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The use of carrier approximation methods for projected capital metrics with an application to the IFRS 17 risk adjustment

Summary

In this paper we explore the use of the carrier approximation for the multi-year projection of risk and capital metrics. The carrier method¹ is attractive as it provides a fast alternative to the calculation of risk and capital metrics that otherwise require nested scenarios. Through the annuity book run-off case study presented here we outline the key factors influencing the performance of the carrier method when applied to the projected IFRS 17 risk adjustment and the projected Solvency II solvency capital ratio (SCR).

The results show that the approximation quality is mediocre when used for the required capital but can be stable and precise when used for the risk adjustment when the true SCR is available. While it is reasonable to expect a greater challenge for more complex balance-sheets, various aspects of the projections such as asset rebalancing and new business may actually help improve the carrier method approximation quality.

¹ To estimate the value $X(T)$ of metric X at future time T the carrier method uses another observable metric Y called the "carrier" to approximate the value of $X(T)$ as follows: $X(T) \sim X(0) * Y(T) / Y(0)$ with the obvious needed precaution of using a carrier for which $Y(0) \neq 0$

CONTENTS

Summary	01
The challenges of calculating projected capital metrics.....	03
The case study	03
Carrier approximation for the required capital.....	04
Extending the use of carrier methods to the projected risk adjustment.....	05
Method 1: Direct carrier	05
Method 2: Double carrier.....	06
Method 3: single carrier	07
Limitations of the carrier-based methods	09
Conclusion.....	09

The challenges of calculating projected capital metrics

The efficient projection of exact risk and capital metrics for insurance balance-sheets usually presents two major challenges. The first challenge is the availability of a projection capability in the asset and liability management (ALM) system. This capability is required to project a base balance-sheet but also to stress from a future base balance-sheet. When the ALM system is able to perform these projections, the second challenge is computation time. The set of nested scenarios required to be run through the ALM model might simply be too large for practical use.

This motivates the use of approximation methods such as the carrier which are relatively easy to implement and can be computed quickly.

The case study

The case study presented here is the run-off projection of an annuity book. The liabilities are modeled as 12 separate model points covering ages 65, 70, 75, 80, 85 and 90 for both males and females. The matching asset is assumed to be a mix of sovereign bonds and corporate bonds. The initial Solvency II style balance-sheet and capital position are in Figure 1:

Figure 1: The initial Solvency II style balance-sheet and capital position

Asset		Liabilities		Own funds	
Cash	2500	Available capital	3535	Required capital	2465
Bonds	16414	Risk adjustment	725	Free surplus	1070
		BEL	14654		3535
	<hr/> 18914		<hr/> 18914		
				Solvency ratio:	143.41%

Using Moody's Analytics Economic Scenario Generator we generated a set of 100 real world scenarios over 40 years. In each scenario and at each point in time, we calculate the following balance sheet items:

- » Bonds value and BEL. These are calculated exactly, without any approximation.
- » Required capital². This is calculated exactly and using a carrier approximation.
- » Risk adjustment³. This is calculated exactly and using three different carrier approximations.

² We used the Solvency II Standard Formula SCR as the measure of required capital. We restricted the constituents to SCR for rates, spread and longevity.

³ The risk adjustment was calculated as the Solvency II risk margin. In this calculation, the unhedgeable SCR is defined as the SCR for longevity risk and market risks beyond 30Y.

Carrier approximation for the required capital

The charts below show a comparison between exact required capital and carrier approximation across all 100 scenarios for three different projection times (1, 10 and 20 years). The chart on the left is for the full SCR, the chart on the right is for the non-hedgeable SCR (defined as the SCR for longevity risk and market risks beyond 30Y).

