

MODELING METHODOLOGY

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Income-Adjusted Risk Contribution-based Capital Allocation

Abstract

Banks commonly use Risk Contribution, or contribution to portfolio Unexpected Loss (i.e., standard deviation), as a risk allocation method. While the method has some very desirable properties, it can also produce seemingly counterintuitive dynamics, whereby high interest income-producing assets are associated with higher risk, all else being equal. This dynamic manifests from the higher interest income assets possessing higher value, leading to higher standard deviation in absolute terms. In reality, financial institutions often use interest income to offset losses, and thus, associate higher interest with lower risk. This paper introduces a new, income-adjusted form of Risk Contribution-based capital allocation, designed so that interest income offsets losses. The measure demonstrates improved properties for exposures with particularly high coupons.

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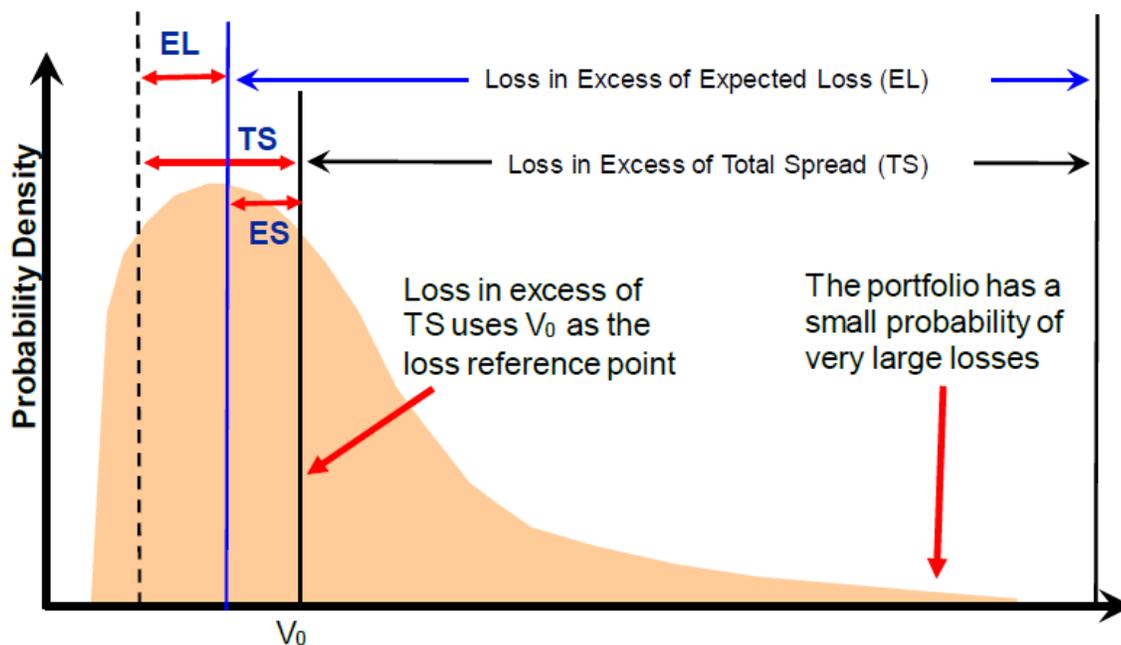
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1. Introduction

Risk contribution (RC) is a measure of an instrument's contribution to portfolio unexpected loss (UL) or standard deviation. In general, RC-based capital allocation shows desirable properties where, for example, it penalizes lower credit quality exposures, recognizing that their risk comprises a greater portion of overall portfolio risk. However, in some cases, the comparative statistics can seem counterintuitive, differing with how financial organizations account for loss, and, as a result, not in-line with their subjective view of risk. In particular, high interest income-producing assets are associated with higher risk, all else being equal. This dynamic manifests from the higher interest income assets possessing higher value, and thus, higher standard deviation in absolute terms. In reality, institutions use interest income to offset losses, and thus, associate higher interest with lower risk. This paper introduces a new, adjusted form of Risk Contribution-based capital allocation, designed so that interest income offsets losses. The measure demonstrates improved properties for exposures with particularly high coupons.

To better understand the design of the new measure, we describe industry standard approaches to measuring losses. Figure 1 depicts a loss distribution along with two commonly used loss reference points: in excess of expected loss and in excess of total spread. Measuring losses in excess of expected loss (EL) corresponds to a buffer measured relative to the portfolio's expected value at horizon. Measuring loss in excess of total spread (TS) corresponds to a buffer measured relative to the analysis date value (V_0) invested at the risk-free rate.

Figure 1 Typical Credit Portfolio Loss Distribution.



The difference between the two reference points is the portfolio expected spread (ES). We can then attribute the overall losses to various portfolio instruments using the corresponding RCs; the sum of RCs for all instruments in the portfolio equals the portfolio's UL.

This paper focuses on measuring the extent to which an economic buffer is required to absorb future losses. Following Bozsoki, et al. (2016), we measure the analysis date as the accounting value (net of loss allowance). We measure the horizon value distribution using the (marked-to-model) economic values. Under this parametrization, loss measured in excess of EL generally increases with fees, all else being equal (including the analysis date value). Mechanically, there is more to lose when fees are higher. On the other hand, when measuring losses in excess of TS, higher fees are associated with lower losses, as fee income can offset losses. With this in mind, Bozsoki, et al. (2016) demonstrate that, when measuring the extent to which an economic buffer is needed to absorb future losses, in addition to the analysis date and horizon valuation choices, one should measure losses in excess of TS.

With this point in mind, RC is a standard deviation-based measure, and it is measured around the mean of the distribution. It has the same property as portfolio capital in excess of EL, where RC generally increases with fees; as fees increase, the horizon value

distribution magnifies driving RC higher. Thus, capital allocated based on RC, even when measured in excess of TS, will have the property that it increases with fees or coupons. The positive relationship between RC and fees can be difficult to manage in the context of business applications such as incentive compensation and risk based pricing. When originating two otherwise identical loans, the RC-based approach will allocate lower capital to the loan with lower fees, rather than the one with higher fees, which can offset losses. To address this shortcoming, the new Income-Adjusted Risk Contribution-based (IA-RC) capital allocation measure offsets the traditional RC-based measure, and recognizes that fee income can indeed be used to offset losses and provide capital relief.

We organize the remainder of this paper as follows:

- » Section 2 defines Tail Risk Contribution (TRC) and RC and provides a more formal discussion of their properties, relating to the chosen loss reference point for capital (in excess of EL or TS).
- » Section 3 defines the new measure.
- » Section 4 provides details on the risk allocation characteristics using a sample portfolio.
- » Section 5 concludes.

2. TRC and RC and the Chosen Loss Reference Point (in excess of EL or TS)

In addition to Risk Contribution (RC), Tail Risk Contribution (TRC) is another common capital allocation measure. It focuses on individual instruments' contributions to portfolio-level tail risk statistics, such as Economic Capital. In this section, we formally define TRC and RC, and we relate their capital allocation properties to the chosen portfolio loss reference point and expected income. Sections 2.1 and 2.2 define the measures, and Section 2.3 relates capital allocation with the chosen loss reference point and expected income.

2.1 Tail Risk Contribution

We define TRC as the expected loss when the portfolio resides in a loss state equal to the capital point. In this case, losses that contribute to conditional expected loss will be in excess of the same loss reference point as the portfolio capital reference point:

$$TRC_{LRP}^i = \mathbb{E} \left[\left(\frac{V_{LRP}^i - \tilde{V}_H^i}{\tilde{V}_0^i} DF_{0,H}^{zero-PD} \mid L^P DF_{0,H}^{zero-PD} = C_{LRP}^P \right) \right] \quad (1)$$

Where V_{LRP}^i is the loss reference point value of instrument i , \tilde{V}_H^i is the distribution of realized values at horizon, \tilde{V}_0^i is the normalized value, L^P is portfolio loss, and C_{LRP}^P is portfolio capital in excess of the loss reference point.

When using TRC to allocate capital, instrument-allocated capital is given as:

$$C_{TRC}^i = \frac{w_i \cdot TRC_{LRP}^i}{\sum_i w_i \cdot TRC_{LRP}^i} C_{LRP}^P \quad (2)$$

Intuitively, TRC allocates capital based on an instrument's contribution to tail event losses and, therefore, will penalize borrowers with high credit quality but with systemic risk.

