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# ASSET CORRELATION, REALIZED DEFAULT CORRELATION, AND PORTFOLIO CREDIT RISK

MODELINGMETHODOLOGY

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## ABSTRACT

Asset correlation is a critical driver in modeling portfolio credit risk. Despite its importance, there have been few studies on the empirical relationship between asset correlation and subsequently realized default correlation, and portfolio credit risk. This three three-way relationship is the focus of our study using U.S. public firm default data from 1981 to 2006. We find the magnitude of default-implied asset correlations is significantly higher than has been reported by other studies. There is a reasonably good agreement between our default-implied asset correlations and the asset correlation parameters in the Basel II Accord for large corporate borrowers.

However, the recommended small size adjustment in the Basel II Accord still produces asset correlation higher than what we observe in our data. More importantly, we find that measuring asset correlation ex ante accurately can improve the measurement of subsequently realized default correlation and portfolio credit risk, in both statistical and economic terms. These results have several important practical implications for the calculation of economic and regulatory capital, and for pricing portfolio credit risk. Furthermore, the empirical framework that we developed in this paper can serve as a model validation framework for asset correlation models in measuring portfolio credit risk.

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# 1 INTRODUCTION

The three most important drivers in determining portfolio credit risk are probability of default (PD), loss given default (LGD), and default correlation. The last one, despite its critical importance, has not received as much attention as the first two. This is mainly owing to the lack of data and a regulatory focus on PD and LGD. The increased acceptance of credit portfolio management practice and of economic capital framework, together with the rapid development of CDS and CDO markets in recent years, has sparked heightened interest in the study of default correlation.

The most common approach to modeling default correlation is to combine default probabilities with asset correlations. The basic idea is that two borrowers will default in the same period if both of their asset values are insufficient to pay their obligations. Asset correlation helps define the joint behavior of the asset values of the two borrowers. This idea, initially conceived by Oldrich Vasicek in the mid-1980s, has become very important because the use of asset values can be supported by a continuous stream of market data. This data advantage overcomes the problem of using default data, for which the amount of historical information is typically limited. The idea has become the basis for many portfolio credit risk models, such as Moody's KMV (MKMV) Portfolio Manager™ and RiskFrontier™, and also the so-called Asymptotic Single-Risk Factor (ASRF) model behind the Basel II IRB credit risk capital charge. It is also the genesis of many of the portfolio models used to price portfolio credit risk in structured products, such as CDS indices and CDOs.

Given the critical role of asset correlation in modeling portfolio credit risk, it is important to assess the relationship between asset correlation and subsequently realized default correlation, and portfolio risk. There are several questions that need be answered with empirical facts. For example, do portfolios with higher asset correlations tend to have subsequently higher realized portfolio risk? Can we really measure asset correlation ex ante? And, are these asset correlations informative in measuring portfolio credit risk? Given the importance of such questions, it is perhaps surprising that there have been few empirical studies that directly address them. The existing literature can be put into two broad categories.[1] The studies in the first category examine default-implied asset correlations. They use observed default data to calculate single and pair-wise default probabilities, and then deduce asset correlations from them. These default-implied asset correlations are further summarized by variables, such as rating, industry, and firm size. The studies in the second category examine the asset correlations calculated from asset return data (or approximated by equity return data). These asset correlations are also grouped according to variables, such as ratings, industry, and firm size, and are usually compared to the default-implied asset correlations from the first group of studies. To our knowledge, there has not been any study that uses both asset correlation data and default data at the same time.

In this paper, we utilize both asset correlation and default data. Our default data consists of U.S. public firm defaults from 1981 through 2006. For asset return correlations, we use output from the Moody's KMV Global Correlation Factor Model™ (GCorr).[2] We first examine the default-implied asset correlations and compare them to the numbers reported in the previously published literature and in the Basel II Accord. We then compare the forecasted default correlations from GCorr with the subsequently realized default correlations. Furthermore, we test whether adding asset correlation information can help differentiate realized portfolio risk. To ensure the robustness of our results, we deploy a number of statistical techniques in our study.

Our analysis sheds interesting light on the magnitude of asset correlations, and the relationship between ex ante asset correlation and subsequently realized portfolio risk. There are six major findings:

- Default-implied asset correlations range from 5% to around 30% in our data, depending on the grouping of the underlying borrowers. The top end of our analysis is much higher than has been previously reported.
- Borrowers with higher ratings, or lower EDF values, tend to have higher asset correlations. This data supports the intuition that larger firms tend to have larger systematic risk, and tend to be more closely correlated with the performance of the economy than do smaller firms.
- Asset correlations manifest themselves more in default clustering during periods of deteriorating credit quality. This reflects the cyclical nature of defaults in an economy.

[1] See Chernish et al (2006).

[2] See Zeng and Zhang (2001) for more information about GCorr.

- If we assess the asset correlation parameters from the Basel II Accord in the context of our results, they are in of a similar magnitude for large corporate borrowers; however, for small firms, we find that the small firm size adjustment in the Basel II Accord yields asset correlations that are higher than we observe in our data.
- When we compare the forecasted default correlations from GCorr with the subsequently realized default correlations, we find a generally close agreement between them, with the realized values inside the 95% confidence bounds of the forecasted values. Furthermore, we find that these forecasted asset correlations yield reasonable estimates for realized default clustering.
- Perhaps most importantly, we find that asset correlation information adds significant economic value to forecasting realized portfolio risk, especially during periods of deteriorating credit quality. Ignoring asset correlations will lead to an underestimation of subsequently realized default correlation and of realized portfolio volatility.

Our results have a number of important implications for credit risk management. Despite the prevalence of asset correlation in modeling portfolio credit risk and its role in the regulatory capital framework, many practitioners continue to doubt its empirical validity and its effect on realized portfolio credit risk. Our results show that it is important to incorporate asset correlation in measuring portfolio credit risk. Our analysis of default-implied asset correlations suggests that asset correlation is manifested in subsequently realized default clustering. The magnitude of the asset correlation can be large, especially during periods of default clustering. It is therefore important for financial institutions to pay attention to asset correlation, in addition to the more traditional areas of focus for credit risk management (i.e., default risk and recovery risk). Furthermore, it is feasible to measure asset correlations ex ante, and to use them in deriving default correlations and for managing portfolio credit risk.

Our study also highlights both the importance and the challenges of validating correlation models used in measuring portfolio credit risk. Asset correlation plays a critical role in measuring portfolio credit risk and determining both economic and regulatory capital. Therefore finical institutions need to ensure the asset correlation assumptions in their internal portfolio models have support in data. The empirical challenges of validating correlation models can be overwhelming. Measuring default correlation is often an exercise of jointly assessing default probability and asset correlation. Extra care needs to be taken to isolate the effect of asset correlation from that of default probability.[3] Given the infrequent and opaque nature of credit event data, sampling variability of any empirical estimate for default correlation can be large and hence need to be accounted for. The empirical framework and statistical techniques that we developed in this paper can lend ideas to those interested in independently validating asset correlation models in the context of modeling portfolio credit risk. The rest of this paper proceeds as follows. Section 2 describes the relationship between asset correlation, default correlation, and portfolio credit risk, in the context of measuring portfolio credit risk. Section 3 describes our dataset and empirical framework. Section 4 presents the main empirical results. Section 5 examines the economic significance of measuring asset correlation for modeling portfolio credit risk. Section 6 provides concluding remarks. For the ease of exposition, all technical details are left in the appendices.

# 2 ASSET CORRELATION IN PORTFOLIO CREDIT RISK

Credit correlations include default correlations and credit migration correlations. Default correlation measures the extent to which the default of one borrower is related to another borrower, while credit migration correlation measures the joint credit quality change short of default for the two borrowers. In practice, these correlations are rather difficult, if not impossible, to measure directly. For example, even though Ford and General Motors have never defaulted, this does not necessarily imply their default correlation zero.

We can, however, infer the default correlation of two borrowers by measuring their individual default probabilities and their asset correlation. The basic idea is very intuitive: a borrower will likely default when its asset value falls below the value of its obligations (i.e., its default point); the joint probability of two borrowers defaulting during the same time period is simply the likelihood of both borrowers' asset values falling below their respective default points during that period. This probability can be determined from knowing the correlation between the two firms' asset values and the individual likelihood of each firm defaulting, as depicted in Figure 1.

[3] Please see Dwyer (2007) for related work on validating PD models.

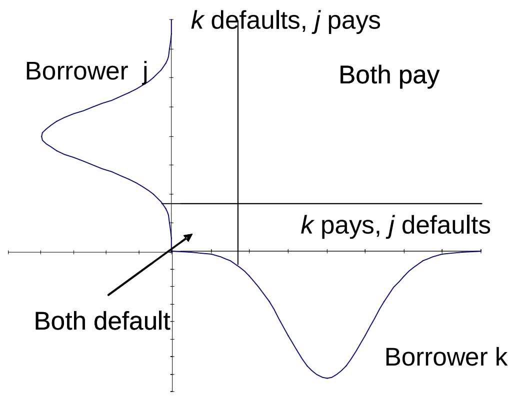


FIGURE 1 Joint Default

With this idea, we can calculate the joint default probability of borrower *j* with borrower *k*, denoted by $JDF_{jk}$, as:[4]

$$\begin{aligned} JDF_{jk} &= \text{Prob}(\text{asset value } j < \text{default point } j \text{ and asset value } k < \text{default point } k) \\ &= N_2\left(N^{-1}(CEDF_j), N^{-1}(CEDF_k), \rho_{jk}\right) \end{aligned} \quad (1)$$

where $N_2$ is the bivariate normal distribution, $N^1$ is the inverse of normal distribution, *CEDF* is the cumulative default probability, and $\rho_{jk}$ is the asset correlation between borrower *j* and borrower *k*.

After we have the joint default probability, the default correlation between borrower *j* and borrower *k* can be derived as:

$$\rho_{jk}^{D} = \frac{JDF_{jk} - CEDF_j \cdot CEDF_k}{\sqrt{\left[CEDF_j \cdot (1 - CEDF_j)\right] \cdot \left[CEDF_k \cdot (1 - CEDF_k)\right]}} \quad (2)$$

It is worth noting that one does not need to use Equations (1) and (2) to explicitly measure all pair-wise joint default probabilities and joint default correlations for portfolio risk calculation. For example, with the idea illustrated in Figure 1 as the foundation, the credit risk capital charge for an exposure under Basel II IRB framework, is given by:[5]

$$capital = LGD \cdot N\left(N^{-1}(PD) \cdot \sqrt{\frac{1}{1-\rho}} + N^{-1}(0.999) \cdot \sqrt{\frac{\rho}{1-\rho}}\right) - PD \cdot LGD \qquad (3) \quad (3)$$

where PD is the default probability, LGD is the loss given default, and $\rho$ can be considered as the average asset correlation of all pair-wise asset correlations in the portfolio.

