

**MODELING  
METHODOLOGY****Modeling Canadian Commercial Real Estate  
Loan Credit Risk: An Overview****Authors**

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**Abstract**

Commercial real estate (CRE) exposures represent a large market share of credit portfolios for many Canadian banks, credit unions, insurance companies, and asset managers. While this segment escaped the most recent financial crisis relatively safely, Canadian CRE loan portfolios may be facing heightened credit risks given the current changing market conditions, which include sliding oil prices and reduced demand for natural resources and possible interest rate hikes. Given this environment, it is critical to use an objective credit risk measurement solution that quantifies CRE loan risks consistently and objectively in order to help assess, stress test, and manage loan portfolios. This paper presents Moody's Analytics Commercial Mortgage Metrics (CMM™) framework, tailored for Canadian CRE loan credit risk, forming the core of our Commercial Mortgage Metrics: Canada (CMM Canada™) product. Based on the well-established CMM U.S. model, our enhanced framework incorporates new factors that capture unique Canada CRE market dynamics and lending practices. We describe our modeling approaches for default probability, loss given default (LGD), Expected Loss (EL), and other related risk measures.

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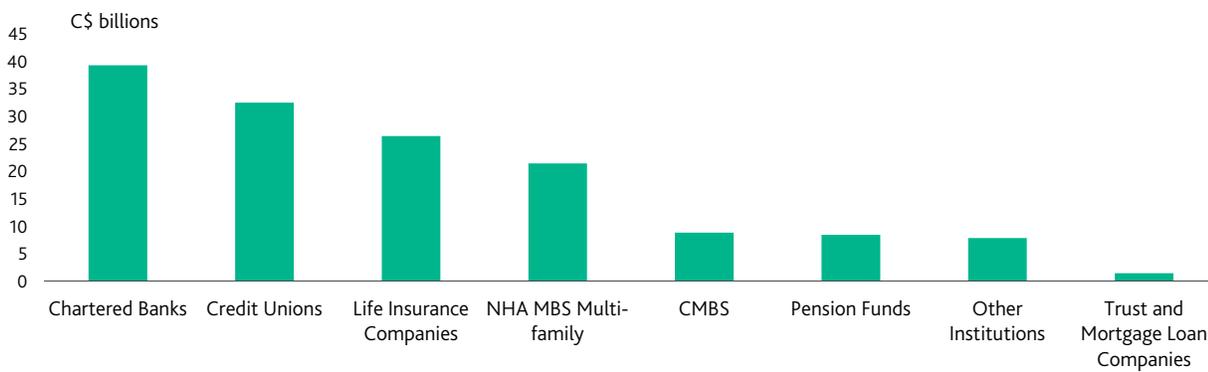
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## 1. Introduction

Commercial real estate (CRE) exposures represent a large share of credit portfolios for many financial institutions, including banks, insurance companies, credit unions, and asset managers. According to the Real Property Association of Canada (REALpac), the total size of the Canadian Institutional commercial mortgage market is \$144.9 billion,<sup>1</sup> as of May 2014, representing a year-over-year growth rate of 4%. Figure 1 shows the breakdown of the CRE market by major market players. Commercial banks claim the largest share of CRE loans, followed by credit unions. Life insurance companies continue to hold sizable market share despite recent declines in CRE exposures. National Housing Act Mortgage-Backed Securities (NHA MBS) in the multi-family category reached \$21.5 billion in 2014. Commercial mortgage-backed securities ("CMBS") represent a smaller proportion of the market, with \$8.9 billion outstanding in 2014.<sup>2</sup> Additionally, pension funds possessed commercial mortgages holdings of \$8.5 billion in 2014.

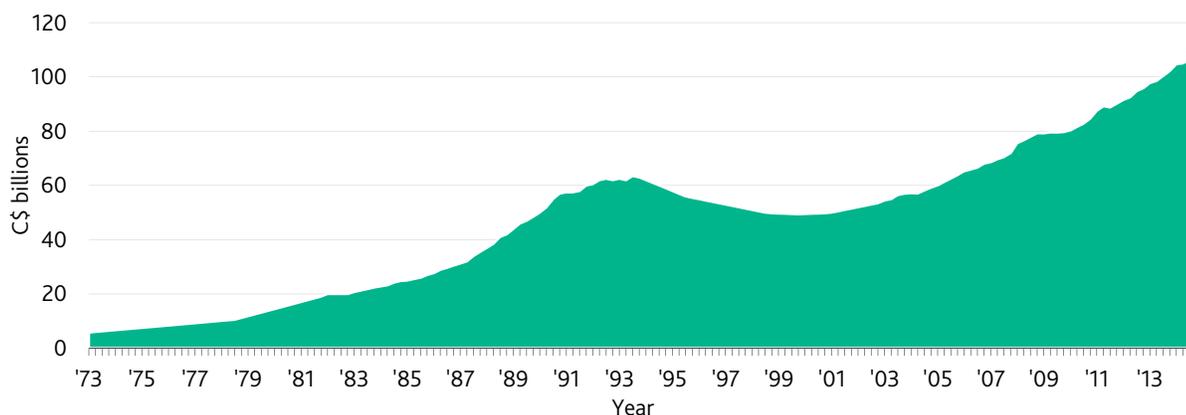
**Figure 1** Commercial mortgage holdings by holder type as of 2014 (\$billions).



