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Incorporating Name-Level Dynamics in Scenario-Based Rating Transition Matrices

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Abstract

This paper introduces a granular, obligor-level, scenario-based model for rating transition matrices. The model recognizes differences in the statistical properties of ratings and forward-looking probabilities of default (PDs), and it deviates from approaches that assume a one-to-one relationship between segment rating and PD or that completely decouple the dynamics of ratings and PDs. Instead, our model describes time-series dynamics ratings as a parallel process alongside the obligor's Moody's EDF™ (Expected Default Frequency) credit measure. In addition, our model captures the impact of the obligor's characteristics, such as industry, region, and its correlation with the credit environment. While calibrated to publicly traded, Moody's rated firms, the model can also be applied to private companies and financial institutions' internal ratings.

Comparing the model's projected downgrade/upgrade rates against actuals, we show the model captures the general pattern of observed rating dynamics across different points along the economic cycle and is thus suitable for use as a stress testing solution to project rating-dependent measures, such as Other Than Temporary Impairment (OTTI), Risk-Based Capital (RBC), and Risk-Weighted Assets (RWA).

Table of Contents

1. Introduction

Ratings are critical for navigating credit markets. Beyond being a cornerstone of credit strategy, they are crucial for regulatory reporting and financial accounting. From banks under the Advanced-IRB approach to insurers that must comply with ORSA or Solvency II requirements, ratings play a critical role in regulatory compliance and reporting. On the accounting front, ratings are often used in IFRS 9 stage classification, as well as evaluating whether a debt security is other-than-temporarily impaired (OTTI).

Because of their critical role, ratings-based statistics are often projected forward along macro scenarios for the purposes of capital adequacy, impairment, and stress testing rating dynamics. This paper introduces a granular, obligor-level, scenario-based model for rating transition matrices. The model recognizes differences in the statistical properties of ratings and forward-looking default probabilities (PDs), and it deviates from approaches that assume a one-to-one relationship between segment rating and PD or that completely decouple the dynamics of ratings and PDs. Instead, our model describes time-series dynamics of ratings as a parallel process alongside the obligor's Moody's EDF (Expected Default Frequency) credit measure. In addition, our model captures the impact of the obligor's characteristics, such as industry, region, and its correlation with the credit environment. While calibrated to publicly traded, Moody's Rated firms, the model can also be applied to private companies and financial institutions' internal ratings.

One important aspect of our approach is using EDF credit measures to provide a probability of default whose dynamics are not tied to a Moody's rating. This technique enables modeling the joint dynamics of ratings with that of forward-looking or point-in-time (PIT) probabilities of default (PD). In effect, the forward-looking nature of the name-level PIT PDs allows us to model rating dynamics more precisely and at the name-level. In particular, Moody's EDF credit metrics are PIT PD measures, which incorporate market information as of a given date in assessing a firm's expected likelihood of default. Meanwhile, a Moody's rating is generally associated as a through-the-cycle (TTC) credit risk measure that shows a high degree of stability over the cycle. Consequently, EDF value movements tend to lead that of ratings, but they are also more volatile.

Our approach recognizes that the relationship between the EDF measure and agency ratings varies cross-sectionally and across time. With this in mind, the model acknowledges their strong connection, both theoretical and empirical, with EDF values alongside other covariates entering into the rating migration model.

Another important feature is our probabilistic approach to modeling rating dynamics. A typical stress testing model produces the "expected" level of the statistics of interest (such as default loss), along a macroeconomic scenario. With ratings being ordinal measures, often having non-linear dynamic relationships with ratings-based statistics, calculating the "expected" rating faces limitations. Our probabilistic model, on the other hand, calculates each obligor's ratings quarterly transition matrices for a given macroeconomic scenario. The ratings distribution implied from these matrices provides a richer set of information. For example, the same "expected" rating may be associated with vastly different expected regulatory capital, because of a larger probability of realizing an extremely bad state. On the other hand, a probabilistic model can produce a more accurate measure of expected regulatory capital by accounting for the entire distribution of ratings.

While we present the model within the context of the publicly traded universe of corporate entities' (with EDF credit measures) and with Moody's rated debt, it is also designed and parameterized to cover the broader corporate loan book with private names and internal ratings. Our model can reasonably capture internal rating transition dynamics along macroeconomic scenarios, provided a mapping between the internal rating and Moody's rating can be established.

We structure the remainder of this paper as follows:

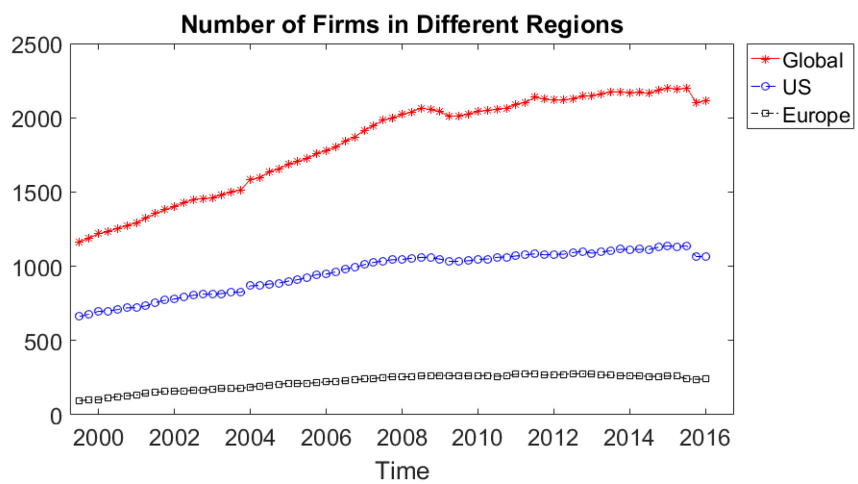
- Section 2 describes the empirical data used to estimate the model.
- Section 3 constructs the motivation behind the model and explains the intuition behind model specification.
- Section 4 explains estimating the model.
- Section 5 details how to use the model to estimate an obligor's Moody's rating transition matrices, given a specific macroeconomic scenario.
- Section 6 shows backtesting results used to evaluate model performance.
- Section 7 concludes.
- Appendix A
- Appendix B

2. Data

Our sample includes all publicly traded obligors with a Moody's rating from 1999 Q3 – 2016 Q1. Data includes each obligor's EDF measure and asset return, in addition to Moody's rating, at a quarterly frequency.[1, 2, 3]

Figure 1 plots the number of distinct obligors during each quarter in our database. The "Global" red line reports the number of covered obligors from all countries. On average, there are about 2,000 global obligors for each quarter. We segment the sample into U.S., European, and the rest-of-the-world (ROW) portfolios, based on country of incorporation. Across the entire sample period, both the U.S. and the ROW portfolios account for slightly less than half of the observations, while the European portfolio accounts for about 10%. On average, there are 800 U.S., 900 ROW, and 300 European obligors for each quarter. We estimate the final model coefficients separately, based on data from each sub-portfolio.

Figure 1 Number of Public Firms with a Moody's Rating



While 800, 900, and 300 obligors may appear as large numbers of observations per period at first look, estimating a Moody's 21x21 rating transition matrix during each quarter typically requires significantly more data, as the transition matrix contains 441 (=21^2) unknown parameters. Due to the insufficient number of observations, we first estimate the rating transition probabilities by 6 coarse ratings instead of 21 fine ratings categories, before transforming the estimated coarse rating transition probabilities into fine rating transition probabilities. Table 1 shows the mapping between coarse and fine ratings.

[1] EDF value is a probability of default measure provided by Moody's CreditEdge™.

[2] Asset return is defined as the log return of the asset value of each obligor. It is closely related to the change in EDF value.

[3] Note, our data does not contain observations of defaulted obligors during the sample period, which means our model describes rating transitions given no default.

Table 1 Coarse Rating Definition

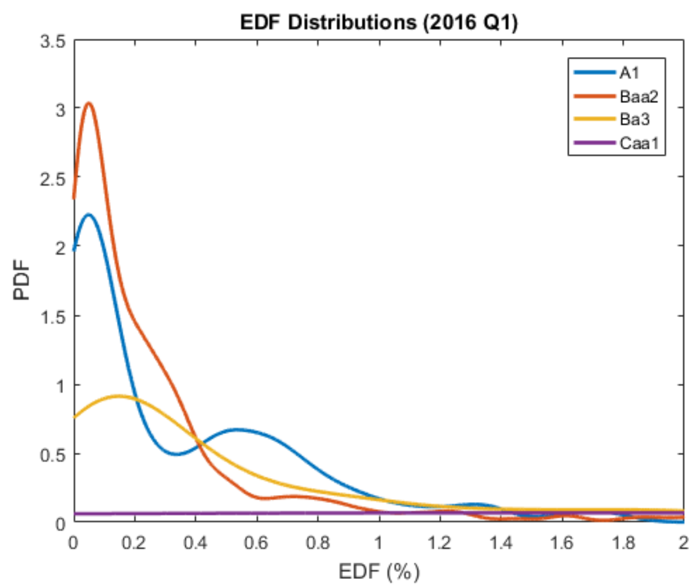
Fine Rating	Coarse Rating	Fine Rating	Coarse Rating
Aaa	Aa	Ba1	Ba
Aa1		Ba2	
Aa2		Ba3	
Aa3			
A1	A	B1	B
A2		B2	
A3		B3	
Baa1	Baa	Caa1	C
Baa2		Caa2	
Baa3		Caa3	
		Ca	
		C	

3. Model Motivation

Rating transitions can differ greatly from market-based, forward-looking PD (e.g., EDF measures) transitions. Differences arise from point-in-time measures reflecting current and forward market conditions, while ratings are intended to capture through-the-cycle effects and are intended to be more stable. In addition, rating change involves actions from rating agencies, and it may be reacting to market information somewhat slower than EDF values, which directly incorporate information from the equity market.

