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MODELING METHODOLOGY

FRTB Marginal Back-Allocation

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Abstract

This paper develops a method to back-allocate to individual positions the market risk capital requirement that a bank must satisfy under the revised standardized approach proposed by the Basel Committee. Our method assesses the contribution of single positions or sub-portfolios to the overall capital charge. One important feature of our method is that it provides insight on which positions, sub-portfolios, and risk factors drive the capital charge and which help mitigate it. A negative contribution indicates that a marginal increase in the position would lead to a decrease in the capital charge, and vice versa.

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1. Introduction

The revised standardized approach that followed the Basel Committee's Fundamental Review of the Trading Book introduced new capital requirements for market risk. Since capital is scarce and costly, it is important for banks to understand how each position contributes to this new capital requirement. Determining each exposure's contribution to required capital is a complex task because, due to hedging and offsetting, the capital charge function is non-linear in the positions.

This paper proposes a method to back-allocate to individual positions a bank's minimum capital requirements for market risk under the Basel Committee's revised standardized approach.[1] Throughout this paper, we refer to the standardized approach of the minimum capital requirements for market risk regulation as "FRTB." Our method follows a marginal back-allocation approach, sometimes referred to as Euler allocation. This back-allocation method allocates capital in proportion to the size of a position. The proportion is determined by the effect on capital of a marginal increase in the size of the trade.

The marginal method has two properties that make it particularly useful. First, it provides insights on which positions and risk factors drive the capital charge and which help to reduce it due to hedging and offsetting effects. Second, the marginal method is informative regarding the effect on overall capital when increasing a particular position or exposure to a risk factor within the portfolio.

We organize the remainder of this paper as follows: Section 2 outlines the general approach for obtaining the contribution of a position to the capital charge using the marginal method; Section 3 provides details regarding the mathematical expressions of the contributions; Section 4 illustrates an example on a test portfolio; Section 5 concludes; the Appendix provides further details on the method.

[1] *Minimum capital requirements for market risk*, Basel Committee on Banking Supervision, January 2016, http://www.bis.org/bcbs/publ/d352.htm.

2. Marginal Capital Charge Contribution

This section provides an overview of the marginal method and discusses its properties. We begin by looking at the contribution to the total capital charge of a position and a trading desk. We then decompose the contribution of a position into its role in the various components of the total capital risk charge (e.g., the delta risk charge, the default risk charge, etc.). We conclude this section by looking at a risk factor's contribution. Each subsection discusses the information we can extract from the capital contributions and their use in portfolio management. Section 3 provides details of our marginal method.

Capital Charge of a Position and of a Trading Desk

The marginal method allocates capital to a position in proportion to its size. The proportion is determined by the position's marginal contribution to the capital charge under the FRTB. The marginal contribution to the capital charge is the derivative of the capital charge, with respect to the size of the position. Formally, we define the contribution of position i to the capital charge as

$$CC_i \equiv \frac{\partial CC}{\partial N_i} N_i \tag{1}$$

where CC is the function that outputs a set of positions' capital charge under the FRTB, N_i is the size of position i, and $\frac{\partial CC}{\partial N_i}$ denotes the partial derivative of the function CC with respect to, and evaluated at, N_i.

The variable N_i is a quantifier of position i's size that is not affected by market conditions. For instance, a bond's face value and the number of contracts for a position in options, are valid examples for the variable N_i. The interpretation of $\frac{\partial CC}{\partial N_i}$ depends upon the chosen variable N_i.

The contributions defined in Equation (1) sum up to the total capital charge under the FRTB, that is $CC = \sum_i CC_i$, by construction. This property derives from the capital charge being a positive, homogeneous function of degree one in the size of the positions.[2] The homogeneity in the size of the positions means that scaling them by a positive factor is equivalent to scaling the capital charge by the same factor. Note, the homogeneity property is one condition for a risk measure to be coherent and, loosely speaking, it means that doubling the portfolio translates to doubling the risk. This property is particularly useful for obtaining risk charges at more aggregate levels. If, for example, we are interested in the risk charge of trading desks, then we can simply sum the capital charges of the positions traded by each desk.

A bank may be interested in understanding how a change in the exposure to a position affects the total capital charge. The contribution CC_i has two features that address this question. First, the sign of the contribution tells us whether the total capital charge increases or decreases, if we increase the size of the position. A positive (negative) contribution CC_i means that total capital is an increasing (decreasing) function of position i's size, so that an increase in it leads to an increase (decrease) in the overall capital charge. Second, the magnitude of the marginal contribution $\frac{\partial CC}{\partial N_i}$ is informative regarding the magnitude of the effect on the total capital charge of changing the exposure to the position. Marginal changes in positions with higher marginal contribution (in absolute value) impact the total capital charge more, and vice versa.

Decomposing a Position's Capital Charge

To gain insight regarding the characteristics of a position that drives the capital charge, it is useful to rewrite the capital charge in Equation (1) as the sum of position i's contributions to the different components of the total capital charge:

$$\begin{aligned} CC_i &= \frac{\partial CC}{\partial N_i} N_i \\ &= \left(\frac{\partial \Delta}{\partial N_i} + \frac{\partial V}{\partial N_i} + \frac{\partial CR}{\partial N_i} + \frac{\partial DRC^{non-sec}}{\partial N_i} + \frac{\partial DRC^{sec\ non-CTP}}{\partial N_i} + \frac{\partial DRC^{sec\ CTP}}{\partial N_i} + \frac{\partial RR}{\partial N_i} \right) N_i \\ &= \Delta_i + V_i + CR_i + DRC_i^{non-sec} + DRC_i^{sec\ non-CTP} + DRC_i^{sec\ CTP} + RR_i \end{aligned} \tag{2}$$

[2] A function $f: \mathbb{R}^n \to \mathbb{R}$ is said to be positive homogeneous of degree 1 if $\forall \boldsymbol{x} \in \mathbb{R}^n$ and $t > 0$, the following holds: $f(t \cdot \boldsymbol{x}) = t \cdot f(\boldsymbol{x})$. A property of such functions is that $f(\boldsymbol{x}) = \sum_i \frac{\partial f(x)}{\partial x_i} x_i$, where $\frac{\partial f(x)}{\partial x_i}$ represents the derivative of $f(\boldsymbol{x})$, with respect to the i-th element of $\boldsymbol{x}$, that is x_i, evaluated at $\boldsymbol{x}$.

In Equation (2), components Δ_i, V_i, and CR_i represent the position's contribution to the risk charges, obtained from the *Delta*, *Vega*, and *Curvature* buckets, respectively, and aggregated across positions.[3] Components $DRC_i^{non-sec}$, $DRC_i^{sec\ non-CTP}$, and $DRC_i^{sec\ CTP}$ represent the position's contribution to the risk charges obtained from the Default Risk Charge for *non-securitisations*, *securitisations (non-correlation trading portfolio)*, and *securitisations (correlation trading portfolio)* risk classes, respectively. Finally, component RR_i is the instrument's contribution to the Residual Risk Add-on.[4]

Position's i capital contribution is thus the sum of the partial derivatives of the capital charge components, multiplied by the quantifier of the size of the position. The above decomposition sheds light on the way position i contributes to the total capital charge's different components. For instance, a position may have a negative contribution to the delta risk charges, a positive one to the vega risk charges, and zero contribution to the curvature charges. This indicates that it is the option's implied volatility driving the contribution, whereas its delta sensitivities act as a hedge for the delta risk charges.

Moreover, using the marginal method, we can achieve a higher level of resolution by looking at the contribution of a position to specific factors within a given SBM bucket. A position's contribution to the delta risk charges, for instance, can be written as $\Delta_i = \sum_k \Delta_i^k$, where $\Delta_i^k = \frac{\partial \Delta}{\partial s_{k,i}} s_{k,i}$ represents the contribution of instrument i to risk-factor k, $s_{k,i}$ is the sensitivity of position i to risk factor k, and the summation is across factors defined for the delta risk charge. In the expression above, each Δ_i^k term measures the importance of a risk factor k to the position's i contribution.

A Risk Factor's Capital Charge

To manage a portfolio's exposure to risk factors, banks can benefit from determining the contribution of each risk factor to the overall capital charge. It is also useful to determine the sensitivity of the capital charge to a risk factor in order to design new positions.