Figure 2:

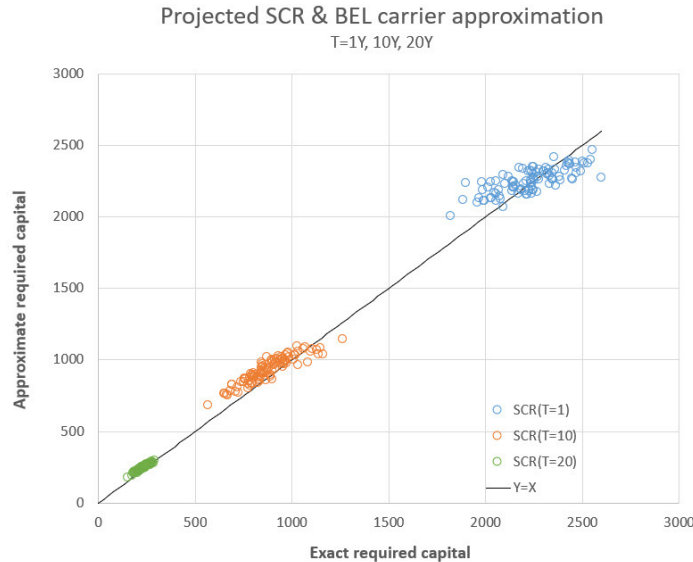
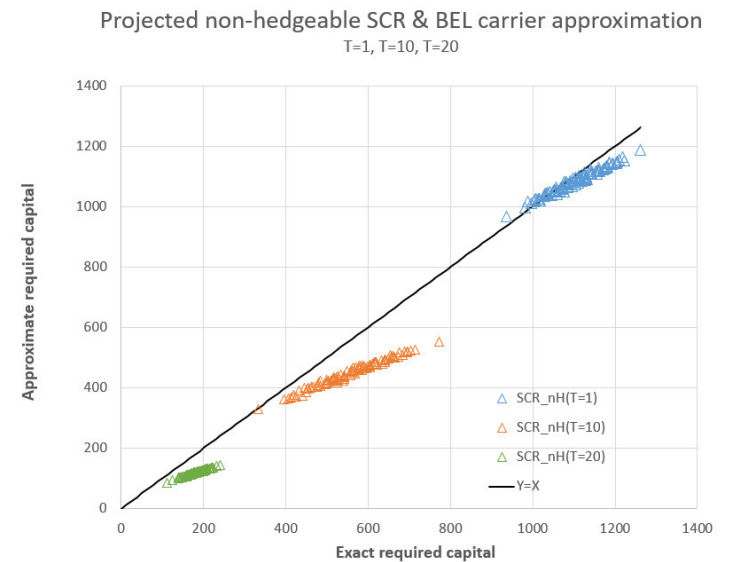


Figure 3:



The performance of the carrier is not good. For the full SCR shown in figure 2 the scatterplots are very volatile around the Y=X line and there is a systematic bias for the non-hedgeable SCR shown in figure 3. To complement the analysis shown above we also looked at the scenario with the largest absolute deviation between actual SCR and carrier estimate in year 1 (which happens to be above 10%). To understand why the error is so large in this scenario we have displayed the evolution of the swap curve and A-rated credit spread curve between t=0 and t=1 in figure 4 and a comparison between the specific rating migrations for year 1 in this scenario and the average transition matrix across all 40 years and 100 scenarios in figure 5. Specific numbers of interest have also been highlighted in the transition matrices.

Figure 4:

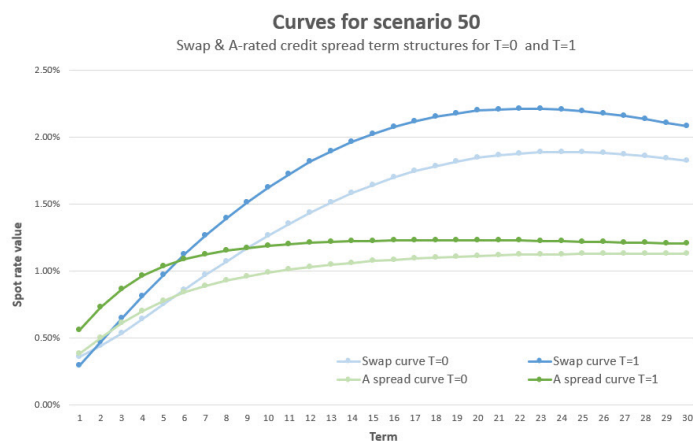


Figure 5:

