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VALUATION OF CORPORATE LOANS: A CREDIT MIGRATION APPROACH

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ABSTRACT

Banks and investors in loan assets have always had difficulty obtaining an unbiased and consistent value for the assets they hold. With the growth of liquidity in the loan market, the demand for a valuation method that can be consistently applied has been growing. However, the problems of loan valuation are complex. In large part this is because of the existence of embedded options and contractual conditions that can significantly affect the value of a loan.

In this paper, we present the Moody's KMV methodology for valuing corporate loans, taking into account both embedded options and credit state contingent cash flows. We have found that our valuation and risk measurement methodologies compare extremely well to quotes from the secondary loan market, making their use in broad portfolios with limited secondary market prices both feasible and valuable.

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1 OVERVIEW

Since the early 1990s, the loan markets have seen tremendous growth in the liquidity of both loans and derivatives that are tied to loans. One element of this growth has been a rapid expansion in the secondary market for leveraged loans (Figure 1). Other elements include the growth of derivatives that are tied to loans (i.e., loan-only credit default swaps, or LCDSs), the growth of structured products with loans in the underlying collateral pool (i.e., collateralized loan obligations, or CLOs), and finally indices of loan derivatives that can form the basis of synthetic structured products (i.e., LCDX). Increasingly, multiple instruments are available to hedge loans, and the value of a specific loan is today much more likely to be directly observable.

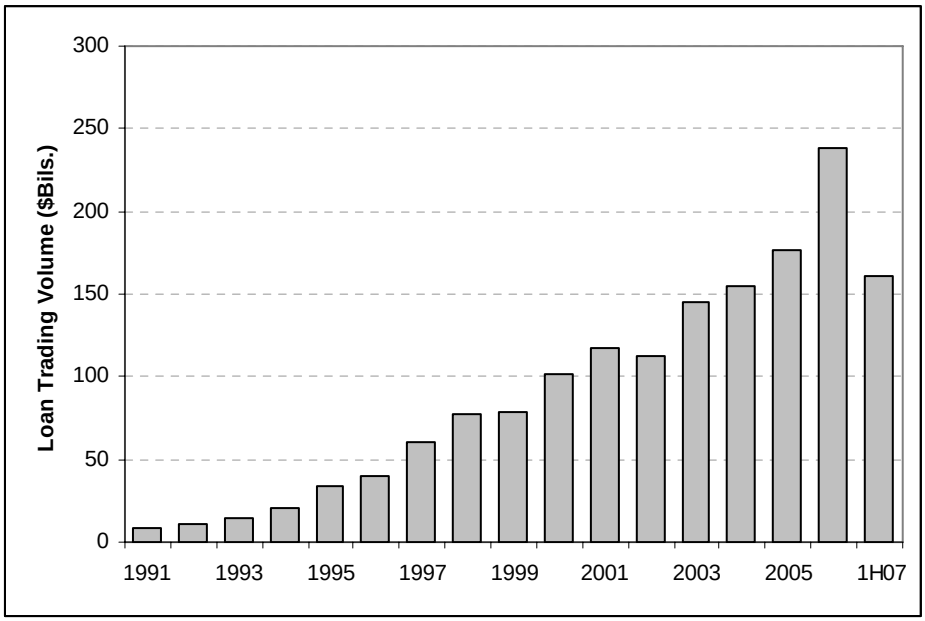

FIGURE 1 Trading Volume in the U.S. Syndicated Loan Market

Source: Reuters LPC Traders Survey.

Historically, loans have typically been carried on the books at their accounting value. Nevertheless, banks do hedge some of these loans using more liquid instruments, such as credit default swaps. The liquidity of the hedging instruments makes it convenient to mark them to market. Marking the hedging instruments to market, but not marking the loan can lead to accounting distortions. For example, if credit quality of a name declined, the value of the hedge would increase while the accounting value of the loan would remain constant, and it would look as though the bank had made a profit from the decline in the credit quality of one of their borrowers. These distortions give rise to both the desire to be able to mark to market loans and the need for a credible methodology to do so.[1]

Loans are different from bonds in several important ways. The first is that most loans are floating rate instruments. Consequently, their value is not highly sensitive to changes in interest rates. Second, loans are typically prepayable. For loans, an improvement in credit quality is a principal driver of the prepayment decision. Consequently, a model of credit migration is required to credibly model the embedded option features in loans. Many bonds are callable and many bond investors have computed an option-adjusted spread on such bonds for many years now. Nevertheless, this option is different from the option found in loans. The typical option-adjusted spreads computed for bonds are based on a model of stochastic interest rates and not credit migration. Consequently, such models are not directly applicable to

[1] For a recent review of industry practices, see Tschirhart, O'Brien, Moise, and Yang (2007).

floating-rate instruments, for which the decision to prepay is driven by improvements in credit quality, rather than changes in the interest rate environment.[2]

Loan agreements can be complex, and these complexities often have an impact on the subsequent value of the loan. Examples of conditions within loan agreements include pricing grids that tie the spread on the loan to various measures of credit quality, such as an agency rating or a set of financial ratios, prepayment penalties, and options to use an alternative base (i.e., LIBOR or Prime) for the pricing resets. In addition, there can be covenants that affect term, and collateral that should impact recovery levels. Revolving lines of credit have commitment amounts, usage fees (a drawn spread), commitment fees (a non-usage fee), and facility fees. Partly as a result of this form of structuring, in the event of default, a loan will typically have a better recovery than a bond. A credible methodology for marking a loan to market needs to account for all of these features.

In September 2006, the Financial Accounting Standards Board issued Statement No. 157, which establishes a framework for fair value accounting in the context of generally accepted accounting principles (GAAP) when fair value accounting is either permitted or required. The statement establishes a hierarchy of valuation methodologies. The hierarchy gives first priority to using actual prices for identical assets in active markets when available for establishing a fair market value (Level 1 inputs). Second priority is valuation methodologies that are based on inputs that include a combination of prices from inactive markets on identical assets, prices of similar assets from active markets combined with observable characteristics of the asset, and finally market-corroborated inputs (Level 2 inputs). The lowest priority is given to unobservable inputs. These include the firm's own assumptions regarding how the market would view a particular asset were it to trade.

The Moody's KMV CreditMark™ methodology—first introduced to the market in 2002—values a loan utilizing Level 1, Level 2, or Level 3 inputs as appropriate.[3] The empirical work of this paper will demonstrate how CreditMark can effectively value a loan using predominantly Level 2 inputs by comparing the model prices to the actual prices on traded loans when available. The start point for CreditMark is the term structure of what we call clean spreads. A clean spread is what the spread would be on a zero-recovery, zero-coupon bond if it were to trade. These spreads can be populated from CDS spreads or bond spreads on the same name should they exist, or alternatively from spreads on names with comparable EDF™ credit measures, agency ratings, or internal ratings, as desired.

From the term structure of zero-recovery, zero-coupon bonds, CreditMark values the loan using a model of credit migration, a forward LIBOR curve, the terms of the loan, the paradigm of risk-neutral pricing, and recursive methods. The modeling of a prepayment option is based on the borrower exercising the option when it is in their best interest under the terms of the loan and a specified transaction cost. The modeling of the usage of a revolver is based on a user-provided usage table that relates usage of the loan to the credit quality of the revolver. The model of credit migration is derived from a long history of firms with publicly trade equity. Our credit migration model is derived from transition matrices that are based on a volatility-adjusted measure of market leverage (distance-to-default).

In this study, we test the ability of this framework to produce loan valuations that are consistent with actual quoted loan prices. We do so by using a data set of over 4,000 loans observed over a five-year period. We show that this model produces loan valuations that correspond reasonably well with the observed loan prices. This finding holds for loans that are valued on the basis of CDS spreads, EDF credit measures, and agency ratings.

Figure 2 presents an intuitive example of our basic results for Goodyear Tire & Rubber. It is the actual loan price against the CDS-implied loan price. The black lines represent the bid-ask spread, the blue circles represent the CDS spreads, and the green solid line represents the CDS-based model price. We highlight both the par value of the loan ($100) and its coupon of the loan on the graph (400 bps). In the middle of 2003, Goodyear Tire & Rubber CDS contracts traded at approximately 1,300 bps, and over the next year they fell to below 400 bps. As the LGD on the loan is approximately

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[2] Of course, changes in credit quality will impact the call decision on a bond. The justification of focusing on interest rate changes as a driver for the call decision of bond issuers is that changes in the interest rate environment are more likely to be the driving factor behind the call decision of bond issuers, or at least for the issuers of investment grade bonds. Moreover, the call decision on bonds is generally on or after a specific date, whereas the prepayment option on a loan is immediate and continuous.

[3] CreditMark is a software tool that enables financial institutions to mark to market loans by marking to model, utilizing market-based inputs whenever possible. McAndrew (2004) provides an introduction to the framework. Wen and Zeng (2003) provide a detailed description of the methodology. The intellectual foundation for the framework grew out of the works of Oldrich Vasicek (e.g., Vasicek 1984) and Stephen Kealhofer (e.g., Kealhofer 2002). As of December 2007, the methodology is also being deployed in the Loan Valuation Web Service™.

half of LGD on the bond, a CDS spread of about 1,300 bps is approximately 250 bps above the loan coupon of 400 bps after adjusting for the LGD difference (0.5*1,300-400). Given that the duration of the loan was approximately 2 years, the loan was trading at about 95 (par minus the duration times spread difference). As the CDS spread fell below 800 bps in early 2004, the CDS-based model price went to par. The actual loan price moved to par a couple of months later. As the CDS spread continued to fall, both the loan price and the CDS-based model price remained somewhat above par until the loan matured. The fact that the price of the loan remains close to par, despite the falling CDS spread, reflects the value of the prepayment option—investors are reluctant to pay more than par for a loan that can be prepaid at par.

