Abstract

With the new CECL and IFRS 9 requirements, we see an increased need for lifetime probability of default models. In this document, we formally investigate and summarize the term structure properties consistently seen across public, private, and rated firms. We observe that the default rate for "good" firms tends to increase over time, while the default rate for "bad" firms decreases over time, an indication of the mean-reversion effect seen with firms' default risk. Also, we observe that annualized, long-term default rates are more stable over time when compared to short-term default rates. This observation aligns with the fact that, given a long-horizon, the business cycle effect tends to be more moderate, and a company's capital structure becomes more important. Hence, the long-run risk levels are less cyclical and more stable. Another important observation: term structure is downward sloping during recessions and upward sloping during expansions.

Furthermore, the rating migration matrix is more stable at a longer horizon when compared to a shorter horizon.

This study uses private firm default information taken from Moody's Analytics Credit Research Database (CRD), public firms from Moody's Analytics CreditEdge™ (CE) database, and rated firms' default information from the Moody's Investor Service (MIS) annual default study.
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1. Overview

Historically, practitioners have focused on the one-year probability of default (PD) calculation using a firm’s financial information, because the default has mostly been modeled as a binary event (except the intensity model), suited for single-period (one-year) considerations within the regulatory framework of a fixed planning horizon. However, the choice of specific period such as one year is more or less arbitrary. This paper provides several stylized facts regarding the default rate term structure.

The finance literature has characterized the behavior of corporate credit risk and its relation to other asset prices. Studies reveal a tendency for risk to revert towards a long-term mean, suggestive of a long-term equilibrium. This phenomena is called mean-reversion. We show that for all three types of firms – private, public, and rated – that are available in Moody’s Analytics’ databases, the aggregate default risk of firms tends to follow the mean-revision phenomena.

We know that over shorter horizons, say one or two years, firms are exposed to the credit cycle effect, while over longer horizons, the business cycle effect tends to impact less, and the company’s capital structure becomes more important. This effect has made long-run risk levels less cyclical and more stable. Intuitively, default risk over a longer time period is less sensitive to the instantaneous default rates in the economy. Hence, quantifying the behavior of a firm’s PD over the longer horizon has always been a challenging problem.

Default rate term structure uses the inherent, time-dependent property of a default event, applying historical data to predict multi-year default probabilities. The reliability of the default frequencies calculated from historical data depends highly on the quality of the underlying data. Furthermore, we know that the amount of data that can be used for calibration decreases with increasing the time horizon. As a consequence, the quality of calibration suffers from a lack of sufficient data for a reliable statistical analysis, especially at longer time horizons. For this reason, we also investigate rating migration. Rating migration is the frequency that corporates experience change in credit quality over the years. Migration matrices contain the likelihood of the transition from a given rating category to another. We observe that the rating migrations are more stable at the longer horizon when compared to the shorter horizon.

Default rate term structure stylized facts provide a basis for calibrating and evaluating term structure in default probability models. A good model should generate probability of default (PD) term structures inline with the stylized facts. Term structure estimations have useful applications. First, in credit assessment, the default risk estimation horizon should match the credit term. To evaluate the risk of a two-year loan, it is better to use the default probability at the two-year horizon. One nuance, organizations, especially banks, often review loans periodically and have the right to take credit action based on review results. In these cases, we see the one-year default risk estimation used on multiple-year loans. Second, loan allowance calculation often requires PD term structures. Under IFRS 9 or Current Expected Credit Loss (CECL) accounting standards, organizations must provision for the life-time expected losses for part or all of their portfolios. The new accounting standards boost the demand for credit loss term structures.

In general, firms experience varying levels of credit risk at various time horizons. Term structure aims to quantify single-name, long-horizon credit risk by taking into account factors such as stage of the credit cycle, region, industry, and firm-specific shocks. Term structure uses all available information at the evaluation time to calculate a PD for longer horizons. This study uses three data sources from private, public, and rated companies to investigate the validity of the following stylized facts on default rates term structure:

- Companies with good ratings tend to worsen over time; companies with bad ratings tend to improve over time.
- Annualized, long-term default rates are more stable over time when compared to short-term default rates.
- Term structure is downward sloping during recessions and upward sloping during expansions.
- Rating migrations are more stable at longer horizons when compared to shorter horizons.
2. Descriptive Statistics of the Data

We consider three types of firms for this study: U.S. private firms, U.S. public firms, and rated firms. Data for private firms comes from Moody’s Analytics Credit Research Database (CRD) (1998–2016). Public firms data are provided by Moody’s Analytics CreditEdge database (1979–2016). Rated firms information comes from Moody’s Investors Service (MIS) default study.