T=1, Scenario #50	AAA	AA	A	BBB	BB	B	CCC	Def
AAA	0.99975	0.00025	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
AA	0.30428	0.69537	0.00035	0.00000	0.00000	0.00000	0.00000	0.00000
A	0.21651	0.20553	0.57778	0.00018	0.00000	0.00000	0.00000	0.00000
BBB	0.21594	0.10945	0.21694	0.45761	0.00006	0.00001	0.00000	0.00000
BB	0.02732	0.08159	0.14548	0.31654	0.42858	0.00048	0.00001	0.00001
B	0.01282	0.01686	0.18852	0.14392	0.23071	0.40650	0.00057	0.00010
CCC	0.01112	0.01394	0.19113	0.13074	0.13415	0.20334	0.31500	0.00059

T=1, AVG all scenarios	AAA	AA	A	BBB	BB	B	CCC	Def
AAA	0.94729	0.05068	0.00176	0.00006	0.00006	0.00006	0.00006	0.00003
AA	0.03208	0.90599	0.05558	0.00228	0.00172	0.00129	0.00020	0.00086
A	0.01855	0.03846	0.89714	0.03829	0.00308	0.00301	0.00021	0.00124
BBB	0.01848	0.01745	0.05741	0.87818	0.01771	0.00397	0.00390	0.00291
BB	0.00110	0.00566	0.01716	0.08019	0.82290	0.05666	0.00454	0.01178
B	0.00044	0.00078	0.01756	0.02446	0.06983	0.80368	0.04959	0.03365
CCC	0.00037	0.00062	0.01752	0.02160	0.03309	0.08535	0.76274	0.07870

In this scenario, risk-free yields and credit spreads both increase while at the same time the realized rating migrations are significantly skewed to the upside (numbers highlighted in green 1 and 2 notches to the left of the diagonal are much higher than the blue equivalent). This large change to the bonds portfolio composition is the main driver of the poor performance here.

Extending the use of carrier methods to the projected risk adjustment

There are many different ways to calculate a risk adjustment⁴. Here we have chosen a definition similar to the Solvency risk margin based on the Cost of Capital formula. The exact definition we have used is the following:

$$RA(t) = 6\% \sum_{i=1}^N \frac{SCR^{NH}_{t+i-1}}{(1+r_{t+i})^i}$$

where SCR^{NH}_{t+i-1} is the required capital for non-hedgeable risks at future time $t+i-1$ and N is chosen to ensure a full run off of the business. There are two key aspects of methodology to decide upon:

- » Which risks to include in the calculation of the SCR for non-hedgeable risks: We have used the risk from interest-rates and credit spreads for maturities beyond 30 years as well as the whole of longevity risk⁵.
- » What is meant exactly by "future SCR": The value of the SCR for future time $t+i-1$ is not known at time t . For the calculation we are using a single deterministic "forward path": for market risks we have used the path given by forward curves at time t and for longevity we have simply used constant mortality tables.

We consider 3 different carrier-based methods to approximate the projected risk adjustment. Method 1 directly approximates the risk adjustment itself using a carrier. Methods 2 and 3 use the Cost of Capital formula, but with a carrier used to approximate the projected non-hedgeable SCRs in this formula.

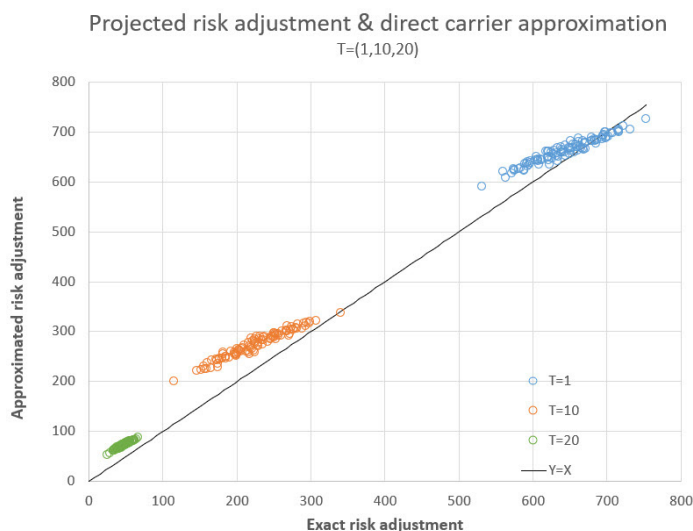
Method 1: Direct carrier

The first method consists in applying the BEL as carrier for the risk adjustment directly. The formula below shows the exact estimator used:

$$RA_{direct\ carrier}(t) = RA_{exact}(0) * \frac{BEL_{exact}(t)}{BEL_{exact}(0)}$$