When there is no closed-form formula like Equation (3), one can use Monte Carlo simulation to construct a portfolio economic capital distribution and calculate an economic capital charge for each facility. Although these calculations can be complicated and computationally intensive, the fundamental idea behind them is the same as illustrated in Figure 1. In fact, this approach is arguably the most common one to modeling portfolio credit risk in practice. The same idea is also behind the industry standard Gaussian Copula model for pricing portfolio credit risk for structured credit products such as CDOs. These models postulate that a borrower will default when the value of its assets falls below a certain threshold. Furthermore, the joint distribution of the asset values of firms can be specified by the marginal distributions and a copula; or alternatively, the asset values of borrowers are driven by a factor model, for example:

[4] More detailed derivations of this section are in Appendix A.

[5] International Convergence on Capital Measurement and Capital Standards, Basel Committee on Banking Supervision, June 2006.

$$r_{it} = \sqrt{\rho}\phi_t + \sqrt{1-\rho}\varepsilon_{it} \quad (4)$$

where $r_{it}$ is the asset return of borrower $i$ at time $t$, $\phi_t$ is the systematic factor, $\rho$ is the correlation between firm $i$ and the systematic factor, and $\varepsilon_{it}$ is the idiosyncratic factor of firm $i$.

Equation (4) can serve as the basis for the Monte Carlo simulation for portfolio credit risk calculation. One can have more a complicated factor model than a single factor model by enlarging the set of systematic factors and assuming various distributions for systematic and idiosyncratic factors. For example, the GCorr model is a multi-factor correlation model with more than one hundred factors.

From the above discussion, we can see that asset correlation plays a pivotal role in portfolio credit risk modeling. Portfolio credit risk calculation is very much hinged on the asset correlations within the portfolio. Thus it is paramount to understand empirically how asset correlations are related to subsequently realized default correlations and portfolio risk. The rest of the paper focuses on this relationship.

# 3 DATA AND EMPIRICAL FRAMEWORK

This section covers information about our data and estimating realized default correlation.

## 3.1 Data

Our dataset consists of 16,268 publicly traded U.S. non-financial firms from 1981 to 2006. The total number of observations is 1,718,333 firm-months. We choose this dataset because we have more comprehensive information about public firm defaults in the U.S. than in other countries. This period has several economic cycles and high default episodes. The default data comes from the Moody's KMV historical default database. The probabilities of default come from the Moody's KMV EDF™ (Expected Default Frequency) model, while the pair-wise asset correlations come from the GCorr model. Because it is more difficult to track default history for small firms and missing defaults can lead to significant downward bias in realized default correlation, unless noted otherwise, small firms are excluded in our study.[6] This reduces the sample size to 5,040 firms with 524,891 firm-month observations. Figure 2 shows the number of firms over time. Table 1 shows the statistics of EDF values, R-squared values, and pair-wise correlations.[7]

[6] We apply a size cutoff of $300 million in sales for 2006. For years before 2006, we use scaling factors to adjust the size threshold downward.

[7] R-squared is a measure of systematic risk of a borrower, as calculated from the GCorr model.

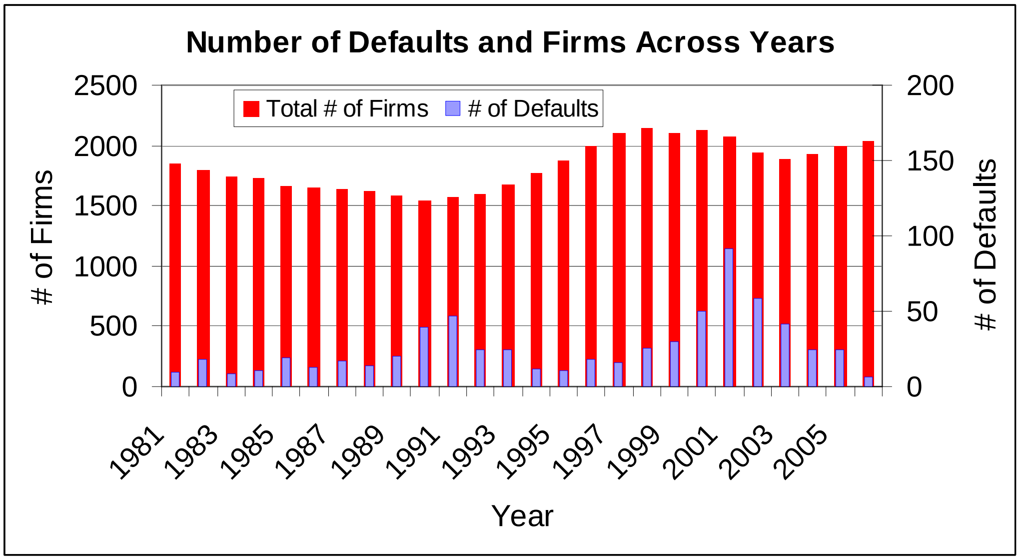


FIGURE 2 The Number of Firms and Defaults

TABLE 1 Sample Statistics

| | Mean | 10%-th | 25%-th | Median | 75%-th | 90%-th | Standard Deviation |
|---|---|---|---|---|---|---|---|
| EDF | 2.30% | 0.05% | 0.11% | 0.28% | 1.03% | 4.59% | 6.41% |
| R-squared | 21.15% | 13.03% | 16.34% | 19.97% | 25.14% | 29.18% | 7.51% |
| Pair-wise Asset Correlation | 17.12% | 12.09% | 14.22% | 16.87% | 19.63% | 22.27% | 4.10% |
| Pair-wise Default Correlation | 1.41% | 0.37% | 0.60% | 1.03% | 1.75% | 2.82% | 1.28% |

## 3.2 Estimating Realized Default Correlation

Given the rare occurrence of joint default for a typical pair of borrowers, it is practically infeasible to estimate its pair-wise default correlation. To overcome the challenge, we can divide borrowers into homogenous groups of similar characteristics. The idea is that borrowers with similar default probabilities and pair-wise correlations would exhibit similar realized default correlations.[8] For a pair of borrowers in two different groups consisting of homogenous borrowers within the same group, we can use Equation (5) to estimate the realized default correlation for borrowers belonging to these two groups:

$$\hat{\rho}^{cd} = \frac{\hat{P}^{cd} - \hat{P}^{c}\hat{P}^{d}}{\sqrt{\hat{P}^{c}(1-\hat{P}^{c})\hat{P}^{d}(1-\hat{P}^{d})}} \qquad (5)$$

where $\hat{P}^{c}$ and $\hat{P}^{d}$ are the estimated default probabilities for borrowers in group *c* and *d*, respectively, and $\hat{P}^{cd}$ is the estimated joint default frequency between borrowers in group *c* and borrowers in group *d*. Here we have the default probability defined for each group rather than for each borrower; or alternatively, we assume all borrowers in the same

[8] Realized default correlation is also referred as historical default correlation or sample default correlation in the literature.

group have the same default probability. Similarly, the joint default probability is defined between two groups rather than between two borrowers. For each time period, we can observe a realization of $P^C$, $P^D$, and $P^{CD}$. If we assume each realization across time is from the same distribution, we can then use the time series of realizations to estimate the expected value of $\hat{P}^c$, $\hat{P}^d$, and $\hat{P}^{cd}$.

The estimation is straightforward for the individual default probability $\hat{P}^c$ or $\hat{P}^d$. For the joint default probability $\hat{P}^{cd}$, we need to consider the ratio of the number of defaulted pairs to the total number of possible pairs between the two groups for each year. Then the joint default probability $\hat{P}^{cd}$ can be calculated as a weighted average of these ratios:

$$\hat{P}^{cd} = \sum_t w_t^{cd} \frac{D_t^c D_t^d}{N_t^c N_t^d} \tag{6}$$

where $w_t^{cd}$ is the weight representing the relative importance of the sample in a given year *t*. (i.e., $w_t^{cd} = N_t^c N_t^d / \sum_i N_i^c N_i^d$, $D_t^c$, $D_t^d$ are the number of defaults in a given year t for group *c* and *d*, respectively, and $N_t^c$ and $N_t^d$ are the number of borrowers at the beginning of year *t* for group *c* and *d*, respectively.)

After we have the estimate of joint default probability from Equation (6), we can invert Equation (1) to derive asset correlation. We call this the default-implied asset correlation.

To help us understand the sampling properties of $\hat{P}^{cd}$, we can rewrite Equation (5) as:

$$\hat{\rho}^{cd} = \frac{Cov(\hat{P}^c, \hat{P}^d)}{\sqrt{Var(\hat{P}^c) Var(\hat{P}^d)}} \tag{7}$$

Under a certain set of assumptions, we can link the true default correlation of the underlying population with the realized default correlation through Equation (8) below for the intra-group (i.e., *c*=*d* in Equation (7)) default correlation:[9]

$$\lim_{t \to +\infty} \rho_{realized} = \frac{Var(P_t)}{E(P_t)(1 - E(P_t))} = \rho + \frac{1-\rho}{N} \tag{8}$$

where $\rho_{realized}$ and $\rho$ are the realized default correlation and the true default correlation, respectively, *t* denotes the time period, and *N* is the total number of borrowers in the group. Thus, if we have a very large number of borrowers in the group, the realized default correlation defined in Equation (8) converges to the true default correlation. The assumptions that underlie the second part of Equation (8) are:

1. All borrowers in the group have comparable default probabilities.
2. All pair-wise default correlations within the group are equal.
3. Defaults are independent over time.
4. The number of borrowers at the beginning of each year is constant.
5. The length of time series is infinite.

The majority of the above assumptions are realistic if we group the borrowers carefully. The last one is naturally difficult to achieve given the finite nature of any dataset. To understand the sampling variability of the estimator, we need to conduct Monte Carlo simulations. Equation (4) serves as the basis of our simulations. In our simulation, we assume both $\phi_t$ and $\varepsilon_{it}$ follow standard normal distribution. For each year, we draw one systematic return and *N* idiosyncratic returns.

[9] See Appendix B for the derivation.