2.1 Private Firms Data Source

We source private firm data using the Moody’s Analytics CRD. We consider the set of active borrowers from the CRD. Table 1 presents the descriptive information of the active borrowers in the CRD-U.S.

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of Firms</th>
<th>Number of Defaults</th>
<th>Number of Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998–2016</td>
<td>156,340</td>
<td>11,505</td>
<td>531,807</td>
</tr>
</tbody>
</table>

Figure 1 illustrates the percentage of statements and defaults per year during 1998-2015 in CRD.

Figure 2 presents the distribution of United States private firms by sector and the proportion of defaults in each sector. Trade is the largest sector in our sample, while Utilities is the smallest sector.

Figure 3 illustrates the distributions by firm size measured by Assets. Our sample includes a suitable cross-section of both large and small firms. For example, more than 20% of total financial statements and more than 15% of total defaults come from firms with total assets greater than $50 million.

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1 A borrower is an active borrower if they have a loan from the bank.
2.2 Public Firms Data Source

We source public firm data using the Moody’s Analytics CreditEdge. Table 2 presents the time period, number of firms, number of defaults, and total number of observations for non-financial firms in the United States (U.S.) among the top-90% of the economy by their total-liabilities in the CE database or rated by credit rating agencies such as Moody’s Investor Service (MIS) or Standard & Poor’s (S&P). The precise definition of top-90% cohort is given in [1]. Observations are recorded in CE database monthly. Figure 4 illustrates the percentage of statements and defaults per year during 1979–2016 in the CE database. We observe two main peaks 2001 and 2009, indicators of the Dot-com and housing bubbles, respectively. Furthermore, we observe approximately 6% default density 2015–2016, primarily due to a crisis in the public energy sector.

Table 2: Union of Top-90% non-financial public firms and rated firms in U.S.

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of Firms</th>
<th>Number of Defaults</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979–2016</td>
<td>3,956</td>
<td>566</td>
<td>444,081</td>
</tr>
</tbody>
</table>
2.3 Moody’s Rated Firm Data Source

We source rated firms default information from the Moody’s Investors Service (MIS) default and migration studies [3, 4]. For more details regarding rated firms data and descriptive statistics, please see [3, 4]. Moody’s default study report [3] documents the statistics on corporate default events among Moody’s rated debt issuers and the performance of Moody’s ratings 1920–2016. Moody’s migration study report [4] outlines average rating migration rates for various time horizons, by both broad and alphanumeric ratings. For the broad rating categories, they look at the time period 1970–2016 and calculate five different average rating migration rates—from the one-year horizon up to the five-year horizon. When calculating the alphanumeric migration rates, they look at the time period 1983–2016, since Moody’s introduced the alphanumeric rating scale above Caa in mid 1982. The data are of monthly cohort frequency.
3. Methodology for Calculating Default Rate

Note, there is no consensus on the best way to compute a default rate. Here, in order to capture various features of the long-run credit risk, we consider calculating the default rate of firms based on two approaches given in the subsequent sections. The main difference between these two approaches is that one of them is robust to adding/dropping firms to/from the cohort during the PD calculation horizon. We argue that the approach that is robust to the change in the cohort makes more sense and is more in-line with the MIS approach.

3.1 Not Correcting for Censorship

The first approach calculates the given population’s default rate during time interval \([t, t + T]\) as

\[
D(t; T) = \frac{\text{number of defaults } \in [t, t + T]}{\text{number of firms at time } t}
\]

Note, this method for calculating default probability does not track changes in the initial cohort of firms. Therefore, if some firms in the initial cohorts drop due to any reasons except default, this action leads to under-estimating the actual default rate. Another issue is that if some new firms enter the database, and they default during the time interval \([t, t + T]\), this action leads to overestimating the default rate using the Not Correcting for Censorship approach. To compensate for the above issues, we can track changes in the cohort of firms, as described in the next section.