2.2 Risk Contribution

We define risk contribution as the marginal contribution to the volatility of the portfolio value. Risk contribution equals the correlation ρ_{ip} between the value of instrument i and the value of the portfolio, multiplied by instrument unexpected loss:

$$RC^i = \rho_{ip} UL_i$$

When using RC to allocate capital, given that properly weighted instrument-level RCs add up to the portfolio UL, instrument allocated capital is defined as:

$$C_{RC(LRP)}^i = \frac{w_i \cdot RC^i}{UL_p} C_{LRP}^P \quad (3)$$

Intuitively, RC allocates capital based on contribution to standard deviation, which can be thought of as everyday losses. Therefore, RC penalizes lower credit quality exposures.

2.3 Relationship Between Capital Allocation and Expected Income

As we discussed, RC capital allocation has the property that higher coupons necessarily give a higher risk contribution-based capital allocation, keeping everything else constant. The same is true for TRC capital allocation when measured in excess of EL.

To understand why a higher coupon creates higher risk contribution, recall that RC equals

$$\rho_{ip} UL_i$$

where ρ_{ip} is the correlation between the value of instrument i and the portfolio value.

Instrument UL increases as the value in a no-default state increases. This trait follows because the valuation differences between the default state and non-defaulted states increase when values in non-defaulted states are higher due to higher interest income. The correlation between instrument value and portfolio value also increases slightly, because higher valuations in non-default states cause higher portfolio values in those states.

To understand why a higher coupon creates higher tail risk contribution when measured in excess of expected loss, recall from Equation (1) that TRC is the instrument's expected losses, conditional on a tail event. Losses measured in excess of expected loss are larger for a higher expected spread instrument than for a smaller expected spread instrument, because the expected value at horizon, which is the reference point in this case, is larger for a higher spread instrument.

On the other hand, the appealing property of TRC in excess of TS is its recognition that higher income assets have a higher buffer (all else being equal) before incurring loss (i.e., before falling below their risk-free, compounded book value), therefore receiving a lower capital allocation under this measure.

While capital allocation based on TRC in excess of TS possesses desirable properties, some institutions prefer RC-based risk allocation, because this method intuitively allocates higher capital to higher risk instruments, and because TRC can potentially penalize their high credit quality, high exposure borrowers.

3. Income-Adjusted Risk Contribution-based Capital Allocation

To address the shortcomings of a traditional RC-based approach to capital allocation, this section defines Income-Adjusted Risk Contribution (IA-RC) based capital allocation. As outlined in the Introduction, IA-RC-based capital allocation is designed to account for interest income that can be used to offset losses. Intuitively, the measure explicitly accounts for the RC-based risk being measured, relative to the instrument's expected value of the horizon distribution, rather than relative analysis date accounting value. Given the objective of allowing fees to offset losses, the income-adjusted risk contribution-based capital is only defined within the context of capital based on losses measured in excess of TS. More formally, instrument i 's IA-RC-based capital is defined as:

$$C_{IA-RC(TS)}^i = RC^i \frac{C_{EL}^P}{UL_p} - ES_H^i DF_{0,H}^{zero-PD} \quad (4)$$

where:

$C_{IA-RC(TS)}^i$ is the Income Adjusted Risk Contribution-based capital in excess of Total Spread

RC^i is the Risk Contribution for instrument i in currency amount

C_{EL}^P is portfolio capital in excess of expected loss

UL_p is portfolio unexpected loss

ES_H^i is instrument-expected spread to horizon in currency amount

$DF_{0,H}^{zero-PD}$ is the zero-EDF™ (Expected Default Frequency) discount factor to horizon

Analyzing the formal definition, the right-hand starts off with multiplying RC^i by C_{EL}^P/UL_p to produce RC-based capital in excess of EL. The next term, $ES_H^i DF_{0,H}^{zero-PD}$, represents use of the interest income to cover for losses via the removal of the instrument's expected spread, so that the measure is consistent with a capital allocation measure in excess of TS. In fact, the measure has the appealing property that it indeed aggregates to portfolio capital in excess of TS:

$$\begin{aligned} \sum_i C_{IA-RC(TS)}^i &= \sum_i \left(RC^i \frac{C_{EL}^P}{UL_p} - ES_H^i DF_{0,H}^{zero-PD} \right) \\ &= \frac{C_{EL}^P}{UL_p} \sum_i RC^i - DF_{0,H}^{zero-PD} \sum_i ES_H^i \\ &= C_{EL}^P - ES_H^P DF_{0,H}^{zero-PD} \\ &= C_{TS}^P \end{aligned}$$

3.1 Comparison to the Traditional RC-based Allocation of Capital in Excess of TS

Traditional RC-based allocation of capital in excess of TS is defined as:

$$C_{RC(TS)}^i = RC^i \frac{C_{TS}^P}{UL_p}$$

Remembering the relationship between the losses (and capitals) in excess of EL and TS discussed earlier, the above can be rewritten

$$C_{RC(TS)}^i = RC^i \frac{C_{TS}^P}{UL_p} = RC^i \frac{C_{EL}^P - ES_H^P DF_{0,H}^{zero-PD}}{UL_p} = RC^i \frac{C_{EL}^P}{UL_p} - RC^i \frac{ES_H^P}{UL_p} DF_{0,H}^{zero-PD}$$

Comparing this representation to the definition of income-adjustment RC-based capital allocation, we see that the fundamental difference is found in the income reference point choice. While the traditional approach essentially allocates the overall portfolio ES using RCs, and then uses it as a reference point, the newly introduced income-adjusted RC capital uses each instrument's own ES. Thus, the difference between the two equations is

$$C_{IA-RC(TS)}^i - C_{RC(TS)}^i = \left(\frac{RC^i}{UL_p} ES_H^P - ES_H^i \right) DF_{0,H}^{zero-PD}$$

and an instrument will have lower capital allocated compared to the traditional capital allocation measure when

$$\frac{RC^i}{UL_p} < \frac{ES_H^i}{ES_H^P}$$

or when an instrument's income (in the form of ES) contribution to the overall portfolio's income is greater than instrument's risk (as expressed by RC) contribution to the overall portfolio risk as expressed by portfolio UL.

4. Allocation Comparison Using a Representative Portfolio

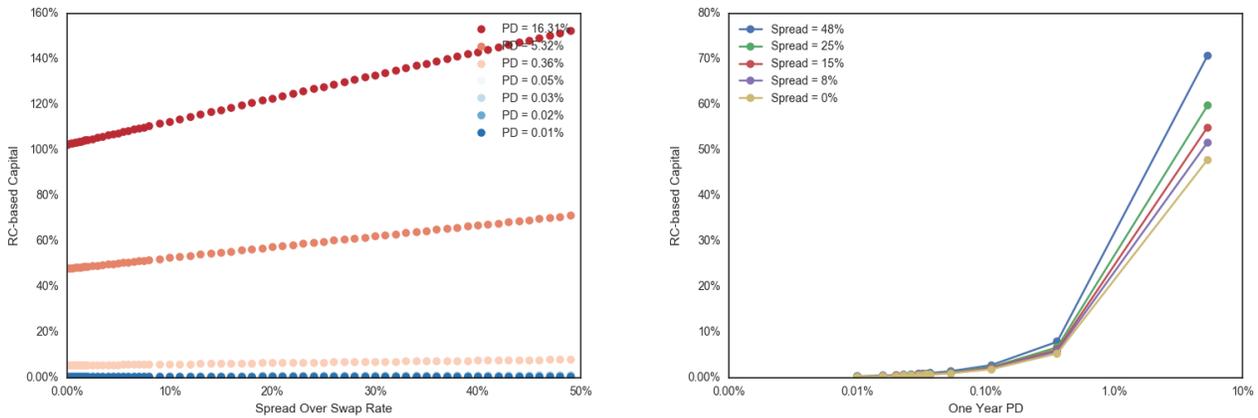
To illustrate the characteristics of the Income-Adjusted-RC-based capital measure and to compare them with more traditional capital allocation methodologies (RC-based Capital in excess of Total Spread and TRC-based Capital in excess of Total Spread), we analyze a representative portfolio of term loans in RiskFrontier™, along with a set of exposures across the dimensions of PD, RSQ, time to maturity, and floating rate spread (the “exposure grid”). Using such a set of exposures allows us to study the capital measure dynamics along a particular dimension, while controlling for all other parameters. The Appendix provides a more detailed description of the representative portfolio, exposure grid, and RiskFrontier parameterization.

We begin with an analysis of the capital measure dynamics across the PD and floating rate spread dimensions.

4.1 Risk Contribution-Based Capital In Excess of Total Spread

Figure 2 shows the dynamics of RC-based Capital across various settings of floating rate spread and PD. Capital is shown as a percentage of MTM exposure.

Figure 2 Risk Contribution-based Capital.