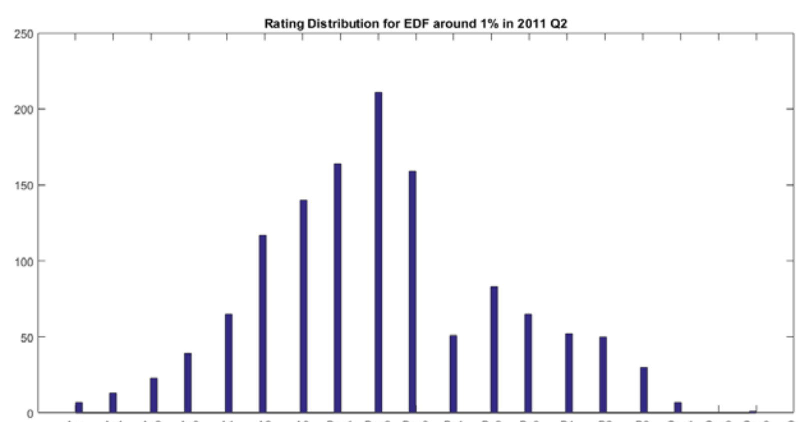
The difference between the nature of rating and PIT PD manifests primarily through two aspects. First, there is no simple static mapping between ratings and PIT PD. As we shall see, the range of EDF values (ratings) for a rating (EDF) category can be large. For example, Figure 2 plots EDF value distributions given a few Moody's rating A1, Baa2, Ba3, and Caa1. We see that, for these ratings, distribution overlaps, meaning obligors with the same EDF level can potentially have very different Moody's ratings at a single point in time.

Figure 2 Empirical Pattern — EDF Value Distribution, Given Ratings



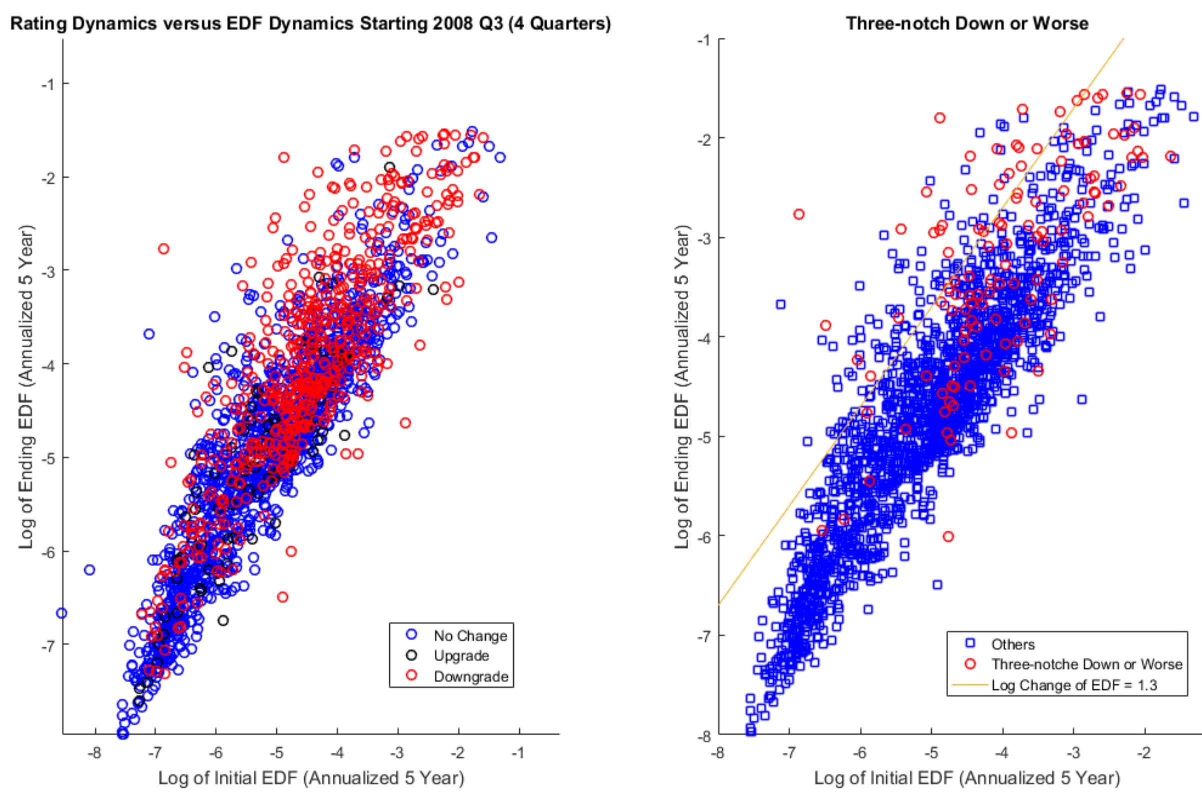
Similarly, Figure 3 plots the rating distributions of obligors with EDF values falling into a small interval, around 1%. The graph shows that, although the most likely rating associated with an EDF level of 1% is Baa, there are names that have similar EDF levels, but they have either a much better rating, such as Aaa, or a much poorer rating, such as Caa.

Figure 3 Empirical Pattern — Rating Distributions, Given EDF Value Interval



Second, rating changes are not necessarily consistent with changes in PIT PD over time. Figure 4 demonstrates this point. Each dot in the figure represents one obligor with a Moody's rating during the period beginning of 2008 Q3 to the end of 2009 Q2. The x and y axes denote the obligor's annualized five-year EDF value at the beginning and end of the period in log scale. An obligor with a large EDF value increase during the period appears on the top range of the figure, and vice versa. In the left chart, dot color represents whether an obligor experienced a downgrade (red), upgrade (black), or no rating change (blue) during the period. In the right chart, the color represents whether an obligor experienced a three-notch or worse downgrade (red) or otherwise (blue) during the period. If the dynamics of rating align perfectly with EDF value, we expect to see clear separation in red, black, and blue dots, with red dots on top corresponding to the largest increase in EDF value. However, while we do see that red dots tend to cluster on top, the separation between red, black, and blue dots is not definitive, indicating some, but not perfect, correlation between rating and EDF value changes.

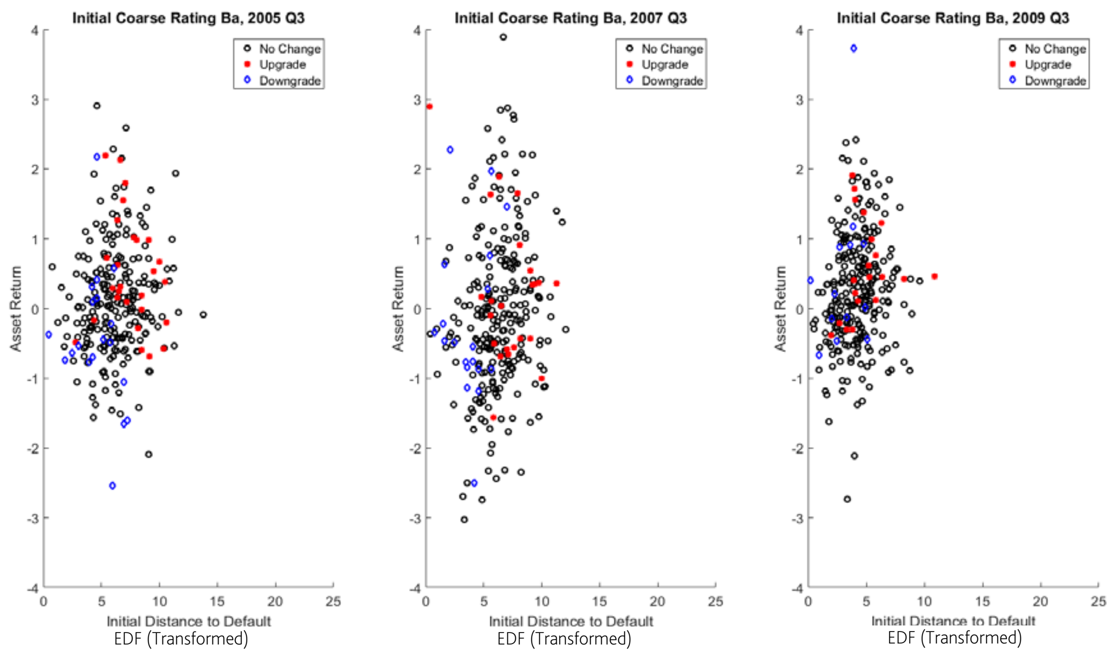
Figure 4 Change in Ratings Versus Change in EDF Value



We proceed with modeling rating dynamics, recognizing that a higher current PIT PD foreshadows a downgrade, increasing its likelihood during the subsequent period. Second, we model the positive, inter-temporal correlation between changes in ratings and PDs, resulting in a higher likelihood of a downgrade with increases in PDs, and vice versa, all else equal.

Empirical data supports both these hypotheses. Figure 5 shows three scatter plots for obligors' annual rating transition starting at three different points in time — at the end of 2005 Q3, 2007 Q3, and 2009 Q3. All of these obligors have the coarse rating Ba, including Ba1, Ba2, Ba3 as their initial rating.

Figure 5 Rating Transition Under Different Initial EDF Value (Transformed), Asset Return, and Macroeconomic Scenario



The x-axis depicts the negative normal inverse-transform of the EDF value asset return over the period.[4] Black dots denote obligors whose rating did not change during the one-year period after the start date. Red/blue dots denote the obligors whose rating was upgraded/downgraded during the same period.

From the graph, we see that the blue dots tend to congregate on the bottom left of the quadrant, indicating that obligors with lower transformed EDF value (i.e., higher initial PD) and lower asset return (i.e., larger increase in PD) are more likely to experience downgrades. Similarly, the red dots tend to congregate on the top right of the quadrant, indicating lower initial PD and smaller increase in PD leads to more rating upgrades.

Another important observation from this graph is that, during different time periods, the probability of upgrade and downgrade can differ, even after controlling for initial distance to default and asset return. This finding confirms the idea that rating transitions depend on not just PD, but other factors such as macroeconomic shocks as well.