The systematic return and the idiosyncratic return are combined according to Equation (4) to generate the total returns for the $N$ borrowers. These returns are then compared to $N^{1}(EDF)$. If a borrower's return is less than $N^{1}(EDF)$, it will default. We then calculate the default rate for each year and finally calculate the realized default correlation from the time series of realized default rates.

We use the simulation exercise to study the effects of various inputs on the sampling variability of realized default correlations. These inputs include the total number of borrowers in the group, the total number of years, the default probabilities, and the asset correlations. We summarize the results of our simulations as follows:[10]

- There is a considerable amount of sampling variability in the estimator even with datasets of thousands of borrowers and decades of years. Care needs to be taken in interpreting the realized default correlations.
- More often than not, the realized default correlations estimated using Equation (5) tend to be larger than the true default correlations.
- As expected, a larger number of borrowers and longer time series lead to more accurate estimates. However, holding everything equal, increasing sample size in the number of years leads to a larger gain in accuracy more than does an increase in the number of borrowers.
- As expected, the realized default correlation increases with default probability and asset correlation.
- If the underlying population has low default probability, a short time window and a small number of borrowers, Equation (5) tends to over-estimate default correlation. However, this overestimation decreases with the increase in default probability.

# 4 RESULTS

This section covers information about our results.

## 4.1 Default-implied Asset Correlation

There have been a number of studies on default-implied asset correlations in the literature. In this section, we present the default-implied asset correlations from our data and compare them to those in the previous studies. We also compare our results to the asset correlation parameters in the Basel II IRB framework.

Table 2 shows the results by industry sectors. Our industry sector classification for non-financial firms includes Utilities, Consumer Goods & Durables, Materials/Extraction, Transportation, Equipment, Cable TV/Telecom, General, Aerospace/Measure, High Tech, and Medical. For these sectors, the default-implied asset correlations range form 9.22% to 29.98%. The Cable TV/Telecom sector has the largest default-implied correlation because of the unprecedented clustering of telecom defaults in 2001 and 2002.

[10] See Appendix C for more details.

TABLE 2 Default-implied Asset Correlations by Industries

| Industry | Average Num. of Firms | Average Default Rate | Realized Default Correlation | Default-implied Asset Correlation |
|---|---|---|---|---|
| Utilities | 102 | 0.56% | 1.88% | 21.17% |
| Consumer Goods & Durables | 318 | 1.27% | 1.02% | 9.43% |
| Materials/Extraction | 223 | 1.19% | 0.95% | 9.22% |
| Transportation | 48 | 2.05% | 2.54% | 15.34% |
| Equipment | 93 | 0.35% | 1.20% | 19.16% |
| Cable TV/Printing/Telecom | 76 | 1.56% | 5.72% | 29.98% |
| General | 251 | 1.55% | 1.63% | 12.39% |
| Aerospace/Measurement | 15 | 0.00% | N/A | N/A |
| High Tech | 96 | 0.91% | 1.59% | 12.15% |
| Medical | 62 | 0.65% | 2.05% | 21.13% |

Next, we present the results by ratings (Table 3) and by EDF values (Table 4). Firms rated by Moody's Investor Service are divided into Aaa– Aa, A1–A3, Baa1–Baa3, Ba1–Ba3, and B1–B3 & Below buckets. For the Aaa–Aa bucket, there is no default in the sample. For the A1–A3 rating bucket, there is only one default in year 1987. This single default leads to a very high default-implied asset correlation. For the rest of the ratings, we observe that default-implied correlation generally increases with better credit quality. We observe similar pattern for the default-implied asset correlation with EDF (Table 4).

TABLE 3 Default-implied Asset Correlation by Ratings

| Rating | Average Num. of Firms | Average Default Rate | Realized Default Correlation | Default-implied Asset Correlation |
|---|---|---|---|---|
| A1–A3 | 144 | 0.03% | 0.65% | 28.74% |
| Baa1–Baa3 | 165 | 0.29% | 0.59% | 13.21% |
| Ba1–Ba3 | 156 | 1.21% | 1.68% | 14.28% |
| B1–B3 & Below | 138 | 6.91% | 2.36% | 7.87% |

TABLE 4 Default-implied Asset Correlation By EDF

| EDF Rank | Average Num of Firms | Average Default Rate | Realized Default Correlation | Default-implied Asset Correlation |
|---|---|---|---|---|
| 80th~100th | 366 | 6.89% | 3.55% | 11.46% |
| 60th~80th | 366 | 0.16% | 0.46% | 14.68% |
| 40th~60th | 366 | 0.06% | 0.30% | 16.74% |
| 20th~40th | 366 | 0.02% | 0.28% | 22.69% |

For comparison, we put our results together with those from other studies in Table 5. Leaving the differences in these results aside, we can see that there is overwhelming evidence for asset correlation, as manifested in realized default data from various sources and different time periods. This highlights the importance and prevalence of asset correlation for credit risk. Examining the differences, we can see our default-implied asset correlations tend to be higher. Potential reasons for this include the following:

- Different datasets and different time periods may have different asset correlation.
- Our dataset is arguably the most comprehensive default dataset of U.S. public firms. Because default events are rare, any missing defaults could lead to an underestimation of default-implied asset correlation.
- Our sample period includes the most recent downturn of 2001–2002, which was a high default episode with high correlation.
- Our results presented in Table 3 and 4 are based on the population excluding small firms. On average, small firms tend to have higher default probabilities and lower asset correlations. It is also more difficult to track the default information of small firms. Thus, including small firms tends to produce lower asset correlations.

TABLE 5 Default-implied Asset Correlations

| Study | Data Source | Default-implied Asset Correlation |
|---|---|---|
| Result in this paper (by EDF) Bucketing) | Moody's KMV 1981–2006 | 11.46%~22.69% |
| Result in this paper ( by Industry ) | Moodys' KMV 1981–2006 | 9.22%~29.98% |
| Result in this paper (by rating) | Moody's KMV 1983–2006 | 7.87%~28.74% |
| Gordy (2000) | Standard and Poor's | 1.5%~12.5% |
| Cespedes (2000) | Moodys' Investor Service | 10% |
| Hamerle et al (2003a) | Standard and Poor's 1982–1999 | Max of 2.3% |
| Hamerle et al (2003b) | Standard and Poor's 1982–1999 | 0.4%~6.04% |
| Frey et al (2001) | UBS | 2.6%, 3.8%, 9.21% |
| Frey & McNeil (2003) | Standard and Poor's 1981–2000 | 3.4%–6.4% |
| Dietsch & Petey (2004) | Coface 1994–2001 | 0.12%–10.72% |
| | Allgemeine Kredit 1997–2001 | |
| Jobst & de Servigny (2004) | Standard and Poor's 1981–2003 | Intra 14.6%, inter 4.7% |
| Duellmann & Scheule (2003) | Deutsche Bundesbank 1987–2000 | 0.5%–6.4% |
| Jakubik (2006) | Bank of Finland 1988–2003 | 5.7% |

## 4.2 Comparison to the Asset Correlations in the Basel II IRB

One of the critical inputs in the capital charge specified in Equation (3) by the Basel II IRB is the asset correlation parameter $\rho$. For corporate borrowers, it is given as function of PD:

$$\rho = 0.12\times\frac{1-\exp(-50\times PD)}{\exp(-50)} + 0.24\times\frac{1-\exp(-50\times PD)}{\exp(-50)} - 0.04\times(1-(s-5)/45) \tag{9}$$

where PD is the default probability and s is the size of the borrower. The last term is a size adjustment to be applied for firms with annual sales between €5 million and €50 million. This correlation function, with and without the size adjustment, is plotted in Figure 3, together with the default-implied asset correlation from our data for the sample excluding small firm. We can see that the Basel II correlation function for large corporate borrowers is roughly in line with out empirical estimates from our default data.

We also plot our estimates of default-implied asset correlation for the entire sample, which include small firms, together with the Basel II correlation function in Figure 4. As discussed before, smaller firms tends to have larger PDs and smaller asset correlation, thus it is not surprising to see the default-implied asset correlations are lower than those in Figure 3. They are also lower than the Basel II correlation function with size adjustment. This raises the question whether the size adjustment in the Basel II asset correlation function is too punitive for small firms in the regulatory capital calculation.

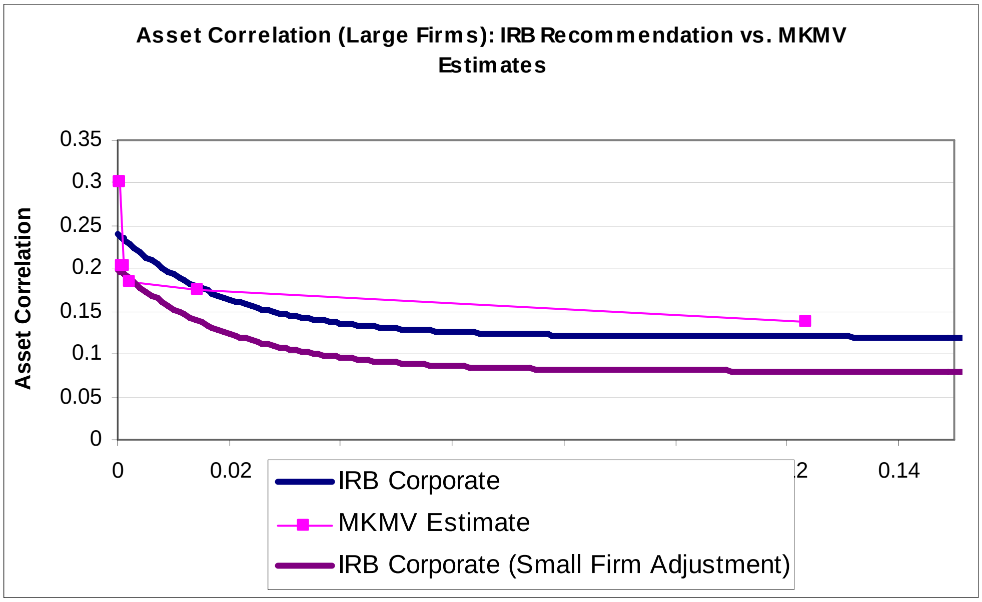


FIGURE 3 Default-implied Asset Correlation and Basel II Correlation Function—Large Firms