We can obtain both the contribution of and the sensitivity to each risk factor by pooling the positions' contributions with respect to the risk factors. Consider, for example, the *Delta* risk charge, denoting k as a particular risk factor; the contribution of risk factor k to the overall capital charge is given by:

$$\frac{\partial \Delta}{\partial s_k} s_k = \frac{\partial \Delta}{\partial s_k} \sum_i s_{k,i} = \sum_i \frac{\partial \Delta}{\partial s_{k,i}} s_{k,i} = \sum_i \Delta_i^k$$
$$\Rightarrow \frac{\partial \Delta}{\partial s_k} = \frac{\sum_i \Delta_i^k}{s_k} \tag{3}$$

where the summation is across positions sensitive to risk factor k and where we use the relation $\frac{\partial \Delta}{\partial s_{k,i}} = \frac{\partial \Delta}{\partial s_k}\frac{\partial s_k}{\partial s_{k,i}} = \frac{\partial \Delta}{\partial s_k}$, because of the chain rule and $s_k = \sum_i s_{k,i} \Rightarrow \frac{\partial s_k}{\partial s_{k,i}} = 1$. The variable $\frac{\partial \Delta}{\partial s_k} s_k$ represents the contribution of risk factor k to the capital charge.

The derivative $\frac{\partial \Delta}{\partial s_k}$ measures the sensitivity of the capital charge to exposure to risk factor k.

We can use this information to design and select new positions. For instance, a bank may be interested in assessing the sensitivity of the capital charge to different vertices of the risk-free yield curve, corresponding to separate risk factors in the GIRR delta bucket, in order to assess which vertices have the lowest impact on the capital charge. Equation (3) shows that the sensitivity of the capital charge to a vertex on the yield curve can be retrieved from the contributions of the positions to that vertex, Δ_i^k. Suppose the sensitivity of the capital charge to the first vertex is 0.5, and for the fifth vertex it equals 0.3, then entering a new position with the fifth vertex as the main risk factor would have lower impact on the capital charge than one with the first vertex as the main risk factor.[5] This reasoning follows, because, for a small change δs_k in sensitivity s_k, the delta risk charge changes $\Delta(s_k + \delta s_k) - \Delta(s_k) = \frac{\partial \Delta}{\partial s_k} \delta s_k$, where $\Delta(s_k + \delta s_k)$ and $\Delta(s_k)$ are the delta risk charges, evaluated at $s_k + \delta s_k$ and s_k, respectively.

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[3] Note, Δ, V, and CR variables represent the capital charges obtained considering the Delta, Vega, and Curvature buckets, respectively, defined in Section 4 of the FRTB. The 'other sector' buckets capital charges are included in these quantities. The variables Δ, V, and CR are therefore different from the variables denoted '*Delta*', '*Vega*', and '*Curvature risk*', defined in Paragraphs 51 (d) and 53 (e) in the FRTB, which do not include the 'other sector' bucket.

[4] In Equation (2) we use the relations $\Delta_i \equiv \frac{\partial \Delta}{\partial N_i} N_i$, $V_i \equiv \frac{\partial V}{\partial N_i} N_i$, $CR_i \equiv \frac{\partial CR}{\partial N_i} N_i$, $DRC_i^{non-sec} \equiv \frac{\partial DRC^{non-sec}}{\partial N_i} N_i$, $DRC_i^{sec\ non-CTP} \equiv \frac{\partial DRC^{sec\ non-CTP}}{\partial N_i} N_i$, $DRC_i^{sec\ CTP} \equiv \frac{\partial DRC^{sec\ CTP}}{\partial N_i} N_i$, and $RR_i \equiv \frac{\partial RR}{\partial N_i} N_i$.

[5] Note, because of the marginal nature of the contributions, the effect of a new trade on the capital charge can only be assessed if the trade is relatively small.

3. Expressions of the Contributions

The following subsections provide the expressions for the contributions Δ_i, V_i, CR_i, $DRC_i^{non-sec}$, $DRC_i^{sec\ non-CTP}$, and $DRC_i^{sec\ CTP}$ introduced in Equation (2). Appendix A provides an alternative representation of these contributions. We do not provide an expression for the contribution of a position to the residual add-on $\frac{\partial RR}{\partial N_i} N_i$ as this contribution is simply the residual add-on of position i.

Contribution to the Sensitivities-based Method Risk Charges

We begin with a position's contribution to the three risk types of the sensitivities-based method risk charges — delta, vega, and curvature. Since the treatment of the delta, vega, and curvature risk charges is similar in the FRTB, we present the marginal back-allocation method for the three risk types side-by-side, highlighting the differences between them when relevant. As delta and vega risk charges are obtained in the same way under the FRTB, we only present the back-allocation for the delta and curvature risk charges, with the understanding that the results derived for the delta also hold for the vega.

More precisely, this section presents the full expressions for position $i's$ contributions Δ_i and CR_i defined in Equation (2). The contribution Δ_i is the sum of position's contributions across the delta risk factors to which the position is sensitive, that is $\Delta_i = \sum_k \Delta_i^k$, with Δ_i^k being the contribution of position i to risk factor k of the Δ. The same is true for the curvature risk CR_i. The contributions of a position i with risk factor k to the Δ and the CR, are obtained according to Table 1.

TABLE 1

Contribution of Position i to Risk Factor k

DELTA RISK CHARGES	CURVATURE RISK CHARGES
$\Delta_i^k = \frac{\partial \Delta}{\partial s_{k,i}} s_{k,i}$	$CR_i^k = \frac{\partial CR}{\partial CVR_{k,i}} CVR_{k,i}$
Δ: aggregate risk charge obtained from the delta buckets, including the "other sector" bucket, and aggregated across risk classes	CR: aggregate risk charge obtained from the curvature buckets, including the "other sector" bucket, and aggregated across risk classes
Δ_i^k: capital charge contribution of position i and risk factor k to the delta risk charges Δ	CR_i^k: capital charge contribution of position i and risk factor k to the curvature risk charges CR
$s_{k,i}$: sensitivity of position i to risk factor k, we have $s_k = \sum_i s_{k,i}$ where s_k is the net sensitivity to delta risk factor k, and where the summation is across positions with risk factor k	$CVR_{k,i}$: sensitivity of position i to risk factor k, we have $CVR_k = \sum_i CVR_{k,i}$ where CVR_k is the net sensitivity to curvature risk factor k, and where the summation is across positions with risk factor k

Using the sensitivity of position i to the risk factor k, $s_{k,i}$ to obtain the contribution Δ_i^k yields the same result as using the size N_i. Equation (1) and the expressions in Table 1 are thus consistent with how we obtain the contributions. To see this point note that, by the chain rule and the linearity of $s_{k,i}$ in N_i, we have:

$$\Delta_i^k = \frac{\partial \Delta}{\partial s_{k,i}} s_{k,i} = \frac{\partial \Delta}{\partial s_{k,i}} \frac{\partial s_{k,i}}{\partial N_i} N_i \tag{4}$$

An analogous relation can be derived for CR_i. See Appendix C for more details on the linearity of $s_{k,i}$ and $CVR_{k,i}$ in the size of position i. The reason for using $s_{k,i}$ and $CVR_{k,i}$ instead of N_i for the computation of the contributions is that, in doing so, we can exploit a lower resolution level, simplifying the final expressions for the contributions. In particular, $s_{k,i}$ and $CVR_{k,i}$ are functions of the size N_i and of other variables, such as the market value of the position and the value of the risk factor. Therefore, the variables $s_{k,i}$ and $CVR_{k,i}$ allow us to compute the contribution of a position using fewer input variables.

To retrieve the marginal contributions of a position to the delta and curvature charges, that is $\partial \Delta / \partial N_i$ and $\partial CR / \partial N_i$, use the relations

$$\begin{aligned} \frac{\partial \Delta}{\partial N_i} &= \frac{\Delta_i}{N_i} \\ \frac{\partial CR}{\partial N_i} &= \frac{CR_i}{N_i} \end{aligned} \tag{5}$$

while noting that $\Delta_i = \sum_k \Delta_i^k$ and $CR_i = \sum_k CR_i^k$. The derivatives in Equation (5) carry the sign of the contributions and quantify the impact on the delta and curvature charges of a marginal increase in the positions.

The remainder of this section provides details regarding the expressions of the contributions Δ_i^k and CR_i^k as functions of the quantities defined in the FRTB. We focus on the contributions of position i belonging to bucket b of a generic risk class. Our focus on a generic risk class is without loss of generality, because there are no diversification benefits between the delta, vega, and curvature risk types and, within each of these, between risk classes.

We begin by summarizing the FRTB's calculation of the risk position for a generic bucket b, of the delta and curvature buckets, that is, not the "other sector" bucket. We then provide the expression for the derivatives $\partial\Delta/\partial s_{k,i}$ and $\partial CR/\partial CVR_{k,i}$, previously introduced in Table 1. We end this section discussing the contribution of a position that belongs to the "other sector" bucket.