With these observations in mind, we choose a regression-style model specification, where the rating dynamics are driven by a set of variables, including the initial distance to default level and asset return, as well as a systematic factor and an idiosyncratic factor:

$$Y_{i,t+1} = \alpha + \beta^{AR} r_{i,t} + \beta^{DD} DD_{i,t} + \beta^{Z} Z_t + \epsilon_{i,t} \tag{1}$$

Where $Y_{i,t+1}$ denotes the rating of obligor i at time $t+1$; $r_{i,t}$ denotes the asset return of obligor i between time t and $t+1$; $DD_{i,t}$ denotes the distance to default of obligor i at time t; Z_t denotes the systematic shock between time t, and $t+1$; $\epsilon_{i,t+1}$ denotes the unexplained idiosyncratic shock. There are nuances with this regression specification worth discussing. First, the dependent variable is rating, which is a categorical rather than numerical variable. This means we cannot use a simple linear regression model. In addition, there is a clear order in rating categories, with Aaa being the best, and C being the worst, which motivates us to use a Probit Order model (Mckelvey and Zavoina) as specified in Equations (2) and (3):

[4] The negative normal inverse transformation converts the EDF value into $EDF^* = -\Phi^{-1}(EDF)$, where Φ is the CDF function for the standard Gaussian distribution.

$$Y_{i,t+1}^{*} = \beta^{AR} r_{i,t} + \beta^{DD} DD_{i,t} + \sum_{n\geq 2} \beta^{FR_n} I(R_{i,t}\ is\ the\ n^{th}\ fine\ rating) + \beta^{Z} Z_t + \varepsilon_{i,\,t+1} \qquad (2)$$

$$Y_{i,t+1} = \begin{cases} C\ if\ Y_{i,t+1}^{*} \leq c_1 \\ B\ if\ \ c_1 < Y_{i,t+1}^{*} \leq c_2 \\ Ba\ if\ c_2 < Y_{i,t+1}^{*} \leq c_3 \\ Baa\ if\ c_3 < Y_{i,t+1}^{*} \leq c_4 \\ A\ if\ c_4 < Y_{i,t+1}^{*} \leq c_5 \\ Aa\ if\ c_5 < Y_{i,t+1}^{*} \end{cases} \qquad (3)$$

In these two equations, $Y_{i,t+1}^{*}$ denotes the latent factor that drives dynamics in rating migration. The model assumes when $Y_{i,t+1}^{*}$ exceeds/drops below a certain level c_j, which are model coefficients to be estimated, the rating Y would migrate to the corresponding coarse rating category, as seen in Equation (3). We also make the additional assumption that $\varepsilon_{i,\,t+1}$has i.i.d standard Gaussian distribution.[5]

Once we estimate the coefficients in Equations (2) and (3), we can then calculate the probability of $Y_{i,t+1}$ being a certain rating by using the formula in Equation (4), where we use numerical values 1, 2,..., 6 to represent coarse rating C, B, ..., Aa

$$P(Y = k) = P(c_{k-1} < Y^{*} < c_k) = P\big(c_{k-1} - X'\beta < \varepsilon_{i,t} < c_k - X'\beta\big) = \Phi(c_k - X'\beta) - \Phi(c_{k-1} - X'\beta) \qquad (4)$$

Where Φ denotes the cumulative density function of the standard Gaussian distribution; and $c_0 = -\infty$, $c_6 = +\infty$,

Note, the current rating $Y_{i,t}$ clearly affects the thresholds c_j. For example, if an obligor's current rating is Aa, it is very unlikely for the obligor to fall into the C rating category during the next quarter. This implies the value of c_1 must be very small, so that the chance of $Y_{i,t+1}^{*}$ being less than it is very small. On the other hand, if the current rating of the obligor is C, it is very likely for the obligor to remain in C over the next quarter. This implies the value of c_1 must be very large, so that the chance of $Y_{i,t+1}^{*}$ being less than it is very large. Because of this, we must allow the thresholds $c_j$to differ, depending on an obligor's starting rating category. We achieve this goal by estimating the Ordered Probit model separately for observations with each one of the six initial coarse ratings — six models in total.

Moreover, not only does the initial coarse rating affect rating migration probability, the initial fine rating should also have a large impact on migration between coarse ratings. For example, migration should be more likely for obligors with borderline fine ratings, such as Baa3 and Ba3, to migrate to the lower coarse rating than for obligors with middle fine ratings, such as Baa2 and Ba2, to have a coarse rating downgrade. Ideally, we should accommodate this feature by estimating a separate Ordered Probit model for each set of observations, with each one of the 21 initial fine ratings. However, this approach is not practical, given the limited observations available, as Section 2 discussed. Instead, Equation (2) introduces a set of additional explanatory dummy variables $I(R_{i,t}\ is\ the\ n^{th}\ fine\ rating)$, that indicate the starting, fine rating of each observation. The dummy variables' effect allows the model to shift the value of thresholds c_j, by a constant amount across j. While this process is a more restrictive specification than estimating the coefficients separately for each fine rating category, it achieves a similar effect, in terms of capturing the impact of the starting, fine rating on migration between coarse ratings.

The second nuance of our model specification is the systematic shock variable (which we refer to as the common rating factor), Z_t, is latent. To overcome this challenge, we design a two-step model estimation approach, where we first estimate the value of Z_t during each quarter in our sample period, and then plug in the estimated Z_t time series into Equations (2) and (3) to estimate the final model parameters. Section 4 describes these two steps in more detail.

[5] We have also explored modeling $\epsilon_{i,t}$ with a logistic distribution, which results in an Ordered Logit model. The overall performance of the Ordered Logit model is similar to the Ordered Probit model.

4. Model Estimation

This section describes our two-step estimation approach: first, we estimate the latent common rating factor during each period; second, we estimate the Ordered Probit model specified in Equations (2) and (3).

4.1 First Step: Estimating the Common Rating Factor Z

Intuitively, we can view the common rating factor *Z* as a gauge of the credit environment: obligors are more likely to receive a rating upgrade when the value of *Z* is high, and more likely to get a rating downgrade when the value of *Z* is low. Mathematically, *Z* can be regarded as a shift factor that shifts the values of thresholds c_i during each quarter: when the shift value is positive, the probability to achieve a poorer rating at the end of the quarter increases, and vice versa. We can estimate this shift factor using the Order Probit model, using the specification seen in Equations (5) and (6), across sample observations with starting coarse rating "RT". Note, Equation (5) differs from Equation (2) in only one term: it replaces $\beta^Z Z_t$ with a set of time dummy variables $Z_{RT,\tau} I(t=\tau)$, equal to the shift factor $Z_{RT,\tau}$ for quarter τ, if the corresponding observation lies in quarter τ and 0 otherwise. From a regression perspective, $Z_{RT,\tau} I(t=\tau)$ can be thought of as the time-varying constant term. Intuitively, $Z_{RT,\tau}$ captures the effect of time periods (and thus macroeconomic scenarios) on rating transitions left uncaptured by obligor-level statistics, and it is constant across all the observations, with the starting coarse rating "RT" during a time period. Hence, $Z_{RT,\tau}$ can be considered as a common credit rating factor.

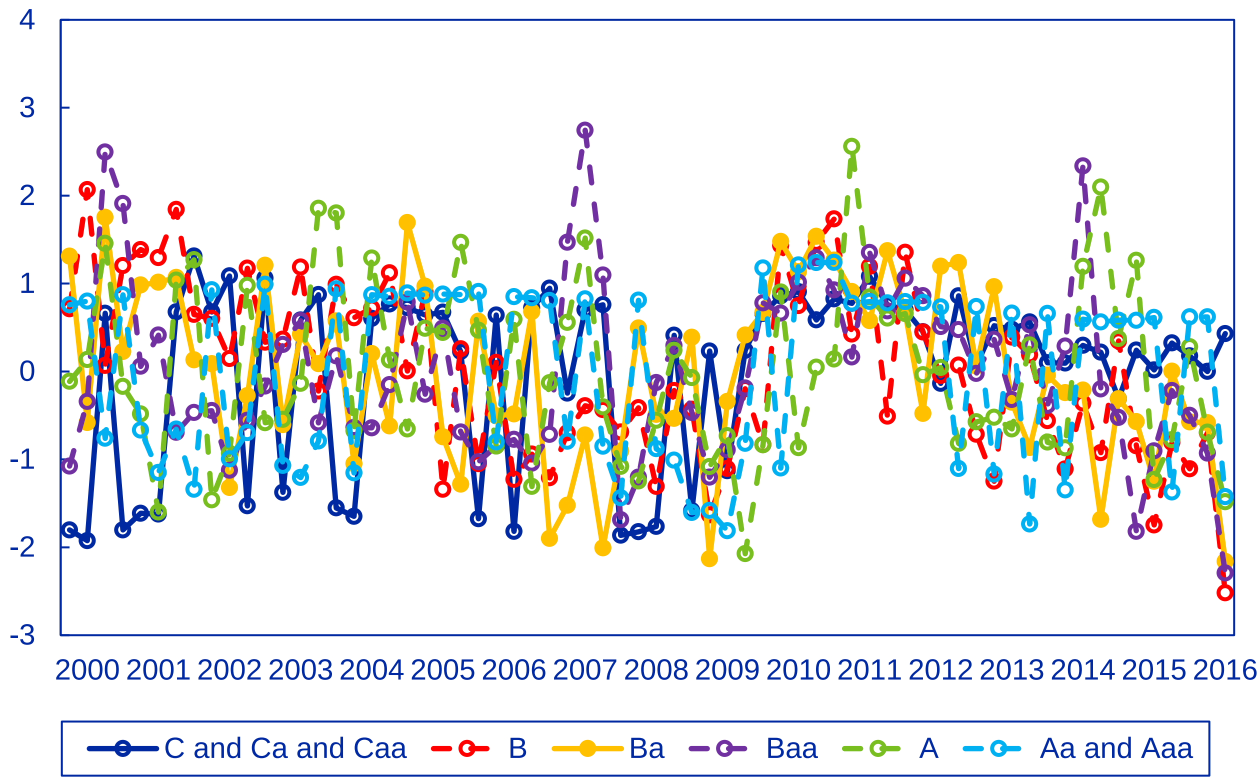
We then pool all the observations with the same starting coarse rating "RT" across time and names and estimate $Z_{RT,\tau}$, together with all the other model parameters ($\beta^{AR}, \beta^{DD}, c_1, c_2, \ldots, c_5$), using the Maximum Likelihood Estimator.

$$Y^*_{i,t+1} = \beta^{AR} r_{i,t} + \beta^{DD} DD_{i,t} + \sum_{n\geq 2} \beta^{FR_n} I\left(R_{i,t} \text{ is the } n^{th} \text{ fine rating}\right) + \sum_{\tau \geq 2} Z_{RT,\tau} I(t=\tau) + \varepsilon_{i,\,t+1} \qquad (5)$$

$$Y_{i,t+1} = \begin{cases} C \text{ if } Y^*_{i,t+1} \leq c_1 \\ B \text{ if } c_1 < Y^*_{i,t+1} \leq c_2 \\ Ba \text{ if } c_2 < Y^*_{i,t+1} \leq c_3 \\ Baa \text{ if } c_3 < Y^*_{i,t+1} \leq c_4 \\ A \text{ if } c_4 < Y^*_{i,t+1} \leq c_5 \\ Aa \text{ if } c_5 < Y^*_{i,t+1} \end{cases} \qquad (6)$$

Figure 6 plots the time series of estimated Z for each coarse rating category, estimated based on the U.S. sub-portfolio. Figure 13 and Figure 16 in Appendix A and Appendix B plot the corresponding time series for the European and ROW portfolios.

