AUGUST 2020

MOODY'S ANALYTICS

MODELING METHODOLOGY

Moody's Analytics EDF-Based Bond Valuation Model Version 2.0

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Acknowledgements

We would like to thank Sergey Maslennikov, Jeffery Ding and Pooya Nazeran for their contribution to this version of the framework. We thank James Edwards, Min Ding and Michael Zeng for their assistance and counsel during the development of this research.

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Abstract

This paper provides an overview of the Moody's Analytics model of bond spreads. The primary input of the model is Moody's Analytics Public Firm EDF™ (Expected Default Frequency) credit measure. Using a risk-neutral valuation paradigm to price bonds, we derive a framework to link the bond spread with the EDF measure, LGD (loss given default), and other market-wide parameters. Calibrated on a sample of relatively liquid bonds, the model generates modeled spreads, which we call the Fair Value Spread (FVS), that are consistent with market spreads. In addition, we find FVS are a valuable signal for relative value investing strategies: model portfolios consisting of bonds with option-adjusted spreads in excess of their FVS outperform the broad market over time, even after allowing for transaction costs. We attribute this outperformance to pricing inefficiencies in the credit markets identified by our approach. Building on its predecessors' established foundation, the Bond Model version 2.0 adds more granularity to the estimation process with highlights in the improvement of fitness in the high yield space. The model achieves improved performance in the mark-to-market pricing of bonds and the identification of investment strategy in mispriced bonds.

Contents

1. Introduction

A model of credit spreads is essential for bond pricing, which can be used for Fair Value Accounting of illiquid assets and for credit portfolio management. In this paper, we discuss the methodology of Moody's Analytics Bond Valuation Model.

The main driver of the credit spread is the probability of default. When a traded or quoted price is not available or is subject to mispricing, it is common to use prices of comparable bonds with similar credit ratings to account for probability of default. However, this approach does not work for unrated bonds. When ratings are available, matching other characteristics along multiple dimensions may not always be possible. Our model of credit spreads solves these problems by employing Moody's Analytics EDF credit measure as the estimated probability of default.

Moody's Analytics public firm EDF credit measure[1] has been the standard metric of credit risk for many years. The public firm EDF metric is a structural model that evolved from the work of Black, Scholes, and Merton.[2] Various studies demonstrate that the EDF metric is a powerful predictor of defaults.[3]

Our model takes the EDF measure of the bond issuer as its primary input and produces the Fair Value Spread (FVS) of the bond as its primary output. We derive an equation linking credit spread with the EDF measure, LGD (loss given default), and market parameters by discounting expected cash flows of the bond in the risk-neutral pricing framework.[4] The model is then calibrated on a sample of liquid bonds by setting sector-level LGDs and market parameters so that modeled spreads on average match market-observed spreads. Once calibrated, the model can compute the Fair Value Spread (FVS), which is the modeled bond spread that is consistent with the EDF measure of the issuer, bond terms, and broad market conditions.

The original Moody's Analytics bond model has been in production since 2004. The model incorporated a number of extensions and enhancements over the years. In 2010, Moody's Analytics launched a CDS-implied EDF model that adapted the bond model framework to the CDS market.[5] This framework produces CDS-implied FVS as well as spread-implied EDF credit metrics. In 2014-2015, Moody's Analytics launched a new version of the EDF model (EDF 9), and we recalibrated the parameters of the Bond Valuation Model by applying the same framework to EDF 9 inputs.[6] We also expanded the model to cover Floating Rate Notes (FRNs).[7] This paper documents new model developments and updates the previous methodology papers.[8]

While the previous version of the model was designed to fit US investment-grade bond spreads, the Bond Model version 2.0 takes into account regional differences. Specifically, model parameters are now calibrated on regional levels: North America, Europe, Japan, and the rest of the world. This ensures that in each individual region, modeled spreads are consistent with observed spreads. We also improved the model to better fit high-yield bonds, secured and subordinated bonds, and bonds issued by smaller companies. The model is now calibrated on a sample that is broader and better aligned with bond indexes, resulting in more accurate pricing for bonds that are likely part of clients' portfolios.

We have significantly increased the number of bonds covered by the model. Previously, we did not cover bonds for which pricing data was not available from vendors. We have now implemented additional valuation steps within the model that allow us to approximate the duration and compute spreads for these bonds. This has increased the average daily FVS counts from roughly 80,000 to approximately 410,000. In addition to expanding our spread universe, this approximation can serve as the primary source of pricing data where it is otherwise not available.

We also improved our unique historical mapping between the bond and the public parent of the issuer. When the bond is issued by a private subsidiary of a public company, we compute the bond spread using Moody's Analytics equity-based probability of default (EDF measure) of the public parent. This requires keeping track of corporate family trees over the years, as they can change due to

[1] Crosbie and Bohn (2003), Dwyer and Qu (2007), Chen et al. (2015), Nazeran and Dwyer (2015), Jeffrey Bohn and Korablev (2005)

[2] Black and Scholes (1973), Merton (1973), Kealhofer (2003a), Kealhofer (2003b)

[3] Nazeran and Dwyer (2015), Kealhofer and Kurbatt (2002), Miller (1998)

[4] Huang and Huang (2012) find that credit risk accounts for only a modest fraction of the observed bond spreads for investment grades using a wide variety of structural credit risk models. Our model, however, employs credit risk drivers to explain a significant fraction of the observed bond spreads. The key differences between the model settings tested in Huang and Huang (2012) and our bond model include the following: our EDF credit measure is granular at the issuer level, and we have granular estimates for market Sharpe ratios and LGDs. For example, our LGDs are estimated by region, sector, and debt seniority. In addition, our model has the size premium factor, which reflects the difference in liquidity to a certain extent. These granular features help establish the explanatory power of our model for observed bond spreads. We also compute bond spreads as option-adjusted spreads over the constructed zero-EDF curves, whereas spreads are constructed relative to treasuries in their study.

[5] Dwyer et al. (2010)

[6] Nazeran and Dwyer (2015), Chen et al. (2016)

[7] Edwards (2015)

[8] Agrawal, Arora, and Bohn (2004), Chen et al. (2016)

M&A activity. While the "as-of-today" snapshot mapping has always been part of our process, model changes and expansion of coverage raise the need for backtesting and recalculations with a historically accurate mapping.

As a result of the improvements mentioned above, the new generation of the bond model achieves significant performance improvement in mark-to-market pricing, portfolio management, and the investment strategies developed based on FVS.

The remainder of this paper is organized as follows:

Section 2 describes the practical applications of the model.

Section 3 presents the model framework.

Section 4 describes the data used to build the model.

Section 5 describes the estimation of model parameters.

Section 6 presents the model performance in consistently estimating bond spreads that align with observed market spreads.

Section 7 introduces an investment strategy by trading on the alpha factor generated by FVS at the portfolio level and discusses the performance of this strategy.

Section 8 discusses related EDF-based valuation models for floating rate notes and credit default swaps.

Section 9 concludes.

2. Practical Applications of the Framework

Bonds are complex securities, and their prices reflect many different drivers. Drivers include interest rates, default risk, recovery risk, the tenor of the bond, and the extent to which the risk of the bond is systematic. Default risk itself is complicated, as the impact of the default risk of a specific issuer on credit spreads may be mitigated by the market's perception of an implicit or explicit guarantee by a parent company and/or the government. Moody's Analytics bond valuation framework enables users to establish what the spread in excess of an appropriate reference rate should be after accounting for the borrower's default risk. The key applications of such a framework include pricing illiquid assets and portfolio management.

2.1 Pricing Illiquid Assets

Pricing illiquid assets in the context of accounting is known as mark-to-market or Fair Value Accounting. For example, Statements of Financial Accounting Standards No.157, Fair-value Measurements (commonly known as FAS 157), establishes a hierarchy of valuation methodologies. The hierarchy gives first priority to using actual prices for identical assets in active markets, when available, for establishing a fair market value (Level 1 inputs). Second priority is given to valuation methodologies based on inputs that include a combination of prices from inactive markets on identical assets, prices of similar assets from active markets combined with observable characteristics of the asset, and market-corroborated inputs (Level 2 inputs). Lowest priority is given to unobservable inputs, including the firm's own assumptions regarding how the market would view a particular asset if it were to trade.

The fair-value spreads produced by our framework can be viewed as Level 2 inputs. The fair-value spread takes information from a liquid market (equity prices) and creates an estimate of what the spread on debt would be if the debt were to trade actively, using the characteristics of the debt and aggregate information on comparable firms. This application can be used for the thousands of firms that have liquid equity prices but illiquid debt. Examples of illiquid debt include debt that never trades, debt that will trade in the future, and debt that has ceased trading.

A fair-value spread can be used to estimate a benchmark price on such debt if it were to trade. A firm may issue a bond to repay a bank debt, in which case, its debt may begin trading. The fair-value spread can also be used to estimate what the debt will trade at when it starts to trade. The market for a specific firm's debt may become illiquid for a variety of reasons and, as a result, the current market price for the debt may not be observable. Finally, a fair-value spread can be used to provide an estimate of what the debt price should be.

In addition to the accounting applications, the pricing of illiquid assets is useful in making business decisions. For example, if a bank lends money to finance a corporate transaction, our framework can be used to determine an appropriate coupon that the bank should earn on such a deal.

2.2 Active Portfolio Management

Portfolio management entails making decisions regarding taking on additional exposures, selling or hedging existing exposures, and the prices at which to do so. Bond investors typically must accept the securities' terms as given. Banks, in contrast, can negotiate loan terms, which affects the tenor, the default risk, and the LGD of a specific exposure. Our framework can be used to determine the FVS under different loan term assumptions. Such a framework helps investors make informed decisions regarding which loans to make, under what terms, and at what price. In addition, the gap between equity-based FVS and observed option-adjusted spreads can be used as a measure of relative mispricing between the equity and CDS markets. Investors can use this gap to construct investment strategies that exploit the relative price differences between the markets.

