|A=DP}} \cdot T)/(T^2 - tT + \rho^2_{r_{RR|A=DP}} \cdot tT)^{1/2}$

3. $R_{r_{RR}} = R_{r_{RR|A=DP}} \sqrt{(1-\rho^2_{r_{RR}} t/T)}\sqrt{1-R^2_{r_A} t/T} + R_{r_A}\rho_{r_{RR}} t/T$

Proof: See Appendix A for details.

The intuition for the first property follows from the conditioning event reducing the volatility for the recovery process; the term in parenthesis on the right hand side is less than or equal to 1. This implies that the unconditional volatility is greater than the conditional volatility. This is not surprising since conditioning on information reduces uncertainty.

The second and third properties are more difficult to intuit since the observed conditional correlation and recovery R-squared combine conditional covariances, and conditional and unconditional standard deviations. As such, we simply point out that $\rho_{r_{RR}} > \rho_{r_{RR|A=DP}}$, and that $R_{r_{RR}}$ can be greater or less than $R_{r_{RR|A=DP}}$.

Armed with Theorem 4, we collect a monthly time series of conditional one-year (12-month) expected recoveries from LossCalc, $E_t[RR_{t+12} \mid A_{t+12} = DP_{t+12}]$. The dataset includes only firms in our global database that have the inputs required by LossCalc to estimate recovery for senior unsecured bonds. We focus on senior unsecured bonds since this dataset is most complete. It is worth pointing out that similar results were obtained when analyzing senior secured bonds.

One wrinkle associated with using the LossCalc dataset is caused by the fact that LossCalc only provides $E_t[RR_{t+12} \mid A_{t+12} = DP_{t+12}]$; it does not provide $E_{t+1}[RR_{t+12} \mid A_{t+12} = DP_{t+12}]$. To proceed we assume that the term structure of recovery is such that $E_{t+1}[RR_{t+12} \mid A_{t+12} = DP_{t+12}] = E_{t+1}[RR_{t+13} \mid A_{t+13} = DP_{t+13}]/(1+r_{RR|A=DP})$. Under this assumption the conditional recovery return process can be computed as:

$$
\begin{aligned}
r_{RR|A=DP_t} &= \frac{V_{RR_{t+1}}}{V_{RR_t}} = \frac{E_{t+1}[RR_T \mid A_T = DP]/(1+r_{RR|A=DP})^{T-(t+1)}}{E_t[RR_T \mid A_T = DP]/(1+r_{RR|A=DP})^{T-t}} \\
&= \frac{E_{t+1}[RR_{t+13} \mid A_{t+13} = DP_{t+13}]}{E_t[RR_{t+12} \mid A_{t+12} = DP_{t+12}]}
\end{aligned} \tag{9}
$$

4 DATA AND PARAMETER ESTIMATION

Following the discussion in the previous section, we estimate three parameters ($R_{r_{RR}}$, $\rho_{r_{RR}}$, and $\sigma_{r_{RR|}}$) for each firm in our recovery series. Parameters are estimated using a combination of recovery data from LossCalc, MKMV's asset return data, and correlation data from MKMV's GCorr.

LossCalc has a statistical model that uses information on collateral, instrument, firm, industry, country, and macroeconomic data to predict estimates for expected recovery and recovery variance under the physical measure conditional on a default in twelve months.[4] The dataset includes 1,424 defaulted public and private firms. The median firm size (measured as sales at annual report prior to default) is \$660 million, and range in value from zero up to \$48 billion. It is based on 3,026 *global* observations of market prices for loans, bonds, and preferred stock one month after default between 1981 and 2004. Using a monthly time series of conditional expected recovery from LossCalc, a time series of conditional recovery returns is constructed using the methodology outlined above. We focus on senior unsecured bonds since our estimates for this class are likely to be the most accurate; the number of observations from this category is largest and collateral data is not needed. For a more complete description of the LossCalc model and its performance, see Gupton and Stein (2005).

MKMV's GCorr is a global asset correlation model. The model associates firms with a standard normal custom index ϕ_i (defined by the firm's associated industry and country weights) and the correlation with this custom index (R_{A_i}). The asset return process has a standard normal distribution and can be represented as: $r_{A_i} = R_{A_i}\phi_i + \sqrt{1-R_{A_i}^2}\,\varepsilon_{A_i}$. Here, ε_{A_i} has a standard normal distribution that is orthogonal to other ε_{A_j} and indices ϕ_j. Therefore, the correlation between any two obligor assets process is $R_{A_i} R_{A_j} Cor_{GCorr}(\phi_i, \phi_j)$, where $Cor_{GCorr}(\phi_i, \phi_j)$ is defined within GCorr. For a more complete description of the GCorr model and its performance, see Zeng and Zhang (2001). For a more complete discussion of the asset return process see Bohn and Crosby (2003).

When modeling the recovery process, we maintain the GCorr structure and model the recovery return process as: $r_{RR_i|A_i=DP} = R_{r_{RR_i}|A_i=DP}\,\sigma_{r_{RR_i|A_i=DP}}\,\phi_{RR_i} + \sqrt{1-R^2_{r_{RR_i|A_i=DP}}}\,\sigma_{r_{RR_i|A_i=DP}}\,\varepsilon_{r_{RR_i|A_i=DP}}$. Notice that $\sigma_{r_{RR_i|A_i=DP}}$ is now included in the process since the recovery returns are not normalized; Section 2 demonstrates that the standard deviation impacts pricing. Since our recovery data is limited to unsecured debt, it is natural to associate the country and industry weights of recovery processes with those of the underlying firms when defining the custom index for the recovery process ($\phi_{RR_i} = \phi_{A_i}$).

[4] LossCalc recommends using these estimates of expected recovery and recovery variance for defaults beyond twelve months.

The merged asset return, recovery, and correlation dataset covers over 9,000 North American firms from January 1999 to April 2005. To minimize sampling errors, firms with less than three-year observations during the sample period are excluded; the final dataset contains 6,001 firms.

For the 6,001 firms, $R_{r_{RR_i|A_i=DP}}$ is estimated by fitting the cross-moments on the GCorr correlation structure:

$$Min\sum_{i\neq j}\left(Cor(r_{RR_i|A_i=DP}, r_{RR_j|A_j=DP}) - R_{r_{RR_i|A_i=DP}} R_{r_{RR_j|A_j=DP}} Cor_{GCorr}(\phi_i, \phi_j)\right)^2 \tag{10}$$

Since the number of equations is large (6,001x6,000/2 =18,003,000), the procedure is conducted on random subgroups of 500 random firms. We found that results do not change in any substantive way when firms are moved across subgroups.