Figure 6 Coarse Rating-Specific Credit Rating Factors' Time Series Estimated Based on the U.S. Sub-portfolio



In principle, we can use these six estimated *Z* time series in the Ordered Probit model defined by Equations (2) and (3) and estimate the final model parameters for the corresponding coarse rating category. However, we opt to use the first principal component of the *Z* as the same explanatory variables across all six coarse rating categories for two main reasons:[6] First, while all six *Z*'s are different values, they seem to follow a common trend over time. Consolidating them into one time series greatly simplifies the model without sacrificing too much accuracy. Second, each *Z* time series by itself is extremely volatile over time, most likely due to the limited sample observations we have for each coarse rating category during each quarter. Consolidating them reduces such data noises significantly.

Figure 7 shows the principal component analysis of all the six *Z* specific to each coarse rating categories. This analysis suggests that there is indeed a common trend among credit rating factors of different rating categories, and their first principal component can explain up to 45% of their total variation.

[6] Note, our model specification still allows for different coarse rating categories to have different exposure/beta coefficients for *Z*.

Figure 7 PCA of Coarse Rating-Specific Credit Rating Factors

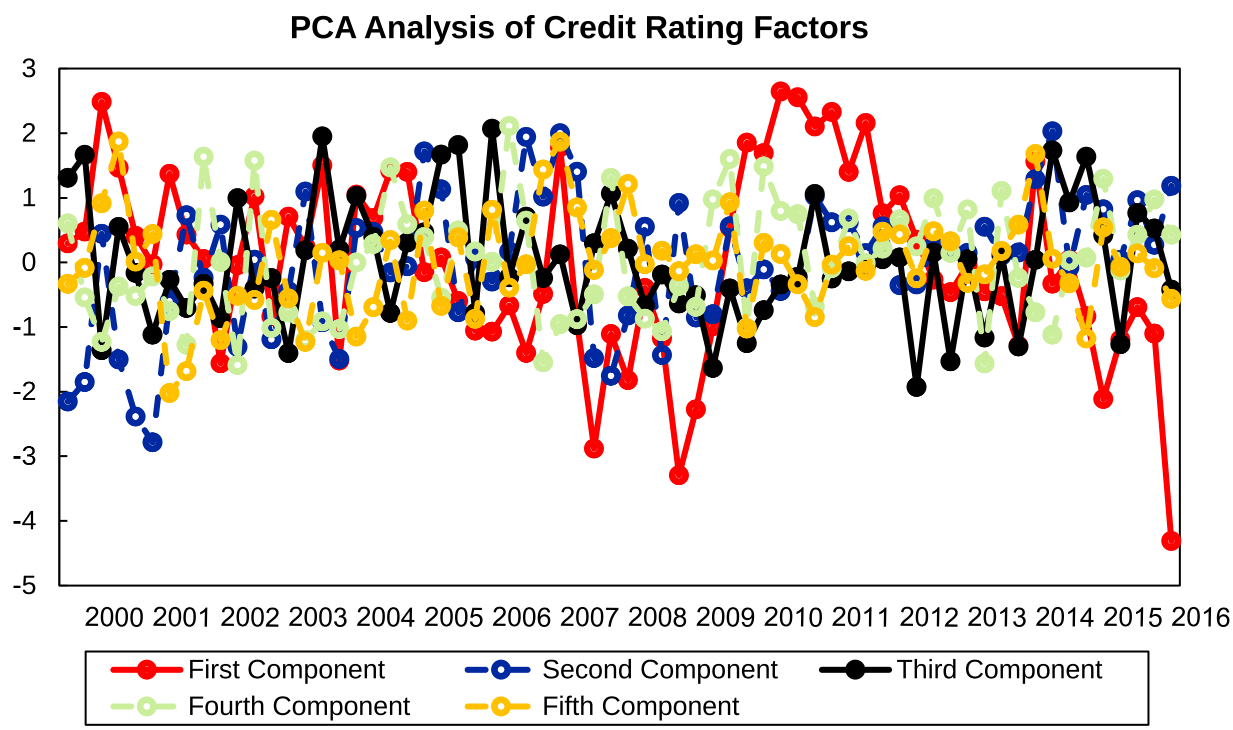
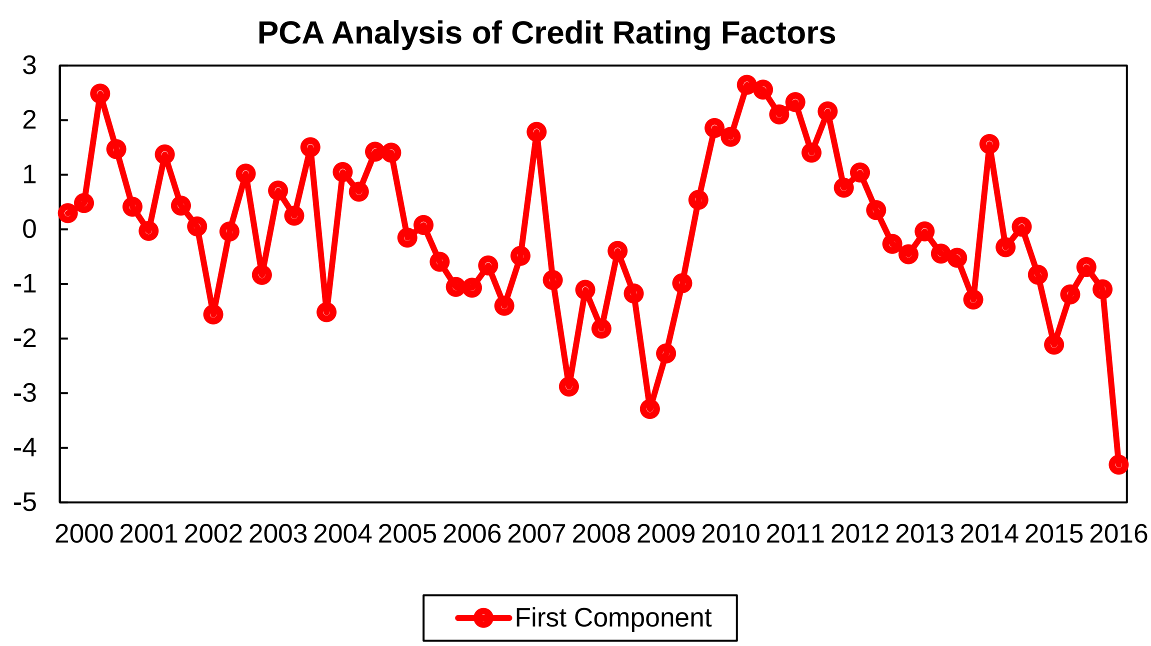


Figure 8 plots the first principal component, denoted as $\hat{Z}_{RT}^{single}$, which captures the main macroeconomic trends over time quite well. For example, the first component drops precipitously during the 2008–2009 financial crisis period and recovers afterward. It also captures the mild recessionary period around 2001. We see that it also declines sharply during 2016 Q1 due to massive downgrades of energy companies caused by a persistent oil price decline.

Figure 8 First PCA of Coarse Rating-Specific Credit Rating Factors



4.2 Second Step: Estimating the Ordered Probit Model

In the first step, we estimate a single credit rating factor across coarse rating categories. It captures the main macroeconomic trend that impacts rating. This single rating factor is common to each coarse rating category, but it potentially affects each rating category differently, as, for example, obligors with A ratings may be more likely to be downgraded than others during an economic crisis, because they have less incentive to maintain the current rating level during hardship compared to an obligor with Baa3 ratings, implying a larger beta coefficient for *Z* for A ratings. In addition, an obligor's exposure to *Z* may depend upon its industry. For example, obligors in the financial industry may be more or less affected by systematic shocks compared to obligors in the corporate sector. Last but not least, an obligor's exposure to *Z* is likely dependent on its correlation with the general economy, driven by the nature of the obligor, such as its assets or sales size. Due to these reasons, we refine the Ordered Probit framework to:

» Differentiate beta coefficients for *Z* across six coarse rating categories

» Differentiate beta coefficients for *Z* for financial versus non-financial sectors

» Set the beta coefficients to be proportional to obligor's squared root of RSQ, which measures each obligor's correlation with general systematic economic shock in Moody's GCorr™ model

Equations (7) and (8) summarize the resulting model specification.

$$Y_{i,t+1}^{*} = \beta^{AR} r_{i,t} + \beta^{DD} DD_{i,t} + \sum_{n\geq 2} \beta^{FR_n} I(R_{i,t}\ is\ the\ n^{th}\ fine\ rating) + \left(\beta^{Z} + \beta^{Z_{financial}} \times I_{financial}\right)\sqrt{RSQ_i}\,\hat{Z}_{RT,t}^{single} + \varepsilon_{i,\,t+1} \quad (7)$$

$$Y_{i,t+1} = \begin{cases} C\ if\ Y_{i,t+1}^{*} \leq c_1 \\ B\ if\ c_1 < Y_{i,t+1}^{*} \leq c_2 \\ Ba\ if\ c_2 < Y_{i,t+1}^{*} \leq c_3 \\ Baa\ if\ c_3 < Y_{i,t+1}^{*} \leq c_4 \\ A\ if\ c_4 < Y_{i,t+1}^{*} \leq c_5 \\ Aa\ if\ c_5 < Y_{i,t+1}^{*} \end{cases} \quad (8)$$

4.3 Estimated Model Coefficients

As discussed, we estimate the Ordered Probit model defined by Equations (7) and (8) separately for observations with different coarse ratings at the beginning of each quarter. Consequently, we have six sets of coefficients. Table 2 reports the coefficients estimated based on the U.S. sub-portfolio. Tables 4 and 5 in Appendices A and B report the coefficients estimated based on the European and ROW sub-portfolios.

We find that asset return has a positive effect. It is statistically significant for initial coarse rating categories C, B, Ba, Baa, and A, which means, for instruments with an initial coarse rating of C, B, Ba, Baa, or A, we observe that the higher asset return of an obligor increases its chances of being upgraded, and decreases its chances of being downgraded. As for the best initial coarse rating category (which includes fine rating at or above Aa3), asset return does not seem to be a significant factor in determining rating transitions.

In addition, the initial distance to default (which gives similar information as initial PD) has a positive effect. It is statistically significant for all initial coarse rating categories. This trait implies that the smaller the distance to default (meaning a higher probability of default), the higher the chance for an obligor to migrate to a lower rating category.