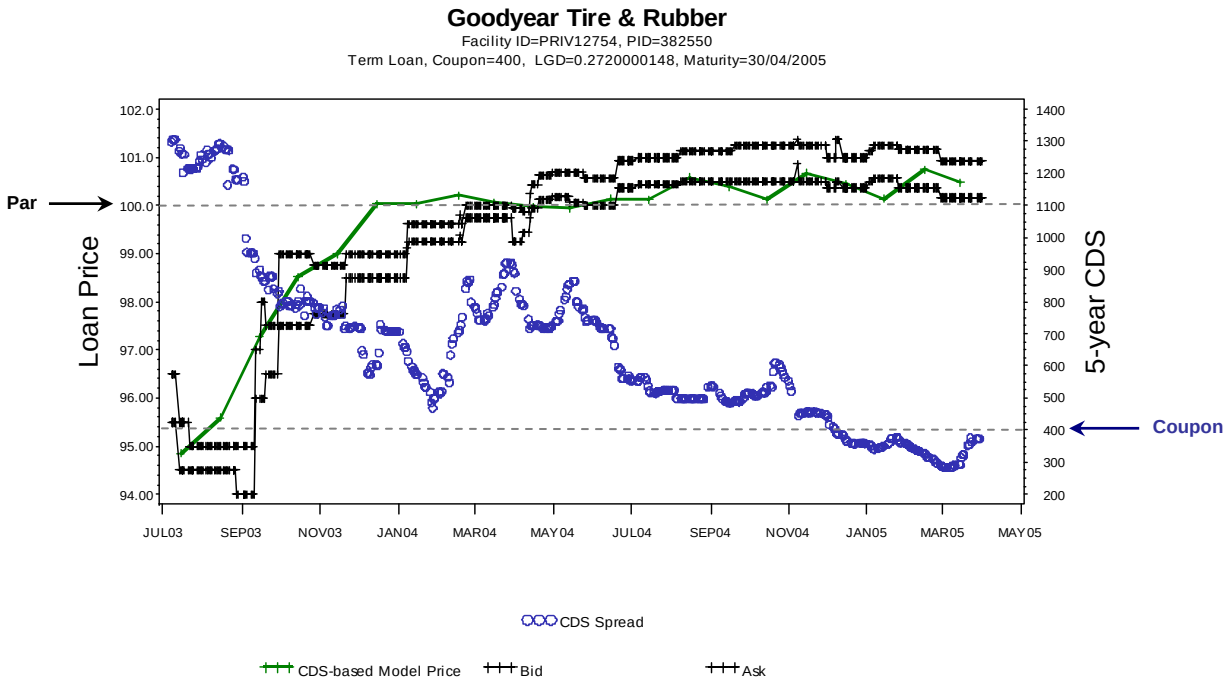

FIGURE 2 Goodyear Tire & Rubber

We also show that the framework allows for the valuation of many types of embedded options, including the usage option in a revolver and the prepayment option in a term loan. We find that the prepayment option is typically worth more than the usage option, and the importance of this option increased over the timeframe of the sample. Finally, we show that loans for which the option values are high are more likely to prepay.

The next section of this paper provides an overview on how CreditMark values a loan. It presents some introductory principals of loan valuation and outlines the embedded options found in loans. It also outlines how we model credit migration and compute a loan value using risk-neutral pricing. The third section provides our empirical implementation. It first discusses the data used and implementation decisions. It then compares model prices to actual prices and CDS-based model prices to model prices based on EDF credit measures. Section four provides concluding remarks.

2 A CREDIT MIGRATION APPROACH TO LOAN VALUATION

In this section, we provide an introductory description of the framework used to value loans. We start with some of the basics of loan valuations that discuss the relationship between prices, spreads and duration as well as the impact of a prepayment option on these relationships. We next turn to a description of all the different types of embedded options found in loans. Then we discuss approach to credit migration and the lattice valuation methodology.

2.1 Loan Valuation Basics

In this section, we discuss the pricing of very simple loan—a term loan without a prepayment option. We then discuss how the prepayment option changes the value of the loan, its duration, and its convexity.

For a simple term loan that pays a coupon plus LIBOR, the value of the loan can be written in the following way.

$$P_t = \frac{QDF_t(1-LGD)+(1-QDF)(c+LIBOR+E^Q(P_{t+1}|no\,default))}{1+r} \qquad (1)$$

where P_t is the price of the loan today, QDF_t is the risk-neutral probability of default over the next period, LGD is the risk-neutral loss given default, $c + LIBOR$ are the payments made on the loan over the next period, r is the risk free rate and $E^Q(P_{t+1}|no\,default)$ is the expected value of the loan (computed under the risk-neutral measure) at the end of the next period given that the obligor did not default and that the payments have been made. Note that we assume the par value of the loan is 1 for ease of exposition.

Written this way, the value of the loan is decomposed into the discounted value in two states of the world: default and non-default. The value of the loan in default is equal to the discounted value of 1-LGD. The value of the loan in the non-default state is equal to the sum of the discounted value of the coupon plus LIBOR, plus the value of the loan after these payments have been made. The value of the loan today is the value of the sum of the value of these two states, weighted by their respective probabilities under the risk-neutral measure. With a simple induction argument, one can show that the price of the loan remains constant at par if the coupon, c, is equal to the risk-neutral expected loss, $LGDxQDF$.[4]

It is useful to express the value of the loan in terms of a second-order Taylor expansion:

$$P \approx 1+\frac{\partial P}{\partial s}(s-c)+\frac{1}{2}\frac{\partial^2 P}{\partial s^2}(s-c)^2 \qquad (2)$$

or

$$\frac{P-1}{P} \approx \frac{1}{P}\frac{\partial P}{\partial s}(s-c)+\frac{1}{2P}\frac{\partial^2 P}{\partial s^2}(s-c)^2 \qquad (3)$$

where s is the current market spread associated with the obligor.[5] The first derivative of the price of the loan with respect to the spread is a measure of duration, and the second derivative is a measure of convexity. It is common practice to

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[4] One sets $P_t = E^Q(P_t|no\,default) = 1$ and LIBOR equal to r. Solving for risk-neutral expected loss reveals $LGD \times QDF = c + QDF(c+r) \approx c$.

[5] This spread should be thought of as the market spread implied by a zero-coupon bond without a call option, and a comparable LGD. Often, this expansion is written with respect to the yield. In this context we are holding the interest rate constant so the spread is playing essentially the same role as the yield.

divide these values by the price of the loan, in which case they are referred to as modified duration and modified convexity. Duration relates the change in the price of the loan to the change in the spread, while a modified duration relates the return on the loan to a change in the spread. In the case of a bullet term loan without a prepayment option, it can be shown that the duration of the loan is equal to the Macaulay's duration, which is the weighted average of the maturities of each cash flow. The weights are determined by the present discounted value of the cash flows.[6]

Figure 3 provides an example of how the value of a non-prepayment loan that pays LIBOR plus a 500 bps coupon changes as the market spreads change. For simplicity, we assume that LGD is 100%. If the market spread is 500 bps, the loan trades at par. If the market spread falls to 250 bps, the value of the loan increases to $110, which is the duration of the loan multiplied by the change in the spread (i.e., the duration of the loan is approximately 4 years). The impact of a change in spread on the value of the loan decreases as the spread increases. One way to think about this is that as the spread increases, the duration of the loan decreases, which implies that the relationship between the market value of the loan and the spread is convex (the second derivative is positive).

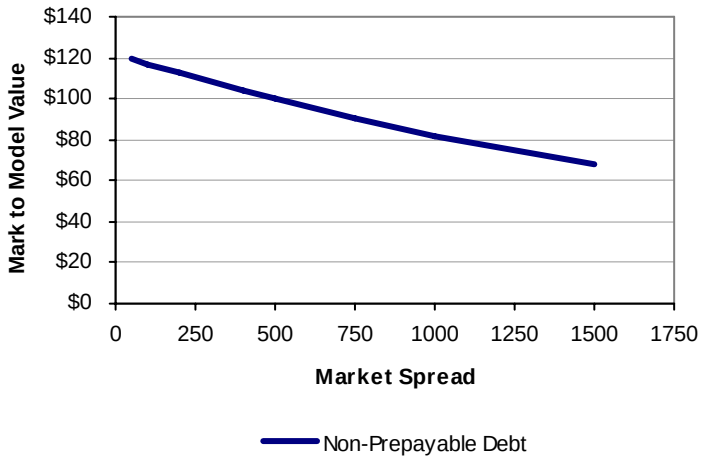

FIGURE 3 Loan Value and the Market Spread without a Pre-payment Option

Introducing a prepayment option changes this relationship. Figure 4 displays the change by valuing the above loan with a prepayment option through Moody's KMV CreditMark. Holding all other factors constant, giving the borrower the option to prepay the loan makes the loan less valuable to the lender. Therefore, the loan is valued below par when the coupon on the loan is equal to the spread. Further, as the spread falls, the value of the loan reaches a maximum value because of the prepayment option that is somewhat above par.[7] This reduces the sensitivity of the value of the loan to changes in the spread—the duration of a prepayable loan is less than the duration of a non-prepayable loan. Further, the sensitivity of the loan to the spread falls as the spread falls. Therefore, for a small spread the relationship between the value of the loan and the spread is concave rather than convex (the second derivative is negative rather than positive). Finally, note that as the spread increases, the value of the prepayment option falls. Consequently, the value of the prepayable loan and the non-prepayable loan converge. Quantifying all of these results regarding the prepayment option requires a model of credit migration, which we discuss in the next two sections.

[6] The concept of a Macaulay duration is typically applied to a fixed-rate bond. In the context of a floating-rate bond, the discounted value of each payment may be calculated using the spread plus a forward interest rate implied by swap curves.

[7] We model the borrower as choosing to prepay if the value of the loan, before prepayment penalties and costs, rises above par plus prepayment penalties, costs, and accrued interest. Consequently, par does not provide a strict upper bound for the value of the loan.

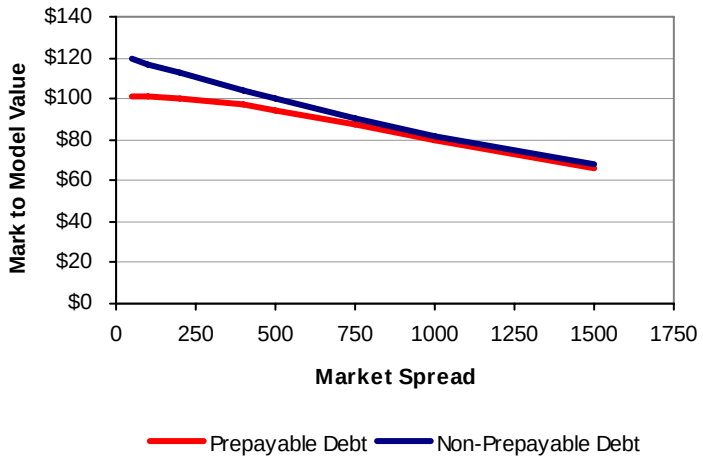

FIGURE 4 Loan Value and the Market Spread with and without a Prepayment Option

2.2 Embedded Contingent Claims in Loan Contracts

Loan contracts are usually designed to be flexible funding arrangements. This is why they have various mechanisms that facilitate flexibility (e.g., prepayment of a loan without penalty or drawings on a line of credit as needed). On the other hand, they also have mechanisms that protect the interests of the lenders (e.g., performance-based pricing grids or protective covenants). As we will see later, the presence of such mechanisms can have a substantial impact on loan values.