Source: Statistics Canada CANSIM Table 176-0023 and REALpac

Figure 2 shows the tremendous growth in the institutional commercial mortgage market from 1973 until May 2013.<sup>3</sup> Throughout the 1990s, we see a decline in the size of the overall commercial mortgage market. If we look closely, we see that during the most recent downturn, Canadian commercial mortgage market growth was almost flat but has picked up in recent years.

**Figure 2** Long-term trend, institutional commercial mortgage market size: 1973 – 2014.



Source: Bank of Canada and REALpac

<sup>1</sup> All figures in Canadian dollars unless otherwise stated.

<sup>2</sup> REALpac Canadian Institutional Commercial Mortgage Market Report 2014.

<sup>3</sup> The data in Figure 2 excludes NHA MBS multi-family mortgages, CMBS, and mortgage holdings held by pension funds.

Significant credit losses from commercial mortgages can often wipe out a financial institution's capital cushion. Globally, we have witnessed the failures of numerous institutions during periods of macroeconomic slowdown and financial crisis, with many failures caused by CRE-specific loan losses. While escaping escaped the most recent financial crisis relatively safely, the Canadian CRE loan portfolios held by financial institutions may be facing heightened credit risks given changing market conditions, including sliding oil prices and reduced demand for natural resources during a global economic slowdown, with possible rate hikes in the near future.

Given these challenges, financial institutions continue to seek better risk management of CRE exposures. Toward this goal, the first step is to measure the credit risk of these CRE portfolios, including the standalone credit risk assessment of individual loans, as well as their correlation and concentration effects at the portfolio level. This paper presents the Moody's Analytics framework for measuring the credit risks of individual CRE loans. Specifically, we describe our modeling approaches for asset volatility, default probability, loss given default, Expected Loss (EL), and other related risk measures. Earlier papers such as Chen, Cai, and Zhang (2011) describe the modeling methodology details in the context of U.S. CRE loans. Patel and Zhang (2009) also documents Moody's Analytics approach for measuring CRE asset correlation within a portfolio context.

In the CMM Canada framework, we begin by modeling the asset process of the underlying CRE collateral. We consider the stochastic evolution of a commercial property's financial performance, including income and market value, as driven by both market-wide and idiosyncratic factors. We first estimate the local market-specific parameters that govern those processes, utilizing extensive historical datasets. We then apply a Monte Carlo technique to simulate the future paths of the collateral's net operating income (NOI) and market value. The Monte Carlo technique enables the model to capture the path-dependency of the survival probability and the remaining credit risks as the future unfolds.

An important model feature is that a CRE loan credit event is doubly triggered by the collateral financial condition at the time of default: both the sustainable NOI falls below the total debt service, and the property market value falls below the total outstanding loan balance. Moreover, since the CRE market operates in an opaque environment that is neither complete nor perfectly efficient, the conditional probability of the default (PD) function is empirically calibrated to large historical datasets in order to capture the actual observed borrower default behavior. We calculate the unconditional EDF™ (Expected Default Frequency) credit measure as the integration of conditional PD values over the many future paths of NOI and market value.

Canadian lenders routinely require recourse and guarantees as part of the loan underwriting process. As such, CMM Canada also allows additional collateral and guarantee features in the model setup, enabling more refined credit analysis by incorporating the prevailing CRE lending practices in Canada.

Finally, we model loss given default (LGD) via the same process. Hence, LGD and PD are structurally correlated and consistently estimated in the same coherent framework. Built upon EDF credit measures and LGD, the model also calculates other measures such as EL, Yield Degradation (YD), Unexpected Loss (UL), and Stressed PD measures and loss. By establishing a strong economic causality relationship between credit risks, the real estate market, and property-specific covariates, the model also enables large-scale scenario analysis and stress testing. Another significant benefit is our model's ability to accurately differentiate the credit risks of senior/junior structure of the multiple loans on the same collateral, as well as that of mezzanine loans. Given that many loans and credit facilities from the same borrower are either cross-collateralized or cross-defaulting in Canada, this model feature aids in accurately assessing the loan-level as well as the borrower-level credit risks.