Figure 6 below shows the approximation quality across all 100 scenarios for three different projection times (1, 10 and 20 years)

Figure 6:



The overall performance of this carrier is similar to the performance obtained when using the BEL as the overall carrier for the SCR.

⁴ For an overview of different methods for calculating the risk adjustment, see 'Calculating the IFRS 17 Risk Adjustment' (Moody's Analytics white paper, 2018).

⁵ For IFRS 17 the risk adjustment would only apply to non-financial risks i.e longevity risk only here.

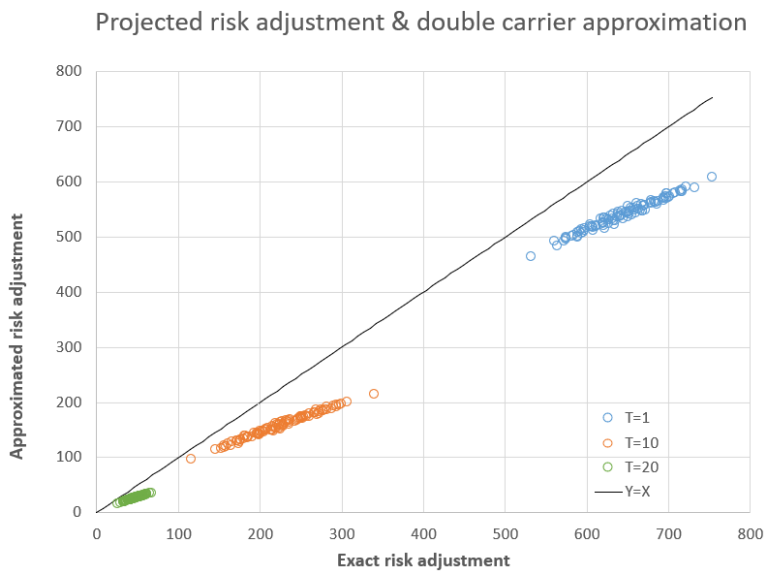
Method 2: Double carrier

For this method, the BEL carrier is applied separately to each non-hedgeable SCR term in the Cost of Capital formula. In addition, Method 2 assumes that for each projection time t the true value of the non-hedgeable SCR at t is unknown. We therefore first approximate the non-hedgeable SCR at time t , and then approximate each forward SCR beyond t conditional on this first approximation. The formula below shows the exact definition of the estimator used:

$$RA_{\text{double.carrier}}(t) = 6\% * \frac{SCR^{NH}_{\text{exact}}(0)}{BEL_{\text{exact}}(0)} \sum_{i=1}^N \frac{\text{ForwardBEL}(t+i-1)}{(1+r_{t+i})^i}$$

In the equation the estimator does not depend on the value of the BEL at time t , which disappears as a result of the “double carrier” structure explained above. In this formula, the “forward BEL” for time $t+i-1$ is the value of the BEL projected $i-1$ periods forward according to the forward curves observable at time t . Figure 7 shows a comparison between exact risk adjustment and the double carrier approximation.