2.2.1 The Prepayment Option

In almost all loans contracts, the borrower has the option to pay the loan, at par, prior to its maturity date (the prepayment option). This is, by far, the most universal and the most important type of embedded option. In most cases, there is no penalty to prepayment. In a small minority of cases, a prepayment penalty may exist and it is likely to be specified in loan terms and conditions. Even when a penalty does not exist, all borrowers are likely to incur some prepayment cost, which may include legal and documentation costs, as well as costs of arranging a replacement loan.

The prepayment option in loans is similar in principal to the call option in callable corporate bonds in the sense that both grant the borrower the right to buy back the debt at a prespecified price (usually the par value). A borrower (issuer) is likely to exercise the prepayment (call) option when the value of the remaining payments on the loan (bond) goes above par, so that it is optimal for the borrower to find a cheaper loan to replace the existing loan. The value of a bond, which is commonly a fixed-coupon instrument, can change substantially with movements in default-free interest rates. In contrast to bonds, the main driver of loan value changes and hence the prepayment decision is the credit migration. If the credit quality of a borrower improves substantially relative to the time of origination of the loan, the borrower may choose to prepay the existing loan and refinance at a lower rate. For a line of credit, the prepayment option is essentially a cancellation option (i.e., it has the effect of canceling the line of credit). When the prepayment option is exercised, the borrower pays back the face value of any drawn amount plus any prepayment penalty and costs. The drawing right of the borrower (the usage option) and all of the fees associated with the line of credit cease at this point.

2.2.2 The Usage Option

The usage option refers to the borrower's right to draw and repay at will. This is embedded in a line of credit. The borrower can choose how much to borrow (up to the commitment amount, and at the prespecified rate known as the usage fee), when to borrow, and when and how much to repay during the lifetime of the line of credit. The right is subject to satisfying certain conditions by the borrower. Typically, draws are a function of a borrower's credit quality and are likely to increase as the credit quality deteriorates. Thus, the usage option can also be modeled using a credit migration framework.

2.2.3 Pricing Grids

Pricing grids, as well as performance-based pricing grids, refer to the contractual provisions under which the coupon rates (fees and spreads) on term loans as well as revolvers are made contingent on some measure of credit quality of the borrower or the level of usage of the loan. Such contingent coupon rates are prespecified in the loan contract and are commonly contingent on the borrower's credit rating, leverage ratio, or another financial ratio. Table 1 displays an example of a ratings-based pricing grid. As one can see, these provisions are in the nature of embedded credit-state contingent claims, and their valuation can also be measured using a credit migration model.

TABLE 1 Example of a Ratings-based Pricing Grid

Rating of the Borrower	Contractual Spread on the Loan
Baa2 or better	87.50 bps
Baa3	112.50 bps
Ba1	137.50 bps
Ba2 or worse	175.00 bps

2.2.4 The Term-out Option

A term-out option is another common type of option available in lines of credit. It grants the borrower the right to convert the drawn portion of a line of credit into a term loan of a prespecified tenor and at a prespecified margin. This option is often found in 364-day lines of credit, which are popular for regulatory reasons.

A credit migration framework is well suited to value loans in the presence of these embedded options. To reduce complexity, we focus on modeling only the prepayment option as a true option—at every node we evaluate the borrower's optimal exercise decision endogenously. We model the usage option by prespecifying a credit-state contingent usage schedule. Pricing grids are also in the nature of prespecified contingent claims. We do not consider the valuation of loans with the term-out options in this paper, but the framework presented is general enough to apply to their valuation as well. In the next section, we outline our credit migration model.

2.3 CREDIT MIGRATION

For the valuation of loans with various embedded contingent claims, we characterize the credit migration of the borrower at different horizon dates.[8] Many practitioners will use agency ratings to represent credit states and a Markov chain model based on a rating transition matrix to characterize the credit migration. Our migration framework, in contrast, proxies for the firm's underlying credit risk with a volatility-adjusted measure of leverage (i.e., its distance to default). We do want to point out, however, that while the concept of a borrower's Distance to Default (DD) comes out of a structural model of default risk (i.e., Moody's KMV Vasicek-Kealhofer (VK) model) an EDF measure of default risk is not required to value a loan. [9] Valuation can be based on the CDS or Bond spread of the name, the RiskCalc EDF, the agency rating or even the bank's internal rating for the obligor associated with a specific loan.

As *DD* values computed from the structural model are continuous, we need to divide them into a finite number of *DD* buckets to take advantage of the ease of discrete-state modeling. We model the evolution of discretized *DD* values as a Markov chain. Because this evolution can be graphically represented by a *DD* lattice (see Figure 5 for an illustration), we refer to our modeling approach as the lattice model. We calibrate the migration probabilities in the lattice to match the realized migration rates obtained from a large historical database of *DD* migrations. Specifically, to compute a borrower's probability of transition from a starting *DD* (at time $t=0$) to an ending *DD* (at the horizon date $t=H$), we track the *DD* values at horizon H for all the firms that fall in the given DD bucket at t=0. A frequency table of how *DD* at time t=0 migrates to various *DD* values at horizon H is calculated, thus yielding an empirical transition probability distribution (for a given horizon H and a given starting *DD* bucket DD_0). Figure 6 displays an example of estimated transition

[8] A more detailed treatment of the approach is found in Wen and Zeng (2003).

[9] For more details on this model, see Crosbie and Bohn (2004) and Kealhofer (2003).

probability function $Pr(DD_H>d|DD_0=d_0)$ for a horizon $H=1$ year and three different initial DD values $DD_0 \in (1.0, 1.5]$, $DD_0 \in (7.0,7.5]$, and $DD_0 \in (12.0,12.5]$. Because the data quantity is large in most DD buckets, the resulting distributions are smooth. It is worth pointing out that a lattice-based approach is preferable for modeling the credit migration behavior compared to a binomial or trinomial tree. First, DD tends to be mean-reverting, so it is more intuitive and tractable to model its migration over a finite set of states. Second, at each point in time, the probabilities for credit to migrate from its originating state to any other state, including the default state remain positive. A tree has non-trivial limitations in characterizing this type of transition behavior.

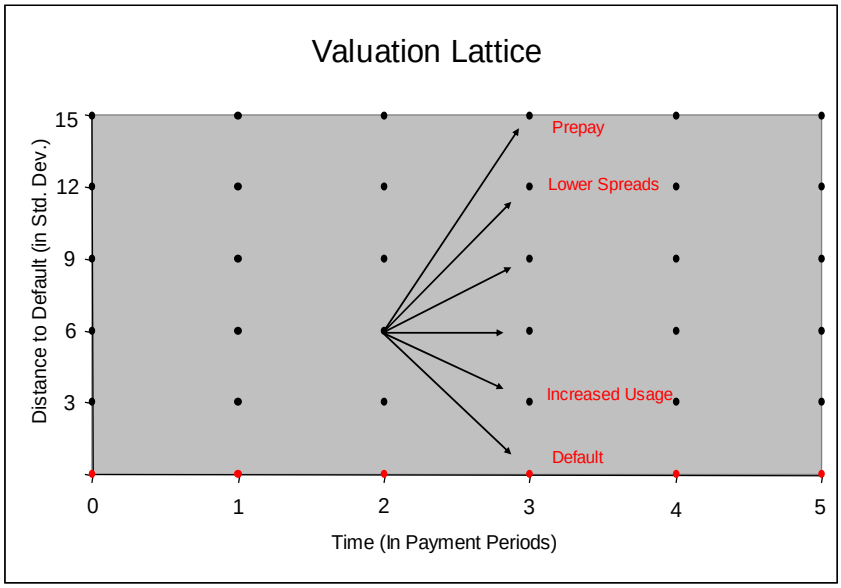

FIGURE 5 An Illustration of a Credit-state Lattice Based on a Firm's DD Measures

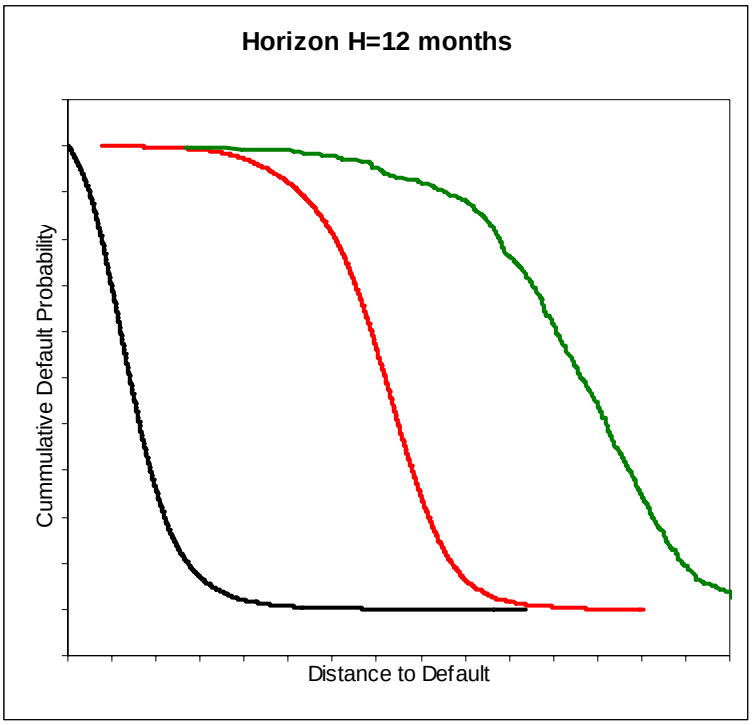

FIGURE 6 Example of Estimated Transition Probability Function for a 1-year Horizon and Three Different DD Values

The DD migration probabilities computed from the historical data above are under the physical probability measure. For valuation purposes, we need to specify the credit migration probabilities under the risk-neutral measure. The transformation of default probabilities (i.e., transition probabilities from a non-default state to a default state) is carried out using the following expression.

$$CQDF_{i,0,T} = N(N^{-1}(CEDF_{i,0,T}) + \lambda_m \rho_{i,m} \sqrt{T}) \qquad (4)$$

Here, $CEDF_{i0T}$ is the cumulative physical probability of default from $t= 0$ to $t=T$ for the ith firm, and is known to us from *DD* transition matrices. $CEDF_{i0T}$ is the cumulative risk-neutral (or "quasi") probability of default from $t=0$ to $t=T$ for the ith firm, λ_m is the market Sharpe ratio for the asset returns, ρ_{im} is the correlation between asset returns of firm i and market asset returns, and T is the time horizon under consideration. The function N and N^{1} refer to normal cumulative distribution function and its inverse respectively. This expression for transformation of physical default probabilities to risk-neutral default probabilities is obtained in a structural model of default by assuming a geometric Brownian motion process for the asset value and then adjusting its drift term to change from the physical measure to the risk-neutral measure.[10] A further assumption is needed that the expected excess returns are described by the Capital Asset Pricing model. We estimate ρ_{im} using asset returns computed from the VK model. The market Sharpe ratio parameter λ_m is estimated by fitting a simple EDF-based model of credit spreads on corporate bonds to the observed bond spreads of a large sample of bond issuers.[11] Risk-neutral transition probabilities to non-default states X can then be computed from taking differences of these computed risk-neutral transition probabilities of transition to state X or below. Risk-neutral transition matrices computed above are further adjusted to match with the borrower-specific CDS spread term structure or EDF term structure before they are used for valuation of loan cash flows.