3. Model: Valuation Framework

Our valuation framework computes values of credit-risky claims such as bonds, floating rate notes, and credit derivatives in the form of Fair Value Spreads, which are produced for instruments issued in the seven major currencies, namely USD, EUR, JPY, GBP, CAD, AUD, and CHF. At a conceptual level, we follow the risk-neutral valuation methodology that is grounded in the No-Arbitrage principle. This principle allows the value of a risky bond to be valued as the expected payoff under the risk-neutral measure, discounted at the risk-free rate. We also impose a two-factor asset pricing model structure on asset returns, which results in a transformation that converts our physical measure of default probabilities (EDF credit metrics) to risk-neutral default probabilities (Quasi-Default Frequencies or QDFs). The main parameter in this transformation is the market Sharpe Ratio, or the market price of risk λ_m. Investors' attitudes toward risk, embodied in λ_m, is reflected in the prices (or spreads) of the credit risky claims–in this case, corporate bonds.

To calibrate Cumulative QDFs (CQDFs), we begin with EDF credit measures, which, as noted, are physical metrics. We assume that the asset process follows a Geometric Brownian motion process with drift μ_i and volatility σ_i:

$$\frac{dA_i}{A_i} = \mu_i dt + \sigma_i dZ_i \tag{1}$$

Due to risk aversion, investors typically require higher returns than the risk-free return r. The risk-neutral pricing principle implies that the asset drift equals r under the risk-neutral measure. CQDF can be obtained from CEDF via the following transformation:[9]

$$CQDF_{i,T} = N[N^{-1}(CEDF_{i,T}) + \frac{\mu_i - r}{\sigma_i}\sqrt{T}] \tag{2}$$

where N is the cumulative distribution function for the standard normal distribution, and N^{-1} is the inverse function of N.

In our current valuation framework, we rewrite this relationship by imposing a two-factor model on asset returns. The first factor is the market factor from the Capital Asset Pricing Model (CAPM). CAPM takes into account the asset's sensitivity $\beta_{i,m}$ to non-diversifiable risk (also known as systematic risk or market risk), as well as the expected market return and the expected return of a theoretical risk-free asset. The second factor is a size factor, which reflects the empirical evidence that firms with lower sales (for industrial and utility issuers) and smaller total asset bases (for financial institutions) typically have higher spreads, holding other variables constant.[10]

$$(\mu_i - r) = \beta_{i,m}(\mu_m - r) + \beta_{i,size}(size_i - E[size]) \tag{3}$$

Denote $\mu_i = E[r_i]$ and $\mu_m = E[r_m]$. Through the first order condition of minimizing the residuals and some algebraic manipulation, it can be shown that

$$\beta_{i,m} = \frac{Cov(r_i, r_m)Var(size_i) - Cov(r_i, size_i)Cov(r_m, size_i)}{Var(r_m)Var(size_i) - Cov(r_m, size_i)^2} \tag{4}$$

Assuming that size and market return are independent, this reduces to

$$\beta_{i,m} = \frac{Cov(r_i, r_m)}{Var(r_m)} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} \tag{5}$$

Similarly, for the size coefficient,

$$\beta_{i,size} = \frac{Cov(r_i, size_i)}{Var(size_i)} = \frac{\rho_{i,size}\sigma_i\sigma_{size}}{\sigma_{size}^2} \tag{6}$$

Plugging this back into 3, we get

[9] Detailed derivation is included in the Appendix.
[10] For a more in-depth discussion, see Agrawal, Deepak, Navneet Arora, and Jeffrey Bohn (2004).

$$(\mu_i - r) = \beta_{i,m}(\mu_m - r) + \beta_{i,size}(size_i - E[size]) \tag{7}$$

$$= \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2}(\mu_m - r) + \frac{\rho_{i,size}\sigma_i\sigma_{size}}{\sigma_{size}^2}(size_i - E[size]) \tag{8}$$

$$\implies \frac{(\mu_i - r)}{\sigma_i} = \rho_{i,m}\frac{(\mu_m - r)}{\sigma_m} + \rho_{i,size}\frac{(size_i - E[size])}{\sigma_{size}} \tag{9}$$

Letting $\rho_{i,m} = \rho$, $\lambda_m = \frac{(\mu_m - r)}{\sigma_m}$, $\lambda_{size} = \frac{\rho_{i,size}}{\sigma_{size}}$, and $\hat{size} = (size_i - E[size])$:

$$\frac{(\mu_i - r)}{\sigma_i} = \rho\lambda_m + \widehat{size}\,\lambda_{size} \tag{10}$$

Finally, plugging this back into 2, we derive the key equation linking CEDF and CQDF.

$$CQDF_t = N(N^{-1}(CEDF_t) + (\rho\lambda_m + \widehat{size}\,\lambda_{size})\sqrt{t}) \tag{11}$$

As noted, $CQDF$ is the cumulative default probability on a risk-neutral basis. $CEDF$ is the physical cumulative default probability derived using our public firm EDF model, and ρ is the correlation coefficient of individual asset returns with market returns. As ρ and λ_m are not separately identified, we follow our convention of setting ρ to $\sqrt{0.3}$.[11]

Many bonds have embedded put and call options, and this structure convolutes credit spread calculation. Therefore, we remove any option-related distortions by calculating the option-adjusted spread (OAS) over the default-free curve. We rely on the industry standard methodology for calculating OAS. We choose an appropriate default-free reference curve for this purpose. We refer to this reference curve as the Zero-EDF curve, which is constructed as a function of the treasury yield and duration.[12]

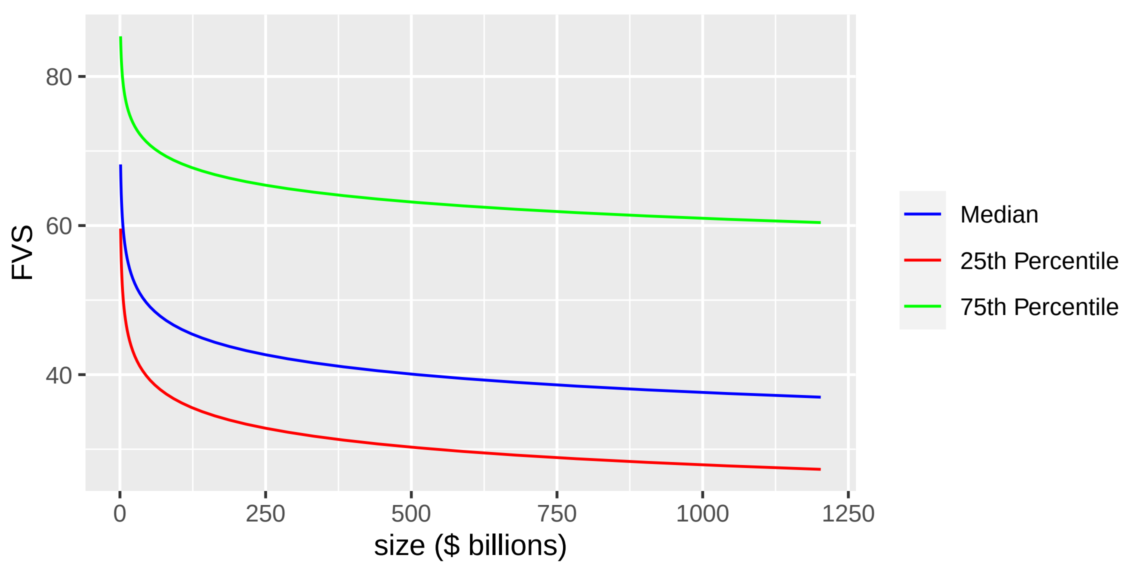

Figure 1: Isolating the size effect on option-adjusted spread

Finally, we model the value of a multi-period coupon paying bond as a zero-coupon bond. The spread on a coupon bond is approximated by the spread on a zero-coupon bond whose tenor is equal to the duration of the coupon bond. This approximation simplifies the estimation and yields intuitive results. We have:

[11] $\sqrt{0.3}$ is the long-run average of correlation with the market portfolio estimated through Moody's Analytics GCorr™ models. This simplification helps the identification of granular market Sharpe ratios, LGDs, and size premiums.

[12] In previous versions of the model, we calculated the zero-EDF curve by employing the LIBOR-swap curve with a small downward adjustment of 10 bp to account for the credit risk known to exist in LIBOR rates. In light of the discontinuation of LIBOR, we update our estimation of the zero-EDF curve for the current version. The detailed estimation process is included in Appendix C.

$$FVS_T = -\frac{1}{T} ln(1 - CQDF_T \cdot LGD) \tag{12}$$

where FVS is our estimation of the OAS and T is the option-adjusted duration of the bond. LGD represents the risk-neutral expected LGD. The OAS is:

$$OAS = Option\ Adjusted\ Yield - Zero\ EDF\ Yield \tag{13}$$

A notable change from the previous version of the bond model is that we now employ a two-factor asset pricing model to include a size factor as a component of the $CQDF$ calculation. Under modern asset pricing theory, investors are compensated for exposure to various risk factors. For bonds with credit risk, size is an important risk factor. For example, it is easier for large firms to raise capital to cover their debts. The size factor is also observed empirically, as larger firms tend to have relatively smaller credit spreads, holding EDF levels constant.

In Figure 1, we show the plot of a firm's size against the OAS, holding CEDF at 2%, duration at 5, LGD at 60%, and the median Sharpe ratio at approximately 0.546. We set the size premium to be either the 25th percentile, the median or the 75th percentile. These values are -0.022, -0.017 and -0.008, respectively. We vary size between $1 billion and $1.2 trillion. Thus, holding all else constant, we can see that a firm with a size of $1 billion will have an FVS that is roughly 20 bps higher than a firm with a size of $1,200 billion.

4. Data

We use several data sources in building the development dataset of the bond valuation model. We obtain the probability of default from Moody's Analytics CreditEdge™ tool.[13] and the corporate bond data from our data vendors. Both the bond and the EDF data have comprehensive global coverage.

A key data step that we perform is linking the bond data with the EDF data. Specifically, for each bond we identify a public firm with an EDF measure as the issuer of the bond. One challenge of such a mapping is that the corporate family tree has to be considered in cases when the bond is issued by a private company that potentially has a public parent. Another challenge of the mapping over a period of time is that the corporate family tree changes over time due to mergers and acquisitions. Over the years of the bond model production, we have accumulated unique historical data that allows us to map a bond to its public issuer or a public parent of its private issuer for our entire model development sample.