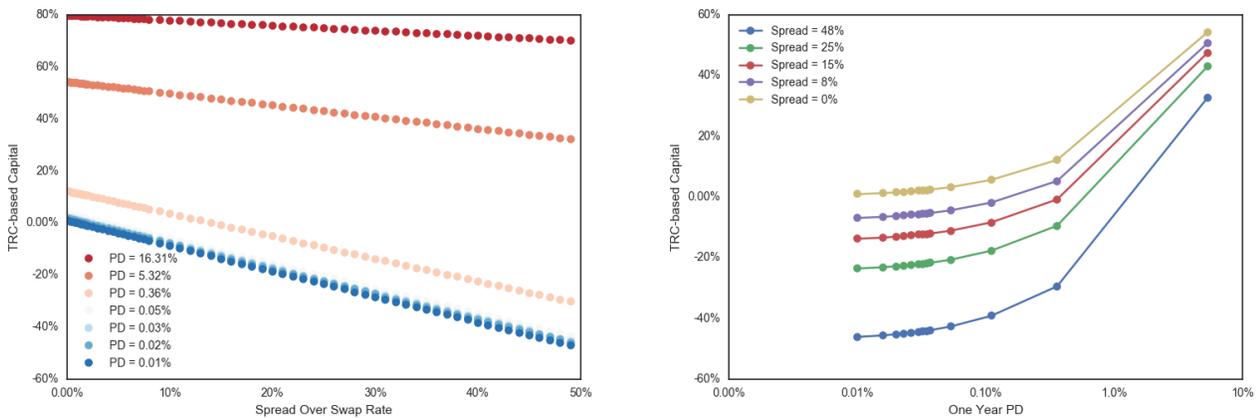
As we see from the left-hand graph in Figure 2, RC-based capital increases when the spread increases for various settings of PDs, in-line with our expectations that RC should increase, as higher spreads lead to higher values in non-default states at horizon, discussed in Section 1. We can also see that increases are more pronounced for higher PD values.

PD increases also lead to increases in RCs for various floating rate spread values (as PD increases up to 50%), as uncertainty between realization of default and non-default states increases.

4.2 Tail Risk Contribution-Based Capital In Excess of Total Spread

Next, we turn our attention to the dynamics of Tail Risk Contribution-based capital across the various settings of floating rate spread and PD.

Figure 3 Tail Risk Contribution-based Capital.



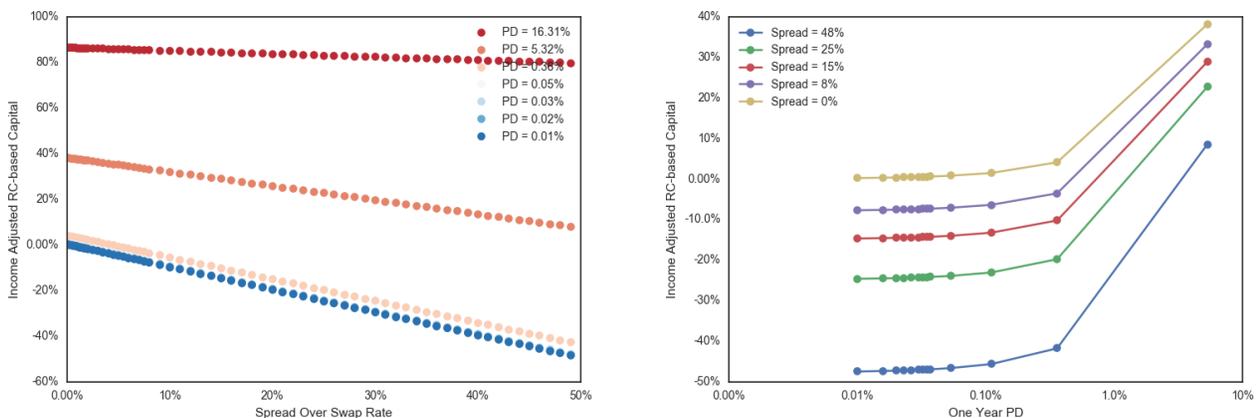
From Figure 3, we can see that tail risk contribution-based capital *decreases* as spread increases for various PD values. As defined by Equation (1), TRC is a conditional expected loss, and, as coupon increases, the conditional expected loss decreases due to the increased interest income during the non-default states for the instrument. Moreover, conditional expected loss can turn negative (i.e. become a gain), if spread is high enough for a given PD. The spread values at which TRCs become negative (leading to negative capital) increase as PDs increase, as more and more interest income is required to cover the increased likelihood of default states.

In addition, the right-hand graph in Figure 3 shows that TRC-based capital increases with PDs for various settings of floating rate spreads as the probability of default states increase, leading to increases in the conditional expected loss.

4.3 Income-Adjusted Risk Contribution-Based Capital In Excess of Total Spread

Figure 4 illustrates the dynamics of Income-Adjusted Risk Contribution-based capital across the various settings for floating rate spread and PD.

Figure 4 Income-Adjusted Risk Contribution-based Capital



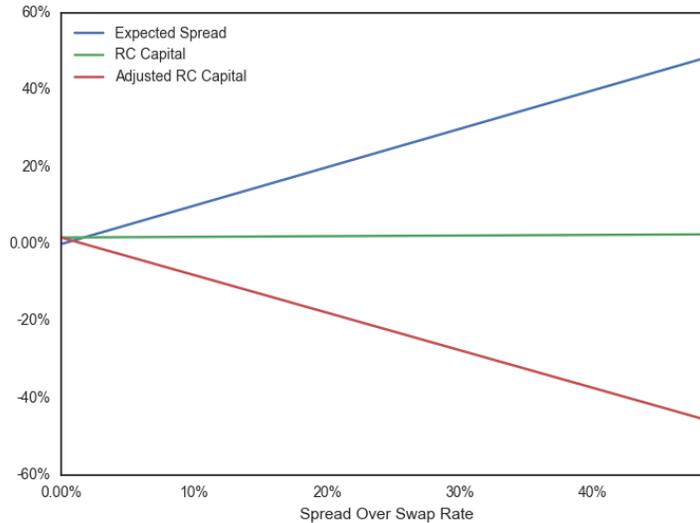
We can see that the Income-Adjusted-RC-based capital measure demonstrates dynamics similar to TRC-based capital for the PDs shown; IA-RC-based capital decreases as spread increases for PDs of 16% and below, but increases as spread increases for very large PDs of 25% – 50% (not shown). To better understand the dichotomy of this dynamic, we take a more detailed look at the Income-Adjusted Risk Contribution-based capital allocation drivers. As we describe in Equation (4), the income-adjusted measure is a function of RC-based capital in excess of EL and expected spread. As a result, the dynamics of the income-adjusted measure when varying spread depend on the relative magnitude of changes in RC-based capital and ES.

We first discuss a low PD example. To illustrate the discussion, Figure 5 presents the dependencies of ES, RC-based Capital in excess of EL, and IA-RC-based Capital on the value of the floating rate spread when setting PD to 11bps. As we can see from the graph, for exposures with low PD, the change in ES drives the change in the Income-Adjusted RC-based capital measure; RC-based

capital increases only slightly over the floating-rate spread when compared to a much larger change in ES and, therefore, in Income-Adjusted-RC-based capital.

$$C_{IA-RC(TS)}^i = RC^i \frac{C_{EL}^p}{UL_p} - ES_H^i DF_{0,H}^{zero-PD}$$

Figure 5 Income-Adjusted RC-based Capital (LowPD) Dynamics.



To see analytically why this result occurs, first recall that Risk Contribution can be expressed as the product of the value correlation of an instrument, ρ_{ip} , and the portfolio and the instrument's UL:

$$RC^i = \rho_{ip} UL_i$$

Furthermore, we can present UL as:

$$UL = \sqrt{PD \cdot \sigma_{h|D}^2 + (1 - PD)\sigma_{h|ND}^2 + PD(1 - PD)(\mathbb{E}[V_{h|ND}] - \mathbb{E}[V_{h|D}])^2}$$

In a simplified example, with a one-year horizon and deterministic LGD, the conditional variances in the first and second terms under the square root are zeroes, as we essentially have a default/no default framework. When recovery is modeled as the proportion of principal, expected value given default is not dependent on expected spread, therefore:

$$\Delta UL = \sqrt{PD \cdot (1 - PD)} \Delta \mathbb{E}[V_{h|ND}]$$

From the above, we can see that the change in Unexpected Loss is proportional to the change in expected value at horizon given no default, where the coefficient of the relationship is a parabolic function with a maxima at 50%. Therefore, the change in UL should be less than half the change in expected value at horizon, given no default for that simplified example. Note, the change in expected value at horizon given no default equals the change in Total Spread.