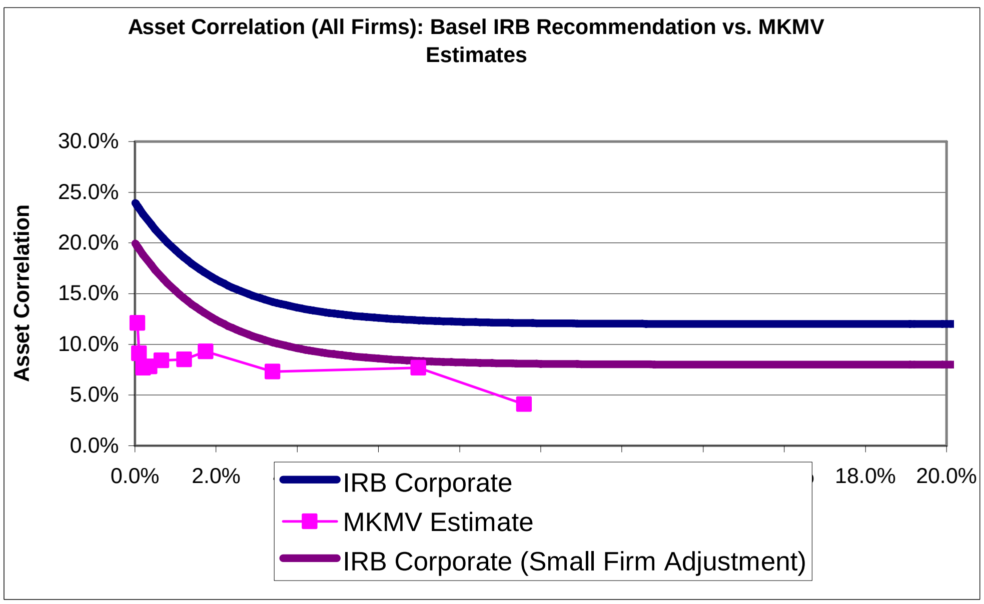


FIGURE 4 Default-implied Asset Correlation and Basel II Correlation Function—All Firms

## 4.3 Modeled Default Correlation vs. Realized Default Correlation

In this section, we present the key result of comparing realized default correlation with modeled default correlation. The following steps describe the procedure of the comparison:

- At the beginning of each year, divide firms into five quintiles based on their EDF values.
- Within each quintile, we calculate a firm's pair-wise asset correlations with other firms in the quintile using the GCorr correlation model.
- Calculate a firm's pair-wise default correlations with other firms using asset correlation and firms' EDF value.

- Calculate the mean modeled correlation for each quintile.
- Track each quintile's performance and calculate the realized default rate for that year.
- Repeat the above steps for each year.
- Calculate a weighted average of mean modeled correlation, weighted by number of firms.
- Calculate realized default correlation with Equation (5).
- Compare the mean modeled default correlation with the realized default correlation.

TABLE 6 Modeled Default Correlation and Realized Default Correlation

| EDF Rank | Average Num Firms | Average EDF | Average Asset Correlation | Average Modeled Correlation | Average Default Rate | Realized Default Correlation | Default-implied Asset Correlation |
|---|---|---|---|---|---|---|---|
| 80th~100th | 366 | 10.81% | 15.50% | 4.70% | 6.89% | 3.55% | 11.46% |
| 60th~80th | 366 | 1.22% | 16.49% | 1.87% | 0.16% | 0.46% | 14.68% |
| 40th~60th | 366 | 0.40% | 17.45% | 1.11% | 0.06% | 0.30% | 16.74% |
| 20th~40th | 366 | 0.18% | 18.14% | 0.73% | 0.02% | 0.28% | 22.69% |

We summarize the relevant statistics in Table 6. For the most risky 20% group, the average modeled correlation is 4.7%, while the realized default correlation is lower at 3.55%. For the other three groups, the realized default correlations are much lower than their respective average modeled correlations.[11] The discrepancies can be caused by several factors. We can see from Table 6 that the average EDF values are higher than the average realized default rates. As discussed in Section 3, a small number of firms and short time period has non-trivial effects on the realized default correlation. To take these effects into consideration, we conduct simulations.

Figure 8a shows the simulated mean default correlation incorporating the effects of the default rate, the small sample, and the short time period. After these adjustments, the simulated modeled correlations are very close to the realized default correlations except for the most risky group. For all groups, the realized default correlations are within the 95th simulated confidence bounds. The overall agreement between modeled default correlation and realized default correlation can also be confirmed by comparing the default-implied asset correlations to the average asset correlations from the GCorr model (Table 6). To ensure our results are not unduly influenced by EDF values, we further divide borrowers in each group into two subgroups according to their R-squared values. The idea is that this will further minimize the impacts of EDF values and allow us to test the significance of asset correlation on realized correlation. The hypothesis is that borrowers with higher R-squared values but similar EDF values would have higher realized default correlation than those with lower R-squareds and similar EDF values. We can see that this is indeed the case as Figure 8b provides the supporting evidence.

[11] The least risky bucket has no default. We exclude it from our analysis.

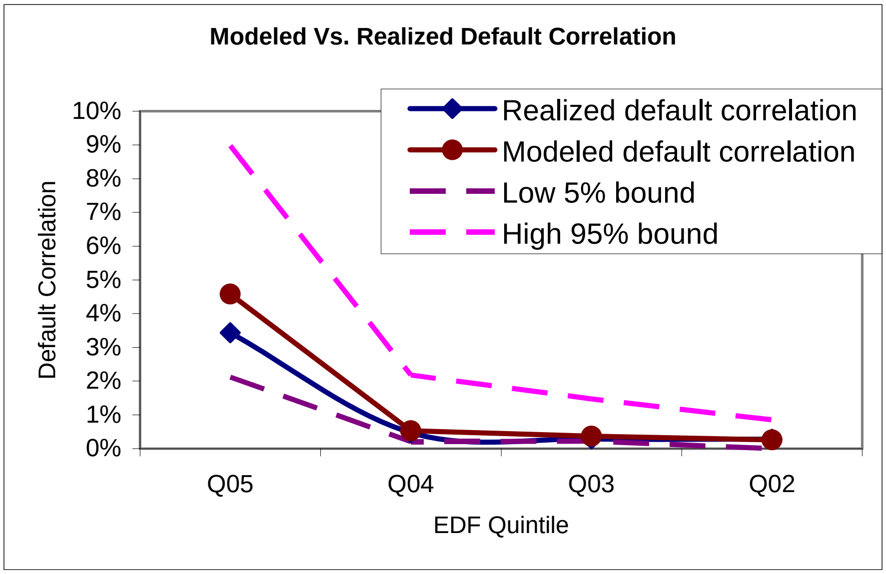


FIGURE 5 Modeled Default Correlations vs. Realized Default Correlations

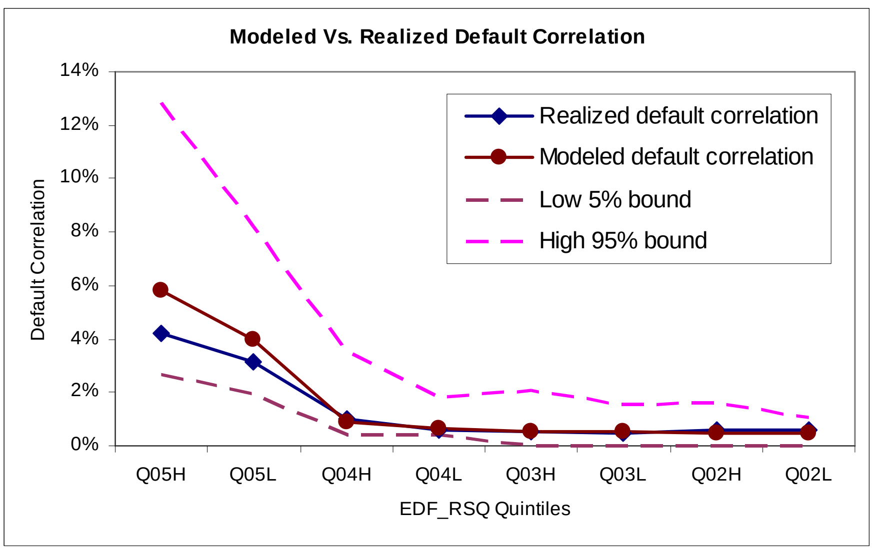


FIGURE 6 Modeled Default Correlations vs. Realized Default Correlations

## 4.4 Distribution of Realized Default Rates

At a portfolio level, a higher average asset correlation can produce a large default clustering in certain years and a smaller number of defaults most of the time. From the perspective of measuring portfolio risk, we need to ensure we adequately capture the total number of defaults with the asset correlation parameter and EDF values we use. Using a too low (or too high) asset correlation could potentially underestimate (or overestimate) the total number of defaults.[12] This risk of underestimation can be especially pronounced during a high default period. To check this, we use simulation to compare the forecasted default rates to the subsequently realized default rate. For each year, the simulation is done using the actual number of firms in each group, the average EDF value, and the average asset correlation of the group. A total of 2,000

[12] See Dwyer (2007) for more details.

trials are drawn for each year. To illustrate, we plot the 95th simulated confidence interval with the realized default rate for the most risky group in Figure 9. We can see that the realized default rates are within the 95th bounds of simulated values.

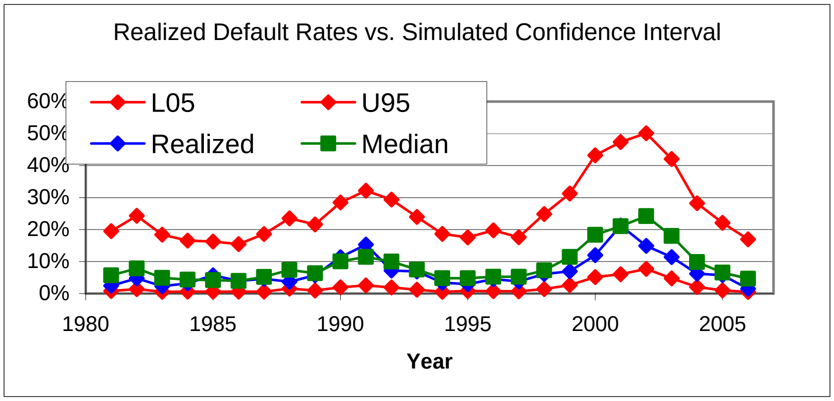


FIGURE 7 Realized Default Rates vs. Simulated Confidence Interval

# 5 ECONOMIC SIGNIFICANCE

This section covers information about our methodology and results.