Our final sample has coverage that starts from 2006. The bond counts are plotted on Figure 2. While the bond data and the EDF data have longer history, we employ the post-2006 sample due to the high quality of our bond-to-public issuer mapping for this period. Note, however, that the sample of bonds on which the model parameters are calibrated is significantly smaller since it is limited to relatively liquid bonds. We observe significant improvement in bond coverage compared to the previous iteration of the bond model. Looking at June 2020, the number of bonds that we produce an FVS for has more than quintupled.

To construct the model calibration sample, we eliminate bonds that may have unusual spread behavior. These types of bonds include convertibles bonds, bonds with sinking funds, payment-in-kind bonds, bonds with embedded put options, bonds issued by Real Estate Investment Trusts (REITs), issues with current durations of less than one year or greater than 30 years, and bonds with OAS exceeding 2,000 bps. For the reliability of calibration, we also restrict the sample to the rated universe and remove bonds with a rating of "C" or "Ca". In addition, we apply exclusions based on the size of the bond issues as well as the size of the issuer to help ensure that the bonds are relatively liquid.[14] These exclusions are helpful in reducing the noise in parameter estimates, and these filters are employed for the purpose of calibrating the market-wide parameters. When using the parameters to compute issue-specific FVS, we use a less restrictive set of filters to include calculations for illiquid bonds and bonds with OAS over 2,000 bps. We also expand the FVS coverage to bonds without the duration information, leveraging a duration approximation mechanism.

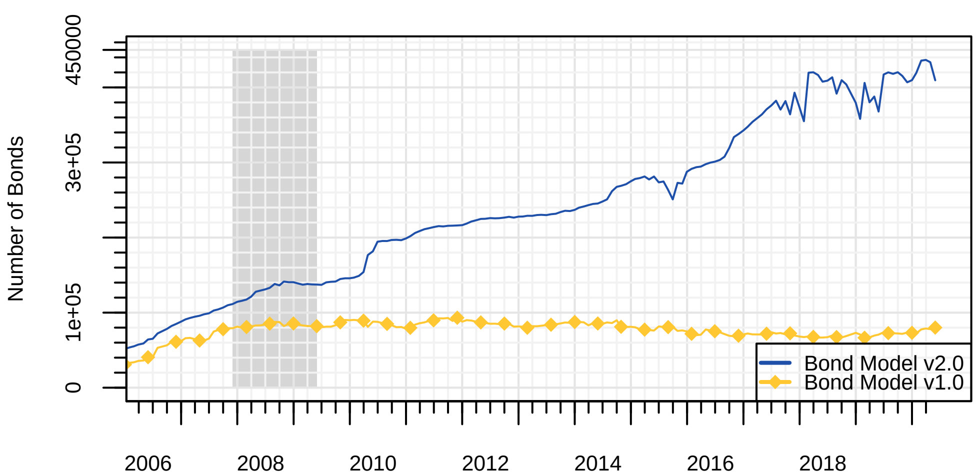

Figure 2: Sample size

[13] Chen et al. (2015), Nazeran and Dwyer (2015), Jeffrey Bohn and Korablev (2005)

[14] For example, we exclude USD denominated bonds with outstanding amount less than $150 million and issuers with size less than 1 million USD. This resembles the criteria employed to construct bond indexes.

5. Estimation of Model Parameters

The primary parameters of the model are the Sharpe ratios and the risk-neutral LGDs. Secondary parameters of the model consist of the size premium and adjustments to LGD for subordinated and secured bonds. This section provides an overview of the parameter estimation methodology.

We account for regional market differences by estimating region-specific Sharpe ratios and risk-neutral LGDs. In defining the regions, we try to strike a balance considering the granularity, sample size, and data availability. There are four main regions defined in our model: North America, Europe, Japan, and the rest of the world. A bond is assigned to a region based on its country of issuance. The rest of the world region is not broken down further due to limited data availability and the relatively small amount of debt outstanding in those markets. Therefore, we do not estimate its own parameters; rather, we use North American parameters to compute FVS for this region.

We estimate Sharpe ratios for investment-grade bonds and high-yield bonds separately. This can be motivated by the market segmentation due to investors with different mandates and risk tolerances operating in these two sub-markets. More importantly, in the results of the estimation we do observe a clear separation between investment-grade and high-yield Sharpe ratios.

LGDs are estimated on the sector level. Our sectors are defined by grouping Moody's Analytics NDY classification industries into 13 sectors listed in Table 10 in Appendix B. In addition to accounting for the LGD differences between sectors, sector-level estimation yields modeled spreads that match observed spreads on average within each sector. We estimate sector-level LGDs on the sample of senior unsecured bonds. For subordinated bonds, we estimate an adjustment to LGD: LGD of a subordinated bond is equal to LGD of senior unsecured bonds in the same sector times the adjustment. The adjustment is a single parameter, the same across all sectors and regions. We do not estimate sector-specific LGDs for subordinated bonds due to the small sample limitations. Similarly, we estimate an adjustment factor for a secured bond LGD that is the same across all sectors and regions.

To summarize, Sharpe ratios are estimated on region-rating class level and LGDs are estimated on a region-sector level, both using the sample of senior unsecured bonds. LGD adjustments for secured and subordinated bonds and size premium are estimated on the whole sample. All steps of estimation were done by minimizing the sum of squared differences between the FVS and the market-observed OAS.

We estimate parameters sequentially: first Sharpe ratios, second LGDs, third LGD seniority adjustments, and finally size premium. In the first step, we estimate Sharpe ratios on senior unsecured bonds, without the size premium parameter and assuming an initial LGD value of 55% and 40% for North American and European corporates and financials, respectively. We also assign 35% and 15% as starting LGD values for Japanese corporates and financials, respectively. The choice of initial LGDs is motivated by historical LGD observations and considerations of culture and government support.[15] In the second step, we estimate LGDs with Sharpe ratios estimated in the first step on the sample of senior unsecured bonds and without the size premium. In the third step, we estimate LGD seniority adjustments, again without size premium. Finally, we estimate size premium using all parameters estimated in previous steps. We adopt sequential estimation because a joint identification of Sharpe ratios and LGDs in a single step is technically challenging. To ensure the economically intuitive property of estimated parameters, we employ the sequential estimation that allows us to anchor the average LGD at the level consistent with observed recoveries.

5.1 Estimation of Sharpe Ratio

The first step in our estimation is to calibrate the market Sharpe ratios for individual region-rating class. With initial LGD assumptions and without the size premium parameter, Sharpe ratio estimates are conducted based on the senior unsecured bond sample. The estimated Sharpe ratios are plotted on Figure 3. The long-term average level of the investment-grade Sharpe ratios is similar in magnitude to the Sharpe ratios estimated in the literature for the equity market and for equity and fixed income funds[16]. The estimates are stable in the time series even though the rolling estimation window is only five days.

All Sharpe ratios go up significantly during the financial crisis. High-yield ratios are significantly higher than the corresponding investment-grade ratios, justifying the additional parameters and providing support for the market segmentation hypothesis. The Sharpe ratio patterns suggest that investors in the corporate bond market are indeed risk-averse. This finding confirms the importance of the conversion of physical default probabilities to risk-neutral probabilities for the accurate pricing of corporate bonds. Japanese investment-grade Sharpe ratios are significantly lower than North American and European ratios. This is potentially a result of low bond spreads in Japan due to the government and banking system support.

[15] For example, Japanese companies have a unique culture of bailout and thus the relatively lower LGD.
[16] Tang and Whitelaw (2011)

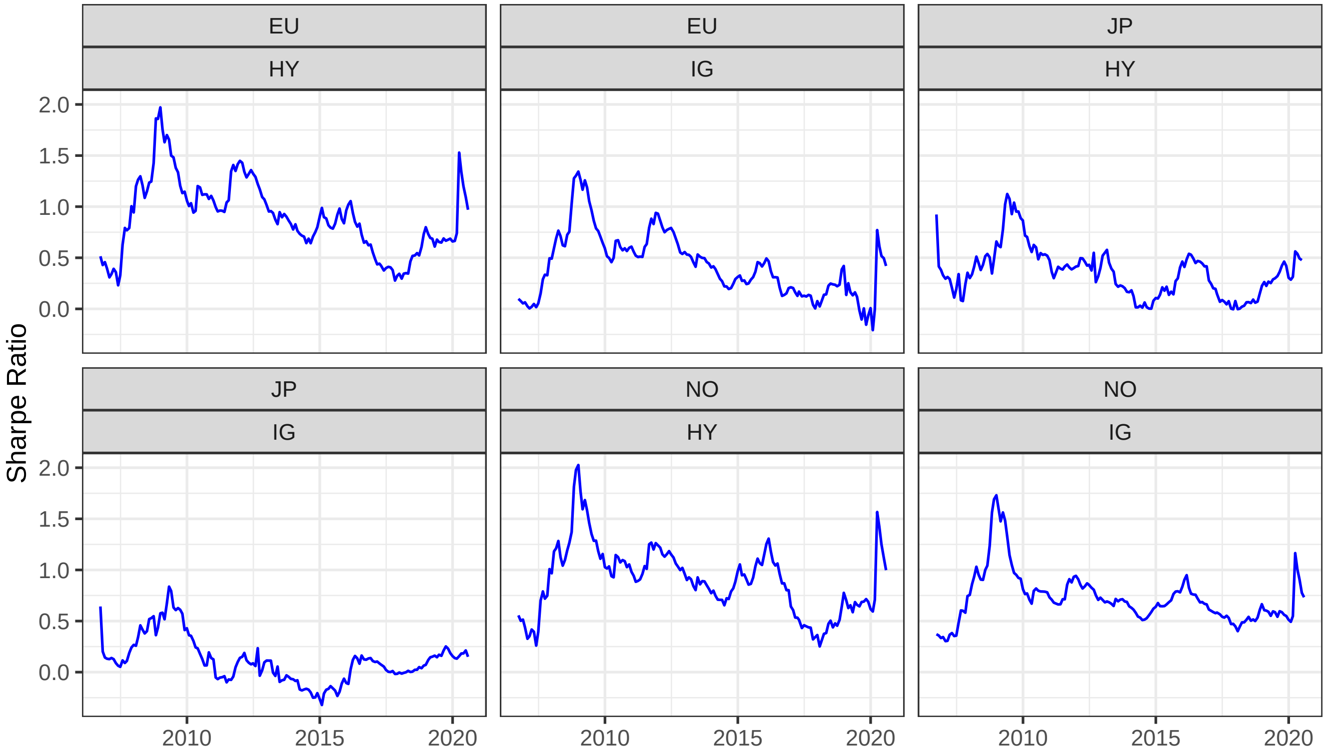

Figure 3: Market Sharpe ratio

5.2 Estimation of Sector LGDs

Risk-neutral LGDs of three major sectors are plotted on Figure 4.Overall, LGDs are stable over the time period. This is consistent with LGD being an expectation, which is less volatile than realized measures. Additionally, LGDs are relatively challenging to predict and, in practice, long-term historical sector-level averages are used. In the model, the stability is achieved by anchoring LGDs at initial levels and estimating the Sharpe ratio in the first step. Thus, the estimated Sharpe ratio is already consistent with the initial LGDs and it absorbs most of the spread variation over time. Long-term average levels of LGDs are consistent with the initial assumption of 55% LGD for corporates in North America and Europe and lower LGD for financials and for Japanese corporates. The estimated values are generally close to the initial LGDs. During the crisis, financial LGDs increased while corporate LGDs decreased. This can be interpreted as the heightened Sharpe ratio not fully accounting for the rise in spreads of financials but overshooting the spreads of corporates, which were relatively less affected by the crisis.