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[10] See, for example, the "Risk Neutral Distribution" section of Vasicek (2002).

[11] For more details on this estimation process, see Kealhofer (2003) and Bohn (2000b).

2.4 Loan Cash Flows and Their Valuation

We compute the value of a loan using the standard risk-neutral valuation methodology. The loan value is simply the expected discounted value of the future cash flows, with the expectation computed under the risk-neutral measure and the risk-free rate used as the discount rate. To actually compute this value, the future cash flows in each time period and for each credit state need to be computed based on the terms of the loan, the credit states, and the choices of the borrower. Finally, a computational method is required to deal with the path-contingent nature of a loan prepayment.[12]

Term loans are mostly floating-rate instruments, and their coupons are specified as a fixed contractual spread over a reference rate, such as LIBOR. The coupons therefore, depend on the future realizations of LIBOR. Here, we use forward LIBOR as a proxy for future expected LIBOR for the purpose of computing the loan coupon payments. The lender incurs two costs when prepaying: the prepayment cost and the prepayment fee. Both are user-provided inputs that are stated as a percent of the principal. The prepayment fee is a fee that is paid from the borrower to the lender upon prepayment, and is contractually specified. It may change over the life of the loan. The prepayment cost is a *friction* that is lost by the borrower upon prepayment and does not accrue to the lender. We view these costs as administrative costs associated with refinancing.

For a revolving line of credit, the cash flows depend on the drawn amount. We model the drawn amount as being state-contingent. Each revolver has a commitment amount, which is the maximum amount the lender committed to lend over the life of the loan. The lender charges various types of fees. First, there is a recurring facility fee on the entire commitment amount.[13] Usually this fee is specified as a fixed fraction of the commitment amount (e.g., in bps/annum). It must be paid over the life of the line of credit even when the line is unused or only partially used. The actual drawn amount on any given date is called the usage level. The lender charges a usage fee (or a drawn spread) on the drawn amount, which is usually a floating rate, specified as a fixed spread over a reference rate, such as LIBOR. Additionally, there may be a commitment fee or a non-usage fee on the undrawn portion of the facility.

With these fees, before the cash flows to the lender can be computed, the usage level for any given credit state must be specified. In CreditMark, the user is required to input the usage level as a function of the borrower's credit quality. A lender is likely to have an expectation of what is *normal usage*, given the terms of the revolver and the nature of the borrowers business, in which case, the user may want to enter this value for the initial credit state. Most lenders expect the borrower's usage to increase as credit quality decreases. Most users will want to specify the usage schedule accordingly. Average usage behavior collected from internal and external usage studies can be utilized as well. Table 2 provides a sample usage schedule.

TABLE 2 Sample Usage Schedule

Debt Rating	Average Usage (%)
Aaa/AAA	0.4
Aa/AA	0.4
A	1.6
Baa/BBB	10.5
Ba/BB	33.2
B	45.1

The valuation procedure is simple in concept, but highly involved in implementation. We break the time interval between the valuation date and the maturity date into a discreet number of time periods. For each time period, there is a finite number of credit states where the borrower can be. For each credit state in each time period, a risk-neutral probability of moving from this state to another can be computed by utilizing the risk-neutral distance-to-default migration matrices. We start at the maturity date and determine the cash flows of the loan for each credit state. At the maturity date, the actual cash flows will depend on: (i) whether the loan is in default, (ii) the usage level associated with the credit state (for revolvers), as well as (iii) any credit state contingent pricing grids. We then step back one time

[12] For example, if we modeled a borrower with a 3-year loan as having 10 credit states and 12 time periods, there would be 1 trillion (10^{12}) possible paths that this borrower could experience over the life of the loan.

[13] There may also be an upfront commitment fee in addition to the recurring commitment fee.

interval, and for each credit state, we compute the cash flows in this period, as well as the expected present discounted value of the next period's cash flows under the risk-neutral measure. At this point, we encounter the first instance of a prepayment decision. The borrower can either prepay and incur one set of cash flows (the principal, the prepayment fees, the prepayment costs, and any outstanding coupons), or continue and pay this period's coupon, as well as the future cash flows. The borrower will prepay if the cash flows associated with prepaying are less than both this period's coupon, and the expected present discounted value of future cash flows under the risk-neutral measure. We continue to *walk backward* until we get to the valuation date. For each time period, we track the value of each loan for each credit state as the sum of the present discounted value of future cash flows under the risk-neutral measure, assuming that cash flows stop following a prepayment. In this fashion, we handle the path-contingent nature of the calculation by converting a complex multi-period choice problem into a matrix of two-period problems that can be solved through backward induction.[14]

The expected value of any prepayment costs are excluded from computing the value of the loan to the lender. As the prepayment cost has the effect of making the borrower less likely to prepay, a high prepayment cost will often increase the value of the loan to the lender by reducing the value of the borrower's prepayment option. For a revolver, we assume that the lender is able to finance the borrower's usage at the forward LIBOR rate.

This completes the description of the theoretical framework used to value loans that contain embedded options, such as prepayment options and usage options. We now turn to our empirical section in which we describe the data used to test the framework, the empirical implementation, and the results.

3 EMPIRICAL VALIDATION

In this section, we first describe the data used in the empirical validation. We then discuss the implementation decisions. We then compare the modeled prices to the actual prices, and the price difference between CDS and EDF valuations. Finally, we show how the model can be used to value an option, and how this value differs over time and across loan types.

3.1 Data on Loan prices

In the last few years, the secondary market for syndicated corporate loans developed rapidly. Trading volume grew, on average, by about 25% per annum from 1991 to 2005. As mentioned earlier, the growth has been driven by banks actively managing their loan books. An increasing number of non-bank institutional investors and CLO vehicles have been actively buying these loans for their investment needs. These investors find the leveraged portion of the market more attractive because of its higher yields. Leveraged loans thus dominate the secondary market activity and are the main focus of the empirical analysis of this paper. More and better data on secondary market prices recently became available. In this paper, we compare our model values against the indicative loan quotes obtained from the mark-to-market (MTM) price service operated jointly by the Loan Syndication and Trading Association (LSTA) and LPC. LSTA/LPC MTM prices (referred to as "LPC quotes," or simply as "loan quotes" for brevity) became the industry standard source of daily, third-party pricing data for the secondary market on loans.[15] Many institutional investors and loan dealers rely on LSTA/LPC MTM prices as their source of secondary loan market pricing data. These quotes thus offer a way to benchmark our model values, though important limitations remain and will be discussed in subsequent sections.

The end-of-the-day price quote data available to us includes mean bid price, mean offer price, and the number of quotes on bid as well as offer side. The means are taken across the quotes submitted by various dealers. We have this data available for a five-year period from January 2002 through December 2006. Table 3 provides counts of the data used in

[14] If we were to model a borrower with a 3-year loan using 10 credit states and 12 time periods, the one trillion possible paths are effectively condensed into 120 two-period problems.

[15] The LSTA/LPC MTM pricing service was created in 1999 to provide a set of indicative benchmark prices of loans active in the secondary market. Pricing analysts of LPC collect daily quotes from more than 30 dealers and then audit and aggregate these quotes to derive indicative prices.

this study. Figure 7 displays how the number of syndicated loans with price quotes available increased rapidly over this period.

TABLE 3 Number of Loans Used in the Study

Time Period	1/2002–6/2004	7/2004–12/2006	Increase
Loans	2,204	4,317	2,113
Borrowers	947	1,574	627
Term Loans	1,474	2,988	1,514
Revolvers	730	1,329	599
Public Firm Loans	1,170	2,232	1,062
Private Firm Loans	1,034	2,085	1,051

For our purposes, a private firm is a firm that does not have public traded equity and consequently, does not have an EDF credit measure that is derived using the stock price.

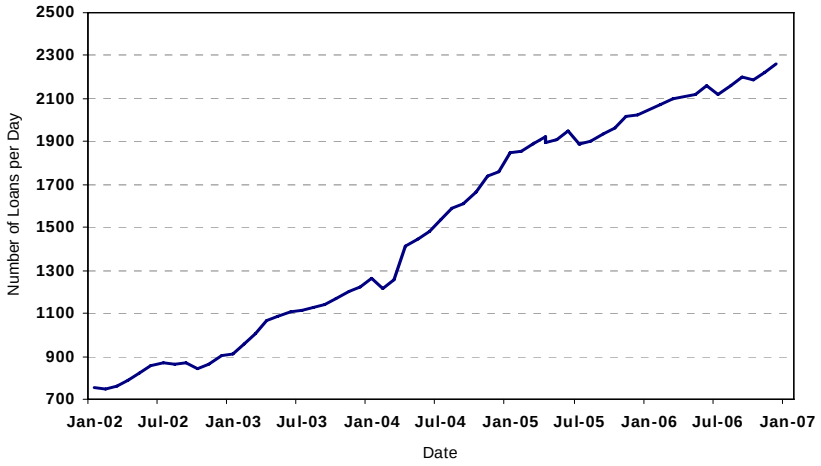

FIGURE 7 Number of Daily LPC Loan Quotes Over Time

Figure 8 presents the distribution of loan quotes over time. Specifically, it presents 10th, 25th, 50th, 75th, and 90th percentiles of the LPC market quotes over time, where the outer blue whiskers are the 10th and 90th percentiles, and the inner red whiskers are the 25th to 75th percentiles. Finally, the center red bar is the 50th percentile. It is clear that after 2004 loans started trading closer to par, which is likely the result of tightening credit spreads during this time period. Note that loan quotes do not rise much above par, even in an improving credit environment. This is clear evidence that the market participants factor in the prepayment option in pricing these loans. Figure 9 presents the distribution of the bid-ask spread over time which tightened substantially over the time period—the 75th percentile bid-ask spread fell from $2 in 2002 to less than $1 in 2001. This tightening of bid-ask spreads is indicative of improved liquidity in this market.