3.2 Correcting for Censorship

Using this approach, we calculate the default rate for a given population during time interval \([t, t + T]\) based on cumulative survival rate, \(S(t; T)\), defined as follows:

\[
S(t; T) = \prod_{k=0}^{T-1} (1 - D(t + k; 1))
\]

where

\[
D(t + k; 1) = \frac{\text{number of defaults } \in [t + k, t + k + 1]}{\text{number of firms at time } t + k}
\]

denotes the default rate based on not correcting for censorship during time interval \([t + k, t + k + 1]\). Since \(D(t + k; 1)\) calculation is done over a short horizon, i.e., one unit of time measure in the given application, the chance of changes in the initial cohort decreases or becomes zero in some cases.

That is, the cumulative survival rate is the product of intervening marginal survival rates. Then, the probability that a firm will default by year \(t + T\) is found as follows:

\[
D^c(t; T) = 1 - S(t; T)
\]

3.3 Annualized Default Rate Conversion

After calculating the default rate, we compute the annulized default rate as

\[
\text{Annualized default rate}(t; T) = 1 - (1 - x(t; T))^\frac{1}{T}
\]

where \(x(t; T) \in \{D(t; T), D^c(t; T)\}\).

We next apply this methodology to verify stylized facts for three populations of companies in our database. In our calculation we only use the correcting for censorship approach, since it is similar to the way Moody's Investor Service calculate the default rate for rated firms. However, in Section 5 we plot the annualized historical default rates calculated based both approaches to clarify their differences.
4. Mean Reversion

This section investigates the mean-reversion phenomena occurrence, based on our available dataset for private, public, and rated firms. To this end, we calculate the average annualized censored default rate for two cohorts of firms. We call the first cohort, "good" firms or "investment grade" firms. The second cohort contains "bad" firms or "speculative grade" firms.

Figures 5a-5f illustrate the average annualized censored default rate for the available dataset. To calculate the value of each point in this graph, we first calculate the $x$-year annualized default rate time series by year. We then compute the average across years for each $x$, where $x \in \{1, 2, 3, 4\}$ for private and public firms, and $x \in \{1, 2, \ldots, 10\}$ for rated firms.

We observe in Figures 5a, 5c, and 5e, for private, public, and rated companies, respectively, that the firms with "good" implied ratings, i.e., investment grade firms, tend to worsen over time. Figure 5b, 5d, and 5f show that private, public, and rated companies, respectively, with "bad" ratings, i.e., speculative grade firms, tend to improve over time (mean-reversion).

Figures 5a and 5b depict the aggregated annualized default rate for private firms. Figure 5a shows that, for "good" implied rating (or investment grades) firms in CRD (namely firms whose EDF-implied ratings are better than Baa3, i.e., $\text{CCA EDF measure} \leq 0.66\%$), their credit risk degrades over the longer horizon. Furthermore, Figure 5b shows that, for "bad" implied rating (or speculative grades) firms in the CRD, namely firms whose EDF implied ratings are worse than Baa3, their credit risk improves slightly over the longer horizon.

Figures 5c and 5d indicate that the mean-reversion holds for public firms as well. Again, we use EDF-implied ratings to classify firms into investment grade and speculative grade. Investment grade firms see their annualized default rates deteriorate over time, while speculative firms see their annualized default rates improve over time. EDF-implied rating is based on dynamic rating. Please refer to [1] on dynamic rating.

Figures 5e and 5f illustrate the mean-reversion effect in the rated firm's domain. The data used to generate these plots are collected from MIS default study [3], Exhibit 35, page 38. Firms with "good" ratings (investment grade firms) tend to worsen over time, while firms with "bad" ratings (speculative grade firms), tend to improve over time. In Figure 5f, we observe that the annualized default rate first increases for some time, but it follows the mean reversion direction. One proper explanation for this behavior can be the rating momentum induced by the rating process of rating agency [3].