Figure 7:

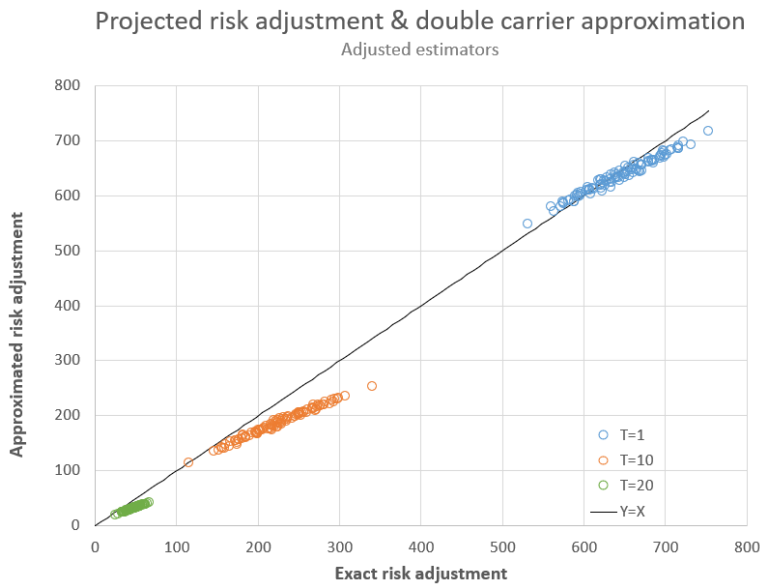


We note that the approximation is biased at all times, with the carrier method consistently underestimating the actual risk adjustment at all times. In particular, this method underestimates the risk adjustment at $t=0$ (unlike Method 1, which exactly matches the $t=0$ risk adjustment by construction). We can however implement a simple correction to the estimator that forces the approximation to match the true risk adjustment value at $t=0$. This ‘adjusted’ estimator is defined below:

$$RA_{\text{double.carrier.adj}}(t) = RA_{\text{double.carrier}}(t) * \frac{RA_{\text{exact}}(0)}{RA_{\text{double.carrier}}(0)}$$

Figure 8 below shows the resulting approximation quality once this correction has been applied. By adjusting the estimator to remove the bias in the t=0 value, we also reduce the bias at future times. Although the bias is reduced (particularly at short horizons) it is not eliminated completely.

Figure 8:



Method 2 presented here was assuming that the true value of the SCR is unknown at the time we are trying to estimate the risk adjustment from. This aspect is critical and method 3 below shows how the approximation quality changes when the true SCR is available for use in the risk adjustment estimation.

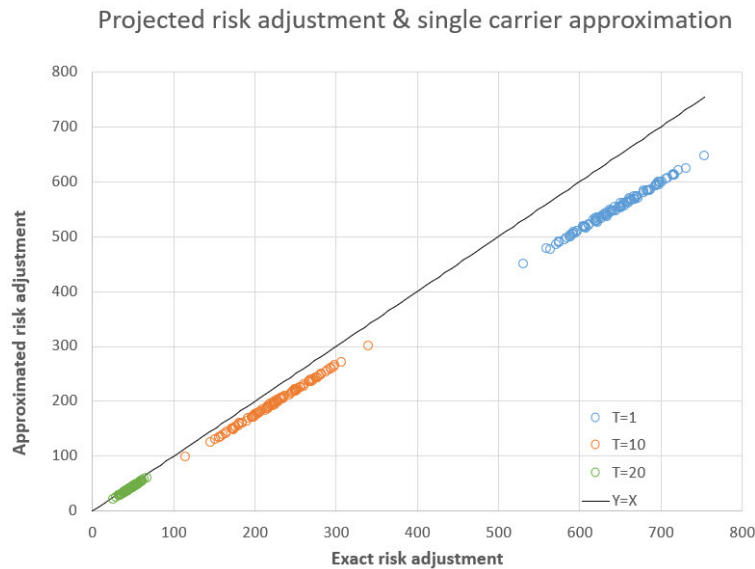
Method 3: single carrier

Method 3 also approximates the future non-hedgeable SCRs in the Cost of Capital formula. However, in contrast to Method 2, here we assume that for every calculation time t, the true value of the non-hedgeable SCR is known. The application of the BEL carrier to each projected non-hedgeable SCR in the Cost of Capital formula leverages this information:

$$RA_{single.carrier}(t) = 6\% * \frac{SCR^{NH}_{exact}(t)}{BEL_{exact}(t)} \sum_{i=1}^N \frac{ForwardBEL(t+i-1)}{(1+r_{t+i})^i}$$

In Figure 9 we can see that the approximation quality is much better than the corresponding estimator using Method 2. This is the direct consequence of the rebasing of the estimator for each time t relying on the true value of the non-hedgeable SCR at time t.