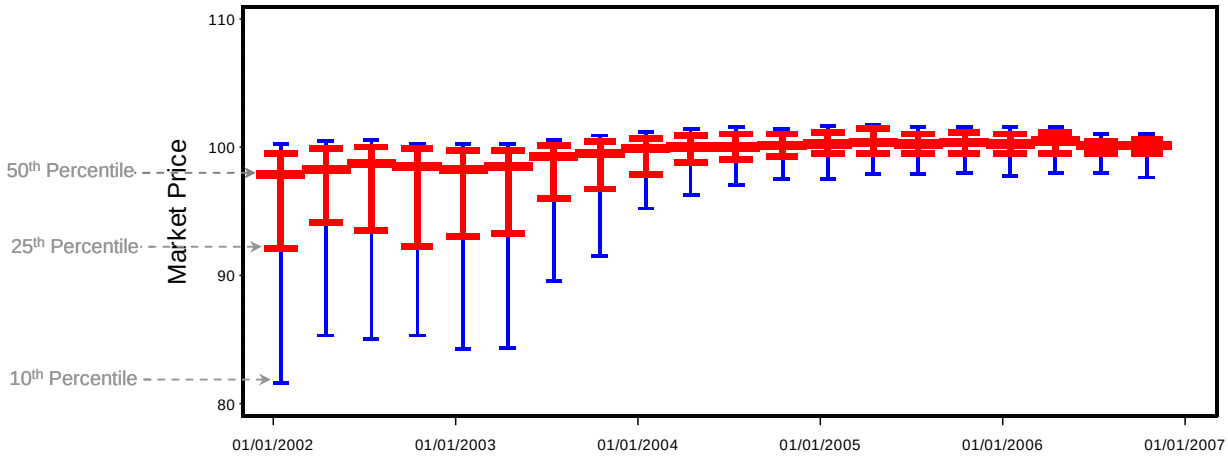

FIGURE 8 Distribution of LPC Quotes Over Time

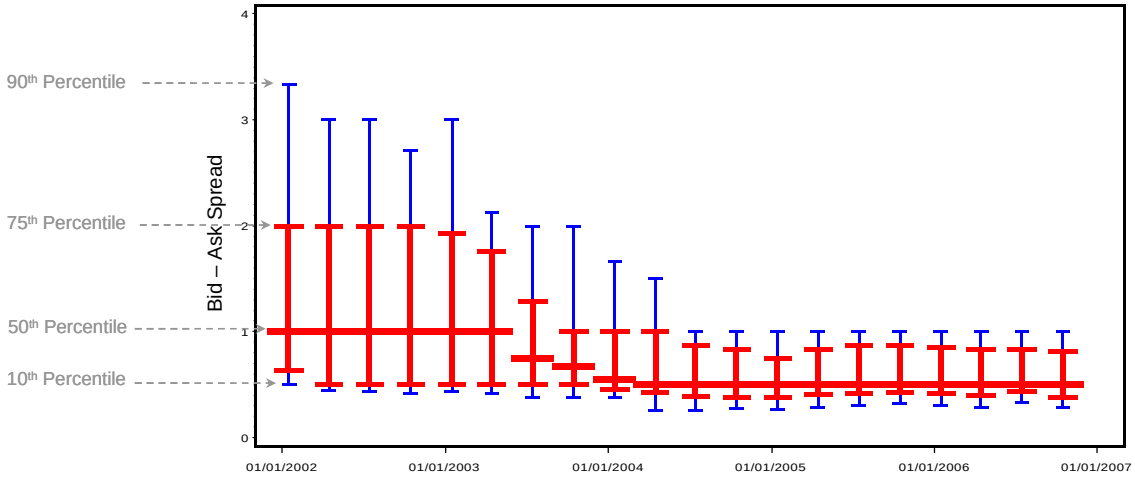

FIGURE 9 Distribution of Bid-Ask Spread Over Time

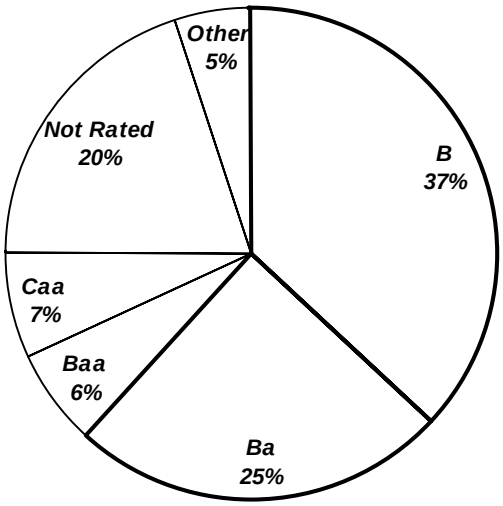

FIGURE 10 Sample Distribution of Moody's Ratings

Figure 10 displays the distribution of Moody's letter ratings of LPC loan quotes over our sample period from 2002 to 2006. It is clear that most of the secondary market activity is concentrated in sub-investment-grade loans, particularly those rated Ba and B. This study, therefore, focuses on sub-investment-grade names. Figure 11 displays the distribution by industry. The most common industries include the General group, followed by Consumer Goods and Durables, Materials/Extraction, and Telecom. There are only a trace of loans to Banks and S&Ls.

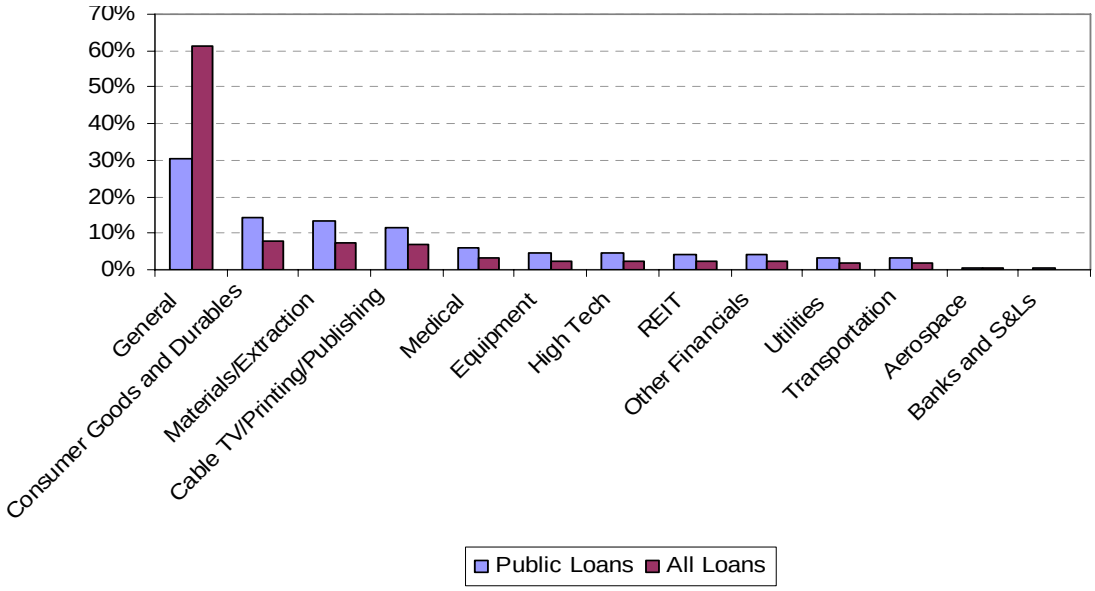

FIGURE 11 Sample Distribution by Industry

The General sector includes the following industries: Agriculture, Business Products Wholesale, Business Services, Construction, Consumer Services, Entertainment and Leisure, Hotels and Restaurants, and Utilities NEC.

3.2 Empirical Implementation

Using loan terms and conditions data from LPC, we implement the above loan valuation model for a large sample of loans.[16] To facilitate comparison, we choose only those loans for which we could obtain market quotes. Our model valuations are performed at monthly frequencies from January 2002 through December 2006, thus covering a five-year period. The first half of the sample captures the bad credit cycle, while the second half captures an improved credit environment. Figure 12 displays the 90th percentile, median, and the 10th percentile of EDF credit measures for the sample period. EDF credit measures started decreasing after December 2004.

We analyze term loans (bullet term loans and custom term loans) as well as revolving credits. We discard specialized loan contracts, such as letters of credits, revolving credits with term-out options, etc. For borrowers having a quoted CDS spread, we use this spread as the anchor point for computing credit curves.[17] We valued 863 loans using CDS spreads from Markit. For borrowers with public equity, we also use EDF credit measures as our estimates of default probabilities. For private borrowers (borrowers without publicly traded equity) we use a rating-implied EDF credit measure, computed using an empirically calibrated rating-to-EDF credit measure mapping. Loss given default for each borrower is estimated using the Moody's KMV LossCalc™ model.[18] Parameters needed to transform the physical transition matrices to risk-neutral transition matrices, such as the market Sharpe ratio, λ_M , and correlation between borrower's asset return and market return, ρ_{im} , are also estimated by Moody's KMV.[19] These parameters are necessary when the loan is valued from EDF credit measures. When the user provides credit spreads, these parameters play only a minor role in the loan valuation. In the absence of any usage information for revolvers, we assume the contingent usage schedule in Table 2 where the usage level increases as the credit quality of a borrower declines.