Considering the correlation component, the change in spread does not change the simulated DD bucket at horizon, so the sign of the instrument values at horizon should not change materially. If the instrument is small in a large, diversified portfolio, the changes to values at horizon do not change significantly the correlation between instrument and portfolio value distributions.

Therefore,

$$\Delta RC \cong \rho_{ip} \cdot \sqrt{PD \cdot (1 - PD)} \Delta \mathbb{E}[V_{h|ND}]$$

To further analyze this relationship, we use the simple example of a floating spread bond that pays a coupon FRS at horizon and matures at the same time. For further simplicity, we assume LGD equals 100% (implying $\mathbb{E}[V_{h|D}] = 0$). Its expected value at horizon equals

$$\mathbb{E}[V_h] = (1 - PD)(1 + FRS + R_f)$$

Therefore, expected spread is given by

$$\begin{aligned} ES &= \mathbb{E}[V_h] - (1 + R_f) = (1 - PD)(1 + FRS + R_f) - (1 + R_f) \\ &= (1 - PD) \cdot FRS - PD(1 + R_f) \end{aligned} \quad (5)$$

Indicating that the change in Expected Spread also depends on PD and is driven by the change in spread for a given PD

$$\Delta ES = (1 - PD) \cdot \Delta FRS$$

Implying that for relatively small PD, the change in Expected Spread is approximately equal to the change in spread. Intuitively, this follows because the discounting of the coupon cash flows due to the expected credit risk is small.

The change in expected value at horizon given no default, which equals change in Total Spread, equals the change in the spread for this illustrative example

$$\Delta \mathbb{E}[V_{h|ND}] = \Delta FRS.$$

Summarizing, the change in Risk Contribution is approximately

$$\Delta RC \cong \rho_{ip} \cdot \sqrt{PD \cdot (1 - PD)} \Delta FRS$$

Therefore, the change in IA-RC-based Capital is approximately

$$\Delta C_{IA-RC(TS)} \cong \Delta RC \cdot CM_p - \Delta ES \cdot DF_{0,H}^{zero-PD}$$

where CM_p is the portfolio's Capital Multiplier (which does not change materially for a relatively small instrument), and $DF_{0,H}^{zero-PD}$ is the zero-PD discount rate. Therefore, the Income-Adjusted RC-based capital measure is likely to decrease when

$$(1 - PD) \cdot \Delta FRS \cdot DF_{0,H}^{zero-PD} > \rho_{ip} \cdot \sqrt{PD \cdot (1 - PD)} \cdot \Delta FRS \cdot CM_p$$

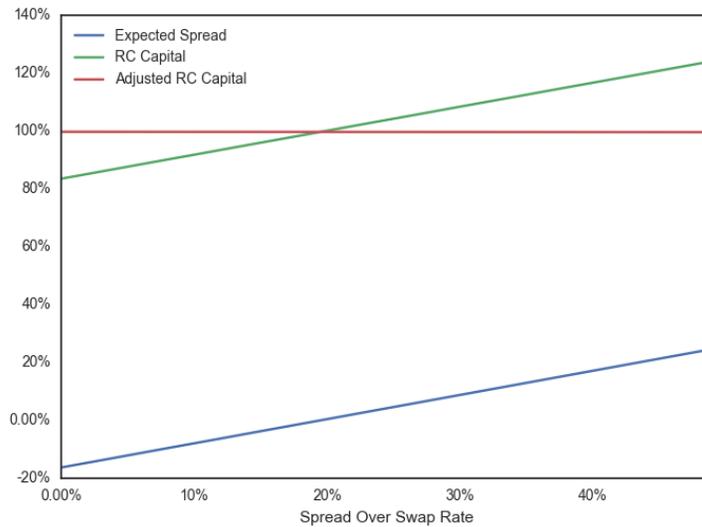
or when

$$\sqrt{(1 - PD)} \cdot DF_{0,H}^{zero-PD} > \rho_{ip} \cdot \sqrt{PD} \cdot CM_p$$

which holds true for a sufficiently low PD, for any combination of zero-PD rate, value correlation, and capital multiplier.

Turning to a discussion of the dynamics of income-adjusted RC-based capital when PD is high, the above inequality also suggests that, as PDs rise, the right-hand side (contribution of RC term) begins to increase, while the left-hand side (ES term contribution to IA-RC-based capital measure) begins to decrease, implying that, for any combination of zero-PD rate, value correlation, and capital multiplier for sufficiently large PDs, Income-Adjusted RC-based capital increases with an increase in floating rate spread. Figure 6 illustrates that observation — the increase in RC-based capital outweighs the increase in ES. Hence, the Income-Adjusted RC-based capital increases over floating rate spread, but by a smaller amount than the increase in RC-based capital.

Figure 6 Income-Adjusted RC-based Capital (High PD) Dynamics.

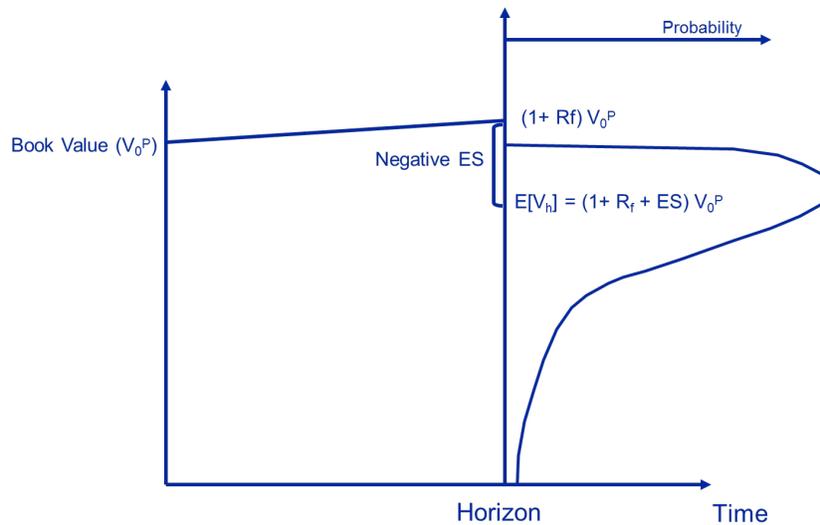


We can also see from Figure 6 that expected spreads are negative when PD is much larger than spread — implying that the contractual income is not large enough to counter the credit risk. Using our earlier case illustration, ES in Equation (5) equals

$$ES = (1 - PD) \cdot FRS - PD(1 + R_f)$$

suggesting that, with the increase in PD, the first term in that expression becomes small, while the second one rises. Intuitively, using Income-Adjusted RC-based capital, when expected spread is negative, the available “income buffer” reduces, thereby requiring additional capital. To understand why it is possible to have negative expected spread, recall, we define the expected spread as the difference between expected value at horizon and the risk-free value at horizon.

Figure 7 Instrument Value distribution with Negative Expected Spread.



Under Book-Lattice valuation, the reference point for calculations is the risk-free value at horizon — the book value compounded forward at the risk-free rate. The horizon distribution, and, consequently, the expected value at horizon, represents the economic value of the exposure. If the expected value at horizon is less than the risk-free value at horizon, then the expected spread is negative. This scenario is more likely to occur for high-risk or low income exposures, or, in other words, when the contractual income is not sufficient to cover and compensate for the underlying credit risk.

We can see from the above discussion, and particularly from Figures 2 and 4, that using income adjusted risk contribution-based capital allocation may well significantly affect the rank ordering of exposures in a portfolio, particularly for exposures with high expected spread.

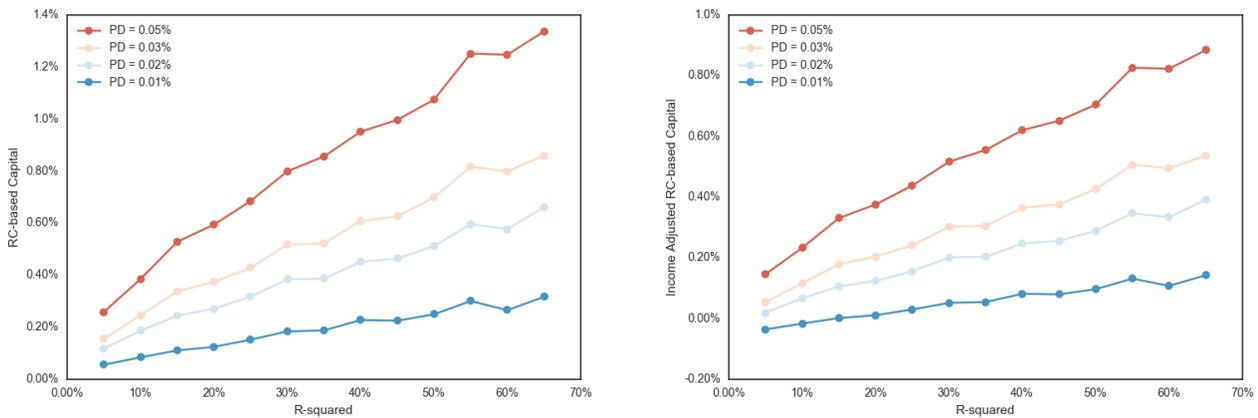
4.4 Further Univariate Analysis

To better understand the behavior of the newly introduced capital measure, this section addresses the dynamics of Income-Adjusted Risk Contribution (IA-RC)-based capital across R-squared and time to maturity dimensions. We compare the results with the dynamics of RC and TRC-based capital under the same conditions.