The risk position for bucket b, K_b, introduced in Paragraph 51.c, for the delta charges, and Paragraph 53.d, for the curvature charges, of the FRTB, has the following expression for any bucket that is not the "other sector" bucket

$$K_b = \sqrt{\max(A_b, 0)} \tag{6}$$

where the variable A_b is defined according to Table 2.

TABLE 2

Details on Risk Positions

DELTA RISK CHARGES	CURVATURE RISK CHARGES
$A_b = \sum_k (WS_k)^2 + \sum_k \sum_{k\neq\ell} \rho_{k\ell} WS_k WS_\ell$	$A_b = \sum_k \widetilde{\omega}_{kk}(CVR_k) CVR_k^2 + \sum_k \sum_{k\neq\ell} \widetilde{\omega}_{k\ell}(CVR_k, CVR_\ell) CVR_k CVR_\ell$
$\rho_{k\ell}$: regulatory correlation between factors $k, \ell \in D_b$ (we define $\rho_{k\ell} = 1$ if $k = \ell$) where D_b indicates the set of delta factors in bucket $b \in B$	$\widetilde{\omega}_{k\ell}(CVR_k, CVR_\ell)$: modified correlation between CVR_k and CVR_ℓ, defined as $\widetilde{\omega}_{k\ell}(CVR_k, CVR_\ell) = \begin{cases} 1 & \text{if } k = \ell \text{ and } CVR_k > 0 \\ 0 & \text{if } CVR_k < 0 \text{ and } CVR_\ell < 0 \\ \rho_{k\ell} & \text{in all other cases} \end{cases}$ $\rho_{k\ell}$: regulatory correlation between factors $k, \ell \in D_b$ where D_b indicates the set of curvature factors in bucket $b \in B$
WS_k: weighted sensitivity of risk factor $k \in D_b$, $WS_k = RW_k s_k$, where RW_k is the risk weight of factor k and s_k is the net sensitivity to risk factor k	CVR_k is the curvature risk charge of risk factor $k \in D_b$

The set B denotes the set of buckets, excluding the "other sector" bucket, for some risk class. The buckets in B enter the calculation of the $Delta$ or $Curvature\ risk$ of the risk class in question and have an implicit dependence on the risk type. The "other sector" bucket is not included in B.

Consistent with the notation introduced in Paragraphs 51 and 53 of the FRTB, we call $Delta$ and $Curvature\ Risk$ the capital charges arising from all buckets in a risk class, minus the "other sector" bucket. The $Delta$ and $Curvature\ Risk$ charges are calculated according to Table 3.

TABLE 3

Delta and Curvature Risk Charges

DELTA RISK CHARGES	CURVATURE RISK CHARGES
$Delta = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c\neq b} \gamma_{bc} S_b S_c}$	$Curvature\ risk = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c\neq b} S_b S_c \gamma_{bc} \psi(\tilde{S}_b, \tilde{S}_c)}$
$S_b = \begin{cases} \tilde{S}_b & \text{if } Q \geq 0 \\ \max[\min(\tilde{S}_b, K_b), -K_b] & \text{if } Q < 0 \end{cases}$	$S_b = \begin{cases} \tilde{S}_b & \text{if } Q \geq 0 \\ \max[\min(\tilde{S}_b, K_b), -K_b] & \text{if } Q < 0 \end{cases}$
$Q = \sum_{b\in D} K_b^2 + \sum_{b\in D} \sum_{c\in D\setminus\{b\}} \gamma_{bc} \tilde{S}_b \tilde{S}_c$ $\tilde{S}_b = \sum_{k\in B_b} WS_k$	$Q = \sum_b K_b^2 + \sum_b \sum_{c\neq b} \gamma_{bc} \tilde{S}_b \tilde{S}_c \psi(\tilde{S}_b, \tilde{S}_c)$ $\tilde{S}_b = \sum_{k\in B_b} CVR_k$
γ_{bc}: regulatory correlation between delta buckets b and c, for the risk class in question	γ_{bc}: regulatory correlation between curvature buckets b and c, for the risk class in question
	$\psi(\tilde{S}_b, \tilde{S}_c)$: function that takes the value 0 if $\tilde{S}_b$ and $\tilde{S}_c$ are both negative and value 1 in all other cases

Note, we compute variable Q at the risk class level for a particular risk type, and the variable S_b depends on the sign of the variable Q for the risk class in question. Consequently, the expressions for the derivatives $\partial\Delta/\partial s_{k,i}$ and $\partial CR/\partial CVR_{k,i}$ depend on the sign of Q.

The contribution to the $Delta$ and $Curvature\ risk$ of a position i with risk factor k in bucket $b \in B$ has the form

$$Contribution_i^k = \beta_i^k X_{k,i} \tag{7}$$

where the coefficient β_i^k is defined as

$$\beta_i^k = \begin{cases} \dfrac{\left(\sum_{\ell\in D_b} \bar{\omega}_{k\ell} X_\ell + \sum_{c\in B\setminus\{b\}} \bar{\gamma}_{bc} S_c\right)}{risk\ charge} y_k & \text{if } \left(Q \geq 0 \text{ or } \left(Q < 0 \text{ and } S_b = \tilde{S}_b\right)\right) \text{ and } A_b \geq 0 \\ \dfrac{\left(\sum_{c\in B\setminus\{b\}} \bar{\gamma}_{bc} S_c\right)}{risk\ charge} y_k & \text{if } \left(Q \geq 0 \text{ or } \left(Q < 0 \text{ and } S_b = \tilde{S}_b\right)\right) \text{ and } A_b < 0 \\ \dfrac{\left(\sum_{\ell\in D_b} \bar{\omega}_{k\ell} X_\ell\right)\left(1 + \frac{1}{K_b}\sum_{c\in B\setminus\{b\}} \bar{\gamma}_{bc} S_c\right)}{risk\ charge} y_k & \text{if } Q < 0 \text{ and } S_b = +K_b \text{ and } |S_b| \neq |\tilde{S}_b| \text{ and } A_b > 0 \\ \dfrac{\left(\sum_{\ell\in D_b} \bar{\omega}_{k\ell} X_\ell\right)\left(1 - \frac{1}{K_b}\sum_{c\in B\setminus\{b\}} \bar{\gamma}_{bc} S_c\right)}{risk\ charge} y_k & \text{if } Q < 0 \text{ and } S_b = -K_b \text{ and } |S_b| \neq |\tilde{S}_b| \text{ and } A_b > 0 \\ 0 & \text{if } Q < 0 \text{ and } (S_b = +K_b \text{ or } S_b = -K_b) \text{ and } |S_b| \neq |\tilde{S}_b| \text{ and } A_b \leq 0 \end{cases} \tag{8}$$

where $B\setminus\{b\}$ indicates the set B without element b, that is the bucket to which factor k belongs. Table 4 defines the variables entering the expressions in Equation (8).

Coefficient β_i^k in Equations (7) and (8) corresponds to the derivative $\partial(risk\ charge)/\partial X_{k,i}$, when this quantity is well-defined. When it is undefined — that is at kinks — we make β_i^k coincide with the derivative defined on one of the two sides of the kink. In particular, if $A_b = 0$ then β_i^k corresponds to the case $A_b > 0$ and when and $Q = 0$, to the case $Q > 0$. When $S_b = \left|\sum_{k\in D_b} X_k\right| = K_b$ or $S_b = -\left|\sum_{k\in D_b} X_k\right| = -K_b$ we take the derivative corresponding to the case $S_b = \tilde{S}_b$.[6] Furthermore, the coefficient multiplying $\sum_{c\in B\setminus\{b\}} \gamma_{bc} S_c$ in the second to last equation is $\frac{1}{K_b}$ when $S_b = K_b$ and $-\frac{1}{K_b}$ when $S_b = -K_b$.

Note, the following relation holds, $Delta = \sum_i \Delta_i$, where the summation is across instruments that enter the $Delta$ calculation, with $\Delta_i = \sum_k \Delta_i^k$ where the summation is across the risk factors of position i involved in the computation of $Delta$. Analogous relations hold for CR_i and CR.