Moreover, all the initial fine rating dummies have a statistically significant impact with intuitive signs, which implies that, for two instruments with the same initial coarse rating, the one with the relatively poorer fine rating has a larger probability of being downgraded.

Finally, the single credit rating factor has a positive effect. It is statically significant for all coarse rating categories, meaning a lower credit rating factor is associated with a higher likelihood of downgrade, as expected.

Table 2 Estimated Coefficients for U.S. Observations

Variables	C, Ca, Caa3, Caa2, Caa1 (Coarse Rating 1)	B3, B2, B1 (Coarse Rating 2)	Ba3, Ba2, Ba1 (Coarse Rating 3)	Baa3, Baa2, Baa1 (Coarse Rating 4)	A3, A2, A1 (Coarse Rating 5)	Aa3, Aa2, Aa1, Aaa (Coarse Rating 6)
Asset Return (β^{AR})	0.0880	0.0892	0.0752	0.0793	0.0000	0.1008
Distance to Default (β^{DD})	0.5017	0.6869	0.8014	0.9387	0.6602	0.5680
Z Factor (β^{Z})	0.2819	0.2641	0.2212	0.2479	0.2596	0.3441
Z Factor Financial ($\beta^{Z_{financial}}$)	0.0447	-0.0467	0.0312	-0.0581	-0.1449	0.0450
Fine rating 1	-0.4378	-0.6578	-0.4707	-0.6157	-0.9846	-1.9355
Fine rating 2	-0.4938	-1.0445	-0.8634	-1.1641	-1.5220	-2.3798
Fine rating 3	-0.4938					-2.9075
Fine rating 4	-0.4938					
c1	2.6347	-1.5360	-1.8667	-2.1991	-3.1093	$-\infty$
c2	4.2574	3.6246	-0.6375	-1.4891	-2.9911	$-\infty$
c3	∞	5.1392	4.1504	-0.5874	-2.6299	-4.6336
c4	∞	5.3472	∞	5.2140	-1.3994	-4.1330
c5	∞	∞	∞	6.3645	4.5855	-3.0455

5. Rating Transitions Across Macroeconomic Scenario

The estimated Ordered Probit model can be used to describe dynamics of rating transition matrices, across macroeconomic scenarios. To do this, we first must project the explanatory variables $r_{i,t}$, $DD_{i,t}$, and $\hat{Z}_{RT,t}^{single}$ in Equation (7), given a macroeconomic scenario. The projection of $r_{i,t}$, $DD_{i,t}$, follows the framework of Moody's GCorr Macro model. The process is omitted here for brevity. Interested readers can refer to Hong et al. (2016) for details. The projection of *Z* follows the same model framework and is described in Section 5.1.

5.1 Projecting *Z* Given A Macroeconomic Scenario

We estimate the *Z* time series obtained in Section 4 using historical data. In order to describe the dynamics of rating transition matrices across macroeconomic scenarios, we project the future *Z* under the corresponding scenario. To achieve this step, we integrate the *Z* factor into the general GCorr Macro framework, first assuming *Z* and all macroeconomic variable shocks have a Gaussian distribution, and then calibrate the modeled correlation between *Z* and different macroeconomic variables to fit their empirical correlation. This process gives us a multinomial distribution including both *Z* and macroeconomic variables, which is then used to project *Z* under a given macroeconomic scenario.

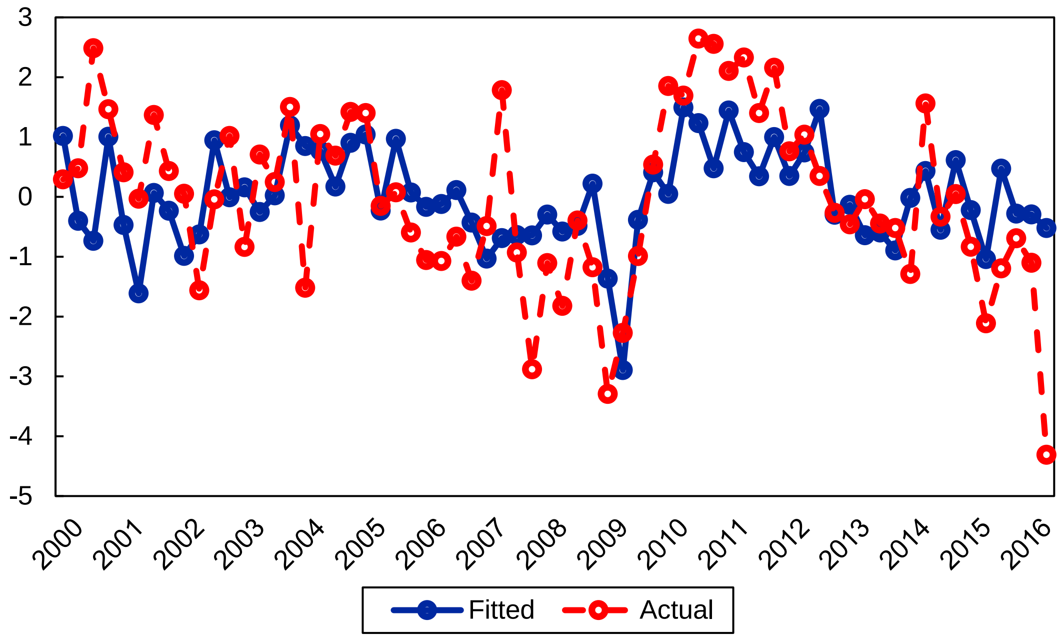
Table 3 shows the empirical correlation between *Z* and several key macroeconomic variables. We see that the *Z* has a significant positive correlation with U.S. Equity, U.S. GDP, and global oil, while having a significant negative correlation with U.S. unemployment. This implies, for example, that a decrease in U.S. equity (which means market conditions deteriorate) is associated with a decrease in *Z*, and is associated with a higher likelihood of a rating downgrades for all obligors.

Table 3 Correlation of Credit Rating Factor with Macroeconomic Variables

U.S. Equity	U.S. GDP	U.S. Unemployment	U.S. VIX	U.S. BBB Spread	Global Oil Price
0.292	0.424	-0.385	-0.135	-0.161	0.45

Based on the *Z*-integrated GCorr Macro model, we calculate the expected (i.e., fitted) value of *Z*, conditional on the value of realized historical macroeconomic variables during the sample period. Figure 9 compares the fitted *Z* against the original *Z* values estimated in Section 4.1. We can see that the fitted value follows the pattern of the actual value reasonably well, capturing the peaks and troughs corresponding to economic booms, busts, and recoveries.

Figure 9 Fitted Versus Actual *Z* Factor



5.2 Coarse Rating Transition Probabilities

With the estimated model coefficients, as well as projected values of explanatory variables $r_{i,t}$, $DD_{i,t}$, and $\hat{Z}_{RT,t}^{single}$, we can now use Equations (7) and (8) to project an obligor's transition probabilities to each coarse rating category at the end of each quarter: $TP_{i,0}^{coarse}, TP_{i,t}^{coarse}, \ldots, TP_{i,T}^{coarse}$. Here, $TP_{i,0}^{coarse}$ is a 1x6 vector that denotes the transition probability to each coarse rating at the end of the first quarter. Where $t \geq 1$, $TP_{i,t}^{coarse}$ is a 21x6 matrix that denotes the transition probabilities from each fine rating at time t to each coarse rating at time $t+1$. We calculate the values of $TP_{i,0}^{coarse}, TP_{i,t}^{coarse}, \ldots, TP_{i,T}^{coarse}$ period by period according to Equation (4).

5.3 Coarse Rating Transition Probability to Fine Rating Transition Matrix

The procedure used in Section 5.2 provides an obligor's transition probability from an arbitrary fine rating at the beginning of a quarter to each coarse rating at the end of the quarter. In practice, however, it is often useful for us to know the full transition matrices in fine ratings. We could have achieved this step by estimating the Ordered Probit model for observations with each fine rating separately. However, as mentioned, due to the limited sample size, this approach is not feasible. Instead, we use the modeled- implied transition probability to coarse ratings, together with the historical composition of fine ratings within each coarse rating, to approximate the fine rating transition matrices.

Specifically, we first estimate an unconditional TTC fine rating quarterly transition matrix based on pooled observations across obligors and time. The transition matrix implies the TTC percentage of observations of each fine rating within each coarse rating category. We then use this TTC fine rating composition to approximate the transition probability to each fine rating, given the transition probability to each coarse rating, according to Equation (9):

$$Prob^{split}\left(R_1^{fine} = l_{fine}^{destination}\right) = \frac{p(R_0^{fine}, l_{fine}^{destination})}{p(R_0^{fine}, l_{coarse}^{destination})} \times Prob\left(R_1^{coarse} = l_{coarse}^{destination}\right) \quad \textbf{(9)}$$

(Where $p(R_0^{fine}, l_{fine}^{destination})$ and $p(R_0^{fine}, l_{coarse}^{destination})$ are the probabilities of transitioning from R_0^{fine} to $l_{fine}^{destination}$ and $p_{l_{coarse}^{destination}}$, respectively, using $TM^{Through\ Cycle}$)

$$Prob\left(R_1^{fine} = l_{fine}^{destination}\right) = \frac{Prob^{split}\left(R_1^{fine} = l_{fine}^{destination}\right)}{\sum_{l_{fine}^{destination}=1}^{21} Prob^{split}\left(R_1^{fine} = l_{fine}^{destination}\right)} \tag{10}$$

Once we obtain the transition matrix (or vector) for a quarter, the fine rating transition probability vector is calculated as the concatenated transition matrices, as represented in Equation (11):

$$P_t^{fr} = TP_{0,1}^{fine} \times TM_{1,2}^{fine} \times \ldots \times TM_{t-1,t}^{fine} \tag{11}$$

6. Backtesting

To evaluate model performance and accuracy, we run a set a backtests by comparing model-implied rating downgrade rates against actual rating downgrade rates. We use the actual macroeconomic scenario, as well as the actual rating of each obligor at the beginning of each quarter, across all three sub-portfolios to calculate the transition probability to each rating category at the end of each quarter during the sample period 1999 Q3 – 2016 Q1, according to the procedure described in Section 5. Note, the rating transition for the U.S., European, and ROW obligors are projected based on corresponding model coefficients, respectively.

We then calculate the probability of downgrade/upgrade for each obligor during each quarter as the sum of probabilities of transitioning to, at the end of the quarter, any of the coarse ratings that are poorer/better than the obligor's actual rating at the beginning of the quarter. We then take the average projected downgrade/upgrade probability across the U.S., European, and ROW across obligors to obtain the projected downgrade/upgrade rates for each of the three sample portfolios.