LGD adjustments for subordinated and secured bonds are plotted on Figure 5. The adjustments are multipliers to the baseline LGDs estimated on senior unsecured bonds. As expected, the adjustment for subordinated bonds is greater than 1, while the adjustment for secured bonds is less than or equal to 1. This means the subordinated LGDs are greater than senior unsecured LGDs in the same sector, while secured LGDs are lower or equal. Secured LGD adjustment is near the imposed boundary of 1 in some time periods, indicating that secured position in the capital structure does not always significantly affect LGD and spread. Subordinated LGD adjustment is always significantly higher than 1, indicating that subordinated position in the capital structure carries significantly higher LGD and spreads. The adjustments are somewhat volatile in part due to the small estimation sample, which includes only bonds that are matched to senior unsecured bonds by issuer and duration.

5.3 Estimation of Size Premium

Size premium coefficient estimates are plotted on Figure 6. The coefficient is negative, in agreement with the observation that smaller issuer size is associated with larger spreads even after accounting for the probability of default and LGD. The magnitude of the coefficient varies over time and the variation seems to relate to the economic cycle. Size premiums are largest at the trough

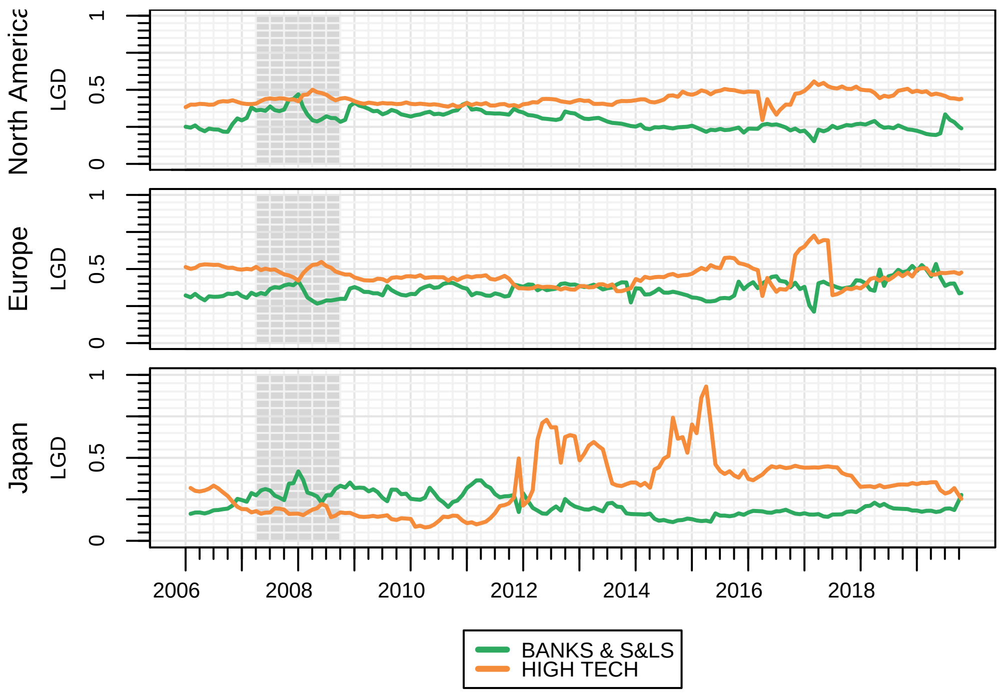

Figure 4: Risk-neutral LGDs

of the cycle, an observation consistent with the general notion that small firms are most affected by recessions. These findings are consistent with the notion that the size effect is a systematic feature of bond spread data and the size premium captures illiquidity due to trading frictions and potentially price effects of informational frictions and unmodeled sources of risk.

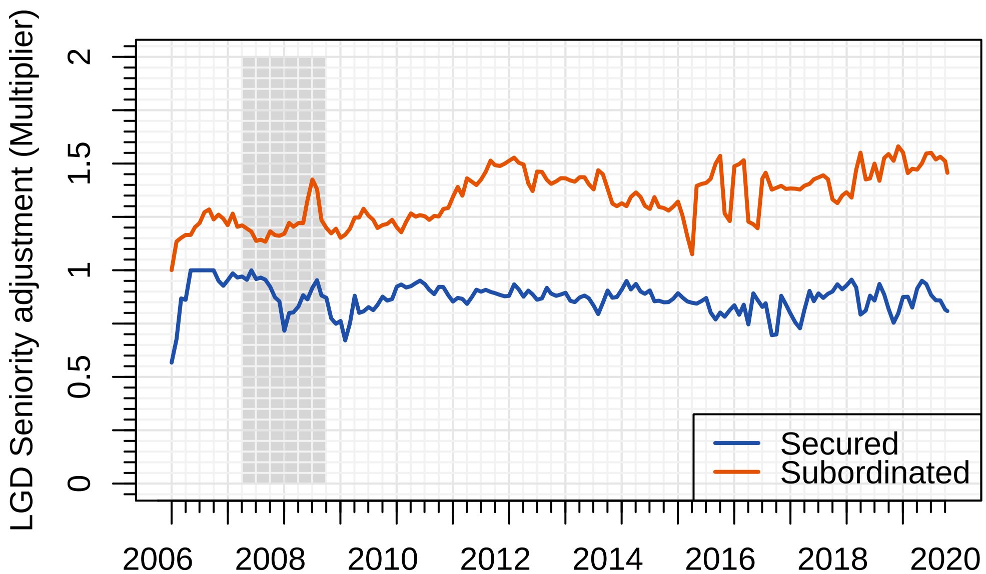

Figure 5: LGD seniority adjustment

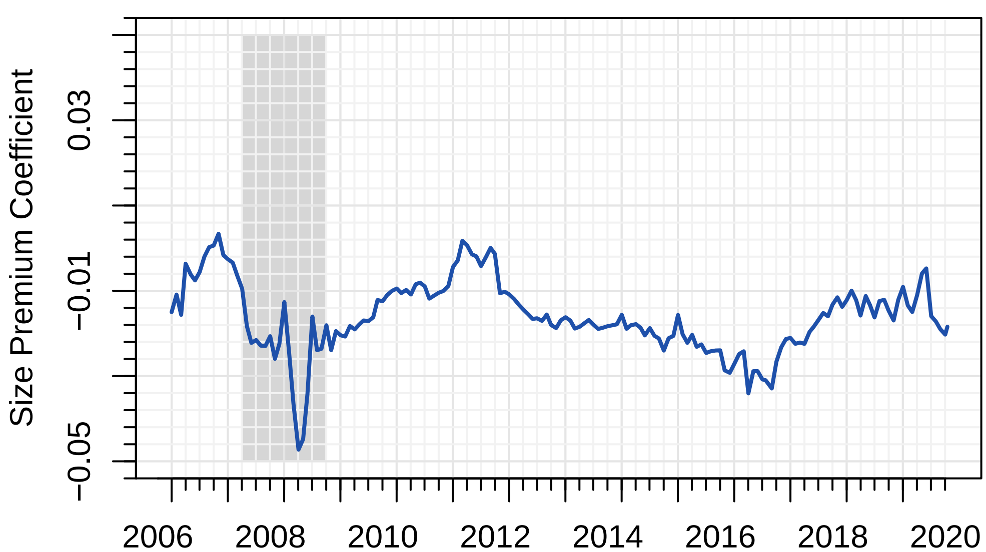

Figure 6: Size premium

6. Consistency of Modeled Spreads with Market Spreads

In this section, we evaluate how well modeled spreads match the market observed spreads. Using regional Sharpe ratios increases the model's complexity and flexibility; thus, we would expect the new bond model to have an improved model fit across regions.

One of the model's strengths is its ability to explain the cross-sectional variation of spreads. To demonstrate this ability, we study the correlation between FVS and OAS. The correlation of modeled and observed spreads is plotted on Figure 7. The correlation varies in the range of 65%-85%,[17] with the value in the top of the range during the crisis. This shows that the model successfully accounts for most of the cross-sectional variation in observed spreads. This level of explanatory power makes the model useful for mark-to-market valuation of bonds without observed market prices. To see the improvement in the model's mark-to-market performance, we also include the previous version of the model as a benchmark. We can see that the new model outperforms Bond Model version 1.0 consistently throughout time.