TABLE 4

Details on Contributions

DELTA RISK CHARGES	CURVATURE RISK CHARGES
$Contribution_i^k = \Delta_i^k$	$Contribution_i^k = CR_i^k$
$risk\ charge = Delta$	$risk\ charge = Curvature\ risk$
$\bar{\omega}_{k\ell} = \rho_{k\ell}$	$\bar{\omega}_{k\ell} = \tilde{\omega}_{k\ell}(CVR_k, CVR_\ell)$
$\bar{\gamma}_{bc} = \gamma_{bc}$	$\bar{\gamma}_{bc} = \gamma_{bc}\psi(\tilde{S}_b, \tilde{S}_c)$
$X_k = WS_k$	$X_k = CVR_k$
$X_{k,i} = s_{k,i}$	$X_{k,i} = CVR_{k,i}$
$y_k = RW_k$	$y_k = 1$

[6] Consider the following example to see a situation in which $S_b = \pm\left|\sum_{k\in D_b} WS_k\right| = \pm K_b$: take $K_b > 0$ and $\sum_{k\in D_b} WS_k > 0$ and assume that $\rho_{k\ell} = 1, \forall k, \ell \in D_b$ (see for instance Paragraph 85 in the FRTB for conditions under which $\rho_{k\ell} = 1$), we then have $(\sum_k WS_k)^2 = \sum_k WS_k^2 + \sum_k \sum_{\ell\neq k} WS_k WS_\ell = K_b^2 = \sum_k WS_k^2 + \sum_k \sum_{\ell\neq k} \rho_{k\ell} WS_k WS_\ell$ and this implies $\sum_k WS_k = K_b$.

For positions contributing to the "other sector" bucket, the expression for the contribution is simpler. In particular, a position i, with risk factor k belonging to the "other sector" bucket $b(other\ bucket) \notin B$, we have

$$Contribution_i^k = \begin{cases} X_{k,i} & \text{if } X_k > 0 \\ 0 & \text{if } X_k = 0 \\ -X_{k,i} & \text{if } X_k < 0 \end{cases} \tag{9}$$

The contributions in Equation (9) are such that the "other sector" bucket risk charge is the sum across positions and risk factors of each position's contribution to a risk factor belonging to the "other sector" bucket, i.e. $K_{b(other\ bucket)} = \sum_k |WS_k| = \sum_i \sum_k \Delta_i^k$. Analogously the curvature risk charge.

Finally, a property of the contributions of individual positions, $\Delta_i = \sum_k \Delta_i^k$, is that they add to the delta risk charge, i.e, $\Delta = \sum_i \Delta_i$. The same is true for the individual contributions to the curvature risk: they add up to the curvature risk charge.

Contribution to the Default Risk Charge

This section presents a position's contribution to the three risk classes of the Default Risk Charge (DRC): non-securitisations, securitisations (non-CTP), and securitisations (CTP). The three risk classes have similar structures but present some differences. One crucial difference in the computation of the risk charge, between non-securitisations and securitisations, is that the former aggregates net jump-to-default (JtD) obtained at the obligor level, whereas the latter aggregates net JtDs at tranche level (non-CTP) and tranche and underlying name level (CTP). In particular, in the CTP, positions correspond to either tranches or single-names.

Note, for a position i, only one of the contributions to the three risk classes of the DRC differs from zero, as a position is included in only one of the DRC risk classes. In the following we do not include replication, and offsetting is only allowed between positions in equal tranches/entities and within the same bucket. The method we propose can easily accommodate replication, more complex offsetting, and other generalizations.

The contribution of position i to either of the risk classes of the DRC has the following form

$$DRC_i^{DRC\ risk\ class} = \frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i} JtD_i \tag{10}$$

- $DRC_i^{DRC\ risk\ class}$: contribution of position i to a particular DRC risk class;
- JtD_i: gross jump to default of position i scaled by the residual maturity of the position;
- $\frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i}$: marginal contribution of jump-to-default of position i to $DRC^{DRC\ risk\ class}$.

The contributions are such that $DRC^{DRC\ risk\ class} = \sum_i DRC_i^{DRC\ risk\ class}$, where the summation is across all positions in the specific DRC risk class.

It follows that $DRC_i^{DRC\ risk\ class} = \frac{\partial DRC^{DRC\ risk\ class}}{\partial N_i} N_i = \frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i} \frac{\partial JtD_i}{\partial N_i} N_i = \frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i} JtD_i$, because of the linearity of JtD_i in N_i. See Appendix C for more details on the linearity of JtD_i in N_i.

The Default Risk Charge for non-securitisations and securitisations is obtained as the (weighted) sum of the Default Risk Charges computed at the bucket level, according to the following equation:

$$\begin{aligned} DRC_b &= \widehat{DRC}_b \cdot \psi_b \\ \widehat{DRC}_b &= m_1^b - WtS^b \cdot m_2^b \end{aligned} \tag{11}$$

- DRC_b: Default Risk Charge for bucket b;
- m_1^b: weighted sum of net long JtDs in bucket b, we have $m_1^b = \sum_{k \in D_b} RW_k^b LnetJTD_k^b$;
- m_2^b: weighted sum of absolute values of net short JtDs in bucket b, we have $m_2^b = \sum_{k \in D_b} RW_k^b \left|SnetJTD_k^b\right|$;
- WtS^b: weighted to short ratio, defined as

$$WtS^b = \frac{\kappa_1^b}{\kappa_1^b + \kappa_2^b}$$

where $\kappa_1^b = \sum_{k \in A} LnetJTD_k^b$ and $\kappa_2^b = \sum_{k \in A} \left| SnetJTD_k^b \right|$.

Some of the variables used in the previous equations differ between risk classes. We define them in Table 5.

TABLE 5

Parameters of Default Risk Charge

NON SECURITISATIONS	SECURITISATIONS (NON-CTP)	SECURITISATIONS (CTP)
RW_k^b: risk weight of obligor k	RW_k^b: risk weight of tranche k	RW_k^b: risk weight of tranche/reference entity k in bucket b
$LnetJTD_k^b$: net long JtD of obligor k	$LnetJTD_k^b$: net long JtD of tranche k	$LnetJTD_k^b$: net long JtD of tranche/reference entity k in bucket b
$SnetJTD_k^b$: net short JtD of obligor k	$SnetJTD_k^b$: net short JtD of tranche k	$SnetJTD_k^b$: net short JtD of tranche/reference entity k in bucket b
WtS^b: Weighted to Short ratio for bucket b	WtS^b: Weighted to Short ratio for bucket b	$WtS^b = WtS^{CTP}$: Weighted to Short ratio for entire CTP
D_b: set of obligors in bucket b	D_b: set of tranches in bucket b	D_b: set of tranches/reference entities in bucket b
$A = D_b$	$A = D_b$	A: set of tranches/reference entities in the entire CTP
$\psi_b = \begin{cases} 0 & \text{if } \widehat{DRC}_b < 0 \\ 1 & \text{otherwise} \end{cases}$	$\psi_b = \begin{cases} 0 & \text{if } \widehat{DRC}_b < 0 \\ 1 & \text{otherwise} \end{cases}$	$\psi_b = 1$

The letter k represents an entity in the case of the *non-securitisations* risk class, whereas it represents a tranche in the case of the *securitisations (non-CTP)* and a tranche or a reference entity (in the underlying index) for the *securitisations (CTP)* risk class.

We obtain the net JtDs in different ways, depending on the risk class, described in Table 6.

TABLE 6

Net JTD Amounts

NON SECURITISATIONS	SECURITISATIONS (NON-CTP)	SECURITISATIONS (CTP)
$LnetJTD_k^b = \sum_s \ell_k^s$	$LnetJTD_k^b = \begin{cases} netJTD_k & \text{if } netJTD_k > 0 \\ 0 & \text{if } netJTD_k < 0 \end{cases}$	$LnetJTD_k^b = \begin{cases} netJTD_k^b & \text{if } netJTD_k^b > 0 \\ 0 & \text{if } netJTD_k^b < 0 \end{cases}$
$SnetJTD_k^b = \sum_s c_k^s$	$SnetJTD_k^b = \begin{cases} 0 & \text{if } netJTD_k > 0 \\ netJTD_k & \text{if } netJTD_k < 0 \end{cases}$	$SnetJTD_k^b = \begin{cases} 0 & \text{if } netJTD_k^b > 0 \\ netJTD_k^b & \text{if } netJTD_k^b < 0 \end{cases}$
ℓ_k^s: net long JtD for seniority s of obligor k	$netJTD_k$: net JtD for tranche k (obtained as the sum of long and short JtD amounts)	$netJTD_k^b$: net JtD of tranche or single-name reference k in bucket b (obtained as the sum of long and short JtD amounts). The netting is done at bucket level only
c_k^s: net short JtD for seniority s of obligor k		

In the non-securitisations risk class, we assume that the variables ℓ_k^s and c_k^s are obtained by maximizing the offsetting between long and short positions — under the constraint that the gross JTD amounts of long and short exposures to the same obligor may be offset where the short exposure has the same or lower seniority relative to the long exposure. See Appendix B for an algorithm that achieves this purpose.