## 5.1 Methodology

Our result in the last section focused on the levels of default-implied asset correlation and realized default correlation in a statistical sense. In this section, we examine the economic usefulness of the forecasted default correlations from asset correlations. We perform a portfolio study to investigate whether our asset correlation model can help explain portfolio realized volatility in terms of level and relative ranking of risk. The idea is that a higher average asset correlation in a portfolio would lead to a higher portfolio unexpected loss (UL) and subsequently a higher realized portfolio volatility. Thus after controlling for default probability, there should be a positive relationship between ex-ante portfolio UL and ex-post realized portfolio volatility.

Portfolio UL can be calculated as following:

$$UL_p^2 = \sum_{i=1}^{N} w_i^2 UL_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_i w_j \rho_{ij} UL_i UL_j \quad \text{or} \quad UL_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij} UL_i UL_j \qquad (10)$$

where $N$ is the total number of exposures in the portfolio, $w_i$ is exposure $i$'s weight, $UL_i$ is exposure $i$'s UL, and $\rho_{ij}$ is the value correlation between exposure $i$ and exposure $j$.

An individual exposure's UL can be calculated as following:[13]

$$UL = \frac{1}{|V_0|}\sqrt{CEDF_H \cdot \sigma_{H|D}^2 + (1-CEDF_H)\cdot \sigma_{H|ND}^2 + CEDF_H(1-CEDF_H)(E\left[V_{H|ND}\right] - E\left[V_{H|D}\right])^2} \qquad (11)$$

[13] This follows from the fact that the variance of a random variable $x$ as the sum of the expectation of its conditional variance and the variance of its conditional expectation.

where $V_0$ is the initial value of the exposure, $CEDF_H$ is the borrower's cumulative EDF value to horizon $H$, $\sigma_{H|D}$ is the value volatility in default state, $\sigma_{H|ND}$ is the value volatility in non-default state, $V_{H|D}$ is the exposure value in default state, and $V_{H|ND}$ is the exposure value in non-default state.

For simplicity, we measure the realized volatility in a default and non-default framework. If a borrower defaults, the value of the exposure to the borrower goes to 50% of the par. If it does not default, the value stays at par. From these assumptions, we have $V_0$=1, $\sigma^2_{H|D}$ =0, $\sigma^2_{H|ND}$ =0, $V_{H|ND}$=1, $V_{H|D}$=0.5. The unexpected loss for an exposure can be calculated as:

$$UL = 0.5\sqrt{CEDF_H(1-CEDF_H)} \tag{12}$$

The pair-wise value correlation in Equation (10) is equal to the pair-wise default correlation as shown in Appendix D.

We apply the analysis to the time period from January 1996 to December 2006 with the following procedures:

- At the beginning of each year, rank all firms based on their R-squared values from the GCorr model.
- Select the first 500 firms to form the first portfolio.
- Move down the rank by 20, and select the 500 firms with R-squared value ranked from 21st to 520th to form the second portfolio.
- Continue doing this until there are fewer than 20 firms left unused.
- For each portfolio, calculate its UL using the EDF values and the pair-wise default correlations.
- To construct a benchmark, calculate portfolio UL using the same EDF values, but zero asset correlation.
- Calculate the quarterly portfolio return by the default experience, that is, if there is a default in the portfolio, the exposure is decreased by 0.5. We then use the 20 quarterly portfolio returns to calculate the realized portfolio volatility.
- Compare the portfolio realized volatility with the ex-ante unexpected loss calculated with the GCorr asset correlation and with zero asset correlation.
- Repeat the above procedures for the years 1997 to 2002.

It is worth noting the rationale behind our choice of portfolios based on R-squared values, instead of randomly selecting 500 firms. A borrower's R-squared value is a proxy of its average asset correlation with other firms. All else being equal, a higher R-squared value leads to a higher asset correlation. By ranking R-squared first, we make sure that different portfolios have notable differences in the average pair-wise correlation (and pair-wise default correlation). If we just randomly picked 500 firms, it is possible that the average pair-wise correlations for different portfolios would be very similar. We would not be able to see the effect of asset correlation on realized portfolio volatility.

## 5.2 Results

First, we compare the realized portfolio volatility with the forecasted unexpected loss. There are three observations worth making from the comparisons in Figure 10. First, the realized portfolio volatilities are generally below the forecasted portfolio unexpected loss calculated with the GCorr asset correlation. This can be explained by the fact that realized default correlation is lower than the modeled default correlation, as we have seen in the previous section. Second, the realized portfolio volatilities are generally above the forecasted portfolio unexpected loss using zero asset correlation. This shows that using the EDF values alone can not make the forecasted portfolio unexpected loss high enough to match the subsequently realized risk level. In other words, failure to incorporate correlation would lead to an underestimate of risk. Third, we observe whether the realized portfolio volatility is close to the forecasted portfolio unexpected loss, which depends on what phase the 5-year period is in the credit cycle. If the 5-year period is in a deteriorating credit quality phase, for example, the periods of 1997 to 2001 and 1998 to 2002, the realized portfolio volatility is closer to the forecasted portfolio unexpected loss. If the 5-year period is in an improving credit quality phase, for example, the period of 2002 to 2006, the realized portfolio volatility is closer to the forecasted portfolio unexpected loss using zero correlation.

Next, we study whether the asset correlations can help differentiate the relative riskiness of portfolios. We examine the Spearman rank correlations between the realized portfolio volatility and the forecasted portfolio unexpected loss, as reported in Table 7. Most of these rank correlations are quite high—greater than 80%. This suggests the importance of measuring the default probability accurately. Although there are periods where the rank correlation between the realized portfolio and the forecasted UL using the GCorr asset correlation is lower than the one using zero asset correlation, the differences tend to be small. For the periods from 2001 to 2005 and 2002 to 2006, the rank correlation between the realized portfolio volatility and the forecasted unexpected loss using the GCorr asset correlation is considerably higher than the one using zero asset correlation.

The above results suggest that adding asset correlation information helps forecast the proper level of realized portfolio risk and differentiate the relative riskiness of portfolios. The added marginal benefits for forecasting portfolio risk is most pronounced during deteriorating credit quality periods, which are the periods that risk managers should care about the most.