Meanwhile, $\rho_{r_{RR_i|A_i=DP}}$ is estimated by directly computing the historic correlation between the asset return and the conditional return on recovery:

$$\rho^*_{r_{RR_i|A_i=DP}} = cor(r_{RR_i|A_i=DP}, r_{A_i}) \tag{11}$$

We use LossCalc estimates of the standard deviation of conditional recovery, $\sigma_{RR|A=DP}$, to compute the standard deviation of conditional recovery returns $\sigma_{r_{RR|A=DP}}$. Alternatively, we could have used estimates from our historical time series of recovery returns. LossCalc estimates were chosen because the model is based on more data. Specifically, LossCalc uses data from 1981 to 2004. Meanwhile, the return series starts in 1999. In addition the standard deviation of recovery returns conditional on default is assumed to be independent of when default occurs. Thus the estimate for each firm is computed using the following relationship:

$$\sigma_{r_{RR_i|A_i=DP}} = \frac{\sigma_{RR_i|A_i=DP}}{E[RR_i \mid A_i = DP]} \tag{12}$$

Since Equation (6) requires a normalized (i.e., annualized) statistic, $\sigma_{r_{RR_i|A_i=DP}}$ must still be divided by $\sqrt{T}$.[5] Within this context the T in Equation (6) can be replaced with $\sqrt{T}$ where $\sigma_{r_{RR}}$ now represents the standard deviation of recovery returns conditional on default (de-annualized):

$$E_Q[RR_T \mid A_T = DP] = E_P[RR_T \mid A_T = DP]e^{(-R_{r_{RR}}\lambda_{RR} + \rho_{r_{RR}} R_A \lambda_A)\sigma_{r_{RR}}\sqrt{T}} \tag{13}$$

Next, the mapping outlined in Theorem 4 is used to map the conditional parameters that were estimated to the unconditional parameters ($R_{r_{RR_i}}$, $\rho^*_{r_{RR_i|A_i=DP}}$ and $\sigma_{r_{RR_i|A_i=DP}}$) needed for the model.

We place a lower bound on the correlation since it is unlikely that the idiosyncratic portions of the asset and recovery processes are negatively correlated; the negative correlation is most likely a result of sample noise. Notice that this restriction implies that $R_{r_{RR}} \leq \rho_{r_{RR}} R_A$ as was referenced in the discussion of Equation (6).

$$\rho_{r_{RR_i}} = \max\left\{\rho^*_{r_{RRi}}, R_{r_{RR_i}} R_{A_i}\right\} \tag{14}$$

5

Histograms of $R_{r_{RR}}$, $\rho_{r_{RR}}$ and $\sigma_{r_{RR}}$ for the sample are presented below. It is worth pointing out that the wide distribution of parameter values indicates a wide range of behavior for recoveries across firms. This implies a wide range for the impact of the risk-neutral adjustment when examining the behavior of bond prices.