Finally, we compare the projected downgrade/upgrade rates against actual coarse rating downgrade/upgrade rates of each portfolio over the period, where the actual downgrade/upgrade rate during a quarter is calculated as the number of obligors experiencing an actual coarse rating downgrade/upgrade, divided by the total number of obligors.

Figures 10, 11, and 12 plot the projected downgrade/upgrade rates against actuals for each of the three sample portfolios. We see, in general, that the model-projected downgrade/upgrade rates match closely the pattern of actuals. In addition, the model-projected downgrade rates during the sub-prime crisis in 2008 and 2009 are of similar magnitude to actuals. Overall, these backtesting results show that our model is well-suited to describe the dynamics of rating transition matrices, across macroeconomic scenarios.

Figure 10 Model-Projected Versus Actual Downgrade/Upgrade Rates of the U.S. Portfolio

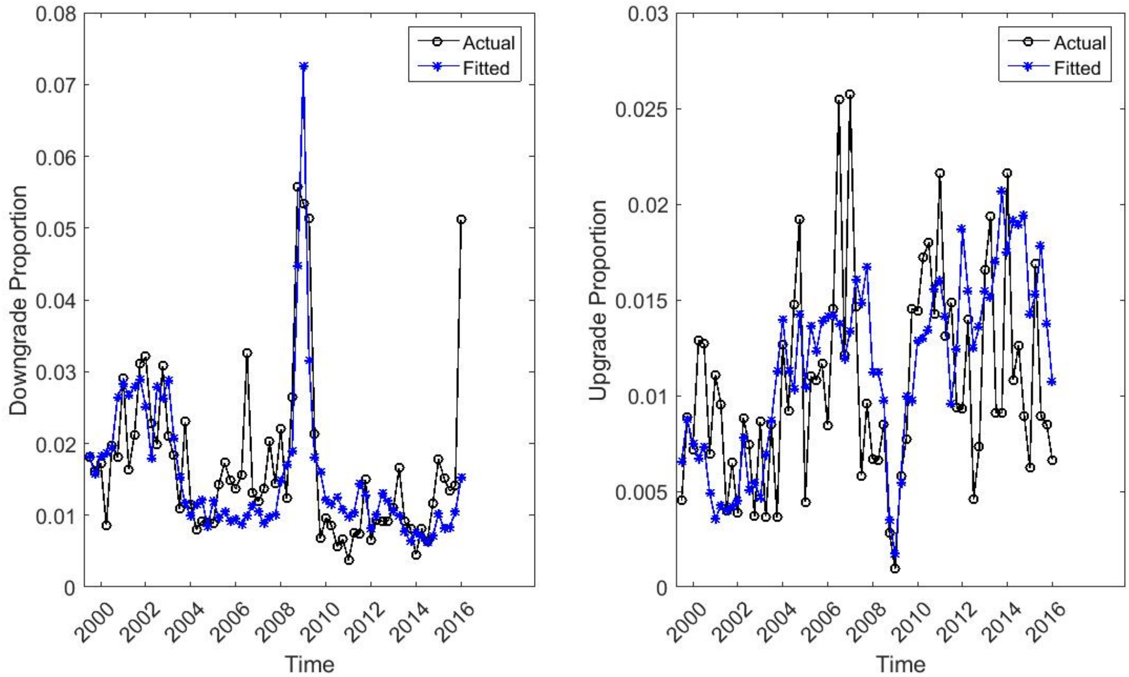


Figure 11 Model-Projected Versus Actual Downgrade/Upgrade Rates of the European Portfolio

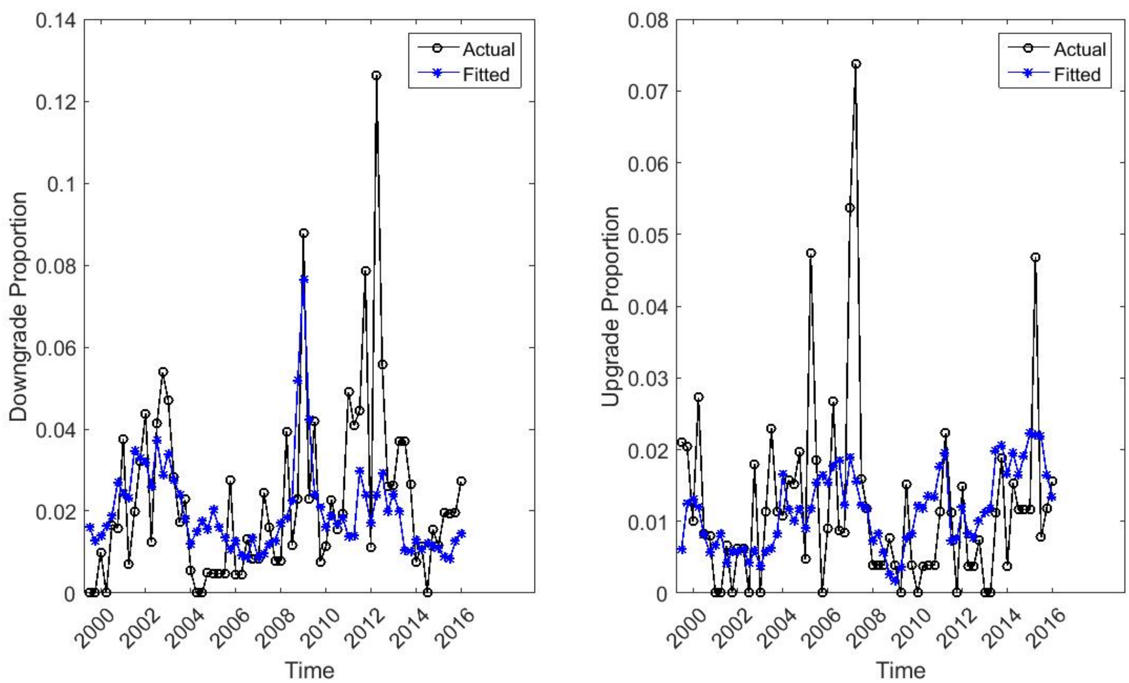
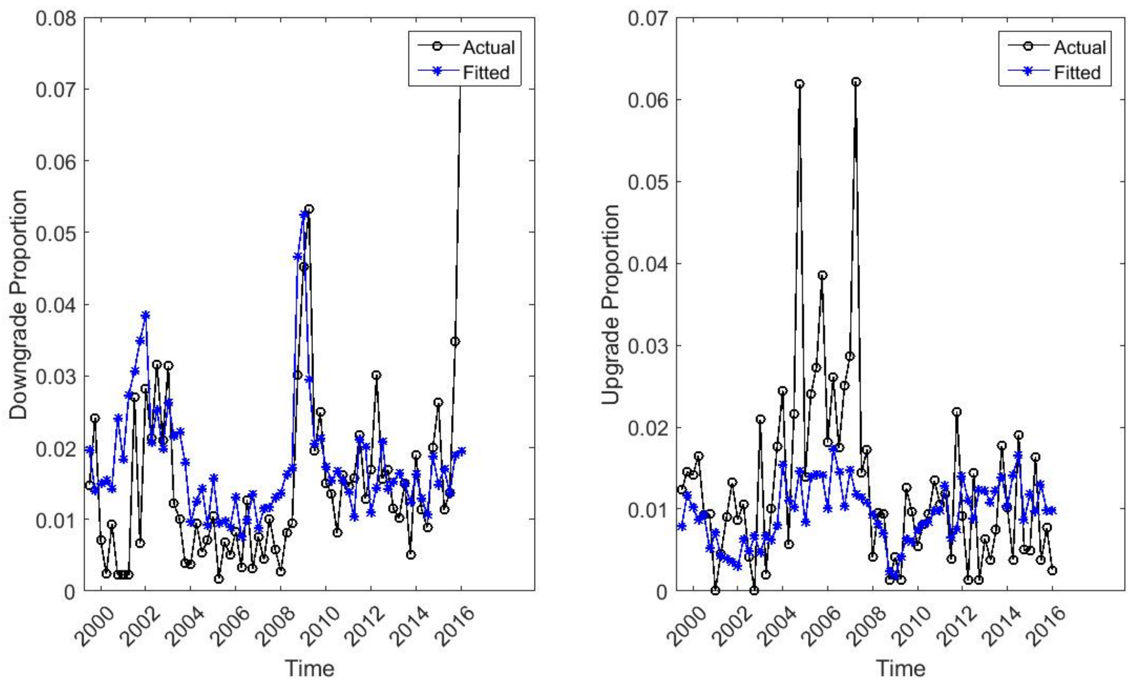


Figure 12 Model-Projected Versus Actual Downgrade/Upgrade Rates of the ROW Portfolio



7. Summary

This paper proposes and estimates an Ordered Probit model that described obligor-level rating transition matrices across macroeconomic scenarios. The approach is unique, in that, it describes time-series dynamics of Moody's ratings as a parallel process alongside the obligor's Moody's EDF (Expected Default Frequency) credit measures. The model can be used as part of a stress testing solution to project rating-dependent measures, including OTTI loss, RBC, and RWA, given a stressed macroeconomic scenario. It can also be used to benchmark the transition matrices of an institution's internal rating, provided a mapping between the internal rating and Moody's rating can be established.

We estimate the model for three separate sets of sample data: the U.S., Europe, and the Rest-of-the-World. The model differentiates the dynamics of rating from PD, while allowing the latter to help in describe rating transition probabilities. It also accounts for obligor-specific characteristics, such as the obligor's industry sector, as well as the obligor's correlation with the general economy. The model backtests well, with model-implied downgrade rates matching well with actuals.