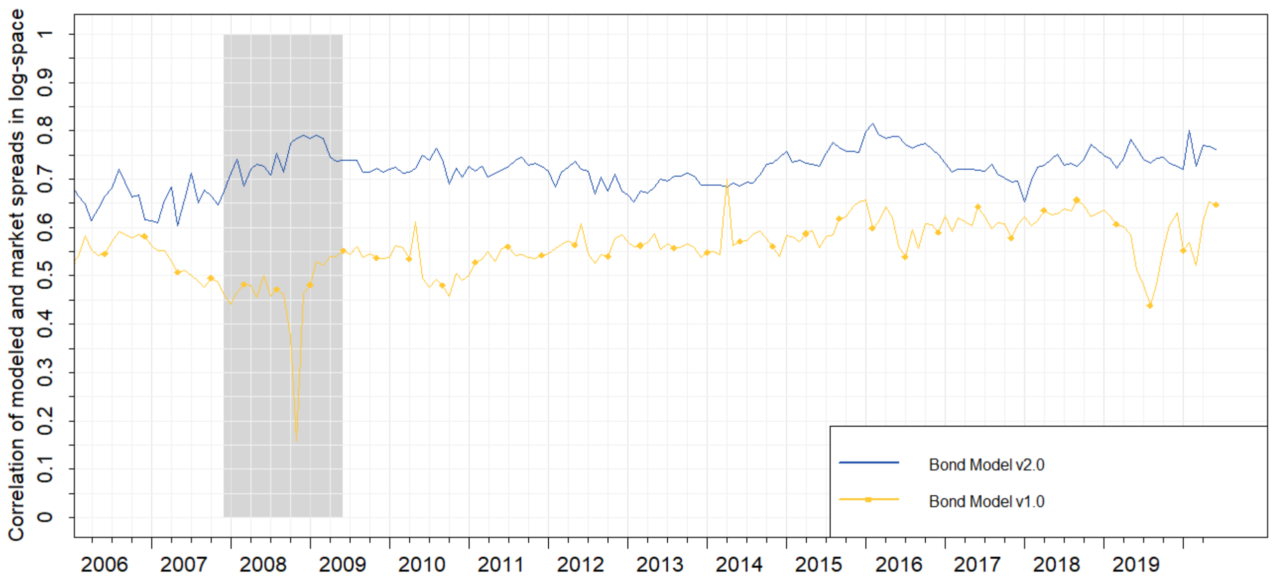

Figure 7: Correlation of modeled and observed spreads

We next examine if the model generates FVS systematically different than observed spreads. The mean of differences between market spread and modeled spread is plotted on Figure 8, and the 25th and 75th percentile of log differences are shown on Figure 9. A positive error indicates that the market pays a return in excess of the risk-adjusted return that the framework estimates as the correct level. For all markets, FVS track the observed market spreads very well, suggested by the small prediction percentage error. Taking North America as an example, both the mean and median percentage errors move around 20%-30% and the mean and median absolute errors move around 30 to 50 basis points, throughout the sample period. Again, when benchmarked against the previous version of the bond model, the improvement in mark-to-market performance of the new model is obvious: the new model has significantly lower prediction error in both the absolute value space and the percentage value space for all markets, measured at the mean, the median, and the quartiles. The small errors associated with the new model predictions indicate that the modeled spread is close to being an unbiased predictor of the market spread.

While certain individual bonds have large deviations of modeled spread from market spread, it is important to note that in a portfolio of bonds these deviations tend to diminish. Corporate bonds are complex instruments with features such as call options, liquidity effects, default risk, recovery risk, and interest rate risk. Our bond valuation model is a parsimonious representation of these complex

[17] This correlation is obtained on the relatively liquid sample of corporate bonds. For the full sample including illiquid bonds, the correlation varies around 60% through the sample period.

instruments and their markets. Considering that no issue-specific parameters are modeled, the model shows encouraging performance given its parsimony. The model also places far fewer demands on the bond data than a reduced-form model that estimates firm-specific parameters from the same source. With minimal information such as a reasonably sized cross-section of bond spreads with representation of various industry and seniority sectors, even if an individual firm's data has gaps and missing information, we can still produce reasonable FVS estimates. This model property is attractive for assets such as bonds and CDS, where continuous trading of particular names is not very common. Our model can generate model spreads for thousands of names for which we have stock trading and thus EDF credit measures, and also for which reliable bond spread data is scarce. In contrast, a reduced-form model calibrated to historical bond spreads would have more limited coverage.

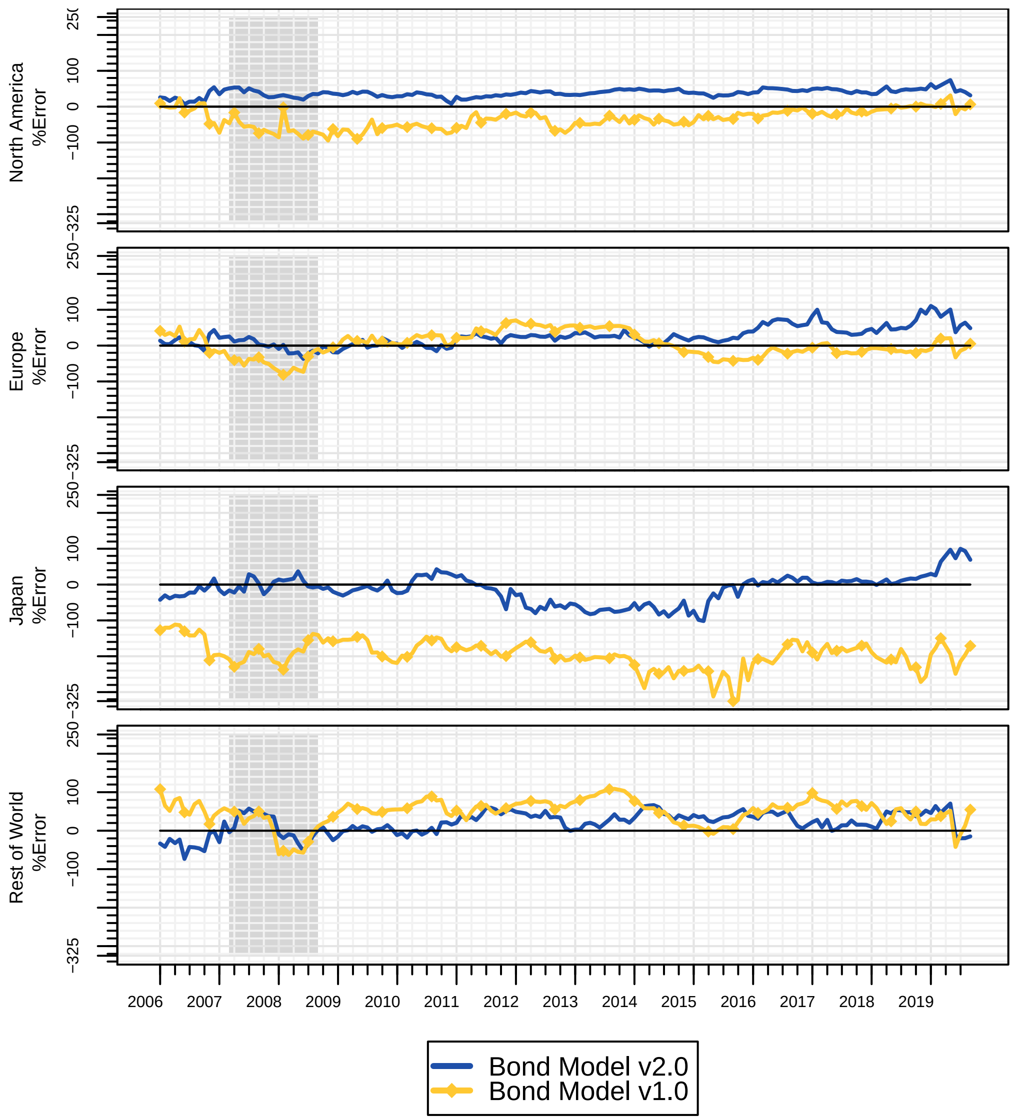

Figure 8: Mean of percentage differences between market spreads and modeled spreads

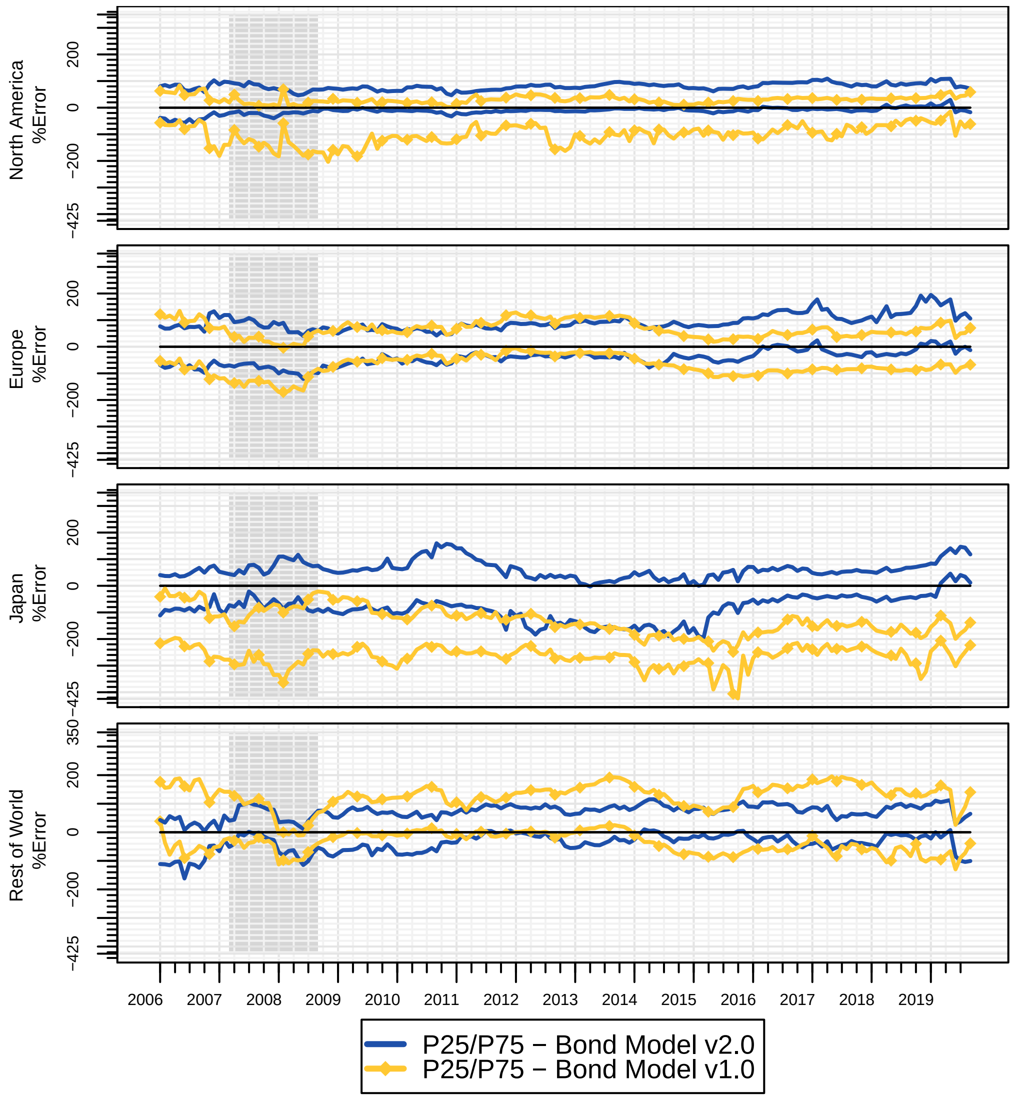

Figure 9: 25th and 75th percentile of percentage differences between market spreads and modeled spreads