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2 Credit Cycle Adjusted (CCA) EDF™ (Expected Default Frequency) measures are impacted not only by a company’s financials, but also by the economy’s general credit cycle and each state’s economic condition. To capture this effect, RiskCalc™ U.S. includes a credit cycle adjustment (CCA) factor. The credit cycle adjustment is designed to incorporate the current position of the credit cycle by combining a regional level index with an industry level-index.
Figure 5: Aggregated annualized default rate for two types of firms: investment grade firms (left-hand side figures) and speculative grade firms (right-hand side figures).
4.1 Term Structure Slope Analysis

In the previous section, we observe that firms with low credit risk tend to become a bit riskier over time, while firms with high credit risk tend to reduce their credit risk over a longer time horizon. This observation seems reasonable, because remaining in a very low risk situation for firms over a long-term horizon is challenging. On the other hand, if firms with high credit risk want to remain active in the market, they must improve their credit risk. Therefore, if we calculate the slope of the annualized default rate from longer horizon to shorter horizon, we should see that, for firms with "good" ratings, the slope is greater than one, and, for firms with "bad" ratings, the slope is less than one.

Figure 6a presents the mean-reversion for private firm default rates. From these results, we see that the slope of the term structure based on a CCA EDF measure implied rating for all firms with an implied rating better than Ba2 is greater than one, and for firms with an implied rating poorer than a Ba2, it is less than one. In all of following figures, we calculate the slope based on following equation:

\[
slope = \frac{\text{Five years annualized default rate}}{\text{One year default rate}}
\]

To analyze the slope of the term structure for public firms, we use the dynamic rating used in CreditEdge to bucket the firms. For more information on dynamic rating, please refer to [1]. We then calculate the one- and five-year annualized default rates for each bucket. Finally, we calculate the slope. Results in Figure 6b show that the mean-reversion point for public firms occurs in the Caa1 bucket. The irregularity of slope for good ratings in Figure 6b is due to thin data for this rating buckets. Similarly, we observe that, for the set of rated firms from the MIS database, the mean-reversion point occurs in ratings poorer than Caa1. The data in Figure 6c are evaluated by division of one- and five-year default rates collected from MIS Migration study [4], Exhibit 6 and Exhibit 10, on pages 4 and 8, respectively. The irregularity of slope for good ratings in Figure 6c is due to thin data for this rating buckets.

![Figure 6a: Mean-reversion for private firm default rates.](image)

![Figure 6b: Mean-reversion for public firm default rates.](image)

![Figure 6c: Mean-reversion for rated firm default rates.](image)

Figure 6: These figures illustrate the slope of term structure for private, public, and rated firms.
5. **Short-Term vs. Long-Term Default Rate**

In this section, we analyze the annualized default rate behavior for different horizon across time. We calculate the annualized default rates using both the Not Correcting for Censorship and the Correcting for Censorship approaches. Figures 7a, 7b, and 7c present the annualized default rates for private, public, and rated firms, respectively. Solid lines represent results calculated using the non-censored method, and dashed lines represent the censored method results in the U.S. Note, the annualized one-year result is the same for both censored and non-censored methods.

From results in Figures 7a, 7b, and 7c, we can see that the one-year default rate curve (black curve) fluctuates more compared to the three-year (blue curve) annualized default rate. The three-year annualized default years is also more cyclical compared to the five-year (red curve). Therefore, we can say that the annualized default rates are more stable over the long-time horizon compared to the short-term default rates.

Another stylized fact we observe from Figures 7a, 7b, and 7c: between 2005–2006, the term structure is upward sloping during the expansion period, while between 2008–2009, the term structure is downward sloping during the recessionary period.

While Moody’s Analytics has access to extensive data on financial statements, default status, and public firm-equity market information for the long-horizon, one inevitable aspect of data collection and management is corrupt or noisy information. Noise in private data can stem from the loss of small-size firms default status issues or delay in financial reporting, etc. In addition, the 2001–2002 recession did not affect private firms as much as public firms. Hence, overall, during 2001–2002, we observe the default series does not reflect the stylized facts as clearly as during 2008–2009. Figure 7a shows that the censored approach generates higher default rates for both the three- and five-year horizons near 2001–2002. This finding is due to dropping/adding some private firms from the database during these time horizons.