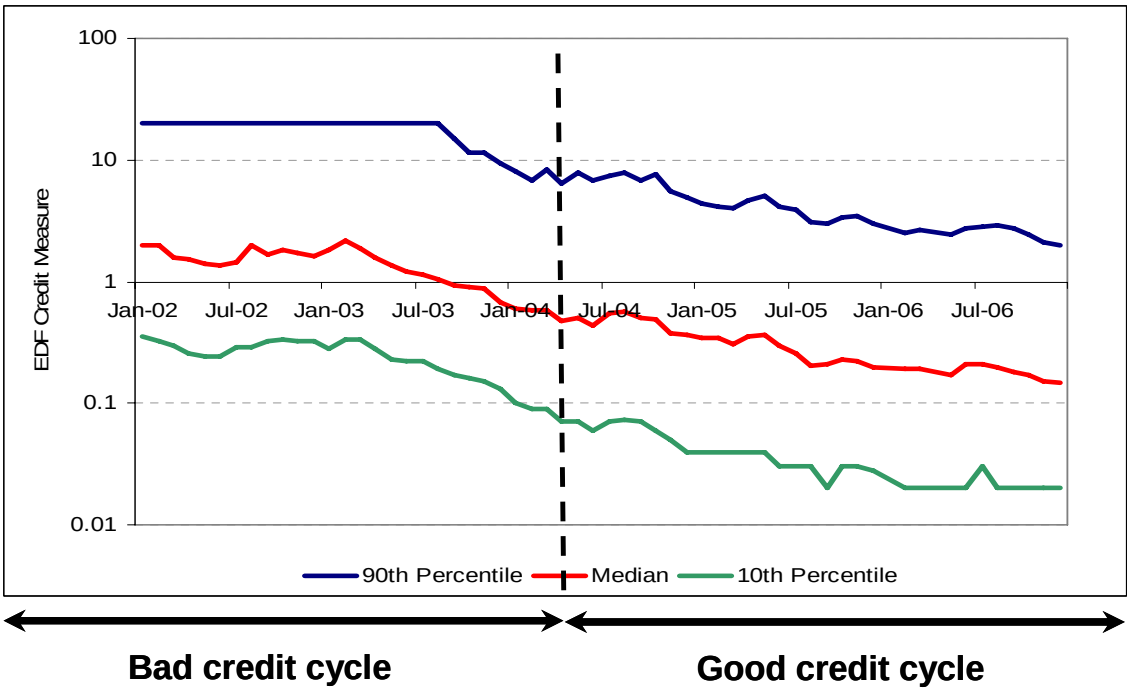

FIGURE 12 EDF Credit Measure During the Sample Period

[16] See Bohn and Zeng (2003) for validation results using data from LoanX.

[17] The CDS spread typically refers to a class of senior unsecured bonds. We do not use the LCDS spreads because there has been very little liquidity in this market until recently.

[18] See Gupton and Stein (2005) for more details of this model.

[19] See, for example, Bohn (2002) and Kealhofer (2003b).

3.3 Results

In this section, we analyze two different variations of the model and their relationship with actual prices. We first compare the CDS-based model price to the LPC price, and then we turn to the EDF-based model price, which we compare to both the LPC price and the CDS price. We present two examples of where the model price is very different than the actual price for different reasons. We then show how the value of the option differs by loan type and changes over time. Finally, we show that the option value—as measured by our model—is related to the actual incidence of loans being repaid.

3.3.1 Comparison of CDS-implied Loan Values and LPC Market Quotes

We calculate the difference between model and market values as the difference between model value and the nearest (bid or ask) LPC quote. If the model value is within bid-ask spread, we set the difference to zero.

We also imposed further restrictions on our sample: we considered only loans that belong to public companies, with time to maturity between one and five years, and EDF credit measures less than 5%. This sub-sample represents more than 70% of the entire sample. We excluded loans for which the EDF credit measure was greater than 5%, because these loans tend to trade on an estimate of the recovery value.

We first compare CDS-based model values for public firms versus LPC quotes. Figure 13 displays the distribution of differences between LPC quotes and CDS-based model values. We break down results by loan type: term loans and revolvers. For the CDS-implied loan values, 72% and 64% of the revolvers and term loans are within $1 of the actual stated price, respectively. Further, 84% and 73% of the revolvers and term loans are within $2 of the actual stated price.

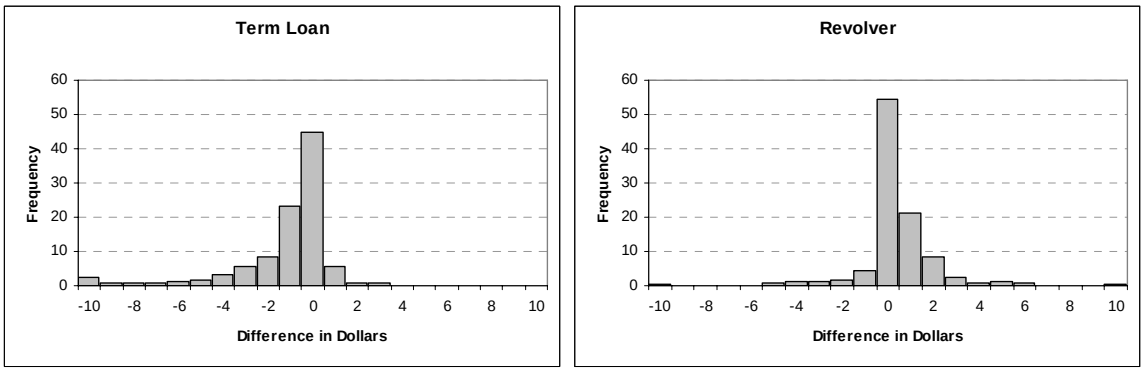

FIGURE 13 Distribution of Differences between CDS Implied Loan Value and LPC Market Prices

3.3.2 Comparison of EDF-implied Loan Values and LPC Market Quotes

In Figure 14, we compare the EDF-based price to the actual price for both term loans and revolvers in our sample. In this figure, we show that 59% of the revolvers and 77% of the term loans are within $1 of the actual price while 77% of the revolvers and 89% of the term loans are within $2 of the actual price.

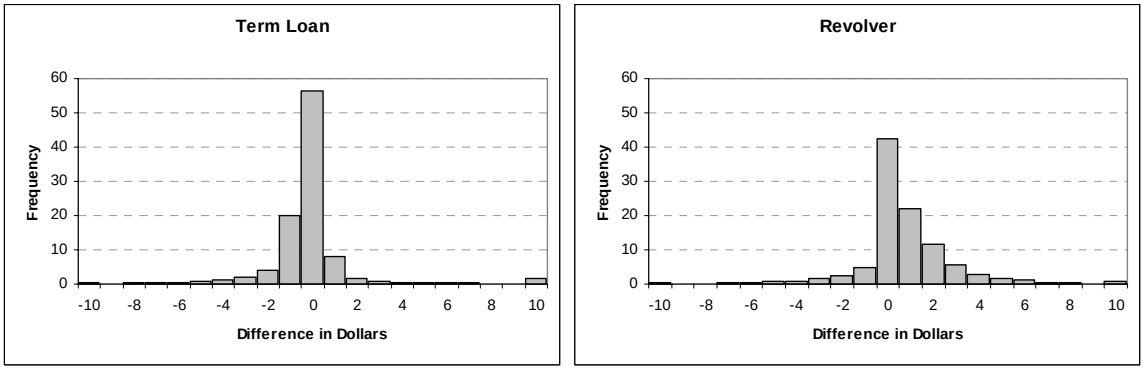

FIGURE 14 Distribution of Differences between EDF-implied Loan Values and LPC Market Prices

3.3.3 Comparison of EDF- and CDS-implied Loan Values

We valued a set of loans using two alternative estimates of default probabilities: EDF credit measures and CDS spreads. The coverage for these alternatives is rather different. The number of loans for public companies valued with EDF credit measures is almost three times more than with CDS. We valued 2,622 facilities with EDF credit measures. However, only 863 facilities had CDS spread data that we could use in the valuation.

We then constructed a common sample to compare the model values. As you can see in Figure 15, the loan model values based on two alternative measures of borrower's credit quality are quite close to each other.

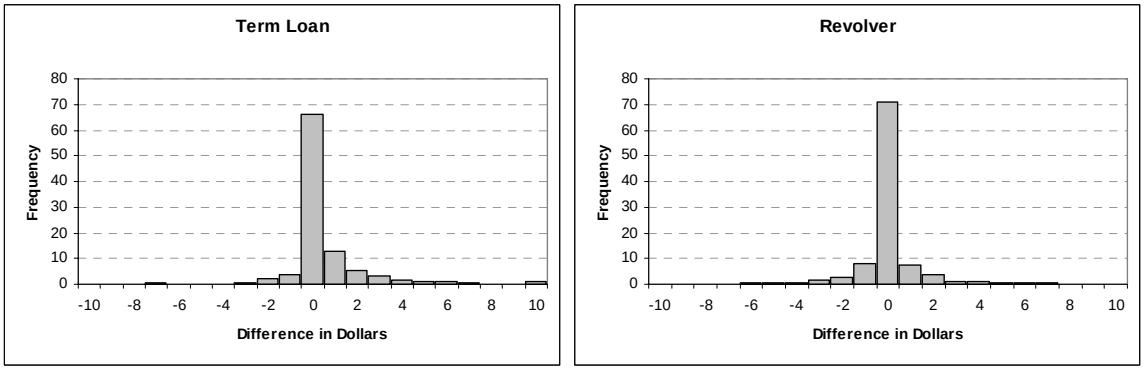

FIGURE 15 Distribution of Price Differences Between EDF- and CDS-implied Loan Values for Term Loans and Revolvers

For the term loans, 76% of the price differences are within $1 and 87% are within $2. The results are similar for revolvers: 78% of the price differences are within $1, and 88% are within $2. It is perhaps surprising how the value of the loan is similar between the EDF-based model price and the CDS-based model price.

3.3.4 Reasons Why Model and Market Prices Will Differ

When we compared LPC quotes against EDF- or CDS-based values, we noticed that there are some large differences between model and market prices. Our further and detailed examination of large differences suggests possible reasons for that phenomenon. The main cause is the staleness of loan prices. Loan values are expected to be more stable than, for example, bond values. This is because loans do not carry interest rate risk, they have lower LGD, and they have prepayment options. Also, the price of a loan can become "stuck at par." If the CDS spread on a loan falls to a low enough value, the value of the loan will hit an upper bound—there is an upper bound to what investors will pay for a loan that embeds a prepayment option at a price of par plus accrued increased and any prepayment penalty.

Figure 16 displays the distribution of monthly changes in loan quotes, which is exactly zero for 47% of the time. There is a high incidence of stale quotes. Market quotes continue to be stale in the second half of the sample. We also checked the loans that are trading at less then $98. Those loans appeared to be even staler than the whole population, with 50% of monthly changes being exactly zero. This suggests that the quotes by themselves will not always be sufficient for Mark to Market purposes.