R-SQUARED

Figure 8 shows the dynamics of RC and IA-RC-based capital measures for exposures with differing R-squared.

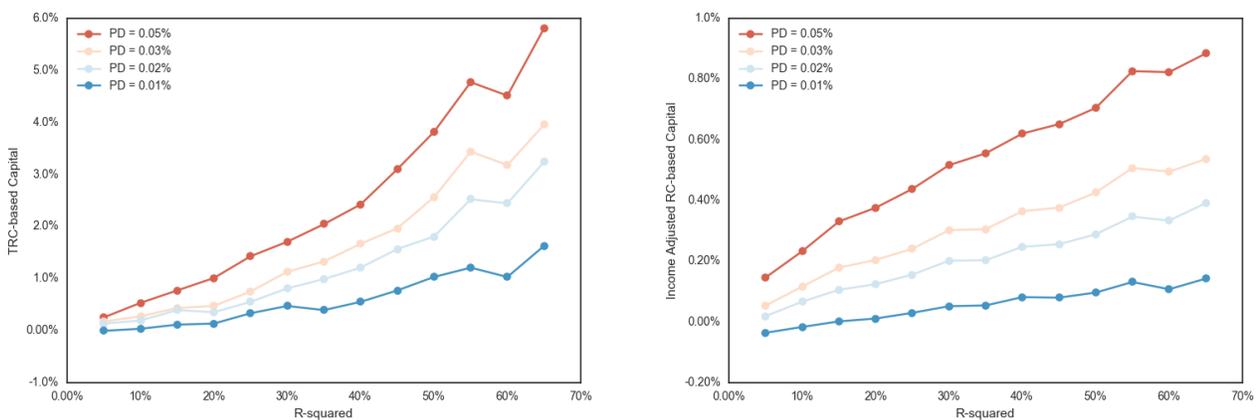
Figure 8 Dynamics of RC (left) and IA-RC (right) Capital with R-squared (One Year Maturity with 20bp Spread).



The relationship between R-squared and RC-based capital is similar to that between R-squared and the IA-RC-based capital measure. As one might expect, and as demonstrated in our earlier examples, allocated capital increases with increases in R-squared in both cases analytically, due to the expected increase in value correlation. However, we can see that the magnitude of capital allocated is generally lower for IA-RC-based capital, due to the 20bp spread income being sufficient to cover the credit risks described by the PDs (corresponding to the high credit rating instruments) used in the graph. Additionally, IA-RC-based capital is negative for low PD/low RSQ combinations, as such instruments produce enough interest income to cover their own credit risk, and they sufficiently diversify the portfolio to serve effectively as hedges.

Figure 9 illustrates the comparison of the TRC-based and IA-RC-based capital dynamics for exposures with differing R-squareds.

Figure 9 TRC (left) and IA-RC (right) Capital Dynamics with R-squared (One Year Maturity with 20bp Spread).



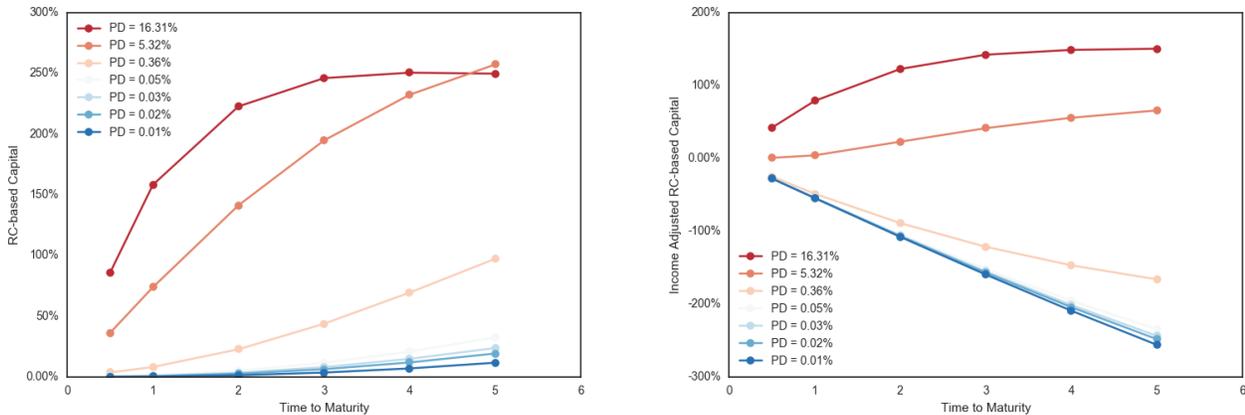
As with the RC-based capital measure, the direction of the relationship between R-squared and TRC-based capital is similar to that between R-squared and the IA-RC-based capital measure. However, in this case, we see that the magnitude of capital allocated

differs vastly, with TRC-based capital allocations much larger than IA-RC-based allocations. This finding is expected and a desirable result, since TRC-based capital, which measures tail risk, emphasizes higher correlations for the low PDs, shown compared to RC-based measures.

MATURITY – HIGH SPREAD

Next, we turn our attention to the RC and IA-RC-based capital dynamics for high-income exposures with differing time remaining to maturity.

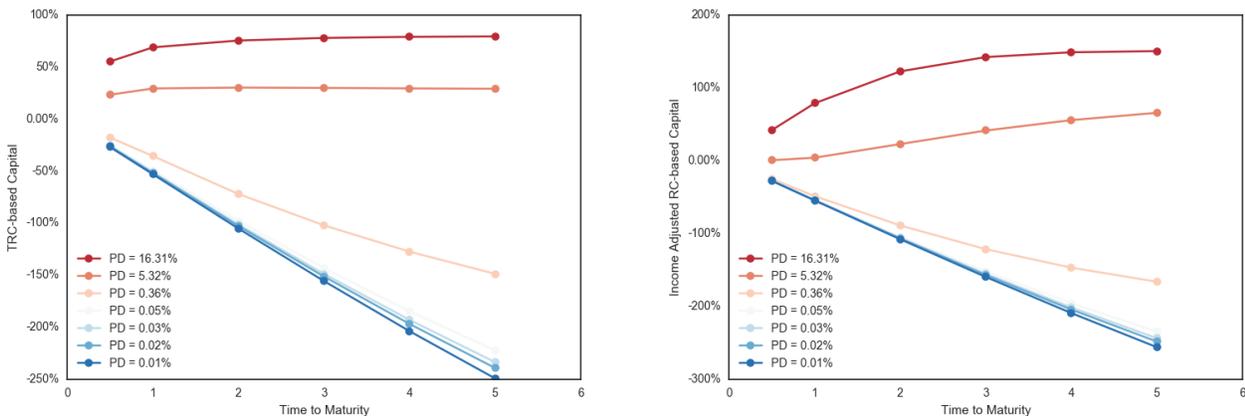
Figure 10 RC (left) and IA-RC (right) Capital Dynamics with Time to Maturity (45% RSQ, 55% Spread)



As shown in Figure 10, the behavior of RC-based capital through time is very different from the behavior of the IA-RC-based capital measure, particularly for low PD, high credit quality borrowers. This distinction follows because, for these instruments, the expected spread is much larger than the corresponding credit risk, leading to a much lower (or negative) IA-RC-based capital. We can see that the magnitudes of capital allocated are very different as well, primarily when IA-RC-based capital allocations are highly negative due to very large expected spread for low PDs with high coupon instruments. We can also see that the effects of high coupons are magnified generally, with increases in time to maturity, as these high coupons provide a larger cumulative effect.

Figure 11 shows the dynamics of TRC and IA-RC-based capital for high-income exposures with differing time remaining to maturity.

Figure 11 TRC (left) and IA-RC (right) Capital Dynamics with Time to Maturity (45% RSQ, 55% Spread).