Note, the same obligor may have a long net position and a net short position in the non-securitisations risk class, whereas when it comes to tranches/reference entities, the net JtD per tranche or reference entity in the index is either long or short. More precisely, if k represents an entity in one of the buckets of the non-securitisations risk class, then $LnetJTD_k$ and $SnetJTD_k$ may both be different from zero. When k refers to a tranche or a reference entity, in the securitisations risk classes, then $LnetJTD_k \neq 0$ implies $SnetJTD_k = 0$ and vice versa.

We obtain the contribution of a position to the three different risk classes of the DRC according to Table 7.

TABLE 7

Contributions to DRC

NON SECURITISATIONS	SECURITISATIONS (NON-CTP)	SECURITISATIONS (CTP)
$DRC_i^{non-sec} = \begin{cases} \beta_i \cdot JtD_i & \text{if } DRC_b > 0 \\ 0 & \text{if } DRC_b = 0 \end{cases}$ b: bucket to which position i is assigned	$DRC_i^{sec\ non-CTP} = \begin{cases} \beta_i \cdot JtD_i & \text{if } DRC_b > 0 \\ 0 & \text{if } DRC_b = 0 \end{cases}$ b: bucket to which position i is assigned	$DRC_i^{sec\ CTP} = \begin{cases} \beta_i \cdot JtD_i & \text{if } DRC^{sec\ CTP} > 0 \\ 0 & \text{if } DRC^{sec\ CTP} = 0 \end{cases}$

We define variable β_i as

$$\beta_i = \sum_{c \in DRC\ risk\ class} \chi_c \cdot \left(m^{c\prime}_{1,i} + \frac{\kappa_1^c m_2^c}{(\kappa_1^c + \kappa_2^c)^2} \kappa^{c\prime}_{1,i} + \frac{\kappa_1^c m_2^c}{(\kappa_1^c + \kappa_2^c)^2} \kappa^{c\prime}_{2,i} - \frac{m_2^c}{\kappa_1^c + \kappa_2^c} \kappa^{c\prime}_{1,i} - WtS^c \cdot m^{c\prime}_{2,i} \right) \quad (12)$$

where the variables κ_1, κ_2, m_1, m_2, and WtS depend on risk class and bucket and are defined above. The derivatives of the variables κ_1, κ_2, m_1 and m_2 with respect to the jump-to-default of a position, are defined according to Table 8.

TABLE 8

Derivatives

NON SECURITISATIONS	SECURITISATIONS (NON-CTP)	SECURITISATIONS (CTP)
$\kappa^{c\prime}_{1,i} = \begin{cases} 1 \ \text{if} \sum_{r \le s_i} c_k^r = 0 \text{ and } c = b \\ 0 \ \text{if} \sum_{r \le s_i} c_k^r < 0 \text{ and } c = b \\ 0 \ \text{if } c \ne b \end{cases}$ k: entity corresponding to position i and belonging to bucket c b: bucket to which position i belongs s_i: seniority of position i c_k^s: net short JtD for seniority s of obligor k	$\kappa^{c\prime}_{1,i} = \begin{cases} 1 \ \text{if } netJTD_k \ge 0 \ \text{ and } c = b \\ 0 \ \text{if } netJTD_k < 0 \ \text{ and } c = b \\ 0 \ \text{if } c \ne b \end{cases}$ k: tranche reference in bucket c corresponding to position i b: bucket to which position i belongs	$\kappa^{c\prime}_{1,i} = \kappa'_{1,i} = \begin{cases} 1 \ \text{if } netJTD_k^b \ge 0 \\ 0 \ \text{if } netJTD_k^b < 0 \end{cases}$ k: tranche reference in bucket b, corresponding to position i b: bucket to which position i belongs $netJTD_k^b$: net JtD of tranche or single-name reference k in bucket b
$\kappa^{c\prime}_{2,i} = \begin{cases} 0 \ \text{if} \sum_{r \le s_i} c_k^r = 0 \text{ and } c = b \\ -1 \ \text{if} \sum_{r \le s_i} c_k^r < 0 \text{ and } c = b \\ 0 \ \text{if } c \ne b \end{cases}$ k: entity corresponding to position i and belonging to bucket c b: bucket to which position i belongs s_i: seniority of position i c_k^s: net short JtD for seniority s of obligor k	$\kappa^{c\prime}_{2,i} = \begin{cases} 0 & \text{if } netJTD_k \ge 0 & \text{and } c = b \\ -1 & \text{if } netJTD_k < 0 & \text{and } c = b \\ 0 & \text{if } c \ne b \end{cases}$ k: tranche reference in bucket c corresponding to position i b: bucket to which position i belongs	$\kappa^{c\prime}_{2,i} = \kappa'_{2,i} = \begin{cases} 0 & \text{if } netJTD_k^b \ge 0 \\ -1 & \text{if } netJTD_k^b < 0 \end{cases}$ k: tranche reference in bucket b, corresponding to position i b: bucket to which position i belongs $netJTD_k^b$: net JtD of tranche or single-name reference k in bucket b
$m^{c\prime}_{1,i} = RW_k^c \kappa^{c\prime}_{1,i}$	$m^{c\prime}_{1,i} = RW_k^c \kappa^{c\prime}_{1,i}$	$m^{c\prime}_{1,i} = \begin{cases} RW_k^c \kappa'_{1,i} & \text{if } c = b \\ 0 & \text{if } c \ne b \end{cases}$ b: bucket to which position i belongs
$m^{c\prime}_{2,i} = RW_k^c \kappa^{c\prime}_{2,i}$	$m^{c\prime}_{2,i} = RW_k^c \kappa^{c\prime}_{2,i}$	$m^{c\prime}_{2,i} = \begin{cases} RW_k^c \kappa'_{2,i} & \text{if } c = b \\ 0 & \text{if } c \ne b \end{cases}$ b: bucket to which position i belongs
$\chi_c = \begin{cases} 1 & c = b \\ 0 & c \ne b \end{cases}$ b: bucket to which position i belongs	$\chi_c = \begin{cases} 1 & c = b \\ 0 & c \ne b \end{cases}$ b: bucket to which position i belongs	$\chi_c = \begin{cases} 1 & DRC_c > 0 \\ 0.5 & DRC_c < 0 \end{cases}$

$DRC\ risk\ class$: set of non securitisations buckets	$DRC\ risk\ class$: set of securitisations (non-CTP) buckets	$DRC\ risk\ class$: set of securitisations (CTP) buckets

Note, the coefficient β_i corresponds to $\frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i}$, when the derivative is defined. In the case of the non-securitisations and the securitisations (non-CTP) risk classes, there are no diversification benefits between buckets, and the contribution of the position to these risk classes corresponds to its contribution to the bucket to which it belongs, i.e. $\frac{\partial DRC^{DRC\ risk\ class}}{\partial JtD_i} JtD_i = \frac{\partial DRC_b}{\partial JtD_i} JtD_i$, where b represents the bucket to which position i belongs. In the securitisations (CTP) risk class, there are diversification benefits across buckets according to the following formula

$$DRC^{sec\ CTP} = \max\left\{\sum_b [\max(DRC_b, 0) + 0.5 \cdot \min(DRC_b, 0)], 0\right\} \tag{13}$$

Equation (13) explains the difference in the expression for the contribution between the non-securitisations and securitisations (non-CTP) risk classes and the securitisations (CTP) risk class. Note, in the CTP case the sum of net long positions $\kappa_{1,i}^c$ is the same across buckets, that is $\kappa_{1,i}^c = \kappa_{1,i}$ and, consequently, ${\kappa^{c}}'_{1,i} = \kappa'_{1,i}$, as shown in Table 8.

In the non-securitisations risk class, the ${\kappa^{c}}'_{1,i}$ and ${\kappa^{c}}'_{2,i}$ values rely upon the assumption that the Default Risk Charge is computed by maximizing the offsetting between long and short positions, that is by utilizing the short positions to offset the long positions in the most efficient way, see Appendix B for an algorithm that achieves this. It is worth noting that the coefficient β_i is the same for all positions belonging to the same seniority and obligor.

Note, in the non securitisations and securitisations (non-CTP), if the risk charge of bucket b is zero then the contribution to the DRC of all positions in that bucket is set to zero.

4. Example

This section presents an example of the marginal back-allocation method for the computation of the SBM capital charge applied to a portfolio consisting of the sub-portfolios 1, 2, 3, 6, 10 detailed in "Analysis of the trading book hypothetical portfolio exercise" published by the BCBS.[7] The portfolio contains sensitivities corresponding to different risk classes and buckets.