7. Investment Strategy

There are cases in our sample where the differences between OAS and FVS are non-trivial. Several possible explanations exist for these pricing errors. They may be the result of bond market inefficiencies due to reduced liquidity vis-à-vis equity markets, for instance, or the result of errors due to non-modeled risk factors and evidence for a value factor in fixed income.

If the mispricing hypothesis is correct, then bond spreads should converge to FVS over time, generating superior risk-adjusted returns. Therefore, strategies that invest in underpriced bonds, i.e., issues with high OAS relative to FVS, should outperform a market value-weighted index. Similarly, if the value factor hypothesis is correct, then more undervalued bonds should earn higher expected returns, in line with their greater exposure to the value risk factor. The performance of trading strategies that invest in undervalued bonds has been studied in various Moody's Analytics papers. One of the earliest examples of this work is the article by Li, Zhang and Crossen (2012) in the *Journal of Fixed Income*.[18]

Our enhanced Fair Value Spread (FVS) is a primary input to the Alpha Factor data point around which we build our investment strategy. The Alpha Factor is calculated at the bond level by dividing the bond's OAS by the FVS. The ratio shows whether the security is overvalued or undervalued based on its level of default risk.

In this section, we present results for major corporate bond markets in the United States, Europe, and Asia. For the United States, we break out results separately for the US investment-grade (USIG) and US high-yield (USHY) markets. In Europe, we focus only on Euro investment-grade (EUIG) bonds. In Asia, we divide our results presentation into two groups: an index of USD-denominated bonds issued by corporations domiciled in a set of APAC ex-Japan countries, and JPY-denominated Japanese corporate bonds. For our APAC ex-Japan index, we include both investment-grade and high-yield debt, whereas for Japan we include only investment-grade bonds, due to the small size of the high-yield bond market in Japan.

In each market, the relevant index against which we judge strategy performance is constructed using a few simple rules. First, in each month we identify the set of MIS-rated corporate bonds whose issuers are also covered in the CreditEdge tool, meaning they possess publicly traded equity. Second, as a simple liquidity screen, we eliminate the subset of bonds whose amount outstanding lies below a threshold that depends on the market. The minimum amount outstanding levels are: USD 300 million for USIG, USD 150 million for USHY, EUR 300 million for EUIG, USD 350 million for APAC ex-Japan, and JPY 35 billion for Japan IG. Third, we apply equal weights to the set of selected index constituents to calculate the index returns each period.

We first present results on investment strategy performance in U.S. and European markets, and then examine results for APAC ex-Japan and Japanese markets.

7.1 Investment Strategy Performance and Characteristics in the United States and Europe

A transparent strategy portfolio can be constructed as follows. We begin with the universe of bonds included in each of three broad market indexes, namely USIG, USHY, and EUIG. In both investment-grade markets, at the end of every month, we first sort the index bonds by their status as a financial or corporate firm. Then, within each of these two firm type buckets, we sort the index bonds by duration and assign each bond to one of five duration buckets, for a total of ten buckets. Within each of these buckets, we rank bonds by each issue's Alpha Factor. For the USHY market, we follow an almost identical procedure, except that we bucket only by duration, and do not consider the issuer's firm type, as the USHY universe contains a smaller number of bonds. This leads to five total (duration) buckets each month in USHY. The minimum duration cutoffs in years for each of the five buckets are defined as follows: USIG: (0, 3, 5, 7, 10); USHY: (0, 3, 4, 5, 6); and EUIG: (0, 2, 3.5, 4.5, 6).

Under either a mispricing or value factor interpretation, high Alpha Factor bonds have the most scope to outperform the market benchmark. To finish the portfolio selection procedure, we choose the top 20% of bonds by Alpha Factor within each duration/firm type bucket for inclusion. For the sake of simplicity, each selected bond is weighted equally in the final portfolio. We refer to this as the Top Quintile AF strategy.

Figure 10 plots cumulative total return curves for the Top Quintile AF strategy in each of the USIG, USHY, and EUIG markets. The Top Quintile AF strategy implemented using the bond model is shown in green, and the benchmark level is shown in red. Each cumulative return curve is normalized to equal 1.0 at the beginning of January 2007. As this exercise is meant to be illustrative, we do not consider transaction costs.

[18] Li, Zhang, and Crossen (2012)

Figure 10: Comparison of the performance between Top Quintile Alpha Factor (AF) and benchmark portfolios (2007-2020, cumulative total return curves)

The FVS-based Top Quintile AF strategy outperforms its benchmark in backtests for each of the three bond markets studied since 2009. The exercises cover the period from January 2007 until June 2020, inclusive, which incorporates the most recent data reflecting the COVID-19 period.

To complement these results, Table 1 reports the annualized monthly mean returns, annualized monthly return standard deviations, and information ratios vs. a zero-return benchmark for the Top Quintile AF portfolios in each bond market. The results of paired t-tests of differences in mean returns of the strategy versus the benchmark index from Figure 10 are also listed.

Table 1: Performance statistics for Top Quintile Alpha Factor portfolios

Characteristic	*USIG*	*USHY*	*EUIG*
Mean	9.92%	13.55%	5.46%
Std Dev	6.70%	13.57%	4.64%
IR vs. zero-return benchmark	1.48	1.00	1.18
T-test Strategy vs. Index Mean Rets (p-val)	0.0000	0.0000	0.0000
N. Obs	160	160	160

From Table 1, we find that we can overwhelmingly reject the null hypothesis that Top Quintile AF strategy mean returns are equal to those of the benchmark. This finding supports the visual results apparent in Figure 10. The risk-adjusted returns of the Top Quintile AF strategy are attractive in all three markets, but especially so in the two investment-grade markets studied.

To see in a bit more detail how yearly returns of our Top Quintile AF strategies compare to those of their benchmarks over time, Table 2 reports yearly total returns for each market during the 2007-2020 period.

Table 2: Yearly performance statistics for Top Quintile Alpha Factor portfolios

Year	*Top20 AF USIG*	*Index USIG*	*Top20 AF USHY*	*Index USHY*	*Top20 AF EUIG*	*Index EUIG*
2007	5.9%	4.8%	4.1%	1.2%	-2.8%	-2.7%
2008	3.1%	-2.4%	-17.2%	-21.0%	-3.5%	-4.1%
2009	37.2%	19.7%	79.3%	46.0%	22.3%	14.4%
2010	12.2%	9.3%	15.0%	14.2%	5.0%	4.8%
2011	11.0%	9.2%	8.7%	6.3%	1.9%	1.8%
2012	12.8%	9.1%	12.4%	11.9%	17.5%	13.0%
2013	0.9%	-1.6%	7.7%	6.4%	5.8%	3.0%
2014	9.5%	8.2%	4.7%	1.1%	8.9%	8.4%
2015	-1.8%	-1.0%	-9.4%	-10.6%	0.7%	-0.5%
2016	11.0%	7.1%	37.8%	23.7%	6.2%	5.7%
2017	6.6%	5.8%	6.0%	6.3%	2.1%	1.3%
2018	-1.8%	-2.3%	0.7%	-1.8%	-0.9%	-0.7%
2019	15.5%	13.6%	15.3%	13.4%	8.5%	6.7%
2020	10.2%	5.9%	15.6%	-3.5%	0.7%	-0.6%

Effectively, all the Top Quintile AF strategies outperform their benchmarks in most years of the 2007-2020 sample. In the USIG market, for instance, outperformance is achieved in 13 out of 14 years; in the USHY market, the tally is also 13 out of 14 years; and in the EUIG market, the Top Quintile AF strategy outperforms in 12 out of 14 years.

To wrap up our discussion of the new bond model investment strategy for the USIG, USHY, and EUIG markets, it remains to compare its selected risk and sector characteristics to those of the respective indexes. Table 3 reports mean LGDs, spreads, durations, and ratings for the Top Quintile AF strategy and each index.

Overall, Table 3 shows that the risk profiles of the Top Quintile AF strategy and the index are quite similar. The mean LGDs, mean durations, and mean ratings are identical or nearly identical for all three markets. The one main area of difference apparent is that mean spreads for the Top Quintile AF strategy are slightly higher than they are for the index. Given similar ratings profiles, this makes sense, as the Top Quintile AF strategy is designed to select bonds whose market spreads are trading wide relative to fair value spreads based on their level of default risk.

Table 3: Mean characteristics for Top Quintile Alpha Factor portfolio and index

Characteristic	*Top20 AF USIG*	*Index USIG*	*Top20 AF USHY*	*Index USHY*	*Top20 AF EUIG*	*Index EUIG*
LGD	0.4	0.5	0.5	0.5	0.4	0.4
OAS	220	162	673	586	204	133
Duration	6.9	7.2	4.8	4.9	5.1	5.0
Rating	Baa1	A3	B1	B1	A3	A3

Finally, Table 4 addresses the issue of sector composition of the Top Quintile AF strategy compared to the index.