The behaviors of TRC and the IA-RC-based capital measure with different time to maturity are much more similar in shape for both high and low PDs, but the magnitudes of capital allocated can still be quite different, particularly where IA-RC-based capital is greater than 100% of MTM due to high PD and negative expected spread.

MATURITY – LOW SPREAD

For the final comparison, we illustrate the dynamics of RC and IA-RC-based capital measures for low-income exposures with differing time remaining to maturity.

Figure 12 RC (left) and IA-RC (right) Capital Dynamics with Time to Maturity (45% RSQ, Zero Spread).

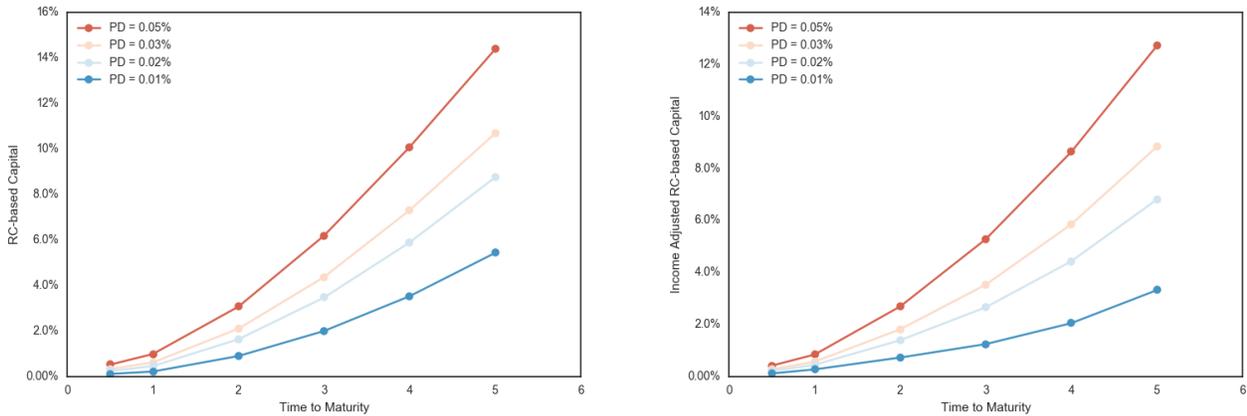
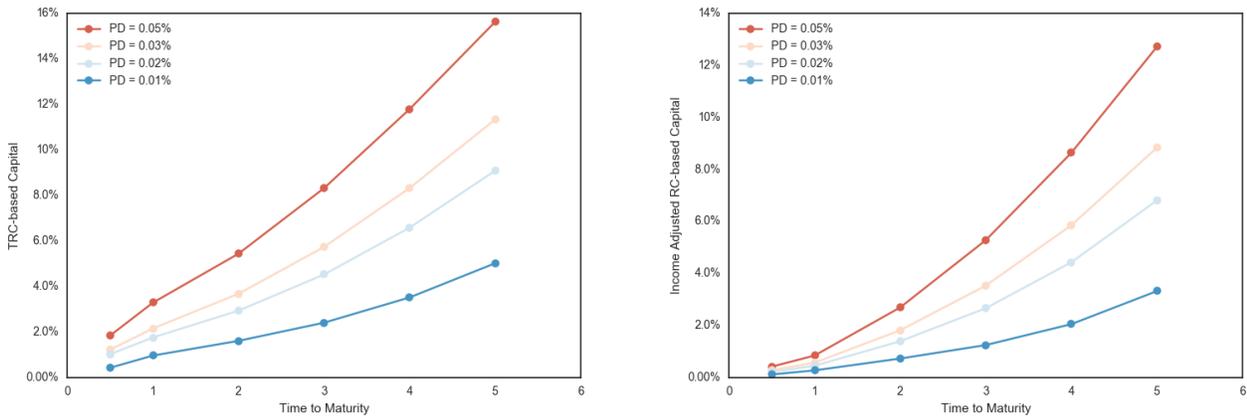


Figure 12 shows that when spreads (and PDs) are small, the behavior of RC-based capital measure with increases in time to maturity is much more similar to the behavior of the IA-RC-based measure. This distinction follows because the expected spread is small, leading to a small income adjustment. Note, even with a zero spread over the swap rate, in our illustrations, the swap rate is 10bp larger than the risk-free rate, implying that income is still larger than expected loss. This result is why we see an upward sloping curve as time to maturity increases, as longer-dated exposures would have “more to lose.”

Figure 13 TRC (left) and IA-RC (right) Capital Dynamics with Time to Maturity (45% RSQ, Zero Spread).



When spreads and PDs are small, the behavior of TRC-based capital measure is still similar to the behavior of both RC and IA-RC-based capital measures. The magnitude of capital allocated is also similar for longer dated deals, but it can vary more significantly at short maturities.

5. Summary

This paper defines and analyzes the dynamics of an income-adjusted risk contribution-based capital allocation measure.

The Income-Adjusted Risk Contribution-based capital in excess of Total Spread (TS) shows the desired property of decreasing capital when increasing floating rate spread, for reasonable combinations of PD and RSQ.

For instruments with high probability of default (PD), the measure may show an increase as floating rate spread increases. This trait occurs when the relative change in Risk Contribution is greater than the change in expected spread.

Using Book-Lattice valuation, instruments with high PD but low income will have negative expected spread. Negative expected spreads are effectively an add-on to the required capital under the adjusted measure.

Maturity effects can be more complex, depending on the risk-return relationship.

The magnitudes of capital allocated can be very different between Risk Contribution (RC), Tail Risk Contribution (TRC), and Income Adjusted RC-based Capital Measure (IA-RC). Therefore, moving to the use of income-adjusted, risk contribution-based capital allocation may significantly affect the rank ordering of exposures in a portfolio, particularly for exposures with high expected spread.

Appendix

RiskFrontier Parameterization

- a. Analysis Date Valuation: Book
- b. Horizon Valuation: Lattice
- c. Deterministic LGD of 100%
- d. Loss reference point set to "in excess of TS"
- e. Credit Migration = on
- f. 5 million standard Monte Carlo trials
- g. 10bp target probability
- h. 0.05% – 0.15% probability interval for TRC calculation (5,000 trials in interval)

REPRESENTATIVE PORTFOLIO CHARACTERIZATION

Moody's created a sample portfolio of floating rate term loans lent to 30,490 public-firm obligors across 72 countries and 61 industry sectors defined by Moody's. In this portfolio, we assign each obligor three loans: one with one-year maturity, a second with seven-year maturity, and a third with a maturity randomly assigned (with a uniform distribution) between one and seven years. At the aggregate level, loans of each maturity type account for approximately one-third of total exposure. The overall commitment amount of the portfolio is \$150 billion USD, with 33% exposure attributed to financial sectors.

OBLIGOR SELECTION FOR THE GLOBAL PORTFOLIO

We select the 30,490 obligors as the entire universe of public companies in the CreditEdge™ dataset as of 03/31/2015, excluding the following:

- » Obligor whose total liabilities as of 03/31/2015 are less than \$10 million USD.
- » Obligor whose one-year EDF value (EDF9) as of 03/31/2015 is not available or exceeds 20%.
- » Obligor whose RSQ values are not available in the Moody's GCorr™ 2014 universe.

COMMITMENT AMOUNT

In general, the commitment amount for a one-year loan is proportional to the underlying obligor's current liability. The commitment amount for a seven-year loan is proportional to the obligor's long-term liability.^{1, 2} The third loan, whose maturity is between one and seven years, has a commitment amount equal to 50% of the total commitment amount of the one-year and seven-year loans. We obtain the current and long-term liabilities as the actual values for each obligor as of 03/31/2015, with the following adjustments for different industries:

- » Banks and S&L (NDY: N06): Insurance Companies (NDY: N29, N30): current and total liabilities reduced by 90% of total liability to account for non-debt liabilities, such as deposits and insurance claims.³
- » Security Brokers and Dealers (NDY: N48): current and total liabilities reduced by 60% of total liability to account for non-debt liabilities such as accounts payable.⁴
- » For all obligors, both current and long-term liabilities floored below by \$5 million USD.

¹ The scaling factor used to convert the liability to commitment amount is selected, such that, the total portfolio commitment amount equals \$150 billion USD.

² The intuition behind this practice: the average composition of the credit portfolios held by all banks should mimic the overall composition of liability.

³ For Banks and S&L, call report data shows that the average deposit-to-total liability ratio for U.S. banks is approximately 90%, with this ratio higher for smaller banks (~95%) than for bigger banks (~85%). For insurance companies, financial statements of companies such as AIG and MetLife show that the debt-to-total liability ratio is around 10%.