Appendix A: Estimated Model Based on the European Sub-portfolio

Figure 13 Coarse Rating-Specific Credit Rating Factors' Time Estimated Based on the European Sub-portfolio

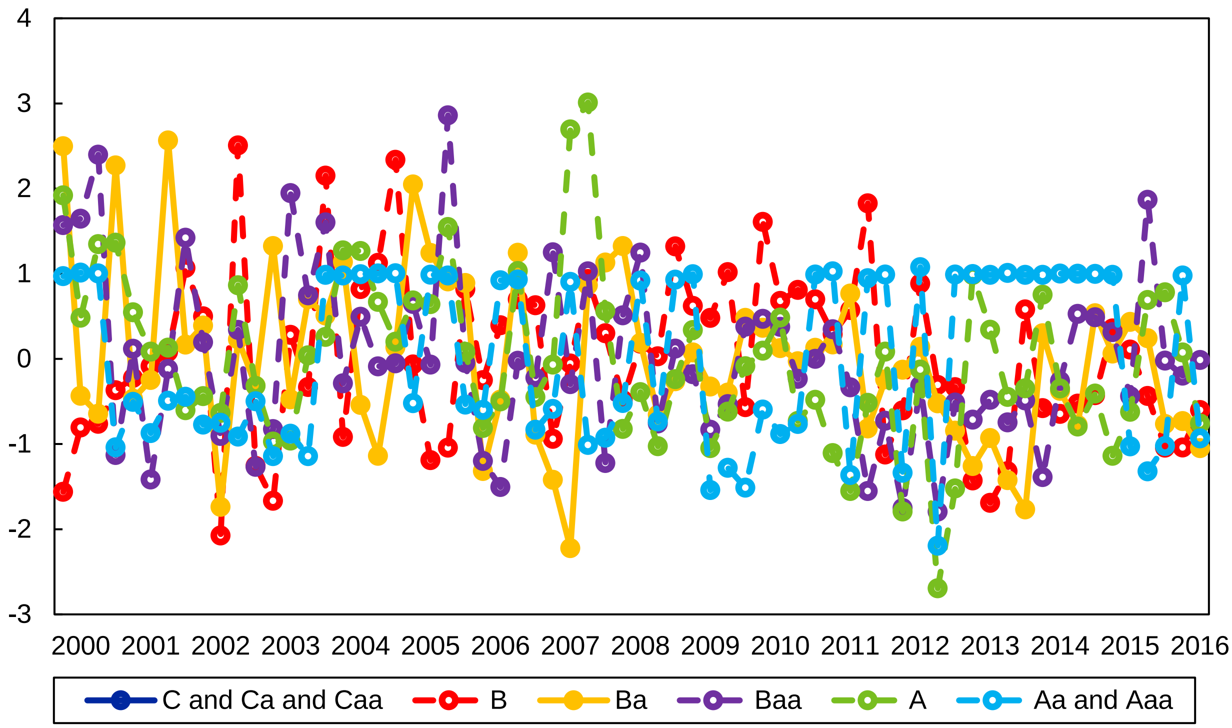


Figure 14 PCA of Coarse Rating-Specific Credit Rating Factors for the European Sub-portfolio

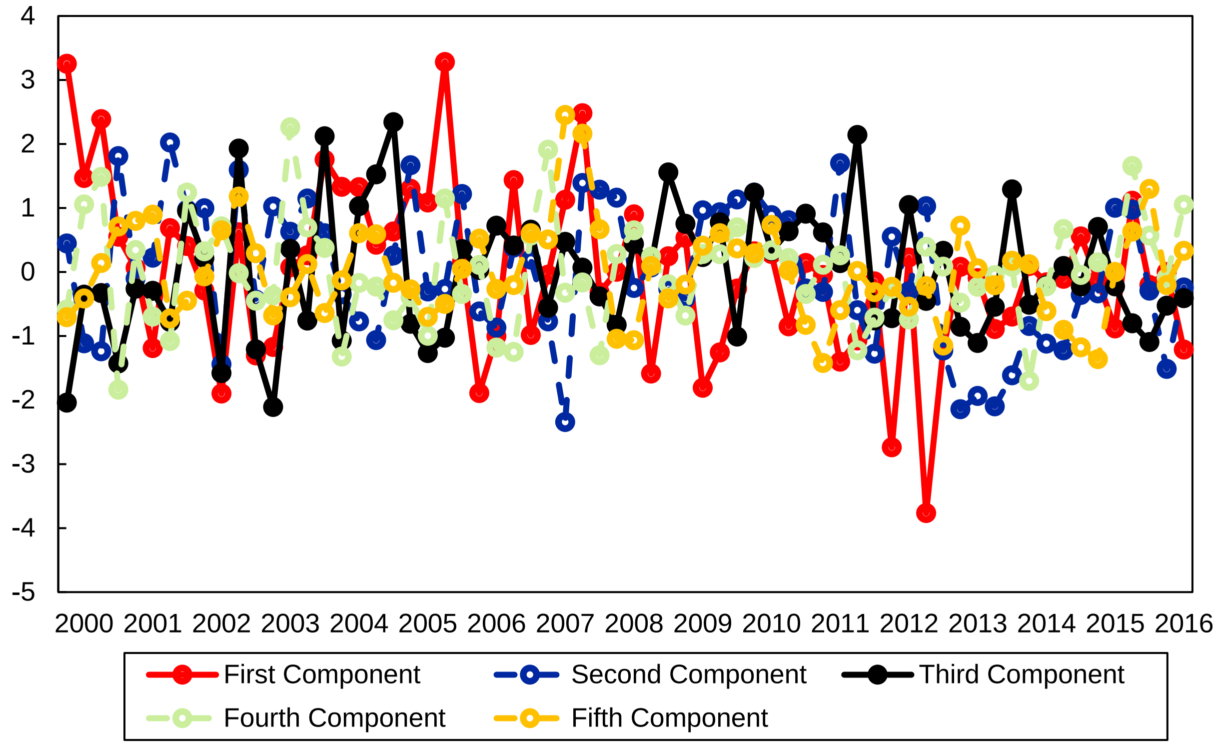


Figure 15 Actual Versus Fitted *Z* Time Series for the European Sub-portfolio

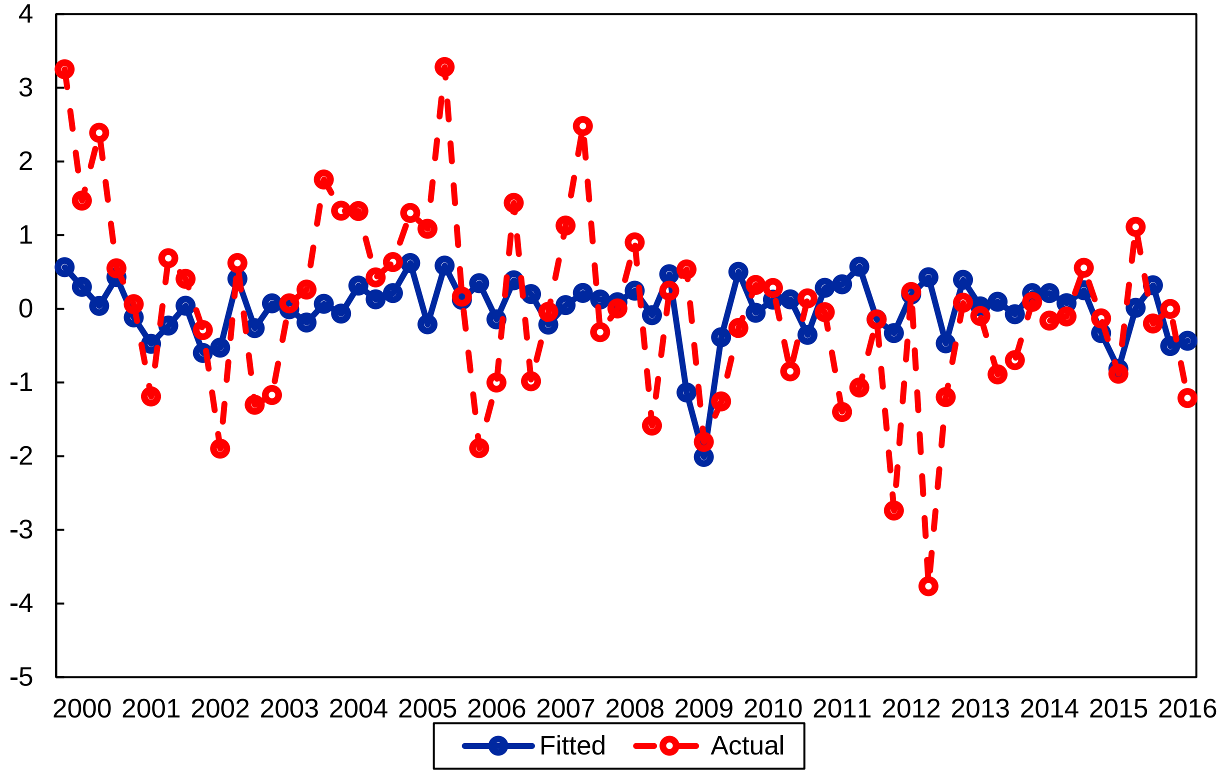


Table 4 Estimated Coefficients for the European Sub-portfolio

Variables	C, Ca, Caa3, Caa2, Caa1 (Coarse Rating 1)	B3, B2, B1 (Coarse Rating 2)	Ba3, Ba2, Ba1 (Coarse Rating 3)	Baa3, Baa2, Baa1 (Coarse Rating 4)	A3, A2, A1 (Coarse Rating 5)	Aa3, Aa2, Aa1, Aaa (Coarse Rating 6)
Asset Return ($\boldsymbol{\beta}^{AR}$)	0.0160	0.1014	0.0000	0.0824	0.0932	0.0000
Distance to Default ($\boldsymbol{\beta}^{DD}$)	0.6452	0.7798	0.7244	0.3954	0.4535	0.3411
Z Factor ($\boldsymbol{\beta}^{Z}$)	0.2567	0.1064	0.1940	0.3004	0.2159	0.2542
Z Factor Financial ($\boldsymbol{\beta}^{Z_{financial}}$)	1.0700	0.3929	0.1580	0.1854	0.3122	0.2697
Fine rating 1	-0.6499	-0.6887	-0.4709	-0.5674	-0.5518	0.0000
Fine rating 2	-0.6499	-1.3002	-1.1105	-1.2499	-1.1146	-0.0958
Fine rating 3	-1.0498					-0.4589
Fine rating 4	-1.0498					
c1	3.1710	-1.1495	-2.5345	$-\infty$	-3.2850	$-\infty$
c2	4.0768	3.6727	-0.9110	-3.1295	-3.2848	$-\infty$
c3	∞	5.2343	3.6229	-2.0227	-2.7669	$-\infty$
c4	∞	∞	5.0662	3.2872	-1.5583	-2.5813
c5	∞	∞	∞	4.4620	3.5854	-1.1734

Appendix B: Estimated Model Based on the ROW Sub-portfolio

Figure 16 Coarse Rating-Specific Credit Rating Factors' Time Series for the ROW Sub-portfolio