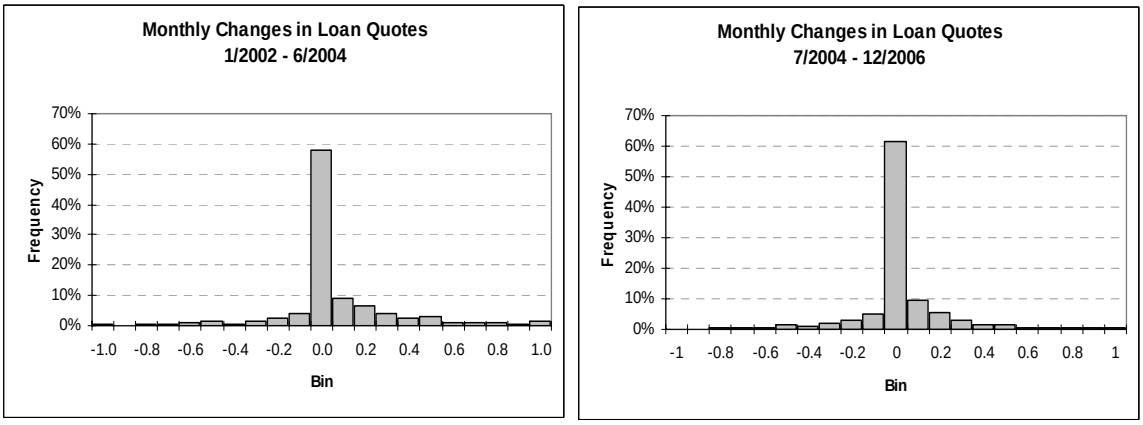

FIGURE 16 Distribution of Monthly Changes in Market Quotes for Two Sub-samples

Most loans in the LPC database are quoted by only one or two dealers. For CDS quotes, such prices are considered ineligible for inclusion in databases, such as from Markit. Figure 17 displays how 60% of composite quotes are based on contributions of only one or two dealers, and 75% are based on three or fewer contributions.

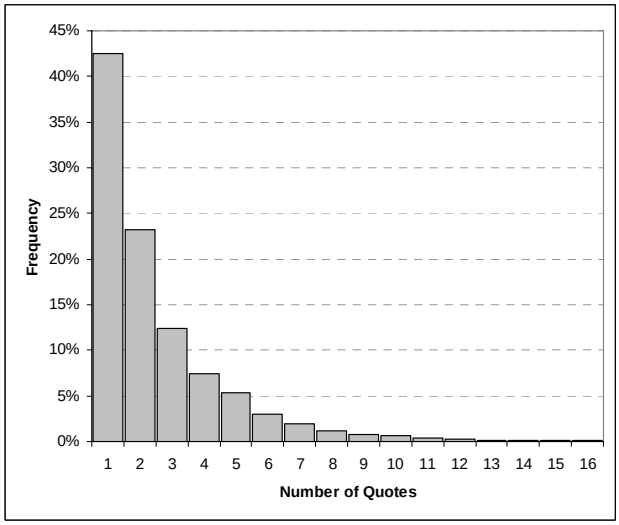

FIGURE 17 Distribution of the Number of Quotes

Large differences tend to be mostly idiosyncratic. They need to be examined on a case-by-case basis. We consider two examples. The first example is Continental Airlines. The facility was a four-year term loan with a maturity date of 1/12/2006. This loan had a floating coupon of 75 bps over LIBOR and an LGD of 51%. Figure 18 presents the time series of the CDS model value, the bid-ask quotes, and CDS spreads for this loan. Both the CDS spread and the loan price are very volatile early on, but from 9/2003, the price becomes stuck at a few dollars below par while the CDS spread was as a high as 2,000 bps. One interpretation of the price trading consistently at close to par despite an elevated CDS spread is a stale price.

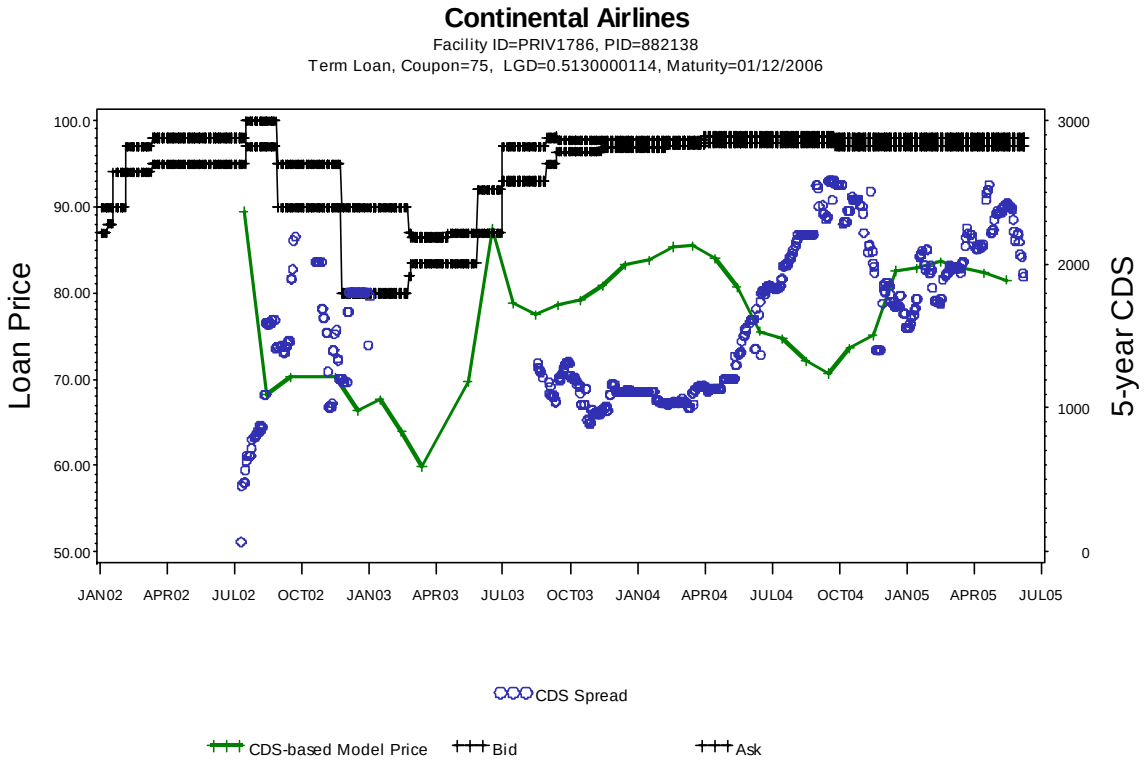

FIGURE 18 CDS-based Model Values, Bid and Ask Quotes, and CDS Spread for Continental Airlines

In addition to stale prices, other possible reasons for divergence of market and model values include: incorrect terms and conditions information, incorrect assumptions about prepayment, usage, or LGD levels. The next example illustrates how generic assumptions about inputs, such as prepayment frictions and usage levels, can cause large differences. Figure 19 displays the CDS-based model values, bid-ask quotes, and CDS spread for Aquila, Inc. The facility that we consider is a three-year term loan with a maturity date of 5/15/2006. The coupon paid 575 bps over LIBOR.

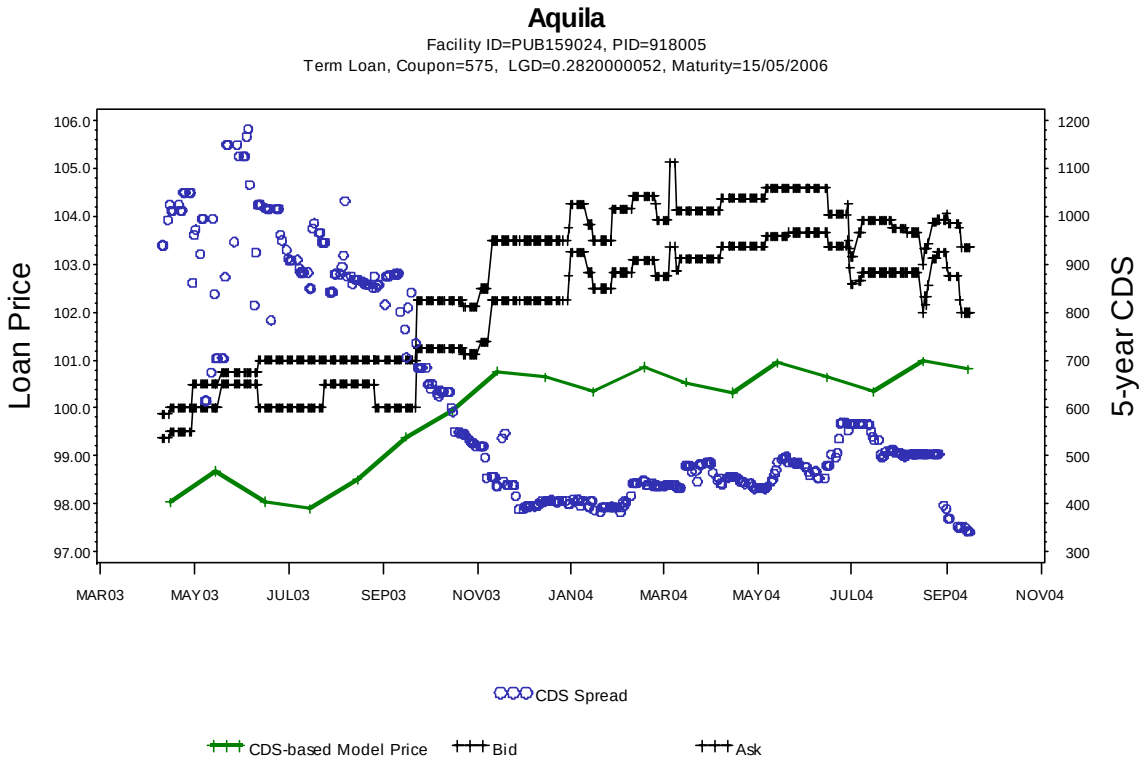

FIGURE 19 CDS-based Model Values, Bid and Ask Quotes, and CDS Spread for Aquila, Inc. Term Loan

In June 2003, Aquila's subsidiary MEP Pleasant Hill, in which Aquila had 50% interest, was unable to refinance or repay $270 million of construction loans prior to their June 26th maturity. In response to the default, the borrower has drawn on letters of credit pledged by Aquila to support the loan. This action was anticipated and did not impact Aquila's near-term liquidity. Nevertheless, as seen from Figure 19, the CDS spread was high—1,200 basis points reflecting the turmoil around the company. However, the loan quotes were above par. At that time (April–June 2003), the difference between market and CDS-based model values was about $2. Credit quality of Aquila improved by February 2004 (CDS spread dropped to 400 bps) and both model and market prices rose. Market price rose to $103 before the loan was prepaid. This fact suggests existence of high prepayment friction.

3.3.5 Value of the Prepayment Option

CreditMark can compute the value of an embedded option in a loan. We valued loans twice: allowing prepayments and disallowing prepayments. We then calculated the value of the prepayment option as the difference between model value without prepayment and model value with prepayment.

