Through-the-Cycle Correlations

Abstract

In some instances, financial institutions prefer to take longer-term views when assessing the risks of their credit portfolio. While forward-looking or Point-in-Time (PIT) parameters might be more reflective of the current economic environment, their frequent updates may create fluctuations in risk measures, such as economic capital and unexpected loss, which may not be desirable in some applications. This paper outlines two approaches that financial institutions can consider to estimate Through-the-Cycle (TTC) correlation parameters. The first approach averages PIT measures across years to obtain a longer-term TTC average. The second approach calibrates a TTC correlation measure that generates a default distribution in-line with the institution’s actual default distribution.
# Table of Contents

1. Introduction 3  
2. R-Squared Values 3  
3. Constructing TTC R-Squared Values 4  
   3.1 Method 1: Calibration using Model R-Squared Values 4  
   3.2 Method 2: Calibration using RiskFrontier™ 7  
4. Summary 9  
References 10
1. Introduction

Credit risk is an important area of focus for financial institutions. Though all credit portfolios are subject to credit losses, the uncertainty of these losses can be reduced using proper credit portfolio management. The correlations between the obligors of a portfolio play a large role in determining the loss distribution of a portfolio.

In terms of model inputs, such as probability of default and correlations, one can either take a Through-the-Cycle (TTC) or Point-in-Time (PIT) approach. PIT measures reflect the current state of the economic environment. During crisis periods, PIT probability of default (PD) and correlations tend to be higher than in benign economic environments, implying higher expected loss, economic capital, and unexpected loss. Alternatively, TTC measures are meant to model parameters over a longer period that contains the entire business cycle (or cycles) and produce less volatile estimates of risk measures over time. The decision of using TTC versus PIT measures vary based on the context of the analysis and an institution's business needs.

GCorr® Corporate is a PIT global correlation model that describes forward-looking asset correlations between publicly traded firms. It provides estimates based on the latest three years of data and applies a forward-looking adjustment incorporating mean-reverting properties observed with correlations.

In this paper, we describe two approaches to computing TTC correlation measures. The approach taken depends on how an institution thinks about TTC parameters. Some institutions consider TTC measures as representing longer-term averages. Their goal might be to obtain a set of parameters that are steady from year to year. The first approach we present averages PIT measures across years to obtain a longer-term TTC average. Some institutions are required to validate their PD and correlation measures to their default history. The second approach calibrates a TTC correlation measure that generates a default distribution in line with the institution's actual default distribution. By construction, the second approach is designed to validate well when coupled with TTC default probabilities.

2. R-Squared Values

Within the portfolio framework considered in this paper, the main stochastic driver is a change in credit quality of a firm $i$, which we model using an asset return variable $r$ as follows:

$$ r_i = \sqrt{RSQ_i} \phi_i + \sqrt{1 - RSQ_i} \epsilon_i $$

The R-squared value of firm $i$, $RSQ_i$, measures the sensitivity of the firm's credit quality changes to its systematic factor, $\phi_i$, and $\epsilon_i$ is the firm-specific idiosyncratic return. Under this framework, the correlation between borrower $i$ and $j$ is modeled as:

$$ corr(r_i, r_j) = \sqrt{RSQ_i \cdot RSQ_j} \cdot corr(\phi_i, \phi_j) $$

The Moody's Analytics GCorr Corporate model provides R-squared values for publicly traded firms around the world, as well as systematic factor correlations for 49 countries and 61 industries, which can be used to compute the asset correlation between two firms.

The objective of the GCorr Corporate model is to provide point-in-time parameters. Empirically, changes in the R-squared values over time are more pronounced than fluctuation in factor correlations. That is why we use more than 10 years of data to estimate the systematic factor correlations, while we let the R-squared values to capture the current economic environment by estimating them with the most recent three years of data and applying a forward-looking adjustment, discussed in more detail in the white paper “Validation of GCorr® 2015 Corporate” (Hong, N., et al). Though GCorr combines both short-term and long-term effects, it works well in predicting correlations over the next year.