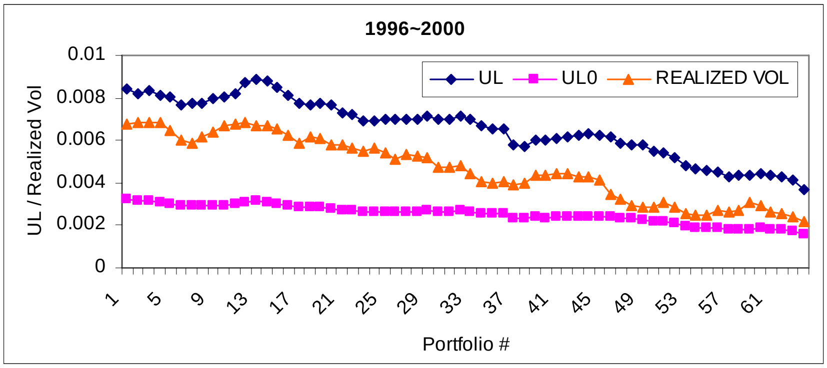


FIGURE 8 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

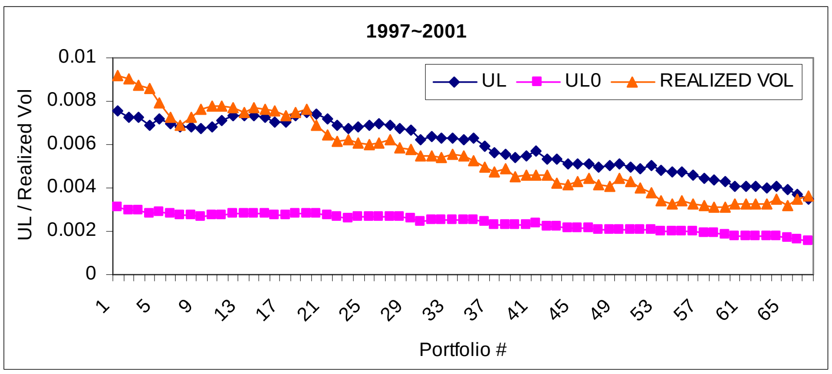


FIGURE 9 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

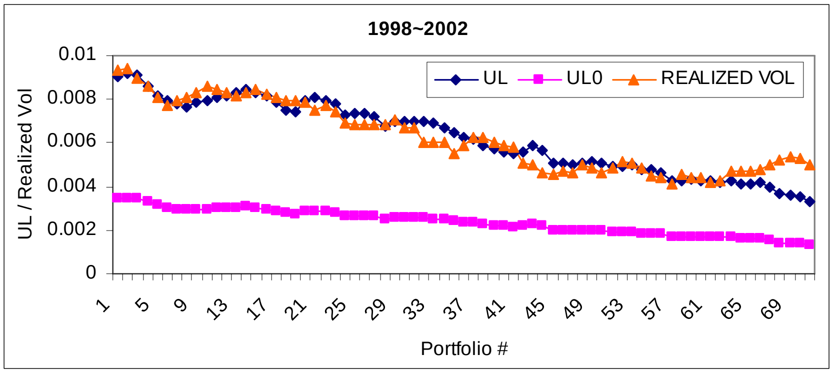


FIGURE 10 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

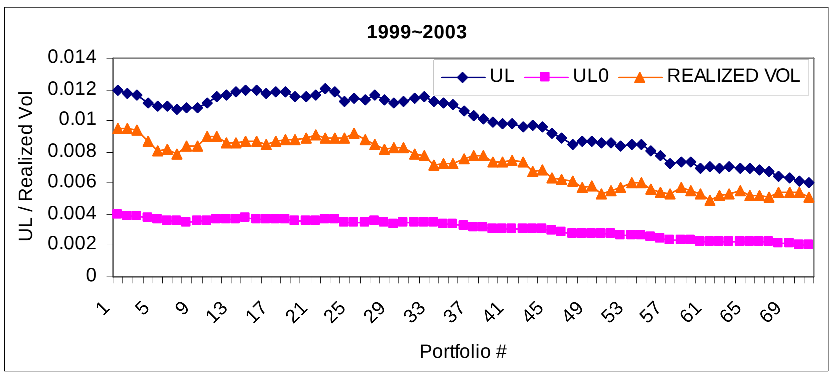


FIGURE 11 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

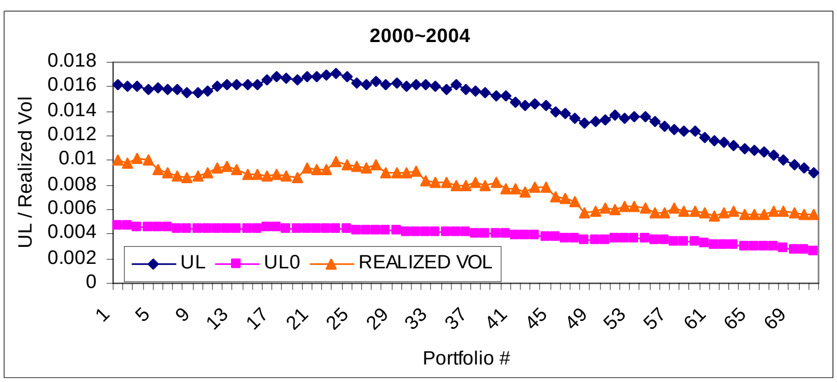


FIGURE 12 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

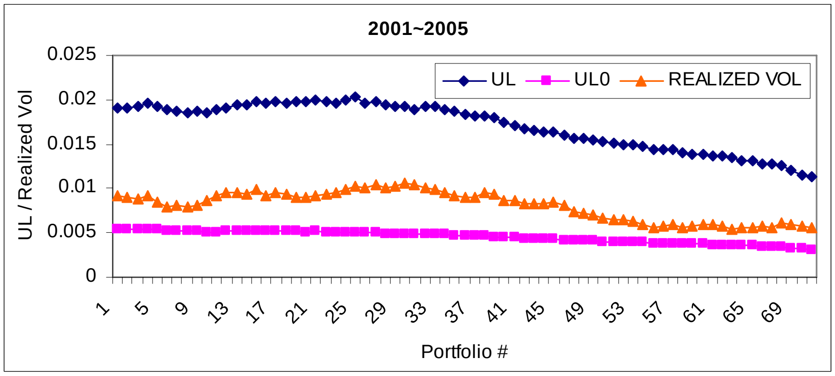


FIGURE 13 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

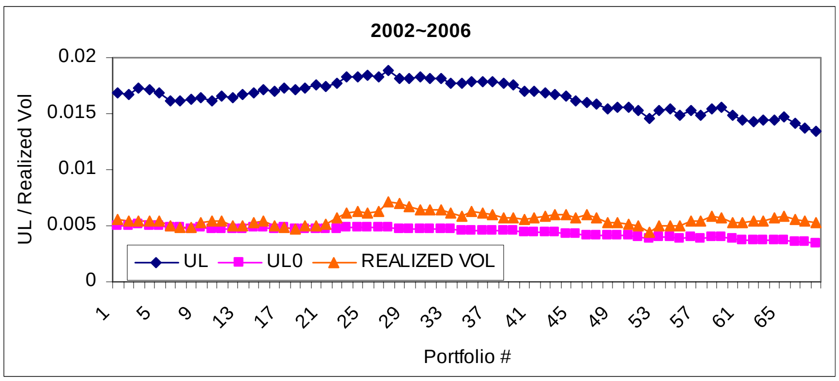


FIGURE 14 Comparison of Portfolio Realized Volatility vs. Unexpected Loss Calculated with and without Asset Correlation

TABLE 7 Rank Correlation Between Portfolio Realized Volatility and Forecasted Unexpected Loss

| | Realized Vol. vs. Unexpected Loss (with GCorr asset correlation) | Realized Vol. vs. Unexpected Loss (assuming asset correlation=0) | Difference |
|---|---|---|---|
| 1996~2000 | 96.74% | 96.97% | -0.23% |
| 1997~2001 | 94.56% | 96.53% | -1.97% |
| 1998~2002 | 88.99% | 88.92% | 0.07% |
| 1999~2003 | 92.59% | 93.70% | -1.11% |
| 2000~2004 | 88.15% | 92.49% | -4.34% |
| 2001~2005 | 86.92% | 71.46% | 15.46% |
| 2002~2006 | 53.97% | 9.41% | 44.56% |

# 6 CONCLUDING REMARKS

Our study has several practical implications for credit risk management. First, we find that asset correlation, the tendency for the asset values of borrowers to move together, is *empirically* significant and cannot be ignored in practice. In fact, the magnitude of default-implied asset correlation from our dataset is larger than previously reported. Failure to incorporate asset correlation will lead to underestimation of default clustering and realized portfolio risk. It is insufficient to focus on measuring only default probability and loss given default when measuring portfolio credit risk.

Second, accurately measuring asset correlation ex ante can improve the measurement of subsequently realized default correlation and portfolio risk. The improvement is significant in both statistical and economic terms. The improvement results in more accurate measurement of realized portfolio risk and better differentiation between the relative riskiness of portfolios.

Third, our study highlights the importance of model validation for portfolio credit risk models. The empirical challenges in validating asset correlation models are significantly greater than those in validating PD and LGD models. Given the critical importance of asset correlation in measuring portfolio credit risk, financial institutions should put more emphasis on assessing and validating the asset correlations used in their economic capital and portfolio risk calculations.

# APPENDIX A: JOINT DEFAULT PROBABILITY AND DEFAULT CORRELATION

We provide details on the derivations of joint default probability and default correlation in this appendix. As discussed in Section 2, a borrower will default if its asset value falls below its default point (DPT). This idea applies to a single borrower as well as to all borrowers in a portfolio. This is the foundation for calculating joint default frequency (JDF) and default correlation from individual default probabilities and asset correlation (see Figure 1 from Section 2 reproduced below).

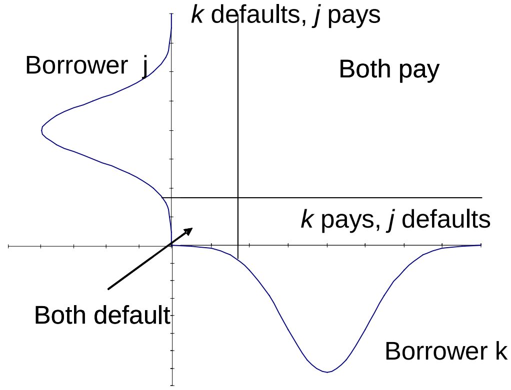


FIGURE 15 Joint Default

The graph is divided into four sections based on the event of default. The probabilities assigned to different regions are functions of the correlation between the asset values of *j* and *k* individual default probabilities. The borrower *i* and *j* default at the same time if both asset values are below their respective default points.

To see how JDF relates to individual borrower's cumulative EDF (CEDF) value, and pair-wise asset correlation, we start from the basis of a structural credit risk model. In this type of model, a firm's asset value is expressed as:[14]

$$A_t = A_0 Exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}\varepsilon_t\right) \quad (13)$$

And a borrower's *CEDF* value is linked to default point through:

$$\begin{aligned} CEDF &= P(A_t < DPT) = P\left(\ln A_0 + \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}\varepsilon_t < \ln DPT\right) \\ &= P\left(\varepsilon_t < \frac{\ln(DPT/A_0) - \left(\mu - \sigma^2/2\right)t}{\sigma\sqrt{t}}\right) = N\left(\frac{\ln(DPT/A_0) - \left(\mu - \sigma^2/2\right)t}{\sigma\sqrt{t}}\right) \end{aligned} \quad (14)$$

[14] We use the cumulative EDF value in this discussion to reflect the possibility that the horizon of analysis may exceed one year. In the case where the horizon is one year the EDF value equals the CEDF value.

Assuming the joint asset value distribution follows a bivariate normal distribution, the JDF can be expressed as:

$$
\begin{aligned}
JDF_{jk} &= P(A_{jt} < DPT_j, A_{kt} < DPT_k) \\
&= P\left(\varepsilon_j < \frac{\ln(DPT_j / A_{j0}) - \left(\mu_j - \sigma_j^2/2\right)t}{\sigma_j \sqrt{t}}, \varepsilon_k < \frac{\ln(DPT_k / A_{k0}) - \left(\mu_k - \sigma_k^2/2\right)t}{\sigma_k \sqrt{t}}\right) \\
&= N_2\left(\frac{\ln(DPT_j / A_{j0}) - \left(\mu_j - \sigma_j^2/2\right)t}{\sigma_j \sqrt{t}}, \frac{\ln(DPT_k / A_{k0}) - \left(\mu_k - \sigma_k^2/2\right)t}{\sigma_k \sqrt{t}}, \rho_{jk}\right) \\
&= N_2(N^{-1}(CEDF_j), N^{-1}(CEDF_k), \rho_{jk})
\end{aligned}
\tag{15}
$$

To calculate the default correlation from the JDF, we can consider the following table:

TABLE 8 Default Calculations from the JDF

| **Observe default for Firm *j* ($D_j$)** | **Probability** | **Observe Default for Firm *k* ($D_k$)** | **Probability** |
|---|---|---|---|
| 1 | CEDFj | 1 | CEDFk |
| 0 | (1-CEDFj) | 0 | (1-CEDFk) |

The default correlation between firm *j* and firm *k* can be derived as the following:

$$
\begin{aligned}
\rho_{jk}^D &= \frac{\operatorname{cov}(D_j, D_k)}{\sigma_j^D \sigma_k^D} = \frac{E[D_j \cdot D_k] - E[D_j]E[D_k]}{\sigma_j^D \sigma_k^D} \\
&= \frac{JDF_{jk} - CEDF_j \cdot CEDF_k}{\sqrt{\left[CEDF_j \cdot (1 - CEDF_j)\right] \cdot \left[CEDF_k \cdot (1 - CEDF_k)\right]}}
\end{aligned}
\tag{16}
$$

It is worth pointing out that the default correlation depends not only on the asset correlation but also on the CEDF values. In fact, as the CEDF values increase, their influence on the default correlation dominates that of asset correlation.

# APPENDIX B: ESTIMATING DEFAULT CORRELATION WITHIN A HOMOGENEOUS GROUP

We provide more details on the derivation of Equation (8) in Section 3.2. First, let's introduce the notations and assumptions.