We observe similar stylized facts for the default term structure from the public firms database. Results in Figure 7b indicate that between 2005–2006 term structure is upward sloping during the expansion period, while 2008–2009 the term structure is downward sloping during recession period. A similar observation holds for the 1998–2000 and 2001–2003 periods. We also see that the one-year default rate curve (black curve) fluctuates more compared to the three-year (blue curve) annualized default rate, and three-year annualized default period shows more cyclicity when compared to the five-year (red curve). Therefore, we can say that the annualized default rates are more stable over the longer time horizon when compared to the shorter term default rates. Interestingly, we see in Figure 7b that the censored approach evaluates a bit higher default rates for both the three-year and five-year horizons. However, compared to private firm results in Figure 7a, the gap between two approaches is much smaller. We can argue that public firms usually are more mature and stable—compared to private firms. Therefore, the change in a cohort of firms between the three- and five-years’ time horizon can be negligible.

Figure 7c illustrates the annualized default rates of MIS rated firms, reported in Exhibit 31, page 35. Note, MIS uses a definition of default similar to the aforementioned second approach. This figure clearly indicates that during 2008–2009, the term structure is downward sloping during recession period, and, during 2005–2006 the term structure is upward sloping during the expansion period. We also see that the one-year default rate curve (black curve) fluctuates more when compared to the three-year (blue curve) annualized default rate. The three-year annualized default years are more cyclical when compared to the seven-year (red curve). Therefore, we can say that the annualized default rates are more stable over the long-term horizon when compared to the shorter term default rates.
Figure 7: Annualized default rates. Dashed lines present the calculation based on Not Correcting for Censorship approach. Solid lines present the calculation based on Correcting for Censorship approach.
6. Rating Migration

One prevalent tool in risk management is a rating migration matrix, which reflects a firm’s transition/persistence probability of credit risk over a time horizon. In other words, we can say that the rating Migration matrix presents the rating migration probabilities. Figures 8a-8d show the rating migration matrix is more stable at longer horizons compared to shorter horizons. This effect is most pronounced for private firms. To approximate a long-horizon rating transition matrix, we can use a shorter time horizon transition matrix. For example, to estimate the four-year transition matrix using one-year matrix, we can multiply the one-year transition matrix four times. Similarly, to estimate two-year using the one-year transition matrix, we simply multiply twice. Note, if the transition matrix evolution behaves closely to a stationary Markov process, then this simple calculation provides a good approximation. However, in practice, the Markovian assumption might not hold for rating transition matrices.

For the private firm sample, we consider the RiskCalc™ EDF-implied rating based on either FSO EDF or CCA EDF. For more information, please refer to [3]. In Figure 8a, we depict the main diagonal elements of the transition matrix. The main diagonal elements represent the probability of staying in the same rating during the considered time interval. For example, for the FSO RiskCalc™ EDF-implied rating A2 in Figure 8a, during a four-year time horizon, firms tend to stay in the A2-implied rating bucket, with a probability of approximately 0.3. Similarly, we can say that for the estimated four-year transition from the one-year time horizon matrix, firms tend to remain in the Baa1 implied rating bucket with a probability of approximately 0.1.

In Figure 8a, we observe that for a transition matrix based on Financial Statement Only [4], with the (FSO) EDF rating, there is a gap between the estimated transition’s matrix main diagonal elements and the actual four-year transition matrix elements. This gap indicates that the Markovian assumption does not hold for this scenario, and that transitions at longer horizon are more stable.

In Figure 8b, we illustrate the sum of the main diagonal, its top, and its below (off-) diagonal neighbors. This summation provides the probability of, at most, a one-notch change in a firm’s rating during the time interval of one, two, and four years. For example, for the FSO EDF-implied rating A2 in Figure 8b, for a four-year time horizon, firms that have had an A2-implied rating tend to stay in A2 or migrate to A1 or A3 buckets with a probability of approximately 0.6, overall. Similarly, we can say for a four-year transition matrix estimated via one-year time horizon, firms tend to stay in Baa1 or migrate to A3- or Baa2-implied rating buckets with probability of approximately 0.4.