In Table 4, the sectors in the left-most column are ordered by decreasing values of the Top Quintile AF strategy weight minus the index weight in the USIG market. The main finding apparent from the table is that in all markets, utilities are over-weighted compared to the index by the Top Quintile AF strategies, with under-weighting applied to most other sectors to compensate for the difference. We see that the materials/extraction sector is under-weighted nontrivially by the Top Quintile AF strategy, for example. Overall, except for the utilities, cable TV and print/pub, and materials/extraction sectors, the sector profile of the selected bonds under the index and the Top Quintile AF strategies is quite similar.

Table 4: Mean sector weights for the Top Quintile Alpha Factor portfolio and index (2007-2020)

Sector	*Top20 AF USIG*	*Index USIG*	*Top20 AF USHY*	*Index USHY*	*Top20 AF EUIG*	*Index EUIG*
Utilities - Low Risk	29%	17%	18%	5%	20%	9%
Cable TV & Printing/Publishing	15%	7%	19%	14%	8%	10%
Banks and S&Ls	12%	11%	1%	1%	24%	24%
Finance Co & Broker/Dealers	12%	13%	5%	5%	11%	8%
Consumer Goods & Durables	9%	13%	11%	13%	12%	14%
Materials/Extraction	8%	12%	18%	26%	6%	11%
Transportation	1%	3%	1%	2%	1%	3%
Equipment	1%	4%	2%	2%	2%	2%
REITS/Finance - High Risk	7%	7%	5%	6%	4%	6%
General Sector	4%	5%	10%	16%	8%	8%
Aerospace & Measuring Equipment	0%	2%	1%	1%	0%	1%
High Tech	1%	3%	1%	3%	0%	1%
Medical	2%	5%	7%	5%	3%	3%

Our comparison of performance and risk characteristics of the Top Quintile AF strategy with an equally-weighted index strategy reveals superior performance of the former, and overall, very modest differences with the index along other relevant risk and sector characteristics.

7.2 Investment Strategy Performance and Characteristics in Asia

We now turn to an examination of Top Quintile AF strategy performance in Asia. For both the APAC ex-Japan strategy and the Japan strategy, we follow a procedure analogous to the one stated earlier for USHY, in that we bucket by duration by not by firm type, due to the smaller sizes of these markets. For duration bucketing in the APAC ex-Japan Top Quintile AF strategy, the minimum duration cutoffs in years for the five buckets are defined as follows: (0, 3, 4, 5.5, 7). For the Japan IG strategy, the minimum duration cutoffs in years for the five buckets are defined as follows: (0, 3.5, 4.5, 5.5, 7.5).

Figure 11 plots cumulative total return curves for the Top Quintile AF strategy in APAC ex-Japan and Japan IG markets. The Top Quintile AF strategy implemented using the bond model is shown in green, and the index levels are shown in red. Each cumulative return curve is normalized to equal 1.0 at the beginning of the period of study for each market. Note that, in contrast to the U.S. and European markets, the APAC ex-Japan results begin in 2013, and the Japan IG results begin in 2016.

The Top Quintile AF strategy outperforms its benchmark index for both markets shown in Figure 11. Note that the magnitude of the outperformance is greater for APAC ex-Japan, consistent with the fact that average total returns in Japan IG are low.

Table 5 reports the annualized monthly mean returns, annualized monthly return standard deviations, and information ratios vs. a zero-return benchmark for the Top Quintile AF portfolios in Asia, along with the results of paired t-tests of differences in mean returns of the strategy versus the benchmark index from Figure 11. We can reject the null hypothesis of equal Top Quintile AF mean returns with the index in the APAC ex-Japan market at the 5% level of significance, and we can reject this null hypothesis in the Japan IG market at well beyond the 1% level of statistical significance. Versus the zero absolute return benchmark, information ratios for the two Asia markets lie in the middle of the range obtained for the USIG, USHY, and EUIG markets, and are generally attractive.

Table 5: Performance statistics for Top Quintile Alpha Factor portfolios

Characteristic	*APAC ex-Japan*	*Japan*
Mean	6.59%	1.08%
Std Dev	4.59%	1.02%
IR vs. zero-return benchmark	1.33	1.06
T-test Strategy vs. Index Mean Rets (p-val)	0.0147	0.0000
N. Obs	88	52

Figure 11: Comparison of the performance between Top Quintile AF and benchmark portfolios (cumulative total return curves)

Table 6 reports yearly total returns for the Top Quintile AF strategy and the index in each market during the backtest period. We see that the strategy outperforms the index in six out of seven years for APAC ex-Japan, and in four out of four years for Japan IG.

Table 6: Yearly performance for Top Quintile Alpha Factor portfolios

Year	*Top20 AF APAC ex JP*	*Index APAC ex JP*	*Top20 AF Japan*	*Index Japan*
2013	0.6%	0.2%	-	-
2014	7.9%	6.9%	-	-
2015	2.6%	3.0%	-	-
2016	9.4%	7.4%	1.0%	0.9%
2017	6.1%	5.0%	1.7%	0.8%
2018	0.6%	-0.8%	1.4%	0.8%
2019	13.6%	11.3%	0.5%	0.3%
2020	7.6%	3.4%	0.1%	-0.1%

To allow for assessment of differences in heuristic risk measures between the strategy and the relevant index, Table 7 reports mean LGDs, spreads, durations, and ratings for the Top Quintile AF strategy and both indexes for the Asia region.

Table 7: Mean characteristics for Top Quintile Alpha Factor portfolio and index

Characteristic	*Top20 AF APAC ex JP*	*Index APAC ex JP*	*Top20 AF Japan*	*Index Japan*
LGD	0.4	0.4	0.3	0.3
OAS	258	225	60	36
Duration	5.7	5.6	5.7	5.5
Rating	Baa1	Baa1	A2	A2

As seen with U.S. and European markets, mean LGDs, durations, and ratings are nearly identical between the strategy and the index for APAC ex-Japan. The same conclusion holds for Japan IG. Also as in the U.S. and European cases, the only salient difference that emerges is that the Top Quintile AF strategy has a somewhat higher mean option-adjusted spread (OAS), consistent with the strategy selecting bonds with a higher average spread for a similar level of credit risk as the index.

To complete our discussion of the investment strategies in Asia, we examine the country and industry composition of the portfolios. Table 8 reports the average country weights in the Top Quintile AF strategy portfolio and the index for APAC ex-Japan. Note that the weights in both columns add up to slightly over 100% (101% and 102%, respectively) due to rounding up to whole percentages. The main salient difference between the Top Quintile AF portfolio and the index for APAC ex-Japan is that the former under-weights China and Hong Kong and over-weights Thailand and Malaysia with respect to the latter. Other weights are similar between the two.

Table 8: Country composition for Top Quintile Alpha Factor portfolios and index

Country	*Top20 AF APAC ex JP*	*Index APAC ex JP*
Hong Kong	23%	30%
China	7%	14%
Singapore	4%	7%
Korea	8%	11%
Thailand	19%	9%
India	21%	17%
Indonesia	3%	2%
Philippines	3%	4%
Malaysia	10%	3%
Taiwan	3%	1%
Macao	1%	1%

Finally, Table 9 reports differences in mean sector weights between the Top Quintile AF strategy and the index for both APAC ex-Japan and Japan.

Upon inspection of Table 9, which organizes sectors in the same order as Table 4, we see that the Top Quintile AF strategy tends to overweight Cable TV & Printing/Publishing, Materials/Extraction, and the General Sector, compared to the index in both Asian markets. It significantly underweights the Banks & S&Ls sector compared to the index as well. A comparison with Table 4 reveals that over/underweight patterns in the two Asian markets is modestly different than in the United States and Europe, with the most importance difference being that Banks & S&Ls are underweighted by the value strategy in Asian markets and market-weighted in Europe and the United States. In contrast, Materials/Extraction is overweighted for the strategy in Asia and underweighted in the United States and Europe.

Table 9: Mean sector weights for the Top Quintile Alpha Factor portfolio and index: Asia markets

Sector	*Top20 AF APAC ex JP*	*Index APAC ex JP*	*Top20 AF Japan*	*Index Japan*
Utilities - Low Risk	13%	6%	13%	12%
Cable TV & Printing/Publishing	12%	6%	17%	8%
Banks and S&Ls	15%	26%	1%	37%
Finance Co & Broker/Dealers	3%	4%	3%	7%
Consumer Goods & Durables	3%	6%	18%	13%
Materials/Extraction	24%	17%	3%	1%
Transportation	2%	2%	0%	1%
Equipment	0%	0%	3%	4%
REITS/Finance - High Risk	2%	12%	1%	4%
General Sector	23%	18%	31%	10%
Aerospace & Measuring Equipment	0%	0%	0%	0%
High Tech	3%	2%	2%	2%
Medical	0%	0%	8%	2%

7.3 Interpretation of Results

Given the outperformance of the Top Quintile AF strategy across all five markets studied, in addition to outperformance of the index for most years studied in each market, the performance advantage conferred by the strategy appears quite robust. Since the metric used to select bonds-the ratio between a bond's OAS and its modeled FVS-is an intuitive measure of undervaluation, our results are highly consistent with the idea of the Alpha Factor ratio as a "value factor" in fixed income.

Factor premiums are meant to compensate for sources of risk distinct from the risk of the market portfolio. In this section, while we report results only for equally weighted portfolios to emphasize the role of bond selection in driving strategy outperformance, we have found that strategies formulated using market weights produce qualitatively similar results. Our results reveal a few years in which the Top Quintile AF strategy underperforms its index in some markets, so a risk-based explanation of our findings as documenting a value factor premium is not out of the question. However, the consistency of the outperformance across time and markets, and the apparent absence of "crash risk" of the value strategy vis-à-vis the index (as one sometimes sees with, for example, momentum and value factors in equities), suggest that we cannot rule out mispricing completely as a source of the measured strategy return premiums.