GCorr Corporate R-squared values are meant to be reflective of the current business cycle. The rest of the document offers methods that can be used to produce TTC R-squared measures.

1 For more details, see “An Overview of Modeling Credit Portfolios” (Levy, A.)
2 For more details, see “Understanding GCorr® 2015 Corporate” (Hong, N., et al)
3. Constructing TTC R-Squared Values

The following section outlines two approaches that financial institutions can use to arrive at a TTC R-squared measure. The first approach averages PIT R-squared values over a window and results in R-squared values that fluctuate little from year to year compared to PIT R-squared values. The second approach estimates a TTC R-squared value calibrated to a financial institution's default history.

3.1 Method 1: Calibration using Model R-Squared Values

Asset R-squared values measure the sensitivity of a firm to its systematic factor. An important step in estimation of these R-squared values involves regressing firms' asset returns onto their systematic factors. The length of the data used in the regression plays a significant role. A shorter window is more reflective of the economic environment but limits the number of data points, which ultimately produces more unstable and noisy R-squared estimates.

Ideally, a TTC R-squared value would be estimated using data from an entire business cycle. Unfortunately, this poses three challenges - (i) the sample of firms is dramatically reduced when we look at only firms that have lasted over an entire business cycle, (ii) firms that have been around for many years introduce survivorship bias since the riskier firms may have defaulted within the business cycle, and (iii) the composition of firms may change through time.

To address the above issues, we instead estimate PIT R-squared values for all the firms using three years of asset return data for each year spanning from 2002 to 2015. Since the composition of firms is changing through time, we cannot simply take the average R-squared value across firms in each year. In theory, firms can be pooled based on their characteristics and TTC averages can be computed within each segment. However, many of the above segments may have very few firms and hence the estimates are not reliable, and there might be some segments which are not represented in the data, and hence R-squared value cannot be estimated for them.

Therefore, in each three-year window, we take all firms with sufficient data and fit an econometric model to estimate model R-squared values. We observe that there is a strong positive relationship between R-squared value of a firm and its size. After controlling for a firm's size, we find cross sectional differences in R-squared across countries and industries. Therefore the empirical R-squared is regressed over size, country dummies, and industry dummies. The model is used to estimate model R-squared values for each year.

We produce a lookup table each year to show the modeled R-squared value for different country, industry, and size combinations. It is important to note that since GCorr Corporate is a forward-looking correlation model, the modeled R-squared values are fitted on the forward-looking R-squared rather than the empirical R-squared. The forward-looking property of GCorr Corporate is based on the observation that in the past, correlations have generally exhibited cyclical (or mean-reverting) short-term patterns, rising during periods of crises and subsiding afterward. Forward-looking R-squared values are higher than the empirical R-squared values during benign economic environments, and conversely during crisis periods. As a result, we see that the forward-looking R-squared values are higher before the 2008–2009 financial crisis than during the actual crisis period.

Figure 1 shows the variation in modeled R-squared based on the size of a firm. Figure 2 shows the variation in modeled R-squared based on the firm's country, and Figure 3 shows the variation in modeled R-squared based on the firm's industry.

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3 Firms with more than 50 observations in the three year time period are kept in the modeling dataset. Data consists of historical weekly asset returns of firms

4 In GCorr, we use annual sales to define the firm size of non-financial firms. For financial firms, we use total assets as size.

5 The model R-squared lookup table values prior to 2015 will be different from the previously released model R-squared lookup tables in GCorr. This is because the asset return calculation was updated in 2015 when EDF9 was released, and the model R-squared methodology has been improved as well. The new lookup tables recalculate all the historical model R-squared values under the enhanced asset return calculation and model R-squared methodology, which will result in differences with the previous GCorr lookup tables. From 2015 and on, the lookup table values are identical to the ones in the GCorr release.

6 We focus on the forward-looking R-squared values rather than the empirical R-squared values for this paper in order to be consistent with GCorr's forward-looking R-squared values. One method of obtaining a TTC R-squared value is to apply a scaling factor based on the ratio of the TTC R-squared to the PIT R-squared. Since many institutions use the forward-looking GCorr R-squared values as the PIT R-squared measure, computing the scaling factor as a ratio of the TTC R-squared to the empirical R-squared and applying it to the GCorr R-squared would result in the wrong level of TTC R-squared.
Figure 1  Variation of model R-squared value by Size.