The overall SBM capital charge of the portfolio amounts to €2,670,535.57. Table 9 provides the contributions of sub-portfolios 1, 2, 3, 6, 10, ranked by sign and size. The contributions indicate in absolute terms which sub-portfolios are contributing more or less to the capital charge. The table also shows the marginal contribution in Euros of the sub-portfolios, representing the change in the capital charge given a change in the exposure of the sub-portfolio. The marginal contribution in Euros allows us to rank the positions in a portfolio according to the impact on the capital charge of a €1 amount change in the exposure to the position.

TABLE 9

Ranked Contributions

PORTFOLIO/ POSITION	CONTRIBUTION (€) $CC_i = \frac{\partial CC^{SBM}}{\partial N_i} N_i$	VALUE OF SUB-PORTFOLIO (€) V_i	MARGINAL CONTRIBUTION EURO VALUE (€) $\frac{CC_i}{V_i}$
PTF01	-55,791.18	2,003,582.99	-0.03
PTF10	9,161.61	94,777.80	0.10
PTF03	430,155.25	494,848.49	0.87
PTF06	677,321.70	261,702.77	2.59
PTF02	1,609,688.19	52,664.02	30.57

Sub-portfolio PTF01 has a negative contribution, which means that marginally increasing its size N_{PTF01} would cause the SBM capital charge to decrease. Marginally increasing any other of the remaining sub-portfolios would cause the SBM capital charge to increase. We further illustrate this point by focusing attention on positions PTF01 and PTF06, detailed in Table 10 .

TABLE 10

Details of Sub-portfolios 1 and 6

PORTFOLIO/ POSITION	CHARACTERISTICS OF POSITION	STRATEGY
PTF01	Long 30 contracts ATM 3-month front running FTSE 100 index futures (one contract corresponds to 10 equities underlying) *Futures price is based on the index level at NYSE Liffe London market close on 21 February 2014*	Equity index futures *long delta*
PTF06	Long 40 contracts of 3-month ATM S&P 500 down-and-in put options with a barrier level that is 10% OTM and continuous monitoring frequency (one contract corresponds to 100 equities underlying). *Strike price is based on the index level at NYSE market close on 21 February 2014*	Long barrier option

Table 11 reports details of the contribution to the SBM capital charge of the long FTSE 100 futures to the long barrier options on S&P 500, together with their size and marginal contribution.

TABLE 11

Contribution of Positions PTF01 and PTF06

PORTFOLIO/ POSITION	NUMBER OF CONTRACTS N_i	MARGINAL CONTRIBUTION (€) $\frac{\partial CC^{SBM}}{\partial N_i}$	CONTRIBUTION (€) $CC_i = \frac{\partial CC^{SBM}}{\partial N_i} N_i$	SBM RISK CHARGE OF COMPLETE PORTFOLIO (€) CC^{SBM}
PTF01	30	-1,859.71	-55,791.18	2,670,535.57
PTF06	40	16,933.04	677,321.70	

[7] Basel Committee on Banking Supervision (September 2014), "Analysis of the trading book hypothetical portfolio exercise," http://www.bis.org/publ/bcbs288.pdf.

Table 11 shows that PTF01 has a negative marginal contribution to the SBM capital charge, whereas PTF06 has a positive one. This means that a marginal increase in the long index futures positions, e.g. a purchase of one more FTSE 100 index future, leads to a decrease in the SBM capital charge. To illustrate this point, we simulate the purchase of an extra FTSE 100 future and report the results in Table 12.

TABLE 12

Effect of Purchasing One FTSE 100 Index Future

PORTFOLIO/ POSITION	NUMBER OF CONTRACTS N_i	MARGINAL CONTRIBUTION (€) $\frac{\partial CC^{SBM}}{\partial N_i}$	CONTRIBUTION (€) $CC_i = \frac{\partial CC^{SBM}}{\partial N_i} N_i$	SBM RISK CHARGE OF COMPLETE PORTFOLIO (€) CC^{SBM}
PTF01	31	-1,782.48	-55,256.84	2,668,714.46
PTF06	40	16,820.17	672,806.77	

The purchase of the future leads to a decrease in the SBM capital charge of €1,821.11, corresponding to a 0.07% drop. The drop in the SBM capital charge is close in size to the marginal contribution of the position, i.e. $\frac{\partial CC^{SBM}}{\partial N_{PTF01}}$, before and after the purchase. The marginal contribution shows the change in the capital charge of a *marginal* increase in the number of contracts, whereas the actual decrease in the SBM charge results from a one unit increase in the number of futures.

Next, we simulate the purchase of another barrier option and report the result in Table 13. The purchase leads to an increase in the SBM capital charge of €17,034.54 corresponding to a 0.64% increase.

TABLE 13

Effect of Purchasing One Barrier Option on S&P 500

PORTFOLIO/ POSITION	NUMBER OF CONTRACTS N_i	MARGINAL CONTRIBUTION (€) $\frac{\partial CC^{SBM}}{\partial N_i}$	CONTRIBUTION (€) $CC_i = \frac{\partial CC^{SBM}}{\partial N_i} N_i$	SBM RISK CHARGE OF COMPLETE PORTFOLIO (€) CC^{SBM}
PTF01	30	-1,972.11	-59,163.20	2,687,570.11
PTF06	41	17,135.53	702,556.80	

5. Conclusions

This paper presents the marginal back-allocation method for the standardized approach outlined in the FRTB. The method computes a contribution of a position in the trading book of a bank to the overall trading book capital charge.

In the SBM capital charges, the effect of a new position on the capital charge depends upon that position's exposure to risk factors and on the marginal effect of each risk factor on the capital charge. Similarly, the effect on the DRC of a position depends on whether the position is helping to offset the other positions, and it is represented by the contribution of the trade to the DRC.

Under the marginal method, contributions can be positive or negative. Marginally increasing the size of a position with a negative (positive) contribution reduces (increases) the capital charge. The marginal effect of the size of the position on the capital charge indicates the magnitude of the impact on the total capital requirement of a change in the position's size. The contributions calculated using this method offer a rank ordering of the positions and sum up to the total capital charge, by construction. These properties make marginal back-allocation a useful tool for risk management.

Appendix A Vector Notation Expressions

A.1 Contribution to the Delta, Vega, and Curvature Risk Charges

The following section provides a vector representation for the expressions of Δ_i^k and CR_i^k, presented in Section 3. This representation of the contributions can be useful for implementing the method and for obtaining computational gains. We limit the exposition to positions that enter the expressions for $Delta$ and $Curvature\ risk$ of some risk class.

We denote with F_k^b the set of positions belonging to factor k in bucket b and with D_b the set of factors k, belonging to bucket b, and with $R = |B|$ the number of buckets in the risk class of interest, where $|B|$ denotes the cardinality of a set B.

The $Delta$ and $Curvature\ risk$ for a given risk class can be written as

$$risk\ charge = \sqrt{\sum_{b \in B} max\left(\boldsymbol{X}^{b\prime} \cdot \bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b, 0\right) + \boldsymbol{S}' \cdot \bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S}} \tag{14}$$

- $risk\ charge$: defined in Table 4;
- $\boldsymbol{S}$: $(N \times 1)$ column vector containing the quantities S_b defined in Table 3, $\boldsymbol{S} = [S_1, \dots, S_R]'$;
- $\bar{\boldsymbol{\Gamma}} = (\bar{\gamma}_{bc})$: symmetric matrix containing the correlations $\bar{\gamma}_{bc}$ defined in Table 4, where we define $\bar{\gamma}_{bc} = 0$ for $b = c$;
- $\bar{\boldsymbol{\Omega}}^b = \left(\bar{\omega}_{ij}^b\right)$: symmetric matrix with elements $\bar{\omega}_{ij}^b$ defined in Table 4, where we add the superscript b to emphasize the dependence on the bucket;
- $\boldsymbol{X}^b$: vector containing the variables X_k^b, $\boldsymbol{X}^b = \left[X_1^b, \dots, X_{|D_b|}^b\right]'$ where X_k^b is defined in Table 4.

Note, $K_b^2 = max\left(\boldsymbol{X}^{b\prime} \cdot \bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b, 0\right)$.