### Notations

*N*: Number of firms in the group

$D_{t,n}$: Default indicator function

$\rho$: Pair-wise default correlation within the same group

### Assumptions

For any *t* and *n*, $E(D_{t,n}) = p$, $\mathrm{Var}(D_{t,n}) = p(1\text{-}p)$

For different *k* and *l*,

$Cov(D_{t,k}, D_{t,l}) = \rho\, p(1-p)$ (Default correlation between firms = $\rho$)

$Cov(D_{k,n}, D_{l,n}) = 0$ (Defaults are independent over time)

Let's define $P_t = \frac{1}{N}\sum_{n=1}^{N} D_{t,n}$

Then, $E(P_t) = \frac{1}{N}\sum_{n=1}^{N} E(D_{t,n}) = p$

$$
\begin{aligned}
Var(P_t) &= E(P_t^2) - p^2 \\
&= E\left[ \left(\frac{1}{N}\sum_{n=1}^{N} D_{t,n}\right)^2 \right] - p^2 \\
&= \frac{1}{N^2} E\left[ \sum_{n=1}^{N} D_{t,n}^2 + 2\sum_{n=1}^{N-1}\sum_{j=n+1}^{N} D_{t,n}D_{t,j} \right] - p^2 \\
&= \frac{1}{N^2} N * E\left[D_{t,n}^2\right] + \frac{N(N-1)}{N^2} E\left[D_{t,n}D_{t,j}\right] - p^2 \\
&= \frac{Np}{N^2} + \frac{N(N-1)}{N^2}(\rho\, p(1-p) + p^2) - p^2 \\
&= \frac{p(1-p)}{N}(1 - \rho + N\rho)
\end{aligned}
$$

The above equation needs the following relationships:

$$E(D_{t,n}^2) = Var(D_{t,n}) + p^2 = p(1-p) + p^2$$

$$E(D_{t,k}D_{t,l}) = Cov(D_{t,k}, D_{t,l}) + p^2 = \rho\, p(1-p) + p^2$$

Thus, the formula for realized default correlation gives the following:

$$\rho_{realized} = \frac{\hat{Var}(P_t)}{\hat{E}(P_t)(1-\hat{E}(P_t))} \rightarrow \frac{Var(P_t)}{E(P_t)(1-E(P_t))} = \frac{1}{N}(1-\rho+N\rho) = \rho + \frac{1-\rho}{N}$$

*if* the length of time series is large enough.

## APPENDIX C: SIMULATION EXPERIMENTS

### The Effect of the Number of Firms in the Group

In this analysis, we test the effect of the number of firms in the group. For the simulations, we change the number of firms while fixing the default probability, default correlation, number of years, and number of trials. Each trial gives us one realized default correlation. We then plot the distribution of the realized default correlations in Figure 16.

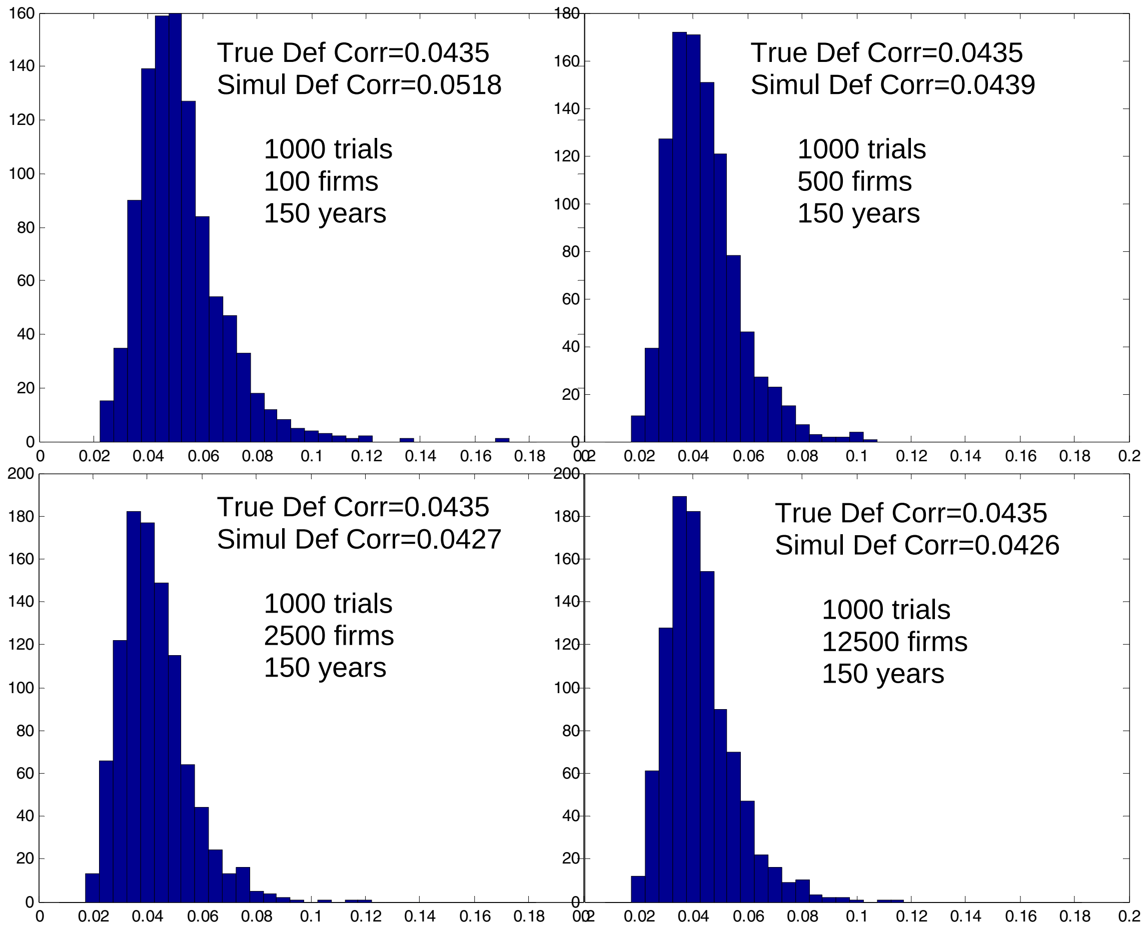


FIGURE 16 The Effect of the Number of Firms on Realized Default Correlation.

Since the true default correlation is 0.0435, Equation (8) suggests that if the number of years is very large, the average simulated default correlation should be close to 0.0531 with N=100.[15] The average simulated default correlation is 0.0518, which is higher than the true value and close to the number from Equation (8). Later on we will see that the difference between 0.0518 and 0.0531 is mainly caused by the limited number of years. As we increase the number of firms to 500, 2500, and 12500, the average simulated default correlations decrease and seem to converge to a fixed value.

---

[15] 0.0531=0.0435 + (1-0.0435)/100

Another observation we can make is that the distributions of realized default correlation are similar with varying numbers of firms. This is probably caused by the fact that the length of the time series is fixed at 150 years.

### The Effect of Number of Years

In this analysis, we assess the effect of the length of time series. For the simulations, we change the number of years while keeping the default probability, default correlation, number of firms, and the number of trials fixed. Each trial gives us one realized default correlation. We then plot the distribution of the realized default correlations in Figure 17.

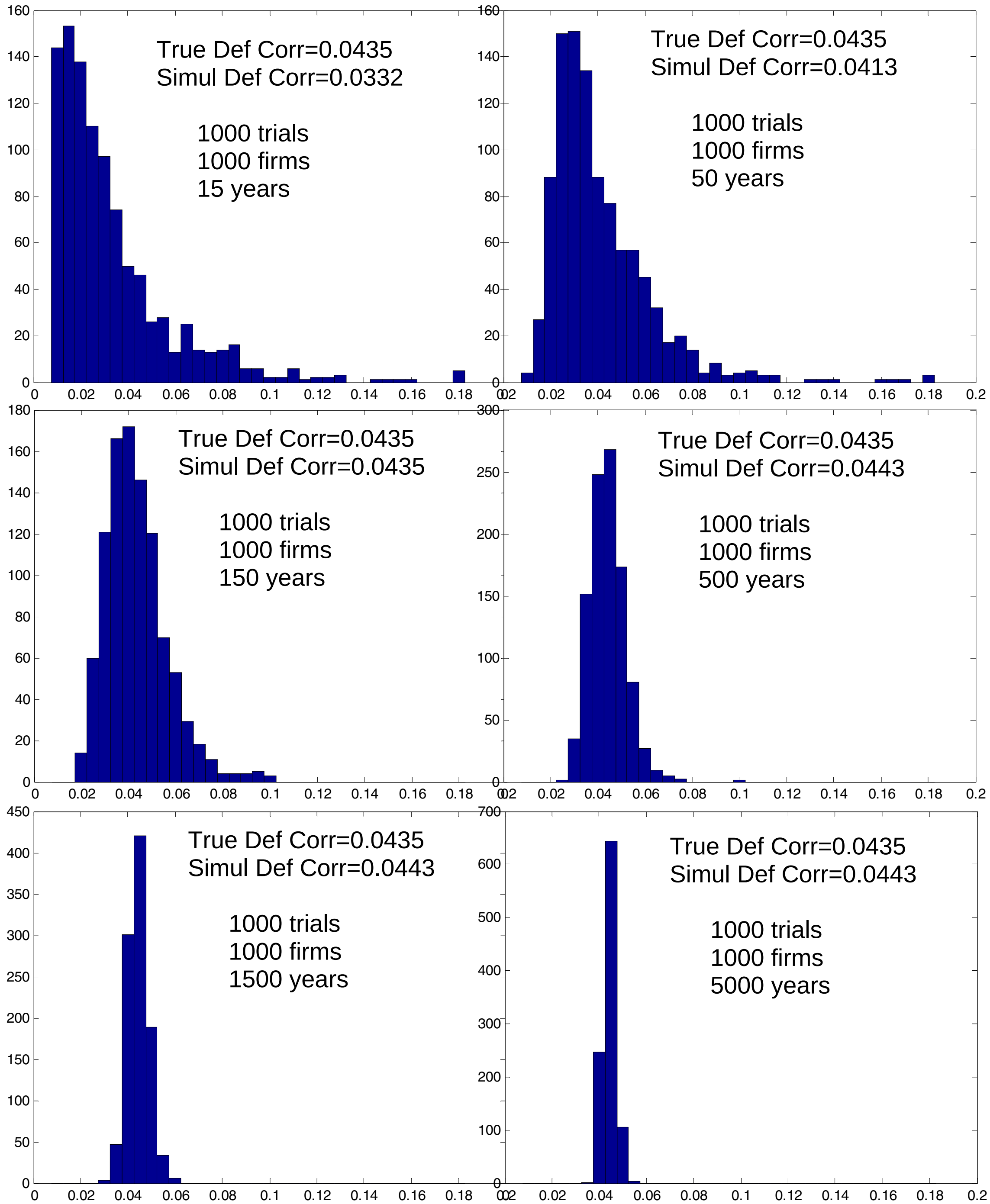


FIGURE 17 The Effect of Number of Years on Realized Default Correlation

When there are 15 years in each trial, the average simulated default correlation is 0.0332, which is lower than the true value of 0.0435. The distribution is very wide with both many very small and large realized default correlations. As we increase the number of years to 50, 150, 500, 1,500, and 5,000, the average simulated default correlations increase and converge to a fixed value. The fixed value of 0.0443 is higher than the true value. The reason is that we only have 1,000 firms in the group, which leads to a 0.001 increase in default correlation according to Equation (8). This exercise demonstrates that increasing the length of time series enables the realized default correlation to converge to the true value with increasingly narrow distribution.