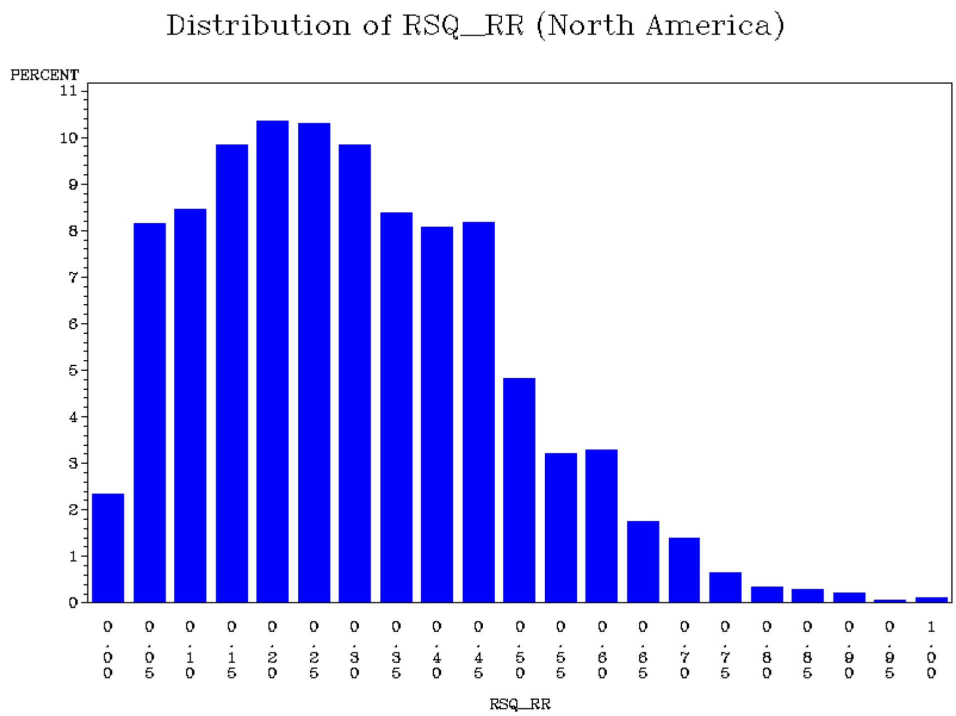

FIGURE 2

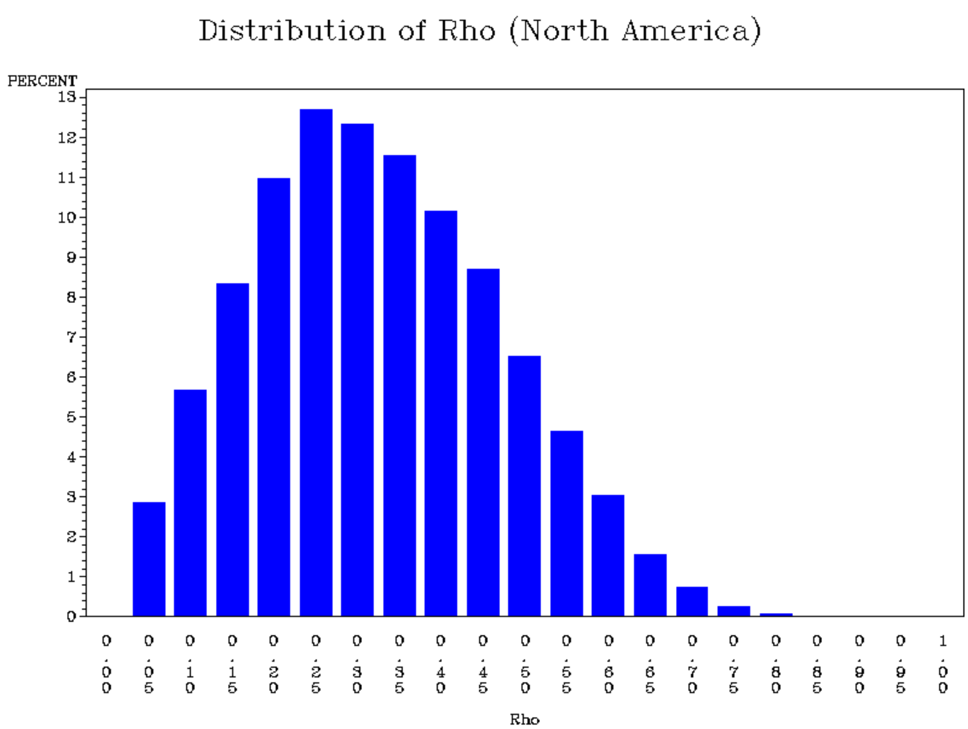

FIGURE 3

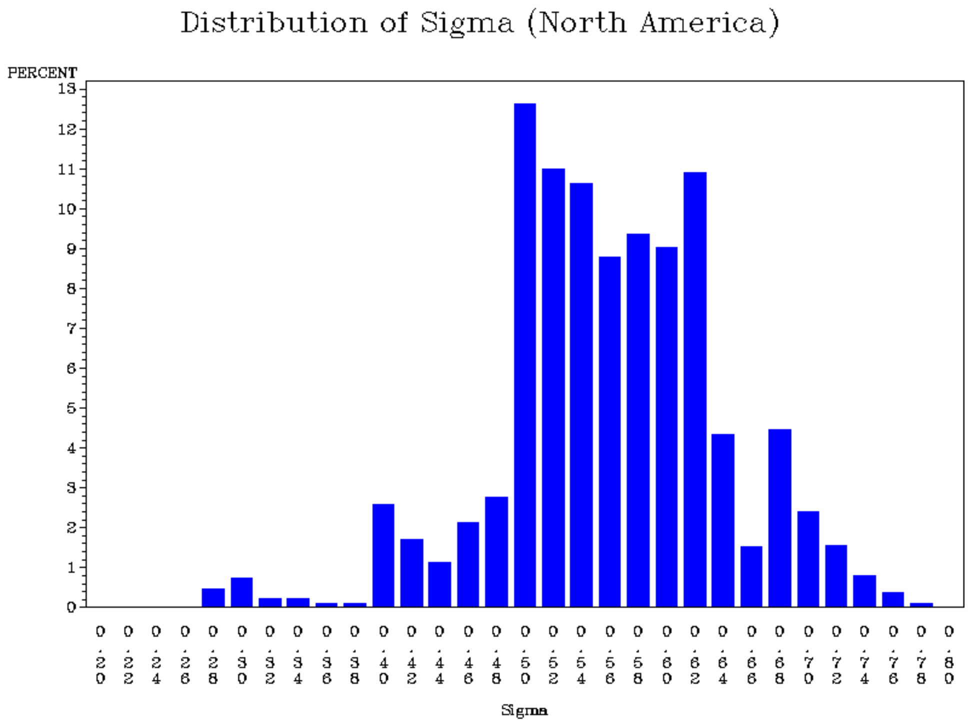

FIGURE 4

Next we illustrate how the model performs in capturing the correlation structures of recovery returns. Specifically the mean level of modeled correlations is compared with realized correlations by sector. We do this by computing the average modeled and realized correlation of each firm with a random sample of 500 firms. Firms are grouped into 13 sectors and average correlations are computed for each sector. As the graph indicates, the model performs reasonably well at distinguishing recovery process when grouped by sector.

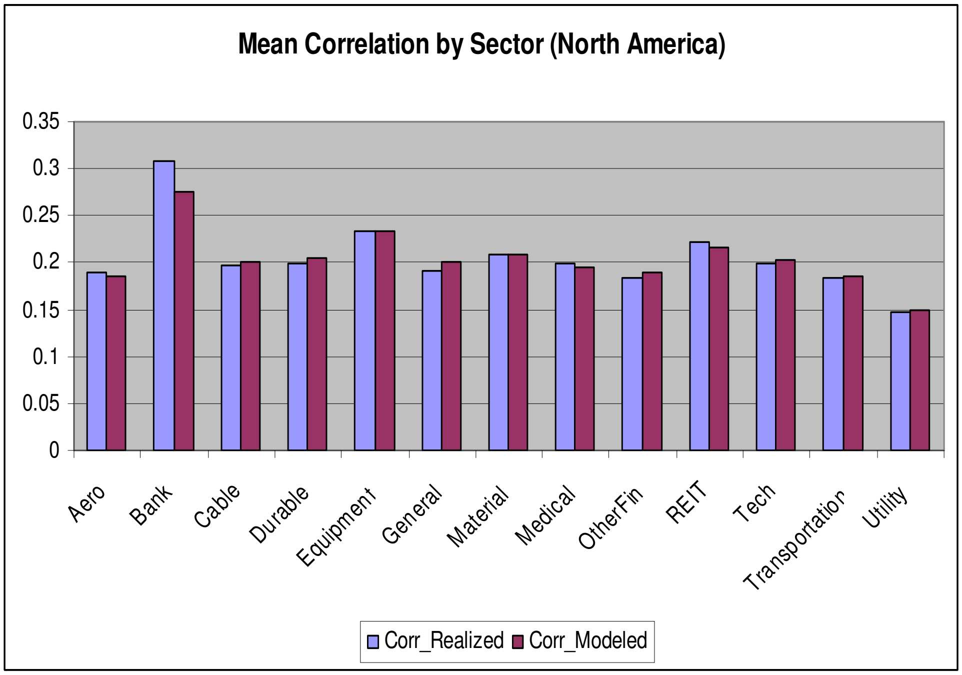

FIGURE 5

5 CALIBRATION AND VALIDATION

This section explores the impact of recovery risk-neutral adjustment on valuation. Two avenues are considered. First, the impact of adjusting recovery for systematic risk on model-implied spreads is analyzed by calibrating the model to parameters observed in the data. Next we consider whether the model allows for a more accurate description of bond spreads that are observed in the data.