In addition to the analytical modeling framework, we have sourced very extensive Canadian commercial real estate market data and forecasts at the national, provincial, census metropolitan area (CMA), and submarket levels, where available. These local market data and forecasts are embedded within our CMM Canada system, making it a transparent, one-stop solution where users can consistently measure credit risks across different property types and geographic locations. In the meantime, the CMM system enables total control over the inputs on the collateral's most recent financial statistics, including NOI and market value (either transaction- or appraisal-based) together with loan characteristics such as coupon rates and amortization terms, etc.

Our model's credit measures have many business applications for CRE practitioners. These include risk assessment and asset selection, risk-based pricing and valuation, risk monitoring and surveillance, regulatory compliance and internal control, loss forecast and provisioning, scenario analysis and stress testing, and portfolio management. For example, loan officers and underwriters can objectively and systematically assess the credit risks of a CRE loan located in any given Canadian market; at the back-end, credit risk managers and portfolio managers can quickly monitor the most recent credit profiles of individual loans, as well as entire portfolios. By establishing a strong economic causality relationship between credit risks and real estate market and property-specific covariates, the model also enables large-scale scenario analysis and stress testing. For example, CMM allows users to compare results from a baseline scenario and a stressed scenario. Users can also input their own commercial real estate market-specific views and test credit risks from those views.

The remainder of this paper is organized as follows.

- » Section 2 describes model outputs and their practical applications.
- » Section 3 describes the modeling framework and inner workings of the model.
- » Section 4 discusses the empirical data.
- » Section 5 documents the model validation findings.
- » Section 6 summarizes the paper and provides concluding remarks.

## 2. Model Outputs and Applications

In this section, we focus on the end results of our CMM Canada model, and we discuss how various practitioners can use these results to make more informed business decisions.

### 2.1 Model Outputs

The Moody's Analytics CMM Canada model estimates the credit risk of commercial real estate loans, combining user-provided portfolios with market-wide data and forward-looking scenarios.

The model provides estimates of the following risk measures, both for individual commercial real estate loans as well as for a portfolio of loans.

- » **EDF (Expected Default Frequency) credit measure:** measures the probability that a commercial real estate loan experiences a default event in the future. We estimate EDF credit measures throughout the loan term, and the model estimates an annual EDF measure for a particular point in time within the loan term. We then calculate cumulative EDF measures to measure the cumulative holding period risks.
- » **Loss Given Default (LGD):** refers to expected loss amount, typically as a percentage of outstanding unpaid loan balance, at the time of the default event, if the default event occurs.
- » **Expected Loss (EL):** measures the expected losses of a commercial real estate loan due to default events. Mathematically, for a given point in time,  $EL = EDF \times LGD$ . This relationship also holds for cumulative holding period measures.
- » **Yield Degradation (YD):** measures the annualized reduction of expected yields from a commercial real estate loan due to losses related to default events throughout the loan term. YD is similar to annualized EL, the main difference being that YD takes into account the timing of expected losses and discounts losses according to the timing, whereas, annualized EL does not involve discounting and time-value of loss.
- » **Unexpected Loss (UL):** defined as one standard deviation of loss from the loss distribution. We estimate the one standard deviation of loss based on a full range of loss distribution derived from Monte Carlo simulations of all possible combinations of systematic market risk factors and non-systematic idiosyncratic loan and property-specific risk factors.
- » **Stressed PD Measure and Loss:** measures the point estimate of PD measures or loss from a full range of PD measure or loss distribution derived from Monte Carlo simulations. Typically, we measure the Stressed PD at a user-specified stressed point, such as a confidence level greater than 50%, for the tail risk at the right-hand side of the distribution.

### 2.2 Business Applications

Underwriters, credit officers, risk managers, and portfolio managers can use Moody's Analytics CMM Canada for a variety of different business applications. For institutions that employ internal rating systems as the foundation of many business decisions, they can either map the EDF credit measures and LGD outputs to their internal rating scales, or combine them with other qualitative inputs to derive an internal rating. Alternatively, they can use CMM Canada to benchmark and calibrate their own internal risk rating systems.

CMM applications include risk assessment and asset selection, risk-based pricing and valuation, risk monitoring and surveillance, regulatory compliance and internal control, loss forecast and provisioning, scenario analysis and stress testing, and portfolio management.

### USING CMM IN INTERNAL RATINGS SYSTEMS

Internal rating systems serve as the foundation of many business decisions within financial institutions, e.g., credit approval, limit setting, regulatory compliance, risk-based pricing, and active portfolio management. An effective internal rating system has the following attributes.

- » Separates default and recovery risk
- » Provides powerful differentiation of relative risk ranking
- » Well-calibrated to provide appropriate risk distinctions
- » Contains well-documented definitions, assumptions, and methodologies
- » Combines qualitative and quantitative assessment where appropriate

We developed CMM Canada with the above attributes in mind. CMM measures CRE loan default risk via EDF credit measures and recovery risk via its LGD. As we show later, the model proves to be powerful, forward-looking, and accurately calibrated to real default and loss experience. For documentation, this introductory paper, together with the more-detailed and comprehensive modeling documents, provides model transparency for users. CMM Canada is ideally suited for use as a quantitatively-based internal ratings system for CRE exposures in Canada. If institutions use internal rating scales not generated in absolute scales, such as PD and LGD, they can map EDF credit measures and LGD outputs to their own internal rating scales.