The contributions of the positions are contained in the following vector (we indicate with $\odot$ the *Hadamard product*, the element-wise multiplication of two matrices or vectors):

$$\boldsymbol{c} = \boldsymbol{\beta} \odot \bar{\boldsymbol{X}} \tag{15}$$

- $\boldsymbol{c} = [\boldsymbol{c}_1', \dots, \boldsymbol{c}_R']'$: vector of contributions, partitioned according to the buckets, where $\boldsymbol{c}^b = \left[\boldsymbol{c}_1^b, \dots, \boldsymbol{c}_{|D_b|}^b\right]'$ and $\boldsymbol{c}_k^b = \left[Contribution_1^k, \dots, Contribution_{\left|F_k^b\right|}^k\right]'$ where subscripts refer to positions with risk factor k in bucket b and $Contribution_i^k$ is defined in Table 4;
- $\boldsymbol{\beta} = \left[\boldsymbol{\beta}^{1\prime}, \dots, \boldsymbol{\beta}^{R\prime}\right]'$: partitioned column vector of elements $\boldsymbol{\beta}^b = \left[\boldsymbol{\beta}_1^b, \dots, \boldsymbol{\beta}_{|D_b|}^b\right]'$ where $\boldsymbol{\beta}_k^b$ is obtained as follows

  $$\boldsymbol{\beta}_k^b = \frac{y_k^b}{risk\ charge}\left(u_k^b + v_k^b\right)\boldsymbol{n}_k^b$$

  where u_k^b and v_k^b are the k-th elements of $\boldsymbol{u}_b$ and $\boldsymbol{v}_b$, respectively, $\boldsymbol{n}_k^b$ is a vector of ones of size $\left(\left|F_k^b\right| \times 1\right)$, where $\left|F_k^b\right|$ is the number of positions in factor k, bucket b, and y_k^b and $risk\ charge$ are defined in Table 4;
- $\bar{\boldsymbol{X}} = \left[\bar{\boldsymbol{X}}^{1\prime}, \dots, \bar{\boldsymbol{X}}^{R\prime}\right]'$: vector of elements $\bar{\boldsymbol{X}}^b = \left[\bar{\boldsymbol{X}}_1^{b\prime}, \dots, \bar{\boldsymbol{X}}_{|D_b|}^{b\ \prime}\right]'$ with $\bar{\boldsymbol{X}}_k^b = \left[X_{k,i}^b, \dots, X_{k,i}^b\right]'$, where $X_{k,i}^b$ is defined in Table 4;
- $\boldsymbol{u}_b$: $(|D_b| \times 1)$ vector with elements

  $$\boldsymbol{u}_b = \begin{cases} \bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b & \text{if } A_b \geq 0 \\ \boldsymbol{0}_b & \text{if } A_b < 0 \end{cases}$$

  where $\boldsymbol{0}_b$ is a $(|D_b| \times 1)$ zero vector;

» $\boldsymbol{v}_b$: $(|D_b| \times 1)$ vector with elements

$$\boldsymbol{v}_b = \begin{cases} (\bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S})_b \cdot \boldsymbol{\iota}_b & \text{if } Q \geq 0 \text{ or } (Q < 0 \text{ and } S_b = \boldsymbol{\iota}_b' \boldsymbol{X}^b) \\ \pm(\bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S})_b \cdot (\bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b) \cdot \dfrac{1}{K_b} & \text{if } Q < 0 \text{ and } S_b = \pm K_b \text{ and } |S_b| \neq |\tilde{S}_b| \text{ and } A_b > 0 \\ \boldsymbol{0}_b & \text{if } Q < 0 \; S_b = \pm K_b \text{ and } |S_b| \neq |\tilde{S}_b| \text{ and } A_b \leq 0 \end{cases}$$

where $\boldsymbol{\iota}_b$ column vector of ones of dimension $(|D_b| \times 1)$ and $(\bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S})_b$ is the b-th element of the vector $\bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S}$.

Note, the vectors $\boldsymbol{X}^b$ and $\bar{\boldsymbol{X}}^b$ contain different variables.

The variable $\beta_{k,i}^b$ equals the derivative of $risk\ charge$ with respect to $X_{k,i}^b$, that is $\frac{\partial risk\ charge}{\partial X_{k,i}^b}$, when the derivative is well-defined. Note, $\boldsymbol{u}_b = \frac{1}{2}\frac{\partial \alpha}{\partial \boldsymbol{X}^b} = \bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b$ if $A_b > 0$ and similarly the variable $\boldsymbol{v}_b$ corresponds to $\frac{1}{2}\frac{\partial \beta}{\partial \boldsymbol{X}^b}$, when this gradient is well-defined, where

$$\begin{aligned} \alpha &= \sum_b max\left(\boldsymbol{X}^{b\prime} \cdot \bar{\boldsymbol{\Omega}}^b \cdot \boldsymbol{X}^b, 0\right) \\ \beta &= \boldsymbol{S}' \cdot \bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{S} \\ risk\ charge &= \sqrt{\alpha + \beta} \end{aligned} \tag{16}$$

Note, in the case $Q > 0$, defined in Table 3, we also have the more compact expression for $risk\ charge$, that is

$$risk\ charge = \sqrt{\boldsymbol{Z}' \cdot \mathbf{W} \cdot \boldsymbol{Z}} \tag{17}$$

» $\boldsymbol{Z} = \left[\boldsymbol{Z}^{1\prime}, \ldots, \boldsymbol{Z}^{R\prime}\right]'$: partitioned vector of elements $\boldsymbol{Z}^b = \left[\boldsymbol{Z}_1^{b\prime}, \ldots, \boldsymbol{Z}_{|D_b|}^{b\prime}\right]'$ where $\boldsymbol{Z}_k^b = X_k^b \cdot \boldsymbol{n}_k^b$ and $\boldsymbol{n}_k^b$ is a vector of ones of size $\left(|F_k^b| \times 1\right)$, where $|F_k^b|$ is the number of positions in factor k, bucket b and X_k^b is defined in Table 4;

» $\mathbf{W}$: square matrix with elements

$$w_{k\ell} = \begin{cases} 1 & \text{if } i, j \in F_k \text{ for some } k \\ \bar{\omega}_{k\ell}^b & \text{if } i \in F_k, j \in F_\ell \text{ such that } k \neq \ell \in D_b \text{ for some } b \text{ and } A_b \geq 0 \\ 0 & \text{if } i \in F_k, j \in F_\ell \text{ such that } k, \ell \in D_b \text{ for some } b \text{ and } A_b < 0 \\ \bar{\gamma}_{bc} & \text{if } i \in F_k, j \in F_\ell \text{ such that } k \in D_b, \ell \in D_c \end{cases}$$

where $\bar{\omega}_{k\ell}^b$ and $\bar{\gamma}_{bc}$ are defined in Table 4.

The vector of contributions is then simply

$$\boldsymbol{c} = \frac{1}{risk\ charge} (\mathbf{W} \cdot \boldsymbol{Z}) \odot \boldsymbol{Y} \odot \boldsymbol{X} \tag{18}$$

» $\boldsymbol{Y} = \left[\boldsymbol{Y}^{1\prime}, \ldots, \boldsymbol{Y}^{R\prime}\right]'$: partitioned vector of elements $\boldsymbol{Y}^b = \left[\boldsymbol{Y}_1^{b\prime}, \ldots, \boldsymbol{Y}_{|D_b|}^{b\prime}\right]'$ where $\boldsymbol{Y}_k^b = y_k^b \cdot \boldsymbol{n}_k^b$ and $\boldsymbol{n}_k^b$ is a vector of ones of size $\left(|F_k^b| \times 1\right)$, where $|F_k^b|$ is the number of positions in factor k, bucket b and y_k^b is defined in Table 4.

The vector representation of $Delta$ and $Curvature\ risk$ of a risk class can be useful to efficiently compute the contributions.

Appendix B Algorithm to Compute Net Long and Short JtDs for *Non-securitizations*

This section illustrates a possible way to obtain the net long and net short jump-to-default amounts (JTDs) at seniority level for a given obligor, that maximizes the offsetting. The conditions for the offsetting between long and short jump-to-defaults are outlined in Paragraph 150 of the FRTB. In particular, a long exposure to an obligor can be offset by short exposures with seniority lower than or equal to the seniority of the long position. Offsetting between long and short positions, for a given obligor, give rise to one net long JTD amount and one net short JTD amount, per obligor.

Suppose we have N seniorities for a given obligor k, ordered in such a way that if positions A and B have seniorities $s = 5$ and $s' = 3$, respectively, then A has higher seniority than B, because $s > s'$. The net jump-to-default of long positions with seniority s for a given obligor k, denoted with ℓ_k^s, and the net short jump-to-default of short positions with the same seniority, denoted with c_k^s, can be obtained with the following algorithm, for $s = 1, \dots, N$.