Calibration

Our calibration exercise utilizes a simplified zero-coupon bond with typical parameters for λ, R_A, T (Macaulay Duration), *CDP*, *RR,* $\rho_{r_{RR}}$, $\sigma_{r_{RR}}$, and $R_{r_{RR}}$. The model spread (*S*) is computed using the following valuation relationship:

$$V = e^{-r_f T}[(1 - CQDP_T) + CQDP_T \times RR] = e^{-(r_f + S)T}$$
$$\Rightarrow S = \frac{1}{T}\ln(1 - (1 - RR) \times CQDP_T) \tag{15}$$

There are two items worth pointing out in the relationship. First, and potentially obvious, *RR* will be populated with either risk-neural or physical-expected recovery depending on whether the spread includes the risk-neutral recovery adjustment. Equation 13 will be used to map physical-expected recovery to risk neutral to risk neutral-expected recovery.

Second, the relationship is given in terms of risk-neutral default probabilities, whereas our dataset from MKMV provides us with physical default probabilities (EDF values). We follow the methodology employed by MKMV to convert physical default probabilities to risk-neutral default probabilities (for further discussion on the *EDF* model see Bohn and Crosbie (2003), and for a discussion of the MKMV approach of mapping physical to risk-neutral default probabilities see Bohn (1999)):

$$CQDP_T = N[N^{-1}(CDP_T) + \lambda\sqrt{T}R_A] \qquad (16)$$

The following table provides the mean values in our sample for each input:[6]

TABLE 1 Mean values for sample inputs

Input	Mean Value
λ	0.4
R_A	0.4076
T	4.0279
4-year CDP	0.0237 (58bps annually)
RR	0.4915
$\rho_{r_{RR}}$	0.3246
$\sigma_{r_{RR}}$	0.5533
$R_{r_{RR}}$	0.5305

The three graphs below represent the model implied spread when recovery is measured under the risk-neutral measure (the sloped line), and when it is measured under the physical measure (the horizontal line). In each graph $R_{r_{RR}}$, $\sigma_{r_{RR}}$, or $\rho_{r_{RR}}$ are varied while keeping all other parameters at their sample mean. Looking at the first and second graphs, when $R_{r_{RR}}$ or $\sigma_{r_{RR}}$ are at their mean value of 0.53 or 0.55 respectively, accounting for systematic risk in recovery increases the spread from approximately 64 bps to 73 bps (14% higher). As expected from the discussion above, the slope of both lines are positive; if we increase $R_{r_{RR}}$ or $\sigma_{r_{RR}}$ the required return increases to compensate for systematic risk. It is worth pointing out that the magnitude of the difference in spreads is very much affected by the choice of default probability. This is not surprising given that the recovery value plays an increasing role in determining spreads as the default probability increases.

In the third graph, the negative relationship between $\rho_{r_{RR}}$ and spreads is consistent with the modeling discussion of Equation (6) which describes how $\rho_{r_{RR}}$ reduces the risk premium associated with recovery uncertainty.

[6] With the exception of λ, duration, and *CDP,* the dataset is the one described above. λ is set at 0.4 which is the long-run estimate provided by MKMV. The mean duration is obtained from a bond sample in the Reuter's EJV database, described in detail below. The mean CDP was obtained from MKMV's Credit Monitor at the time this study was conducted.