Furthermore, from Figures 8a and 8b, we observe that using shorter time horizon transitions to estimate the transition in a longer time horizon leads to underestimating the actual transition matrix. In other words, results suggest that rating migration are more stable over longer horizons than under shorter horizons.

Results in Figures 8a and 8b show that the rating transition matrix for private firms based on the CCA EDF measure is also more stable at longer horizons when compared to shorter horizons.

Figures 8c and 8d present the estimation of the four-year (longer time horizon) transition matrix using the one-year (shorter time horizon) transition matrix for public firms. Here, to bucket the public firms, we use the dynamic rating table-based public firms’ EDF values, updated each month based on CreditEdge outputs.

Results in Figure 8c and 8d show that the rating transition matrix for public firms based on EDF-implied rating is more stable at longer horizons when compared to shorter horizons, although the difference is smaller than that of the private firm sample. Interestingly, in these curves, we observe that the estimation of the longer-horizon transition matrix from the shorter-time transition matrix experiences very small errors. However, we should emphasize that this observation neither proves nor disapproves the Markovian property of rating transition matrix for public firms’ rating evolution.

In Figures 9a and 9b, we plot rating migration for MIS ratings. The data for these figures are collected from MIS Migration study [4]. Exhibits 6 and 10, on pages 4 and 8, respectively. Interestingly, in Figures 9a, we see that the actual five-year transition and the estimated five-year transition based on the one-year time horizon transition are quite similar. In Figure 9b, we even observe that the estimated five-year transition is more stable than the observed one. The explanation is that MIS rating measures long-run default risk of firms by assessing several qualitative factors regarding a firm’s financial health.

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3 The difference between the FSO EDF and the CCA EDF is that we adjust the CCA EDF to reflect our assessment of the credit cycle’s current stage, while the FSO EDF only considers financial statement to predict the EDF measure.
Figure 8: Probability of no rating change. Black line is based on actual four year time horizon, red line is based on an approximation of four-year transition via two-year time horizon, and green line is based on an approximation of four-year transition via one-year time horizon.
Figure 9: Probability of at most one-notch change. Black line is based on actual four year time horizon, red line is based on an approximation of four-year transition via two-year time horizon, and green line is based on an approximation of four-year transition via one-year time horizon.
7. Summary

Our research investigates the term structure properties consistently observed across public, private, and rated firms. The stylized facts of default rate term structure provide a basis for calibrating and evaluating term structure in default probability models. A good model should generate probability of default term structures inline with the stylized facts.

Term structure estimations provide useful applications. In credit assessment, the default risk estimation horizon should match the credit term, e.g., to evaluate the risk of a multi-year loan, it is better to use the default probability at the multi-year horizon. One nuance: organizations, especially banks, often review loans periodically and have the right to take credit action based on the review results. Under IFRS 9 or Current Expected Credit Loss (CECL) accounting standards, organizations must provision for the lifetime expected losses for part or all of their portfolios. The new accounting standards boost the demand for credit loss term structures. In general, firms experience varying levels of credit risk at various time horizons. Term structure aims to quantify long-horizon credit risk for single names by taking into account factors such as credit cycle stage, region, industry, and firm-specific shocks. The key stylized facts on default rate term structure observations presented in this document:

- We observe that the default rate for "good" firms tends to increase over time, while it decreases over time for "bad" firms.
- The rating migration matrix is more stable at longer horizon compared to shorter horizons.
- The annualized longer-term default rates are more stable over time when compared to shorter-term default rates.
- The term structure is downward sloping during recessionary periods and upward sloping during expansionary periods.

These facts regarding default events over time imply that a good term structure must account for the current stage of the credit cycle. Furthermore, they indicate that the evolution of the firms' ratings over time is not a simple Markov process.

In this study, we use U.S. public and private datasets to generate the stylized facts on default rate term structure. We expect most of the results hold for non-U.S. firms. There may be exceptions to the stylized facts. For example, in countries where the average default rate is much higher (for example, some emerging market countries), we might observe that the effect of mean-reversion to lower default rates in the sample is so strong that it overtakes the cyclical changes of the term structure slope. In such cases, it is possible that while the term structure is less downward sloping during expansionary period, it never becomes upward sloping, and, therefore, the last stylized fact would not hold.
References

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