Figure 2  Variation of model R-squared value by Country.
Figure 3  Variation of model R-squared value by Industry.

Using the model R-squared lookup tables, one can then calculate $R_{TTC}^{i,j,k}$, the TTC average R-squared value for country $i$, industry $j$, and size $k$, by averaging $R_{T}^{i,j,k}$, the PIT RSQ from $t_0$ to $t_1$.

$$R_{TTC}^{i,j,k} = \frac{\sum_{t=t_0}^{t_1} R_{T}^{i,j,k}}{t_1 - t_0}$$

An important question is how long of a window should be used to construct the TTC average R-squared value. Different institutions may have their own view on an economic cycle. Some may want to include more of the pre-crisis years to reflect a longer window, while others may want to use a shorter window to place more weight on the financial crisis. Windows covering an economic cycle will also differ by country.

Table 1 shows an example of how a financial institution can use the lookup tables to calculate the TTC R-squared value. First, the TTC average R-squared value is calculated for each relevant country-industry-size combination over the appropriate window. From here, there are two approaches. The first approach is to classify firms into each of the size, country, and industry segments and directly apply the TTC R-squared, $R_{TTC}^{i,j,k}$, for the respective segment. The drawback of this approach is that all firms in the same segment would have the same TTC R-squared. For this example, all US Aerospace firms in the portfolio would have a TTC R-squared of 15%.

The second approach would introduce a single scaling factor $k$ that can be applied to the PIT R-squared values. The scaling factor would be computed as a ratio of the weighted average TTC R-squared values to the weighted average PIT R-squared values of the portfolio.

$$R_{TTC}^{portfolio} = \sum_{i,j,k} w_{i,j,k} R_{TTC}^{i,j,k}$$
When the PIT R-squared values are scaled by $k$, the portfolio average R-squared will equal the TTC R-squared value. In the example, $RSQ_{\text{TTC}}^{\text{portfolio}}$ is 25%. For example, a financial institution is using GCorr 2015 and the exposure-weighted GCorr 2015 R-squared for the portfolio is 30%. Each counterparty’s R-squared can then be scaled upwards by $30%/25% = 1.2x$ so that the portfolio’s average R-squared value is equal to the $RSQ_{\text{TTC}}^{\text{portfolio}}$. The benefit of this approach is that the rank-ordering of firm-level GCorr 2015 R-squared values are preserved.

<table>
<thead>
<tr>
<th>COUNTRY_NAME</th>
<th>INDUSTRY_NAME</th>
<th>SIZE</th>
<th>WEIGHT</th>
<th>20XX</th>
<th>…</th>
<th>2014</th>
<th>2015</th>
<th>TTC R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA/CARIBBEAN</td>
<td>AEROSPACE &amp; DEFENSE</td>
<td>5000</td>
<td>20%</td>
<td>10%</td>
<td>…</td>
<td>19%</td>
<td>20%</td>
<td>15%</td>
</tr>
<tr>
<td>USA/CARIBBEAN</td>
<td>AGRICULTURE</td>
<td>5000</td>
<td>20%</td>
<td>15%</td>
<td>…</td>
<td>24%</td>
<td>25%</td>
<td>20%</td>
</tr>
<tr>
<td>USA/CARIBBEAN</td>
<td>AIR TRANSPORTATION</td>
<td>5000</td>
<td>20%</td>
<td>20%</td>
<td>…</td>
<td>29%</td>
<td>30%</td>
<td>25%</td>
</tr>
<tr>
<td>USA/CARIBBEAN</td>
<td>APPAREL &amp; SHOES</td>
<td>5000</td>
<td>20%</td>
<td>25%</td>
<td>…</td>
<td>34%</td>
<td>35%</td>
<td>30%</td>
</tr>
<tr>
<td>USA/CARIBBEAN</td>
<td>AUTOMOTIVE</td>
<td>5000</td>
<td>20%</td>
<td>30%</td>
<td>…</td>
<td>39%</td>
<td>40%</td>
<td>35%</td>
</tr>
</tbody>
</table>