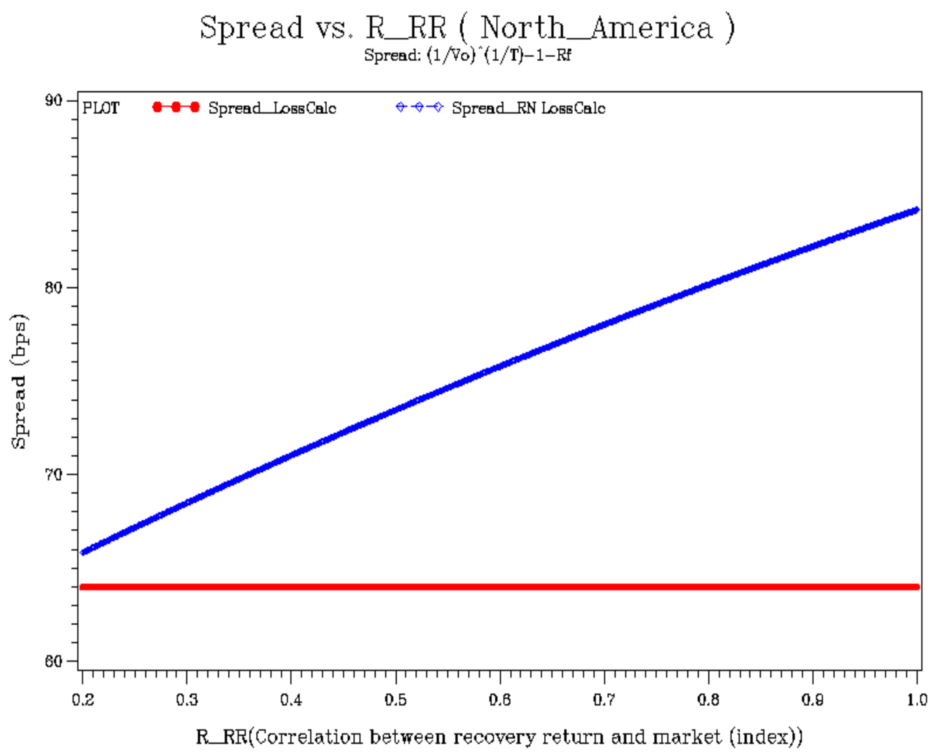

FIGURE 6

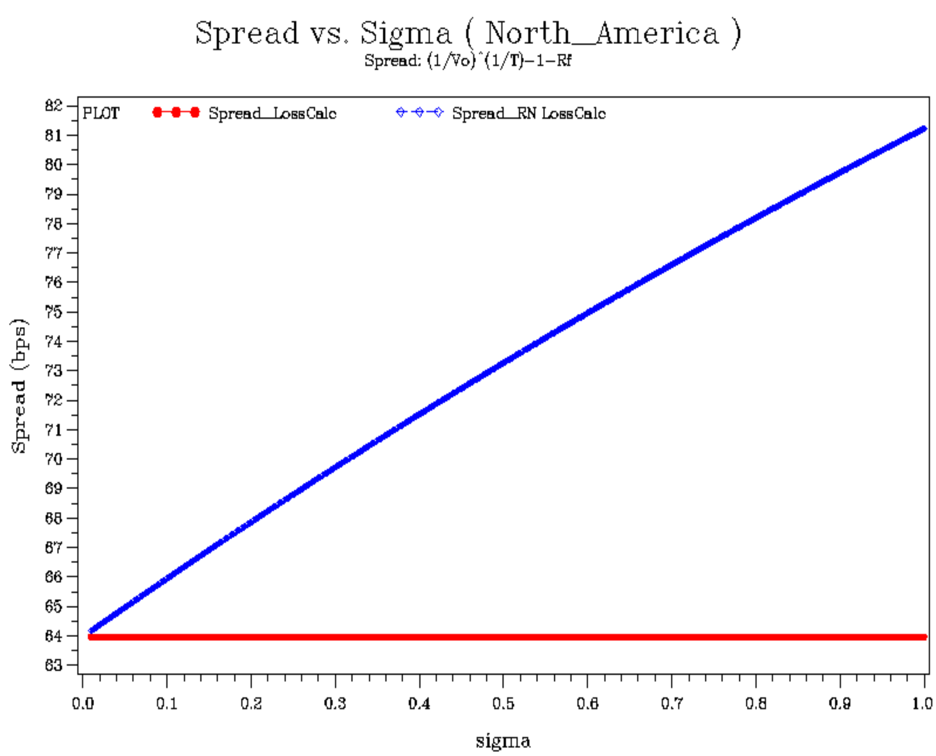

FIGURE 7

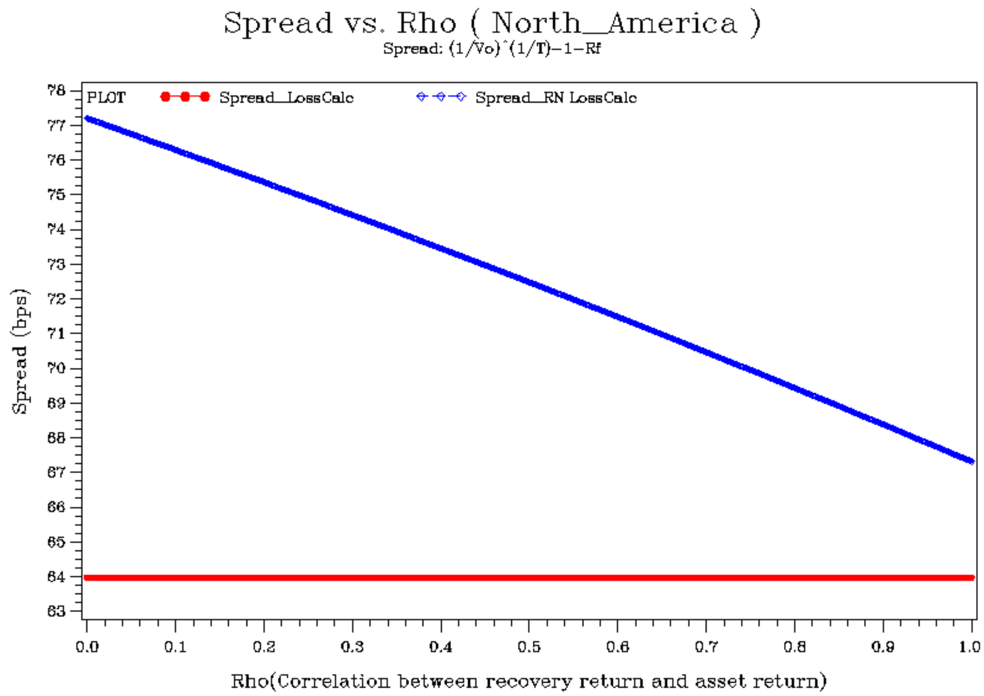

FIGURE 8

Validation

Next, we examine the model's ability to describe corporate bond spreads bonds available through the Reuter's EJV database. Data filters are imposed to remove EJV bonds with less accurate pricing information such as bonds with extreme option-adjusted spread (OAS), less liquidity, small issue size, and odd features. The resulting database consists of 6,134 bonds issued by 1,153 firms from January 1999 to April 2005. We focus on a single observation (specifically, the most recent) per issue to avoid overweighing the observations toward a few particular firms and issues; each bond had between 1 and 1,521 observations. As above, the zero-coupon bond spread equation is applied, Equation (15) where Macaulay duration and OAS (Option Adjusted Spread as quoted by EJV) are used to measure tenor and spread.

We conduct two validation exercises. In the first we compare two measures the *AE* (Absolute Error) of the difference between the OAS and the model implied spread, S_i. The spread is computed with recovery measured under the physical measure, $S_i(E_P[RR_i])$ first, and the recovery measured under the risk-neutral measure, $S_i(E_Q[RR_i])$ second:

$$AE_P = \left|OAS_i - S_i(E_P[RR_i])\right| \quad \text{vs.} \quad AE_Q = \left|OAS_i - S_i(E_Q[RR_i])\right| \qquad (17)$$

Overall, approximately 80% of our sample is estimated with a lower root-square error when using the risk-neutral adjustment to compute model-implied spreads. To get a sense of the magnitude of improvement, the *AE* is under the physical and risk-neutral measures are presented on the x- and y-axes respectively for differences in spreads between 0 and 200 bps. As indicated by the graph, the improvement in fit is evident with most of the observations falling below the 45-degree line.