Set $\tilde{\ell}_k^r(0) = \tilde{\ell}_k^r$, $\tilde{c}_k^r(0) = \tilde{c}_k^r$ for $r = 1, \dots, N$, where $\tilde{\ell}_k^r$ and $\tilde{c}_k^r$ are the aggregate long and short jump-to-default amounts for seniority r and obligor k, respectively, before any offsetting has occurred;[8]

» Set $n = 1$. For $j = 0, \dots, N-1$ set $i = 0$ and run the following recursions.

a) until $\tilde{\ell}_k^{N-j}(n) = 0$ or $i = N - j - 1$, iterate on the following expressions by jointly increasing n and i, one unit at a time:

$$
\begin{aligned}
k_j^i(n) &= \min\left(\tilde{\ell}_k^{N-j}(n-1), \left|\tilde{c}_k^{N-j-i}(n-1)\right|\right) \\
\tilde{\ell}_k^{N-j}(n) &= \tilde{\ell}_k^{N-j}(n-1) - k_j^i(n) \\
\tilde{c}_k^{N-j-i}(n) &= \tilde{c}_k^{N-j-i}(n-1) + k_j^i(n) \\
\tilde{\ell}_k^{N-j'}(n) &= \tilde{\ell}_k^{N-j'}(n-1) \text{ for } j' \neq j \\
\tilde{c}_k^{N-j'-i'}(n) &= \tilde{c}_k^{N-j'-i'}(n-1) \text{ for } i' \neq i \text{ and } j' \neq j
\end{aligned}
$$

» Set $\ell_k^{N-j} = \tilde{\ell}_k^{N-j}(n^*)$ and $c_k^{N-j} = \tilde{c}_k^{N-j}(n^*)$ for $j = 0, \dots, N-1$, where n^* represents the last iteration reached by the algorithm.

The above algorithm ensures that the offsetting between short and long positions is maximized, under the restriction that long gross JTDs can only be offset by short gross JTDs with equal or lower seniority, and that net long positions are non-negative, and the net short ones are non-positive. The algorithm offsets the gross long JTDs sequentially, starting from the highest seniority one and proceeding with the long gross JTD one seniority lower. Each long gross JTD is offset using the short gross JTDs, starting with the one with equal seniority and proceeding with the short gross JTD one seniority lower and so on.

[8] The variable $\tilde{\ell}_k^r$ is the simple sum of the JtD_i-s of long instruments of seniority r and obligor k. Similarly, the variable $\tilde{c}_k^r$ is the simple sum of the JtD_i-s of short instruments of seniority r and obligor k. In both cases, we assume that the variable JtD_i is the jump-to-default of the instrument, scaled by the maturity of the instrument, see Paragraph 146 in the FRTB.

Appendix C Linearity of $s_{k,i}$, $CRV_{k,i}$, and JTD_i in the Size

C.1 Linearity of $s_{k,i}$ in N_i

The variable $s_{k,i}$ has the following functional form

$$\begin{aligned} s_{k,i} &= \frac{V_i(x'_k) - V_i(x_k)}{h} \\ &= \frac{v_i(x'_k)N_i - v_i(x_k)N_i}{h} \\ &= \frac{v_i(x'_k) - v_i(x_k)}{h} N_i \end{aligned} \tag{19}$$

- » $V_i(x_\ell)$: value function of position i, evaluated at risk factor level x_ℓ;
- » $v_i(x_k)$: is the value of the position per unit of size N_i, it is constant given the level of the risk factor x_k; we have $v_i(x_k) \equiv \frac{V_i(x_k)}{N_i}$;
- » x'_k: k-th risk factor of position i shocked by a 'small' amount;[9]
- » h: small positive number, determined by the FRTB standard.

From the expression above it follows that $\frac{\partial s_{k,i}}{\partial N_i} N_i = \frac{v_i(x'_k) - v_i(x_k)}{h} N_i = s_{k,i}$.

C.2 Linearity of $CRV_{k,i}$ in N_i

The variable $CVR_{k,i}$ is defined as follows

$$CVR_{k,i} = \begin{cases} -\left\{V_i\left(x_k^{(RW^{(curvature)+})}\right) - V_i(x_k) - RW_k^{(curvature)} \cdot s_{k,i}\right\} & \text{if } \min(C_k, B_k) = C_k \\ -\left\{V_i\left(x_k^{(RW^{(curvature)-})}\right) - V_i(x_k) + RW_k^{(curvature)} \cdot s_{k,i}\right\} & \text{if } \min(C_k, B_k) = B_k \end{cases} \tag{20}$$

- » $V_i\left(x_k^{(RW^{(curvature)+})}\right)$: value of position i at risk factor level $x_k^{(RW^{(curvature)+})}$;
- » $V_i(x_k)$: value of position i at risk factor level x_k;
- » $RW_k^{(curvature)}$: risk weight of factor k;
- » $s_{k,i}$: delta sensitivity of position i with respect to the delta risk factor that corresponds to curvature risk factor k.

The two variables C_k and B_k are intermediate quantities needed for the computation of the $Curvature\ risk$ for risk factor k, that is $CVR_k = -\min(C_k, B_k)$, and are defined according to equation (21).

[9] The value x'_ℓ of the shocked risk factor x_ℓ is defined by the regulation as either $x'_\ell = x_\ell + h$ with $h = 0.0001$ or $x'_\ell = (1+h)x_\ell$ with $h = 0.01$, depending on the risk factor x_ℓ, see Paragraph 67 in the FRTB.

$$
\begin{aligned}
C_k &= \sum_{i \in F_k} \left\{ V_i\left(x_k^{(RW^{(curvature)+})}\right) - V_i(x_k) - RW_k^{(curvature)} \cdot s_{k,i} \right\} \\
B_k &= \sum_{i \in F_k} \left\{ V_i\left(x_k^{(RW^{(curvature)-})}\right) - V_i(x_k) + RW_k^{(curvature)} s_{k,i} \right\}
\end{aligned} \tag{21}
$$

- F_k: set of positions with risk factor k.

From the equations above, we can see that the variable $CVR_{k,i}$ has the general form $CVR_i = -[V_i(x') - V_i(x) - RW \cdot s_i]$. This general expression is useful in interpreting the sign of the contribution. In particular, we have

$$
\begin{aligned}
CVR_{k,i} &= -[V_i(x_k'') - V_i(x_k) - RW_k s_i] \\
&= -\left[v_i(x_k'')N_i - v_i(x_k)N_i - RW_k \frac{v_i(x_k') - v_i(x_k)}{h} N_i \right] \\
&= -\left(v_i(x_k'') - v_i(x_k) \left(1 - RW_k \frac{1}{h}\right) - RW_k \frac{v_i(x_k')}{h} \right) N_i
\end{aligned} \tag{22}
$$

- RW_k: risk weight that depends on the risk factor.

As for $s_{k,i}$ the variable $CVR_{k,i}$ can be thought of as a linear function of the size amount of the position with slope given by $-(v_i(x_k'') - v_i(x_k)(1 - RW_k h^{-1}) - RW_k v_i(x_k') h^{-1})$.

C.3 Linearity of JTD_i in N_i

The jump-to-default of position i for non-securitisations, when different from zero, has the following form

$$
\begin{aligned}
JtD_i &= \alpha_i(LGD_i N_i + P\&L_i) \\
&= \alpha_i(LGD_i N_i + MV_i - N_i) \\
&= \alpha_i(LGD_i N_i + \tilde{v}_i N_i - N_i) \\
&= \alpha_i(LGD_i + \tilde{v}_i - 1)N_i
\end{aligned} \tag{23}
$$

- α_i: scaling factor depending on the maturity of the position, see for instance Paragraphs 146 and 150 of the FRTB;
- LGD_i: loss given default of position i;[10]
- N_i: size of position i;
- $P\&L_i$: profit and loss of position i, corresponding to the mark-to-market loss (or gain) already taken on the exposure, $P\&L_i = MV_i - N_i$ where MV_i is the current market value of the position;
- $\tilde{v}_i$: market price of position i per unit of size quantifier, such that $MV_i = \tilde{v}_i N_i$.

It also follows that, in this case, we can write $\frac{\partial JtD_i}{\partial N_i} N_i = \alpha_i(LGD_i + \tilde{v}_i - 1)N_i = JtD_i$.

[10] In the case of positions entering the Default Risk Charge for securitisations, the JtD_i is set equal to the market value, so that $JtD_i = MV_i$.

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Basel Committee on Banking Supervision (January 2016), “Minimum capital requirements for market risk”.

Basel Committee on Banking Supervision (September 2014), “Analysis of the trading book hypothetical portfolio exercise”.

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