### The Effects of EDF Values and Asset Correlations

We present the results from more simulation exercises in Table 8. The goal of these exercises is to understand how the realized default correlation behaves under different scenarios. The key points of these exercises are already summarized at the end of Section 3.2.

TABLE 9 Average Simulated Realized Default Correlation

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| EDF1=0.005 | | A_Corr=0.10, D_Corr=0.58% | | | | A_Corr=0.14, D_Corr=0.93% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 1.58% | 1.67% | 1.71% | 1.70% | 1.84% | 2.01% | 2.10% | 2.13% |
| | N=200 | 1.06% | 1.11% | 1.14% | 1.14% | 1.36% | 1.43% | 1.52% | 1.51% |
| | N=400 | 0.78% | 0.81% | 0.86% | 0.87% | 1.04% | 1.13% | 1.19% | 1.19% |
| | N=800 | 0.65% | 0.68% | 0.70% | 0.72% | 0.92% | 0.98% | 1.03% | 1.07% |
| EDF1=0.005 | | A_Corr=0.18, D_Corr=1.35% | | | | A_Corr=0.22, D_Corr=1.87% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 2.12% | 2.38% | 2.53% | 2.57% | 2.56% | 2.79% | 2.95% | 3.08% |
| | N=200 | 1.61% | 1.81% | 1.89% | 1.91% | 1.94% | 2.10% | 2.37% | 2.45% |
| | N=400 | 1.38% | 1.50% | 1.58% | 1.64% | 1.74% | 1.93% | 2.08% | 2.15% |
| | N=800 | 1.23% | 1.34% | 1.41% | 1.50% | 1.59% | 1.75% | 1.87% | 1.94% |
| EDF1=0.02 | | A_Corr=0.10, D_Corr=1.47% | | | | A_Corr=0.14, D_Corr=2.23% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 2.40% | 2.53% | 2.60% | 2.64% | 3.10% | 3.33% | 3.37% | 3.47% |
| | N=200 | 1.93% | 2.01% | 2.03% | 2.05% | 2.60% | 2.73% | 2.78% | 2.81% |
| | N=400 | 1.68% | 1.69% | 1.73% | 1.75% | 2.27% | 2.43% | 2.48% | 2.52% |
| | N=800 | 1.47% | 1.57% | 1.60% | 1.59% | 2.14% | 2.28% | 2.34% | 2.37% |
| EDF1=0.02 | | A_Corr=0.18, D_Corr=3.09% | | | | A_Corr=0.22, D_Corr=4.08% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 3.91% | 4.12% | 4.30% | 4.36% | 4.62% | 4.95% | 5.28% | 5.35% |
| | N=200 | 3.29% | 3.50% | 3.64% | 3.71% | 4.14% | 4.39% | 4.65% | 4.65% |
| | N=400 | 3.05% | 3.32% | 3.33% | 3.44% | 3.83% | 4.03% | 4.28% | 4.29% |
| | N=800 | 2.86% | 3.06% | 3.23% | 3.25% | 3.71% | 3.91% | 4.10% | 4.21% |
| EDF1=0.08 | | A_Corr=0.10, D_Corr=3.30% | | | | A_Corr=0.14, D_Corr=4.80% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 4.26% | 4.39% | 4.52% | 4.54% | 5.62% | 5.94% | 6.03% | 6.06% |
| | N=200 | 3.79% | 3.86% | 3.89% | 3.89% | 5.10% | 5.39% | 5.38% | 5.46% |
| | N=400 | 3.35% | 3.49% | 3.64% | 3.57% | 4.80% | 5.00% | 5.06% | 5.11% |
| | N=800 | 3.29% | 3.36% | 3.42% | 3.40% | 4.68% | 4.82% | 4.88% | 4.92% |
| EDF1=0.08 | | A_Corr=0.18, D_Corr=6.40% | | | | A_Corr=0.22, D_Corr=8.10% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 7.22% | 7.54% | 7.59% | 7.64% | 8.54% | 9.10% | 9.24% | 9.43% |
| | N=200 | 6.49% | 6.86% | 6.96% | 7.04% | 8.01% | 8.60% | 8.60% | 8.74% |
| | N=400 | 6.31% | 6.50% | 6.53% | 6.66% | 7.85% | 8.16% | 8.27% | 8.34% |
| | N=800 | 6.28% | 6.33% | 6.44% | 6.51% | 7.56% | 7.91% | 8.11% | 8.22% |
| EDF1=0.2 | | A_Corr=0.10, D_Corr=5.07% | | | | A_Corr=0.14, D_Corr=7.20% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 6.07% | 6.29% | 6.26% | 6.31% | 8.00% | 8.29% | 8.43% | 8.48% |
| | N=200 | 5.42% | 5.58% | 5.66% | 5.72% | 7.50% | 7.68% | 7.77% | 7.78% |
| | N=400 | 5.16% | 5.33% | 5.31% | 5.38% | 7.16% | 7.36% | 7.44% | 7.47% |
| | N=800 | 4.95% | 5.16% | 5.16% | 5.20% | 6.93% | 7.26% | 7.34% | 7.34% |
| EDF1=0.2 | | A_Corr=0.18, D_Corr=9.39% | | | | A_Corr=0.22, D_Corr=11.64% | | | |
| | | T=25 | T=50 | T=100 | T=200 | T=25 | T=50 | T=100 | T=200 |
| | N=100 | 10.43% | 10.56% | 10.70% | 10.70% | 12.47% | 12.78% | 12.90% | 12.94% |
| | N=200 | 9.77% | 9.91% | 9.96% | 10.00% | 11.72% | 12.00% | 12.27% | 12.34% |
| | N=400 | 9.39% | 9.60% | 9.67% | 9.72% | 11.42% | 11.76% | 11.91% | 11.91% |
| | N=800 | 9.06% | 9.33% | 9.49% | 9.47% | 11.16% | 11.55% | 11.61% | 11.70% |

A_corr: Asset Correlation

D_Corr: Default Correlation

# APPENDIX D: VALUE CORRELATION

We illustrate the derivation that the value correlation is equal to default correlation if credit migration is ignored. For simplicity, we assume a portfolio with only two exposures. Furthermore, we assume: $V_0$=1; $V_{H|D}$=0.5. The derivation is as follows:

Assume D is the default indicator function. When the borrower defaults, D is 1; otherwise it is 0.

$V_i = 1 - 0.5D_i;$

$V_j = 1 - 0.5D_j;$

Portfolio value: $V_p = 1 - 0.5w_i D_i - 0.5w_j D_j$

$UL_i = \sigma(V_i) = 0.5\sigma_{D_i};$

$UL_j = \sigma(V_j) = 0.5\sigma_{D_j};$

Portfolio UL: $UL_p = \sqrt{(0.5 * w_i)^2 \sigma_{D_i}^2 + (0.5 * w_j)^2 \sigma_{D_j}^2 + 2 * 0.5 * 0.5 w_i w_j \rho_{ij}^D \sigma_{D_i} \sigma_{D_j}}$

From Equation (11), we have:

$UL_p^2 = w_i^2 UL_i^2 + w_j^2 UL_j^2 + 2w_i w_j \rho_{ij} UL_i UL_j$

Under the current assumptions, $\rho_{ij}$ can be calculated as

$$\rho_{ij} = \frac{UL_p^2 - w_i^2 UL_i^2 - w_j^2 UL_j^2}{2w_i w_j UL_i UL_j}$$

$$= \frac{0.25(w_i^2 \sigma_{D_i}^2 + w_j^2 \sigma_{D_j}^2 + 2w_i w_j \rho_{ij}^D \sigma_{D_i} \sigma_{D_j}) - 0.25w_i^2 \sigma_{D_i}^2 - 0.25w_j^2 \sigma_{D_j}^2}{2w_i w_j (0.25\sigma_{D_i} \sigma_{D_j})}$$

$$= \rho_{ij}^D$$

# REFERENCES


1. J. Cespedes (2002). Credit Risk Modelling and Basel II. Algo Research Quarterly, 57, Vol. 5, No. 1.
2. A. Chernih, S. Vanduffel & L. Henrard (2006). Asset Correlations: A Literature Review and Analysis of the Impact of Dependent Loss Given Defaults, Working Paper.
3. D. Dwyer (2002). The distribution of defaults and Bayesian model validation, Journal of Risk Model Validation Volume 1/Number 1.
4. M. Dietsch & J. Petey (2004). Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. Journal of Banking and Finance, 28, pp 773–788.
5. K. Duellmann & H. Scheule (2003). Determinants of the Asset Correlations of German Corporations and Implications for Regulatory Capital. Deutsches Bundesbank.
6. R. Frey & A. Mcneil (2003). Dependent Defaults in Models of Portfolio Credit Risk. Journal of Risk 6(1), pp 59–92.
7. M. Gordy (2000). A Comparative Anatomy of Credit Risk Models. Journal of Banking and Finance, Vol. 24, No. 1–2, pp 119–149.
8. A. Hamerle, T. Leibi & D. Roesch (2003a). Benchmarking Asset Correlations, Risk 16, 77–81.
9. A. Hamerle, T. Leibi & D. Roesch (2003b). Credit Risk Factor Modeling and Basel II IRB Approach. Deutsches Bundesbank Discussion Paper.
10. P. J. Jobst & A. de Servigny (2005). An Empirical Analysis of Equity Default Swaps II : Multivariate Insights. Working Paper, Standard & Poor's.
11. A. De Servigny & O. Renault (2002). Default correlation : empirical evidence. Standard & Poor's.
12. A. De Servigny & O. Renault (2003). Default Correlation Evidence. Risk, July 2003, pp 90–94.
13. B. Zeng & J. Zhang (2001). An Empirical Assessment of Asset Correlation Models. Moody's KMV Research Paper.