The results presented on Figure 20 show how the value of a prepayment option can be substantial. The upper two panels present the CDS-based pricing, while the lower two panels use the EDF-based pricing. The two left panels present the option value for revolvers, while the right panels present the option value for term loans. The EDF sample has a much larger proportion of term loans compared to the CDS sample. Term loans are likely to have larger option values. In our EDF sample, 71% of term loans have a prepayment option of 1.5% of face value or more. In the CDS sample, 58% of term loans have a prepayment option value of 1.5% of face value or more. For revolvers, the percentage of loans with a prepayment option value of 1.5% of face value or more is smaller: 11% in CDS sample and 27% in EDF sample. The value of a prepayment option is smaller for a revolver than for a term loan, because even without the prepayment option, the borrower can reduce his usage if his credit quality improves.

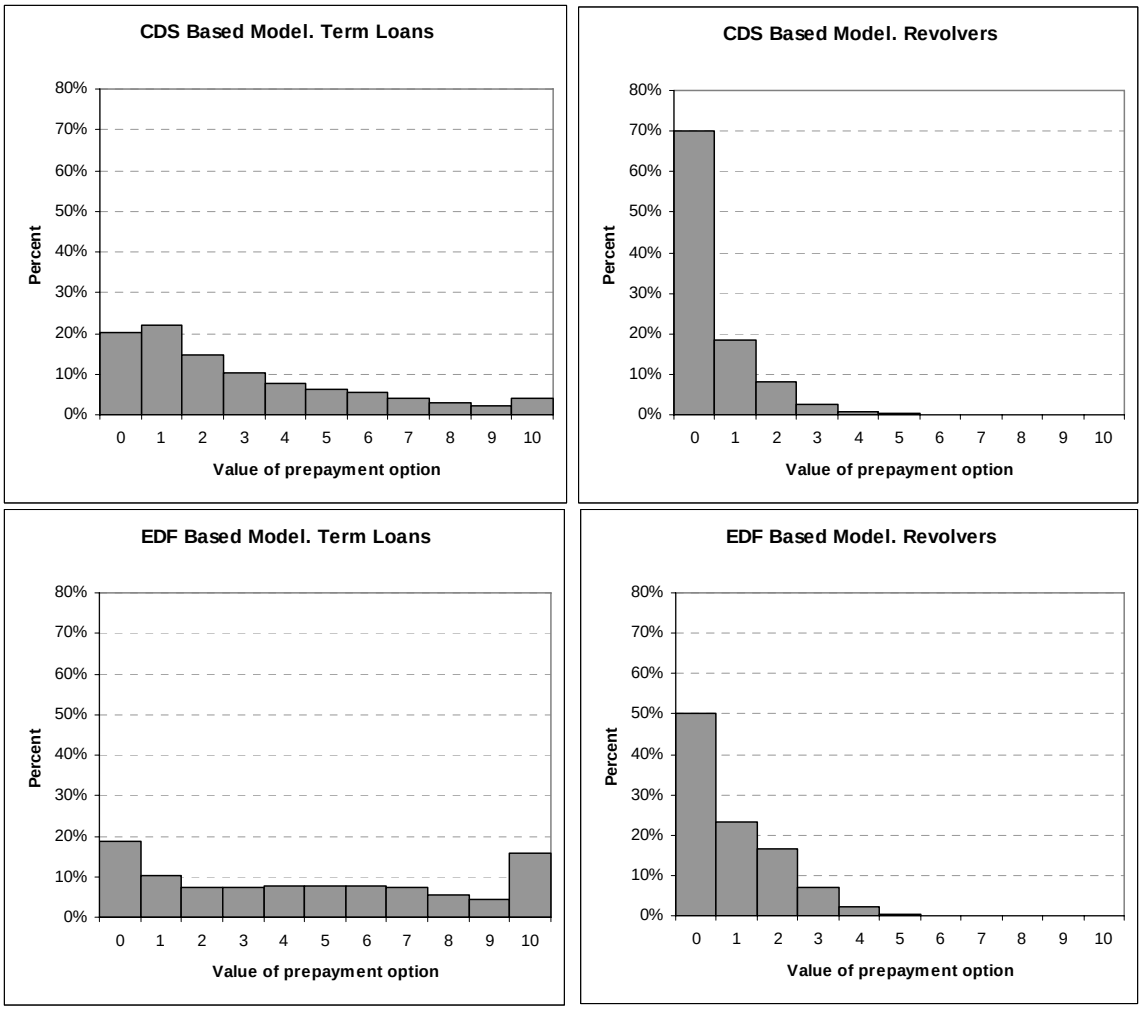

FIGURE 20 Value of Prepayment Option for EDF- and CDS-based Model Values by Loan Types

We can see from Figure 21, which displays median values of prepayment options for term loans and revolvers over time, that the prepayment option is much more valuable in a good credit environment than in a bad credit environment. Loans were priced according to the credit state at the origination date, which was during a bad credit environment (January 2002–April 2003) but EDF/CDS spreads improved since then, so valuation is giving us significant value of the prepayment option. Revolvers usage drops in a good credit environment, and as a result, the increase in the option value is relatively small.

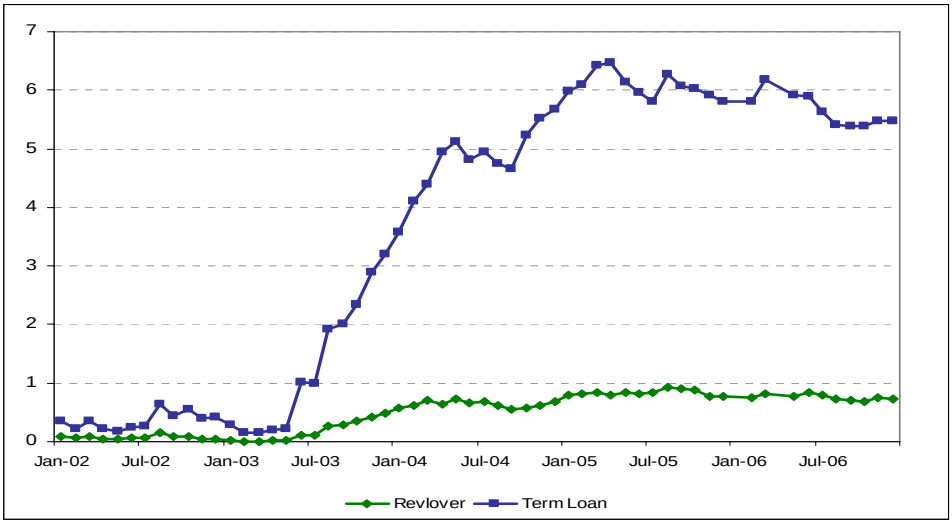

FIGURE 21 Median Values of the Prepayment Option for Term Loans and Revolvers Over Time

We then ask the following question: Are the loans with high prepayment options in fact more likely to prepay? While we do not directly observe whether a loan was prepaid in our data set, we can infer that the loan was prepaid under the following scenario. Suppose we observe a term loan with a price of greater than $95 and years to maturity greater than 12 months. Further, we observe liquidity as measured by more than three quotes on the loan. If the loan suddenly disappears from our data set, we infer that it was prepaid.

In Figure 22, we bucket the loans into 20 groups according to the value of the option. The horizontal axis represents the percentile of the option value, while the vertical axis represents the prepayment rate as inferred using the criteria enumerated above. Each dot represents the prepayment rate for each bucket. A clear upward pattern is discernible. We fit a line through the relationship using a statistical technique known as local regression (or LOESS). The loans with the lowest option values prepay at a rate of about 5%, whereas the loans with the highest option value prepay at a rate of roughly 12%. Despite the crudeness of our measure of the sensitivity, we do see loans that we measure as having a high prepayment option value are in fact more likely to be prepaid.

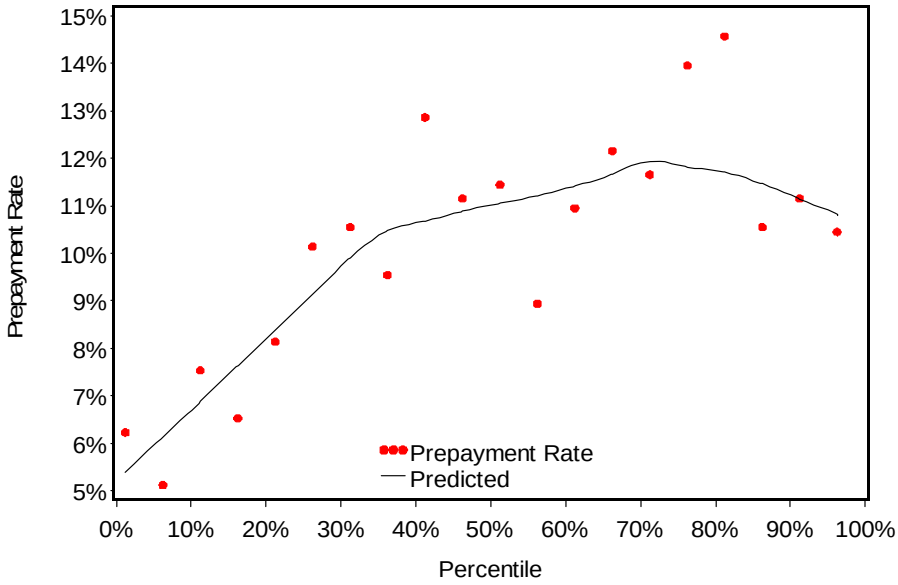

FIGURE 22 Prepayment Rate and the Value of the Embedded Option

4 CONCLUSION

Loans are structured in many different ways. Terms loans pay a coupon on top of a floating interesting rate for a specified period of time, but the borrower has the option to prepay. Revolvers are lines of credit for which the borrower pays a usage fee and a facility fee. Revolvers can be canceled. There are pricing grids and term out options. Sometimes there is a prepayment penalty associated with a prepayment option. To rigorously model these options, a concept of credit migration is required.

In this paper, we showed how these options can be rigorously modeled using DD dynamics. We have shown that the model prices produced by the framework line up reasonably well with actual traded prices. We also showed how the option value can represent a significant portion of the loan value. The option is more important for term loans, and the option is more important as credit quality improves. Finally, we showed how loans with a high option value are in fact more likely to be prepaid.

Such a framework is highly valuable for marking a loan to market. As more options to hedge loans develop, an accurate mark-to-market framework will be essential for hedge accounting. Further, when negotiating the terms and conditions of a loan, the lender now has various tools for assessing the value. These tools include, but are not limited to, the following: a prepayment penalty, the option to change the mix of usage and facility fee, and the incorporation of a pricing grid.

We also demonstrated how CreditMark, used with appropriate care, is effective at marketing to market a loan that does not have identical assets trading in active markets. We demonstrated how the model prices generated by CreditMark are very sensible when compared to actual prices on traded loans when available.

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