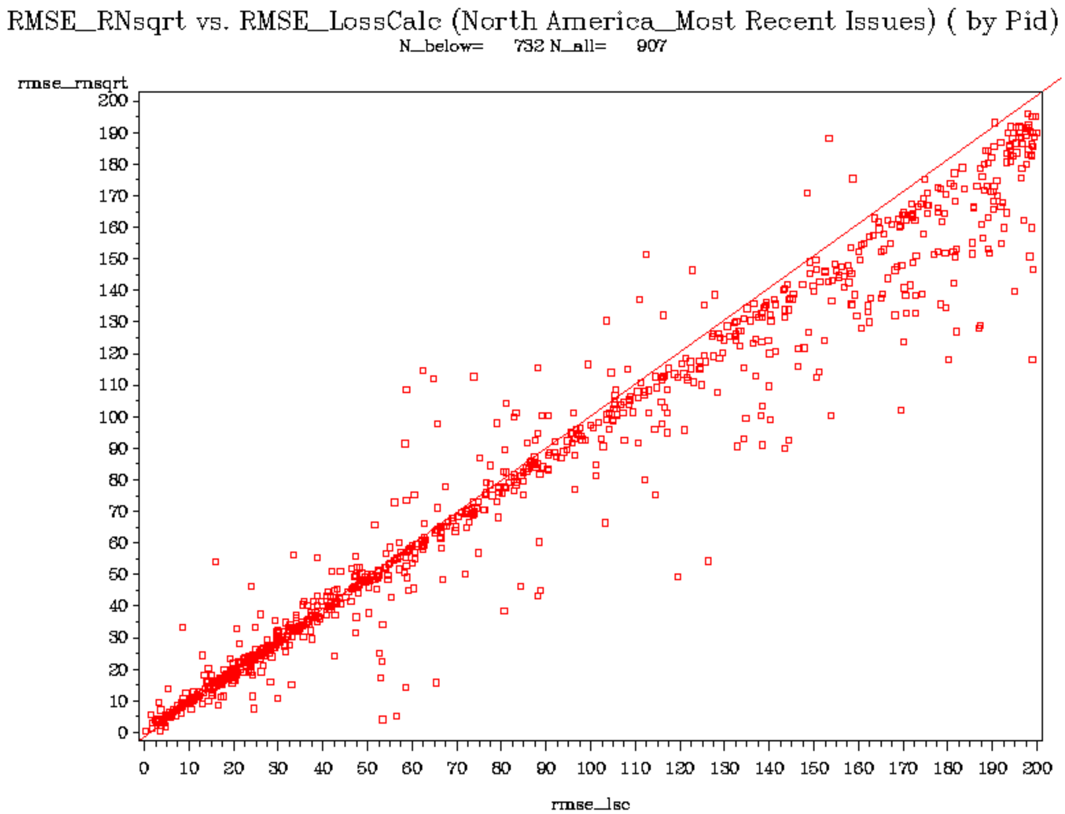

FIGURE 9

The graph above provides evidence that the performance of the model is improved when considering the risk-adjusted recovery. However, we found that $S_i(E_P[RR_i])$ is generally understated, and so it is not surprising that the *AE* typically decreases when accounting for systematic risk in recovery. We deal with this issue in the next validation exercise which focuses on the ability of the recovery adjustment to help describe the cross-sectional variation in the difference between OAS_i and $S_i(E_P[RR_i])$.

In this next validation exercise we analyze the relationship between the risk-neutral recovery adjustment ($Adj_i = e^{(-R_{r_{RR_i}} + \rho_{r_{RR_i}} R_{A_i})\lambda\sigma_{r_{RR_i}}\sqrt{T_i}}$) and the difference between the OAS_i and $S_i(E_P[RR_i])$ to better understand the model's ability to help describe the cross-sectional distribution of spreads. We begin by placing each issue into one of 28 evenly spaced bins ranging from (0.72, 0.73] to (0.99, 1] based on the value of Adj_i. The average OAS_i and $S_i(E_P[RR_i])$ is then computed for each bin. As the graph below indicates, the average $S_i(E_P[RR_i])$ is relatively unchanged (except at the fringes) with changes in Adj_i indicating that the overall sample of instruments is relatively homogeneous for the modeled required spread. Meanwhile, there is a clear negative relationship between the average OAS_i line and Adj_i. This indicates that the market requires a higher spread for instruments associated with low value for Adj_i and is consistent with the model predictions. It is worth pointing out that the number of observations (the humped curve) decreases dramatically at the fringes making the results somewhat difficult to interpret.