8. Related EDF-Based Valuation Models

In addition to the valuation model for fixed rate bonds, we also model FVS for floating rate notes (FRNs). FRNs are a sizable component of the corporate bond universe. To value FRNs within the Bond Valuation framework, we take advantage of the fact that a correctly priced FRN can be seen as the summation of two instruments: a fixed-rate note of the same issuer and maturity plus a fixed-for-floating interest rate swap on the FRN's underlying reference rate. This relationship allows us to at first treat an FRN as a fixed-coupon bond and then to run the Bond Valuation Model to find a provisional FVS for the issue. As a subsequent step, we subtract from the provisional FVS the expected yield on a fixed-for-floating interest rate swap on the reference rate. This process ultimately produces a Fair Value Discount Margin (FVDM) for the FRN-our pricing of the FRN, given its underlying credit risk and market conditions.[19]

We also adapt the Bond Valuation Model to credit default swaps (CDS) using a similar risk-neutral framework. The CDS model calculates the Fair Value CDS spread implied by the equity-based EDF measure, as well as the CDS-implied EDF value (CDSiEDF). CDSiEDF helps increase default risk coverage for firms with CDS that do not have traded equity, such as private companies, subsidiaries of public firms, and sovereigns. Furthermore, this modeling duality, from EDF to Fair Value CDS and from CDS to CDSiEDF, allows practitioners to make credit risk and relative value assessments based on the information reflected in the CDS and EDF values. In particular, the equity-based Fair-Value CDS can be compared with observed CDS spreads to generate insights. This helps investors conduct long-short CDS relative-value trading strategies. Our studies show that investors can use information from both markets to construct outperforming portfolios.[20]

[19] Edwards (2015)
[20] Du and Zhang (2014)

9. Summary

In this paper, we present a model for the valuation of credit risky claims. Using the default risk prediction based on stock market information, we developed a framework to produce the fair value spread for less liquid assets such as corporate bonds. The model leverages EDF credit measures derived from the Moody's Analytics structural model of default and builds a mechanism to convert EDF measures to risk-neutral measures using an estimated market Sharpe ratio. We calibrate the market Sharpe ratio using corporate bond market prices. The derived risk-neutral default probabilities can then be used to price various credit-risky instruments using the standard risk-neutral valuation methodology.

We apply our valuation method to a large sample of bond spreads and found that the modeled spreads consistently match well with the market-observed spreads. With the average correlation being about 75% on average for the model calibration sample over the past decade, our model explains a significant portion of the cross-sectional variation in bond spreads. We provide evidence that the remaining unexplained variation is partially due to the equity-based FVS measure leading the observed bond spreads. Furthermore, FVS-based relative value trading strategies for corporate bonds show the ability to outperform the market in all major markets. The explanatory power of our model makes it a competitive candidate for the pricing of bonds and loans and the identification of potential investment opportunities.

Compared to its predecessor, the new bond model accounts for regional differences in calibration and ensures model performance for each individual region. In addition to investment-grade bonds, the new bond model has improved spread prediction for high-yield bonds and bonds issued by relatively smaller companies. The model also demonstrates improved performance for secured and subordinated bonds, in addition to unsecured bonds.

With the improved historical mapping between the bond and the public parent of the issuer, we can use the EDF credit measure more accurately for bond valuation. We also implemented a new valuation mechanism to approximate the duration and spreads for bonds where pricing and duration information are not available. These improvements contribute to the significantly expanded coverage of our bond model. Furthermore, in light of the upcoming retirement of LIBOR by 2021, we remove the dependency of zero-EDF curves on LIBOR swap curves by establishing a relationship between zero-EDF curves and treasury yields for major currencies.

In summary, the model framework integrates equity market information and bond market information. Empirical evidence shows that such integration is valuable in providing useful signals for risk management and credit investment strategies.

10. Appendix A: Linking the risk-neutral and physical probability of default

Consistent with our EDF model, we assume that the asset process is given by:

$$\frac{dA}{A} = \mu dt + \sigma dZ \tag{14}$$

Using Ito's Lemma on the above asset process, we obtain:

$$d\,lnA = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dZ \tag{15}$$

This can be integrated to obtain:

$$ln(\frac{A_t}{A_0}) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma\sqrt{t}\epsilon \tag{16}$$

Assuming that default can happen at time t if asset value falls below the default point DPT, one can derive the following condition on the random shock term e, for default to occur:

$$\begin{aligned} &If\ A_t < DPT, \\ &\epsilon < -\frac{ln(\frac{A_t}{A_0}) + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = N^{-1}(CEDF) \end{aligned} \tag{17}$$

One can also look at the above process in a risk-neutral setting in which the asset drift is equal to the riskless rate r.

$$\frac{dA}{A} = rdt + \sigma(dZ + \frac{\mu - r}{\sigma}dt) = rdt + \sigma d\tilde{Z} \tag{18}$$

$d\tilde{Z}$ represents the Brownian motion in a risk-neutral framework and is related to the Brownian motion dZ in the physical framework as:

$$d\tilde{Z} = dZ + \frac{\mu - r}{\sigma}dt \tag{19}$$

$$\tilde{Z}_t = \tilde{\epsilon}\sqrt{t} = \epsilon\sqrt{t} + \frac{\mu - r}{\sigma}t \tag{20}$$

$$\implies \tilde{\epsilon} = \epsilon + \frac{\mu - r}{\sigma}\sqrt{t} \tag{21}$$

Recalling relationship 17 above, we obtain:

$$\tilde{\epsilon} = N^{-1}(CEDF) + \frac{\mu - r}{\sigma}\sqrt{t} \tag{22}$$

We can therefore define the cumulative quasi-EDF as:

$$CQDF = N(\tilde{\epsilon}) = N[N^{-1}[CEDF] + \frac{\mu - r}{\sigma}\sqrt{t}] \tag{23}$$

11. Appendix B: Additional Figures and Tables

Table 10: Sectors

Sector
Aerospace and Measuring Equipment
Banks and S&Ls
Cable TV and Publishing
Consumer Goods and Durables
Equipment
Finance Co and Broker/Dealers
General Sector
High Tech
Materials/Extraction
Medical
REITs and Other Finance
Transportation
Utilities

12. Appendix C: Update on Zero-EDF Curve

To detach our Zero-EDF Curves from the LIBOR Swap Curves in light of LIBOR rates retirement by 2021, we construct a new version of the Zero-EDF Curves from a panel regression with the Treasury/government bond yield and duration as independent variables. The regression model is as follows:

$$Zero\ EDF\ Rate = \alpha + \beta_1 \times Treasury\ Yield + \beta_2 \times Duration + \epsilon \tag{24}$$

The original Zero EDF Rate as the response variable preserves the property of a small downward adjustment of the LIBOR Swap Curve. We run this regression for each of the seven major currencies, with adjusted R^2 in general higher than 95%.

To output the above granularized Zero EDF Rate, the Treasury/government bond yield curve is interpolated and extrapolated using a cubic spline algorithm, for each currency. Figure 12 and Figure 13 provide examples of such constructed curves.

The final step is that for each tenor and currency we set the Treasury yield curve as the lower bound of the Zero EDF Curve as the liquidity premium of a "default-free" corporate is not equal to or less than 0, making the Zero EDF Curve lie above the Treasury yield curve at all time.

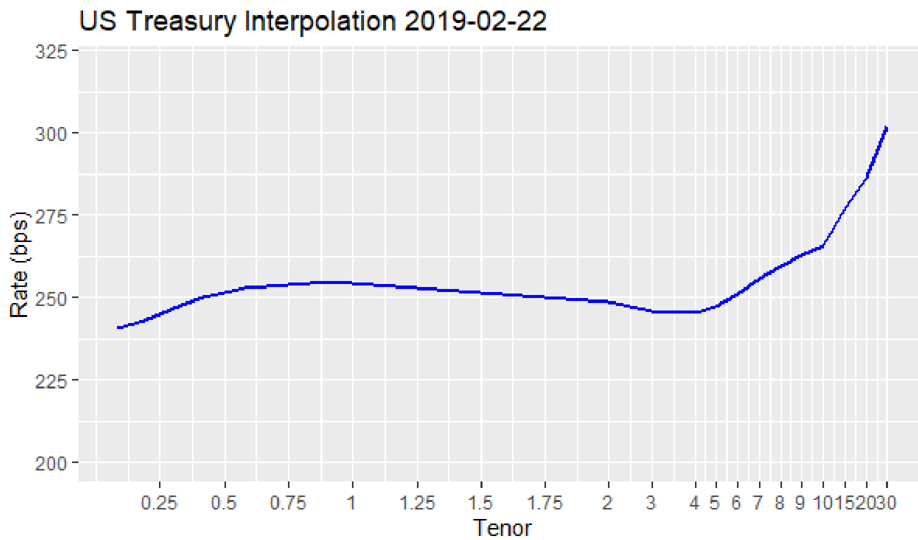

Figure 12: US Treasury curve

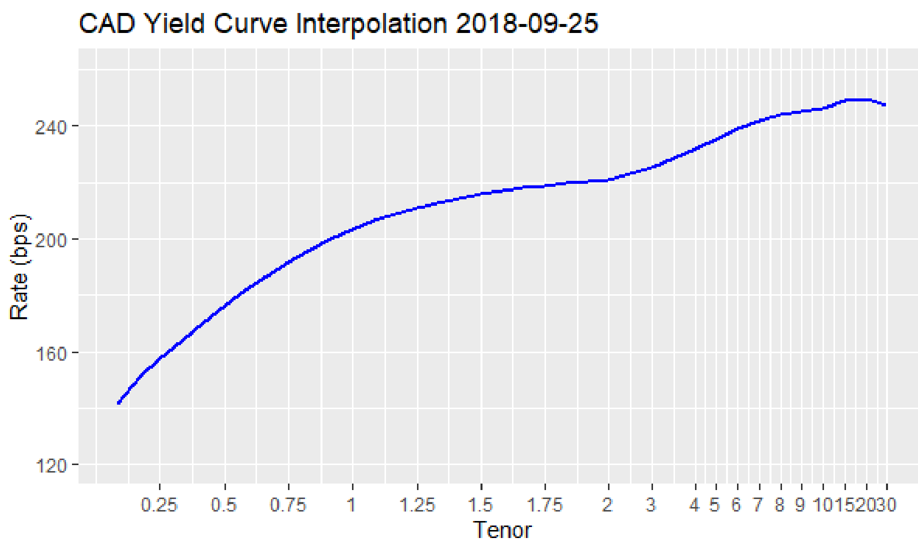

Figure 13: Canada yield curve

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