⁴ Financial statements of firms such as Morgan Stanley and Goldman Sachs show that the debt-to-total liability ratio is approximately 40% for Security Brokers and Dealers.

INDUSTRY/COUNTRY WEIGHT AND RSQ SELECTION

We use GCorr 2014 values.

PORTFOLIO CHARACTERIZATION

Figures 14 and 15 show an overview of the portfolio exposure amounts and capital characteristics by country and industry.

Figure 14 Representative Portfolio Exposure by Country.

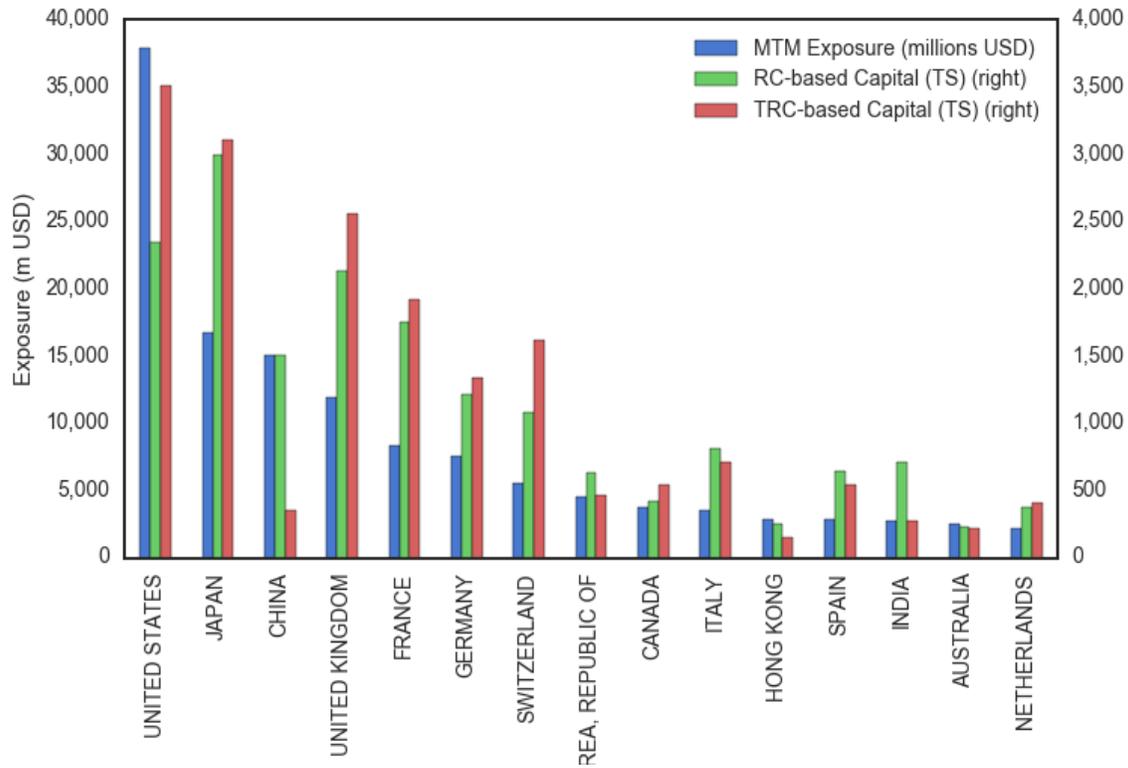
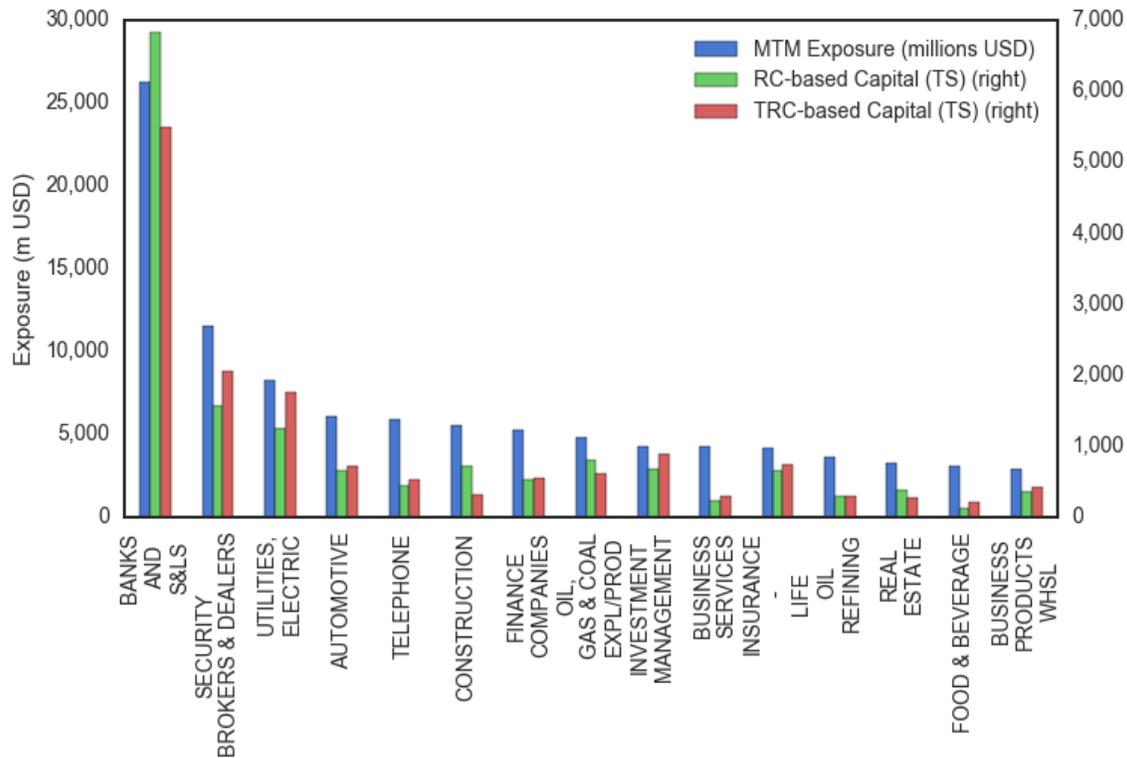


Figure 15 Representative Portfolio Exposure by Industry.



The portfolio's largest exposure is in the U.S., but with higher allocated capital rates in Japan. The portfolio is concentrated in Financials.

EXPOSURE GRID

In order to assess the Income-Adjusted RC-based capital measure across the dimensions of credit quality, RSQ, and spread, we create a grid of exposures, each with \$1.00 USD holding amount, with the characteristics:

- » PDs from AAA (1bp) to C (50%)
- » Spread from 0% – 200% over the swap rate (assumed to be 10bp higher than risk-free rate)
- » RSQ from 5% – 65%
- » Maturity from six months – five years
- » All with the same custom factor (U.S., Banks and S&Ls)

We select PD, spread, RSQ, and maturity because, in our view, they cover the overwhelming majority of real exposures. The correlation factor remains the same in order to control for the effects of the correlation model.

All possible combinations of these dimensions create 108,810 exposures.

CAPITAL MULTIPLIER

Recall, we define RC-based capital allocation as

$$C_{LRP}^i = \frac{w_i \cdot RC^i}{UL_p} C_{LRP}^P$$

which can be rearranged as follows

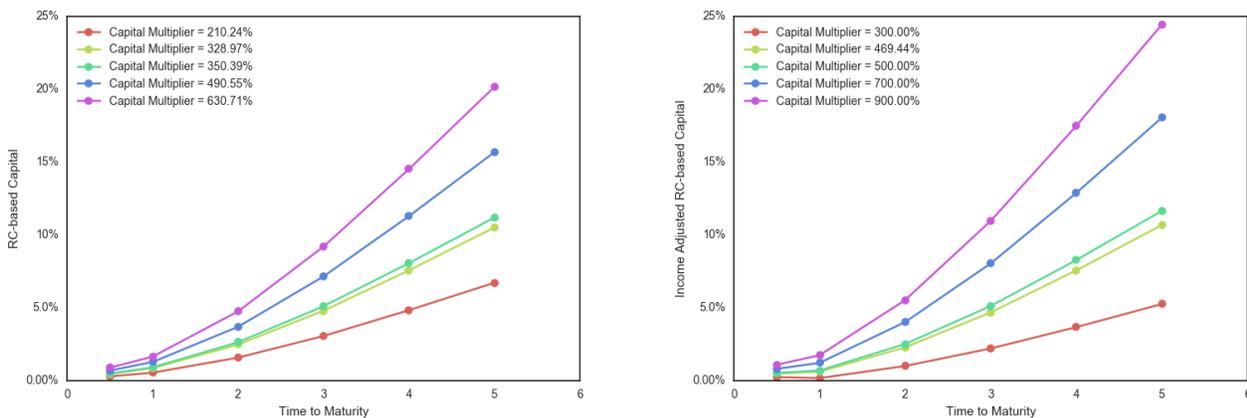
$$C_{LRP}^i = w_i \cdot RC^i \frac{C_{LRP}^P}{UL_p} \tag{6}$$

We see that the risk contribution is multiplied by the ratio $\frac{C_{LRP}^P}{UL_p}$, defined here as the capital multiplier. The capital multiplier is unique for each unique portfolio, and it depends on portfolio composition. Moody's has observed that the capital multiplier generally ranges from about 3 – 10 for typical client portfolios at the traditional 10bp target probability.