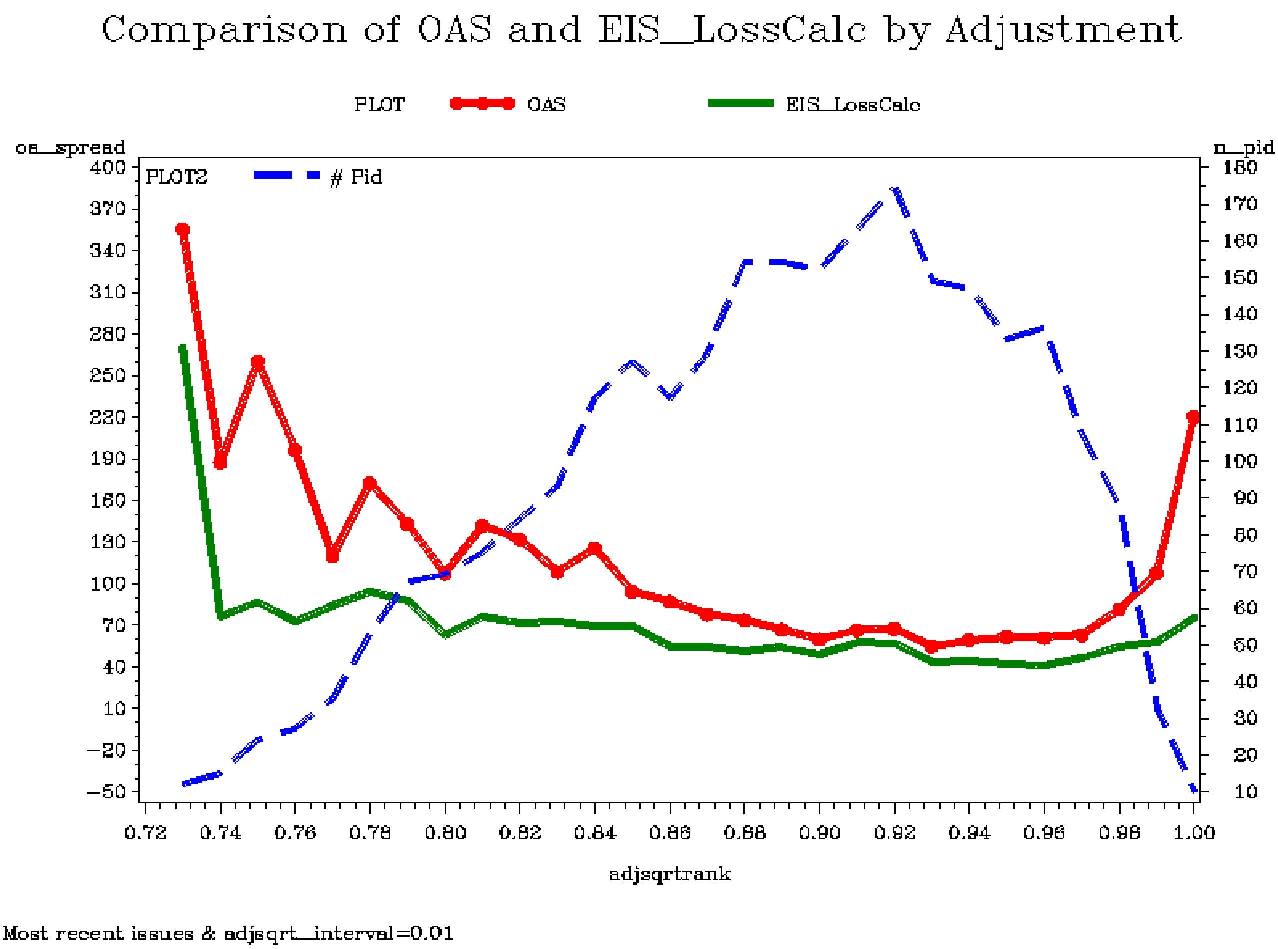

FIGURE 10

6 CONCLUSION

This paper develops a theoretical framework that incorporates systematic risk when valuing recovery. The final relationship between the expected recovery amount and the risk-neutral recovery amount is shown to be linear. This allows for a simple relationship between expected recoveries observed under the physical measure (i.e., those reported in LossCalc), and expected risk-neutral recoveries that can be used in risk-neutral valuation methods. This paper provides detailed information on how one can parameterize the model given available data. Moreover, evidence is presented that spreads can be described more accurately if the model accounts for systematic risk in recovery.

APPENDIX A THEOREM PROOFS

Proof of Theorem 1:

To derive the expected value under the physical measure, note that the two processes have "instantaneous" drifts and diffusions of r_{RR}, r_A, $\sigma_{r_{RR}}$ and σ_{r_A}. Moreover, their natural logs have means and variances of $r_{RR}-\sigma_{RR}^2/2$, $r_A-\sigma_A^2/2$, $\sigma_{r_{RR}}$ and σ_{r_A} respectively. Therefore, the recovery and asset process have the following bivariate probability density function at time *t*:

$$
\begin{aligned}
&pdf_0[RR_T, A_T]=\frac{1}{RR_T}\frac{1}{A_T}\frac{1}{2\pi\sigma_{r_{RR}}\sigma_{r_A}t\sqrt{1\text{-}\rho_{r_{RR}}^2}}e^{-z/2(1-\rho_{r_{RR}}^2)},\\
&z=\frac{(\ln(RR_T/RR_0)-(r_{RR}T-\sigma_{r_{RR}}^2T/2))^2}{\sigma_{r_{RR}}^2T}\\
&-\frac{2\rho_{r_{RR}}(\ln(RR_T/RR_0)-(r_{RR}T-\sigma_{r_{RR}}^2T/2))(\ln(A_T/A_0)-(r_AT-\sigma_{r_A}^2T/2))}{\sigma_{r_{RR}}\sigma_{r_A}T}\\
&+\frac{(\ln(A_T/A_0)-(r_AT-\sigma_A^2T/2))^2}{\sigma_A^2T}.
\end{aligned}
\tag{18}
$$

The probability density function of recovery conditional on the asset process hitting the default point (*DP*) is equal to:

$$
\begin{aligned}
&pdf_0[RR_T|A_T=DP]=\frac{pdf_0[RR_T,A_T=DP]}{pdf_0[A_T=DP]}\\
&=\frac{1}{RR_T}\frac{1}{\sigma_{r_{RR}}\sqrt{t}\sqrt{2\pi}\sqrt{1\text{-}\rho_{r_{RR}}^2}}e^{-w^2/2},\\
&\textit{where}\\
&w=\frac{\ln(RR_T/RR_0)-(r_{RR}T-\sigma_{r_{RR}}^2T/2)-\rho(\ln(DP/A_0)-(r_AT-\sigma_{r_A}^2T/2))\sigma_{r_{RR}}/\sigma_{r_A}}{\sigma_{r_{RR}}\sqrt{T}\sqrt{1\text{-}\rho_{r_{RR}}^2}}
\end{aligned}
\tag{19}
$$

The expected value can now be computed directly:

$$
\begin{aligned}
&E_P[RR_t\mid A_t=DP]=\int_0^\infty\frac{1}{\sigma_{r_{RR}}\sqrt{T}\sqrt{2\pi}\sqrt{1\text{-}\rho_{r_{RR}}^2}}e^{-w^2/2}\partial RR_t,\\
&\textit{where}\\
&w=\frac{\ln(RR_t/RR_0)-(r_{RR}T-\sigma_{r_{RR}}^2T/2)-\rho(\ln(DP/A_0)-(r_AT-\sigma_{r_A}^2T/2))\sigma_{r_{RR}}/\sigma_{r_A}}{\sigma_{r_{RR}}\sqrt{T}\sqrt{1\text{-}\rho_{r_{RR}}^2}}
\end{aligned}
\tag{20}
$$

The integral can be easily computed since it represents the expected value of a lognormal random variable with a natural log that has a variance of $\sigma_{r_{RR}}\sqrt{T}\sqrt{1\text{-}\rho^2_{r_{RR}}}$, and a mean of $\ln(RR_0)+(r_{RR}T-\sigma^2_{r_{RR}}T/2)+\rho_{r_{RR}}(\ln(DP/A_0)-(r_AT-\sigma^2_{r_A}T/2))\sigma_{r_{RR}}/\sigma_{r_A}$. Specifically, the integral is equal to $e^{\ln(RR_0)+(r_{RR}T-\sigma^2_{r_{RR}}T/2)+\rho_{r_{RR}}(\ln(DP/A_0)-(r_AT-\sigma^2_{r_A}T/2))\sigma_{r_{RR}}/\sigma_{r_A}+\sigma_{r_{RR}}T(1\text{-}\rho^2_{r_{RR}})/2}$. Replacing the integral and rewriting the equation yields the following result:

$$E_P[RR_T \mid A_T = DP] = RR_0 \times \left(\frac{DP}{A_0}\right)^{\rho_{r_{RR}}\sigma_{r_{RR}}/\sigma_{r_A}} \times e^{(r_{RR}T-\sigma^2_{r_{RR}}t/2)-\rho_{r_{RR}}(r_At-\sigma^2_{r_A}T/2)\sigma_{r_{RR}}/\sigma_{r_A}+\sigma^2_{r_{RR}}T(1\text{-}\rho^2_{r_{RR}})/2} \qquad (21)$$

Proof of Theorem 4:

We prove this by first changing the measure to a normal distribution. Specifically, we define $\begin{pmatrix} A_t^N \\ RR_t^N \\ A_T^N \end{pmatrix}$ to be:

$$\begin{pmatrix} A_t^N \\ RR_t^N \\ A_T^N \end{pmatrix} = \begin{pmatrix} \log(A_t) \\ \log(RR_t) \\ \log(A_T) \end{pmatrix} \sim N\left(\begin{pmatrix} (r_A-\sigma^2_{r_A}/2)t+\log(A_0) \\ (r_{RR}-\sigma^2_{r_{RR}}/2)t+\log(RR_0) \\ (r_{r_A}-\sigma^2_{r_A}/2)T+\log(A_0) \end{pmatrix}, \begin{pmatrix} \sigma^2_{r_A}t & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t & \sigma^2_{r_A}t \\ \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t & \sigma^2_{r_{RR}}t & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t \\ \sigma^2_{r_A}t & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t & \sigma^2_{r_A}T \end{pmatrix}\right)$$

Using the fact that $Cov(RR_t^N \mid A_T^N = DP, A_t^N) = Cov(RR_t^N \mid A_T^N = DP, A_t^N \mid A_T^N = DP)$, we can use the property that for two sets of multivariate normal distributions:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right].$$

which implies that:

$$X^* = X_1 \mid X_2 \sim N(\mu^*, \Sigma^*),$$

where

$\mu^* = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2-\mu_2)$ and $\Sigma^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

Applying this to result to our multivariate normal, we get:

$$\begin{pmatrix} A_t^N \mid A_T^N \\ RR_t^N \mid A_T^N \end{pmatrix} \sim N\left(r^N, \Sigma^N\right),$$

where

$$r^N = \begin{pmatrix} (r_A - \sigma_A^2/2)t + \dfrac{(DP - (r_A - \sigma_{r_A}^2/2)T)t}{T} + \log(A_0) \\ (r_{RR} - \sigma_{r_{RR}}^2/2)t + \dfrac{(DP - (r_A - \sigma_{r_A}^2/2)T)\rho_{r_{RR}}\sigma_{r_{RR}}t}{\sigma_{r_A}^2 T} + \log(RR_0) \end{pmatrix}$$

$$\Sigma^N = \begin{pmatrix} \sigma_{r_A}^2\left(t - \dfrac{t^2}{T}\right) & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}\left(t - \dfrac{t^2}{T}\right) \\ \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}\left(t - \dfrac{t^2}{T}\right) & \sigma_{r_{RR}}^2\left(t - \dfrac{\rho_{r_{RR}}^2 t^2}{T}\right) \end{pmatrix}$$

This implies that $\sigma_{r_{RR}}^2$ and $\rho_{r_{RR}}$ adhere to the following equalities:

$\sigma_{r_{RR|A=DP}}^2 = \sigma_{r_{RR}}^2\left(1 - \dfrac{\rho_{r_{RR}}^2 t}{T}\right)$ and $\rho_{r_{RR|A=DP}} = \dfrac{\rho_{r_{RR}}\sigma_{r_A}\sigma_{RR}\left(t - \dfrac{t^2}{T}\right)}{\sigma_{r_{RR|A=DP}}\sigma_{r_A}\left(t - \dfrac{t^2}{T}\right)^{1/2}}$. Solving we get $\sigma_{r_{RR}}^2 = \sigma_{r_{RR|A=DP}}^2\left(1 - \dfrac{\rho_{r_{RR}}^2 t}{T}\right)^{-1}$ and $\rho_{r_{RR}} = (\rho_{r_{RR|A=DP}} \cdot T)/(T^2 - tT + \rho^2_{r_{RR|A=DP}} \cdot tT)^{1/2}$.

Similarly, one can construct the mapping for $R_{r_{RR}}$. First, define $\begin{pmatrix} \phi_t^N \\ RR_t^N \\ A_T^N \end{pmatrix}$ to be:

$$\begin{pmatrix} \phi_t^N \\ RR_t^N \\ A_T^N \end{pmatrix} = \begin{pmatrix} \log(\phi_t) \\ \log(RR_t) \\ \log(A_T) \end{pmatrix} \sim N\left(\begin{pmatrix} (r_\phi - \sigma_\phi^2/2)t \\ (r_{RR} - \sigma_{RR}^2/2)t + \log(RR_0) \\ (r_A - \sigma_A^2/2)T + \log(A_0) \end{pmatrix}, \begin{pmatrix} \sigma_\phi^2 t & R_{r_{RR}}\sigma_\phi\sigma_{r_{RR}}t & R_{r_A}\sigma_\phi\sigma_{r_A}t \\ R_{r_{RR}}\sigma_\phi\sigma_{r_{RR}}t & \sigma_{r_{RR}}^2 t & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t \\ R_{r_A}\sigma_\phi\sigma_{r_A}t & \rho_{r_{RR}}\sigma_{r_A}\sigma_{r_{RR}}t & \sigma_{r_A}^2 T \end{pmatrix}\right)$$

As above, we recognize $Cov(RR_t^N \mid A_T^N = DP, \phi_t^N) = Cov(RR_t^N \mid A_T^N = DP, \phi_t^N \mid A_T^N = DP)$, and solve for the conditional distributions:

$$\begin{pmatrix} \phi_t^N \mid A_T^N \\ RR_t^N \mid A_T^N \end{pmatrix} \sim N\left(r_\phi^N, \Sigma_\phi^N\right),$$

where

$$r_\phi^N = \begin{pmatrix} (r_\phi - \sigma_\phi^2/2)t + \dfrac{(DP - (r_A - \sigma_A^2/2)T)t}{T} \\ (r_{RR} - \sigma_{r_{RR}}^2/2)t + \dfrac{(DP - (r_A - \sigma_{r_A}^2/2)T)\rho_{r_{RR}}\sigma_{r_{RR}}t}{\sigma_{r_A}^2 T} \end{pmatrix}$$

$$\Sigma_\phi^N = \begin{pmatrix} \sigma_\phi^2 t(1 - R_{r_A}{}^2 t/T) & \sigma_\phi \sigma_{r_{RR}} t(R_{r_{RR}} - R_{r_A}\rho_{r_{RR}} t/T) \\ \sigma_\phi \sigma_{r_{RR}} t(R_{r_{RR}} - R_{r_A}\rho_{r_{RR}} t/T) & \sigma_{r_{RR}}^2 t(1 - \rho_{r_{RR}}^2 t/T) \end{pmatrix}$$

Finally, solving for $R_{r_{RR}}$ we get:

$$R_{r_{RR}} = R_{r_{RR|A=DP}} \sqrt{(1 - \rho^2{}_{r_{RR}} t/T)} \sqrt{1 - R^2{}_{r_A} t/T} + R_{r_A}\rho_{r_{RR}} t/T \ .$$

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