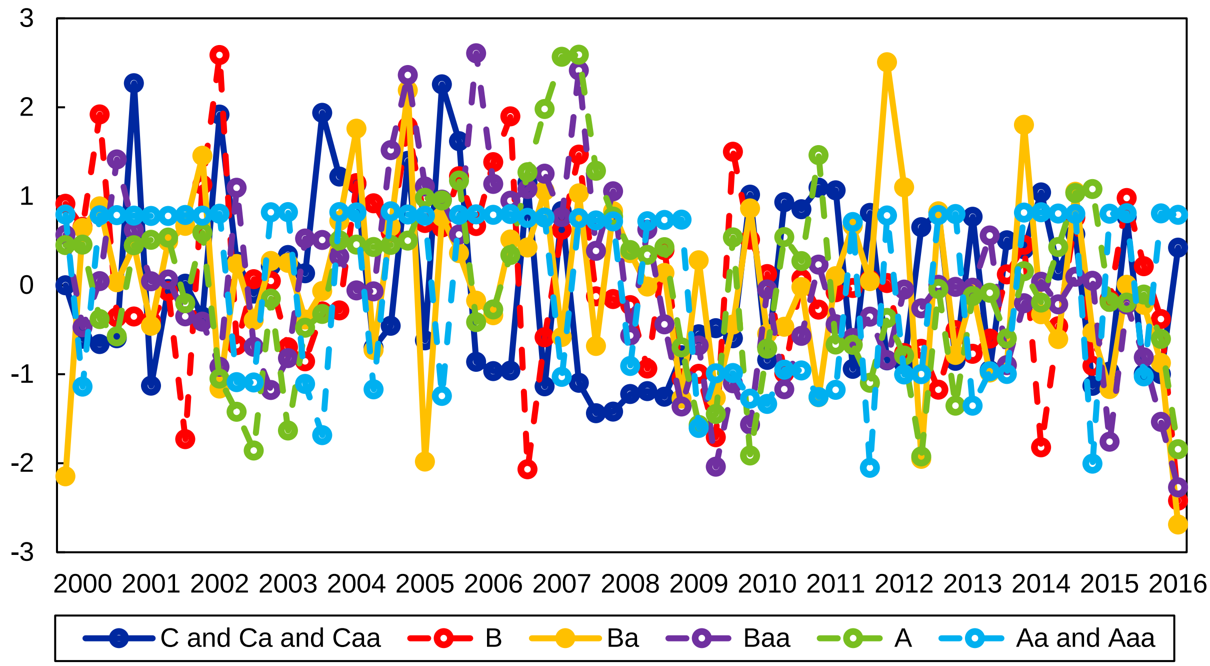


Figure 17 PCA of Coarse Rating-Specific Credit Rating Factors for the ROW Sub-portfolio

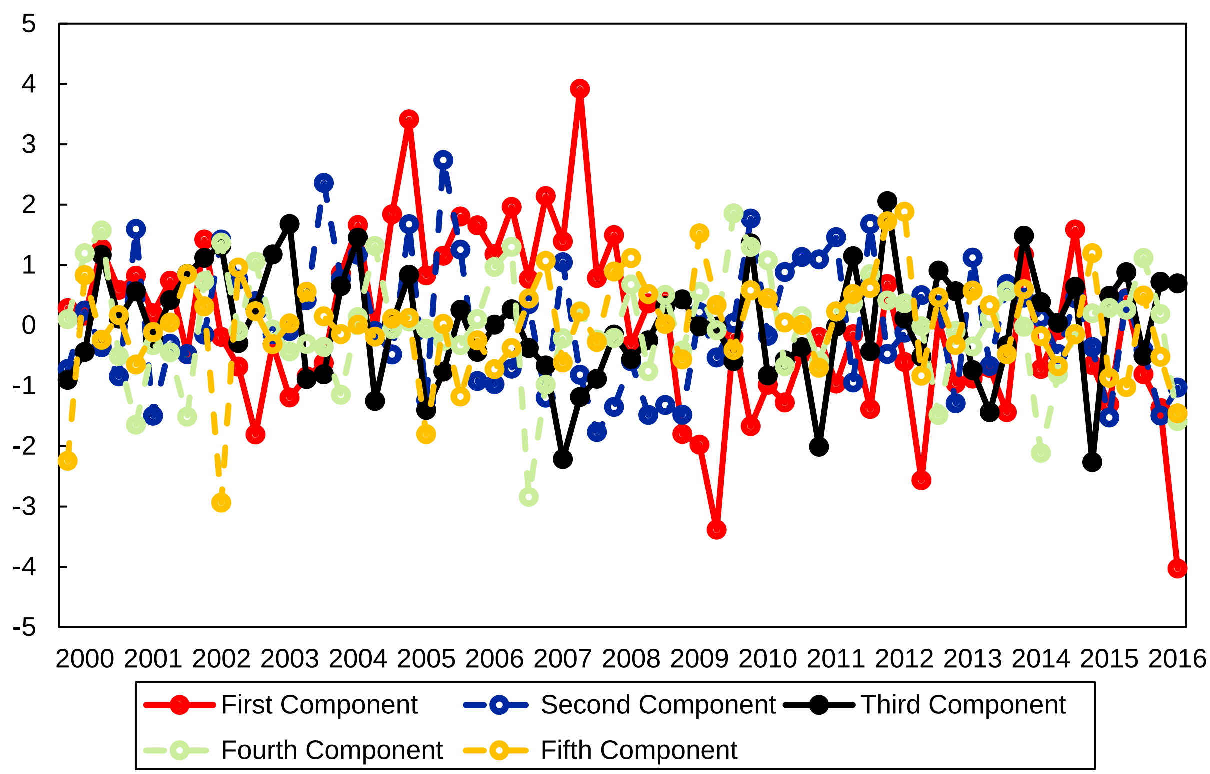


Figure 18 Actual Versus Fitted *Z* Time Series for ROW Observations

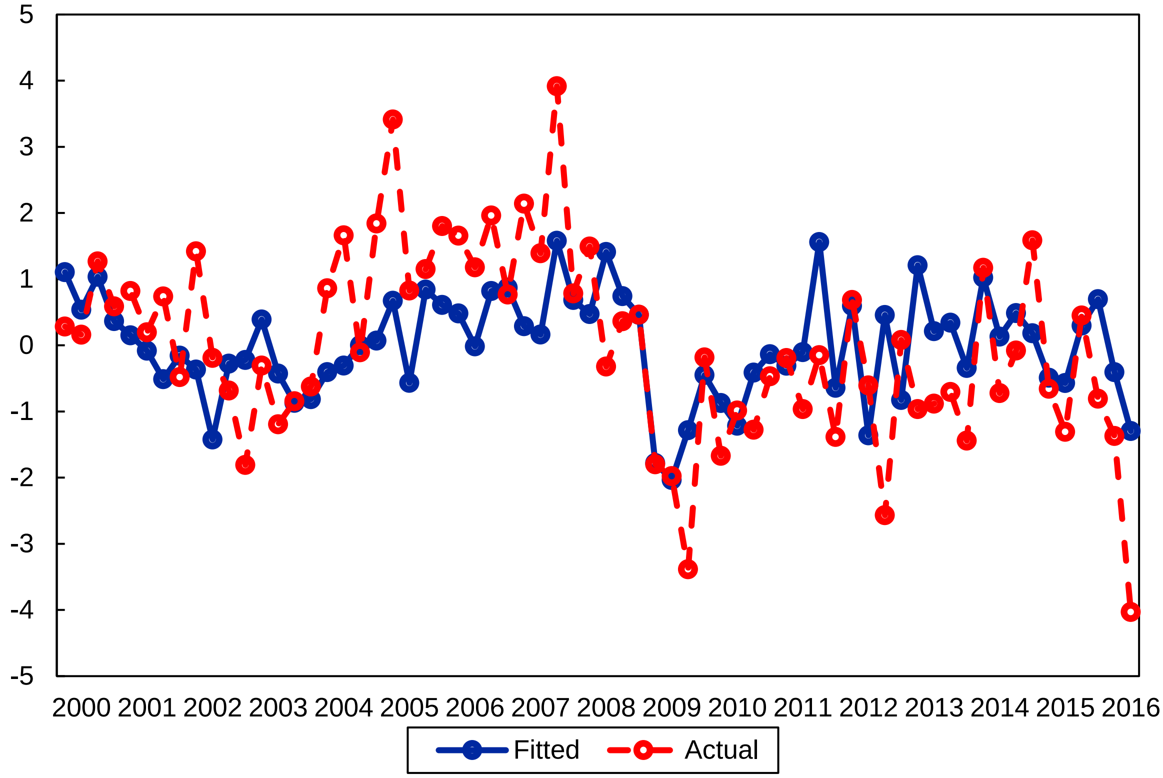


Table 5 Estimated Coefficients for the ROW Sub-portfolio

Variables	C, Ca, Caa3, Caa2, Caa1 (Coarse Rating 1)	B3, B2, B1 (Coarse Rating 2)	Ba3, Ba2, Ba1 (Coarse Rating 3)	Baa3, Baa2, Baa1 (Coarse Rating 4)	A3, A2, A1 (Coarse Rating 5)	Aa3, Aa2, Aa1, Aaa (Coarse Rating 6)
Asset Return ($\boldsymbol{\beta}^{AR}$)	0.0181	0.0519	0.0065	0.0442	0.0396	0.0000
Distance to Default ($\boldsymbol{\beta}^{DD}$)	0.6295	0.5427	0.3561	0.2625	0.3211	0.0476
Z Factor ($\boldsymbol{\beta}^{Z}$)	0.1130	0.3908	0.3120	0.3486	0.3159	0.2991
Z Factor Financial ($\boldsymbol{\beta}^{Z_{financial}}$)	0.0944	0.0853	-0.1264	0.0702	0.0871	-0.0434
Fine rating 1	-0.2814	-0.6685	-0.5129	-0.4535	-0.5631	0.0000
Fine rating 2	-0.2814	-1.1671	-1.0200	-0.9851	-1.2796	-2.5051
Fine rating 3	-0.6970					-2.7991
Fine rating 4	-0.6970					
c1	3.0569	-1.7722	-3.0157	-4.0075	$-\infty$	$-\infty$
c2	3.8382	3.0566	-1.9871	-3.5594	$-\infty$	$-\infty$
c3	4.5088	4.1826	2.5050	-2.4724	-3.7013	$-\infty$
c4	∞	∞	4.3260	2.8527	-2.4884	-6.0585
c5	∞	∞	∞	∞	3.2057	-4.6982

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