As follows from Equation (6), allocated RC-based capital is linear across the dimension of capital multiplier. We show below⁵ that RC, TRC, and IA-RC-based capital measures display non-linear behavior over the dimension of time to maturity, depending on the capital multiplier level.

Figure 16 illustrates the dynamics through time of a low PD exposure maturity (11bps, which corresponds to Ba1 rating), with the capital multiplier ranging from three – nine for RC-based capital (left) and IA-RC-based capital (right).

Figure 16 RC and IA-RC Capital Dynamics with Time to Maturity for a Low PD Exposure (45% RSQ, 11bp PD).

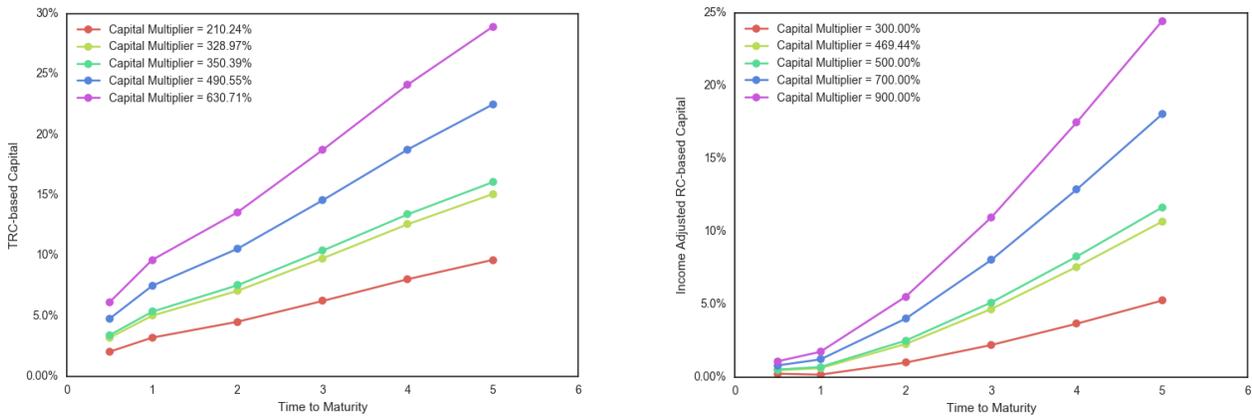


When PDs (and, therefore, mark-to-par spreads) are small, the behavior and magnitude of the RC-based measure are both similar to the behavior and magnitude of the IA-RC capital measure. This trait follows because, when PD is small, the mark-to-par spread is small, and, therefore, makes less difference in the income adjustment.

Figure 17, shows the dynamics through time of a low PD exposure, with the capital multiplier ranging from three – nine for TRC capital (left) and IA-RC-based capital (right).

⁵ Results shown use instruments whose coupon is set at exactly at the right level to value the instrument at par using RiskFrontier lattice valuation methodology.

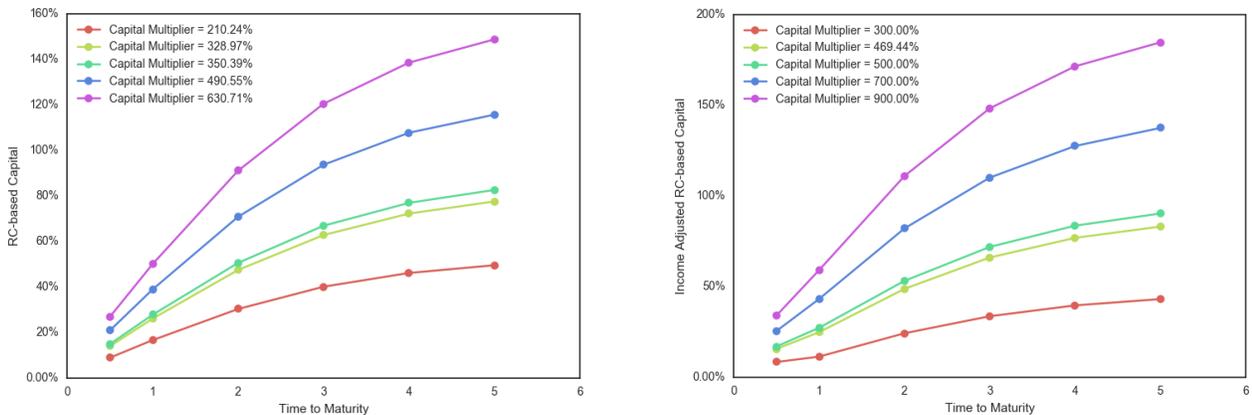
Figure 17 TRC and IA-RC Capital Dynamics with Time to Maturity for a Low PD Exposure (45% RSQ, 11bp PD).



As we might expect from the previous results, the dynamics of TRC capital differ from IA-RC capital, particularly in the magnitude of the amount of capital allocated, but also in the non-linearity at short maturities.

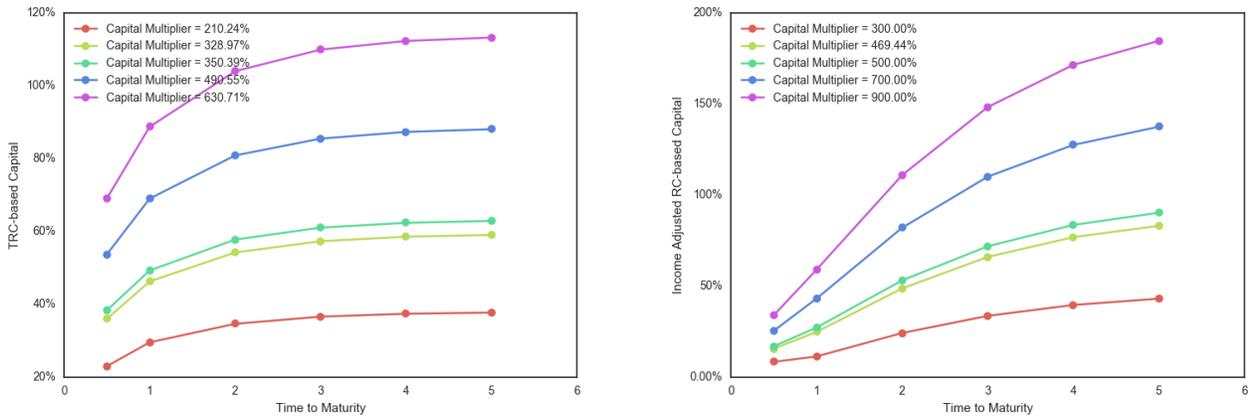
Figures 18 and 19 show the dynamics through time of a high PD exposure (5.32%, which corresponds to Caa2 rating), with the capital multiplier ranging from three – nine for RC/TRC-based capital (left) and IA-RC-based capital (right).

Figure 18 RC and IA-RC Capital Dynamics with Time to Maturity for a High PD Exposure (45% RSQ, 5.32% PD).



With a large PD, the behavior with increasing time to maturity remains similar between the RC and IA-RC-based capital methods. However, the magnitude of capital allocated now differs more, because the expected (mark-to-par) spreads are larger.

Figure 19 TRC and IA-RC Capital Dynamics with Time to Maturity for a High PD Exposure (45% RSQ, 5.32% PD).



Here, we see that the magnitude of allocated capital differs vastly between the Tail Risk Contribution and the Adjusted Risk Contribution methods. As we have mentioned, the TRC measure should not often produce capital allocations greater than 100%, so the “capital multiplier” approach makes less sense for TRC.

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Bozsoki, Peter, Amnon Levy, Thomas Tosstorff, and Mark Wells, "Measuring Required Economic Capital and Parameterizing the Loss Reference Point." Moody's Analytics White Paper, 2016.

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