Figure 9:

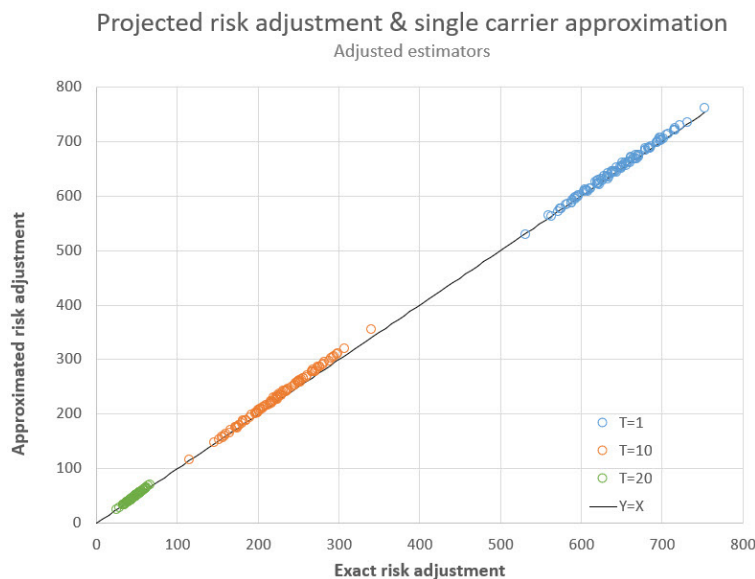


As with Method 2 however we have the same problem of not matching the true value of the risk adjustment at time 0. To fix this problem, we apply the same adjustment as for Method 2:

$$RA_{single.carrier.adj}(t) = RA_{single.carrier}(t) * \frac{RA_{exact}(0)}{RA_{single.carrier}(0)}$$

The resulting quality of fit is shown in Figure 10. This estimator provides an excellent quality of fit at all times in all scenarios, and is the best performing estimator of all considered. However it is important to note that we have assumed the true non-hedgeable SCR is known at each valuation point within the projections.

Figure 10:



Limitations of the carrier-based methods

Our case study illustrates that the quality of approximation provided by carrier-based methods can vary quite a lot depending on the details of the metric being estimated and the estimator used. In the annuity-centric run-off case study presented the approximation obtained for the SCR is arguably just mediocre, while for the risk-adjustment it is only good when the underlying SCR is known and can be used to improve the approximation quality. We note however that the assessment of quality of fit is somewhat subjective, and the required level of accuracy dependent on the particular application.

It is important to note that the practical use of the carrier methods presented in this paper and the performance associated are specific to the type of balance-sheet analyzed. For use with other types of balance-sheet or applications other than high-level capital planning, the following aspects would need to be assessed carefully:

1. Is there a strong asymmetry of risk exposure between the assets and the liabilities? As this asymmetry grows, it becomes harder to capture the behavior of the SCR simply by using the BEL as the carrier.
2. Is the projection a run-off or are assets and liabilities rebalanced? Rebalancing may result in carrier methods performing more accurately than presented here (where we assumed a run off).
3. How complex are the assets and the liabilities? For more complex balance-sheets the calculation of the true projected SCR would typically require the calculation of more sub-modules of the Standard Formula. This increased complexity of the SCR could lead to a reduction of the overall approximation quality for carrier-based methods (or the need for individual carriers for each SCR sub-module, which makes the approximation less attractive).

Conclusion

In this paper we have presented the application of the carrier method to the calculation of projected required capital and projected risk adjustment. We illustrated the performance of these approximations with an annuity-centric balance-sheet with a traditional bonds matching portfolio. Our results show that the approximation quality is mediocre when used for the required capital but can be stable and precise when used for the risk adjustment when the true SCR is available. While it is reasonable to expect a greater challenge for more complex balance-sheets, various aspects of the projections such as asset rebalancing and new business may actually help improve the carrier method approximation quality.

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