**Weighted Average** 20% … 29% 30% 25%

### 3.2 Method 2: Calibration using RiskFrontier™

A financial institution can also estimate the TTC R-squared so that the portfolio default distribution matches the actual default distribution. This can be done by leveraging RiskFrontier. For a given portfolio, RiskFrontier is used to estimate the default distribution over the next year. Both the PD and R-Squared parameterization impact the default distribution. The PD affects the overall level of the defaults, and the RSQ affects the shape of the distribution. The TTC R-squared value can be calibrated such that the simulated distribution in each year is in line with the distribution of the realized historical defaults. For example, the simulated distribution might show that there is a 10% chance for more than 100 defaults. If the portfolio does not change, then a financial institution should see the number of obligors defaulting to exceed 100 in 10% of the years. In reality, a financial institution’s portfolio changes through time, so the simulated default distribution changes as well. With a correctly calibrated TTC R-squared, the number of years with defaults exceeding $N_{xt}$ should be $x\%$, where $N_{xt}$ is the $x\%$ of the default distribution for year $t$ and it will depend on the portfolio composition and PD parameterization.

To perform the calibration, an institution can run its portfolio through RiskFrontier using a historical year as the analysis date. The actual number of defaults that occurred over the next year can be translated into a percentile of the predicted distribution. Using the above example, if 100 defaults actually occurred, this would correspond to the 10th percentile for that year. Next, simulate the default distribution for the next year and determine the percentile corresponding to the realized number of defaults for that year. Repeat this for each year and this would provide a time series of the mapped percentiles. If the TTC R-squared is correct, every percentile value should occur in $[0,100]$ with the same probability. In other words, the percentiles should be independent and identically uniformly distributed.

If there is large dispersion between percentiles for different years, it is an indication that the TTC R-squared underestimates the correlation between obligors, as the model fails to capture the extreme co-movements corresponding to joint defaults of underlying credits. In Figure 4, for hypothetical portfolio distributions, we can see that the percentile values are widely scattered around the top and bottom of the range, indicating the level of the parameterized R-squared values is lower than the level implied by the realized defaults.
Figure 4  Loss distribution where R-squared value is too low.

On the other hand, if the TTC R-squared values are too high, the loss distribution curve will have a thicker and wider tail and the simulated distribution would predict an extreme number of defaults — either too many or too few. Figure 5 shows the percentile results for a higher TTC R-squared value. The dispersion of percentile values is limited with the realized values falling within a narrow range around 50 percent.

Figure 5  Loss distribution where R-squared value is too high.

The goal is to come up with a TTC R-squared measure such that the percentiles are uniformly distributed. Figure 6 shows an example of a pattern in the mapped percentiles reflecting the optimal TTC R-squared. On average, the realized defaults lie around the 50th percentile of the predicted default distribution. The percentiles are expected to lie between the 5th and 95th percentiles 90% of the time.
4. Summary

GCorr Corporate estimates the R-squared values of publicly traded firms using the most recent three years of asset return data, with a forward-looking adjustment. Financial institutions may wish to take a longer term view of credit risk and may want to use a long-term window to parameterize their PD and correlation parameters. In this paper, we outline two approaches of doing so. One method utilizes model R-squared values over a multi-year window to formulate a TTC average R-squared value. Another method uses RiskFrontier to calibrate a TTC R-squared measure that produces a default distribution that matches the financial institution’s default distribution.

We can also run statistical tests to determine if the distribution is uniform. Using percentile values, we can conduct a two-sided Kolmogorov-Smirnov Goodness-of-Fit test for uniform distribution \([0,100]\). The null hypothesis states that the true distribution is a uniform distribution with a significance level of \(\alpha\). The statistical power of the Goodness-of-Fit test depends on the amount of available data to the financial institution; longer historic data results in more reliable statistical tests.
References