Many institutions find that market-based information, when available, is particularly relevant and powerful in internal risk rating assessment. CMM credit measures utilize a significant amount of CRE market information. We construct them to reflect all the relevant property type and location-specific market information. Thus, if a user finds it appropriate to combine qualitative assessment with quantitative components, CMM credit measures are particularly useful as the market-based quantitative component of an internal rating system. In fact, CMM makes it feasible and efficient to implement such an approach using Moody's Analytics RiskAnalyst™ system. An internal rating system must provide sufficient differentiation of default risk. To calibrate such a system, regulators typically expect a sizeable amount of realized default events and loss severity data spanning at least a full economic cycle. Many institutions do not possess enough internal data and can benefit from using CMM credit measures to benchmark and calibrate their internal risk systems.

### RISK ASSESSMENT AND ASSET SELECTION

CMM Canada can be very effective during an institution's credit underwriting process. Commercial mortgage underwriters and credit officers can benefit significantly by using CMM to directly measure and compare credit risks at loan origination for given loan and property characteristics. For example, CMM allows users to run multiple "what-if" analyses to compare how credit risks, including PD, LGD, and EL, would change if either the Debt-service Coverage Ratio (DSCR) and/or the loan-to-value (LTV) change. Underwriters can use this information to risk-base price loans according to a specific combination of DSCR and LTV.

Another invaluable feature is CMM's embedded local real estate market data and forecasts, which make it possible to compare loan risks across different property types and locations. For example, a national financial institution often conducts CRE lending in many locales throughout the country, and individual underwriter expertise and assessment may vary significantly between local offices. CMM enables centralized credit risk management to objectively and consistently measure credit risk without overly relying upon individual judgments from dozens of or even hundreds of underwriters.

### RISK-BASED PRICING AND VALUATION

A financial asset such as a commercial mortgage must be appropriately compensated for its given risks. Because commercial mortgages fit well in held-for-investment portfolios, the long-term credit risks become, de facto, the most important source of risk. As such, CMM can help determine the trade-offs between loan pricing and future risks. This type of risk-based pricing and valuation can be performed at a geographic level, where market data is available, including submarket, metropolitan area, and provincial or national levels.

### RISK MONITORING AND EARLY WARNING

Commercial mortgages' credit quality can change quickly as the market environment or property-specific conditions change. Because CMM's risk measure outputs are objective and forward-looking, risk managers can target their risk assessment and mitigation resources toward cases where they can be the most effective. Annual reviews and other traditional credit processes cannot maintain the same degree of speed, consistency, and objectivity. Within CMM, accurate and timely market information

can be applied consistently across the entire portfolio, an often difficult and expensive task to achieve using traditional credit analysis processes.

#### REGULATORY COMPLIANCE AND INTERNAL GOVERNANCE

The probability of default associated with an internal rating plays a central role in calculating capital requirements within the Basel framework. Banks may use external PD models, such as EDF measures from CMM, as part of their internal ratings, either for regulatory capital calculations or for fulfilling internal governance and external regulatory requirements.

#### LOSS FORECAST AND PROVISIONING

Loan loss provisions are expenses charged to a bank's earnings and affect their capital position. In estimating the provisioning amount, one can use a credit risk model to estimate the potential credit losses on loans. The model should respond to changes in the risk environment across the economy as a whole. In other words, a provisioning calculation should be as forward-looking as possible. International Accounting Standard Board (IASB) has issued a new standard IFRS 9 in July 2014, which moves from an "incurred loss" model to an "expected loss model" and is expected to be implemented starting from 2018 by financial institutions like banks and insurance companies who hold large portfolio of loans on their books. Existing FASB loan loss reserve model, which applies to U.S., is also based on "Incurred Loss" model, and FASB is finalizing the forward looking expected loss model, i.e. the FASB CECL model. CECL model requires the firms to set aside the lifetime expected credit loss for the financial assets in scope. Similarly, Basel has also come up with guidance on calculating "expected credit losses<sup>[1]</sup>". All CMM credit measures are forward-looking assessments that respond to changes in the CRE market cycle and produce accurate estimates of credit losses over a long period. Consequently, these credit measures are appropriate for expected loss-based provisioning calculations.

#### SCENARIO ANALYSIS AND STRESS TESTING

The future remains inherently uncertain. No single person or entity, nor the market as a whole, possesses a crystal ball that predicts exactly what will occur. We built the CMM system so that it contains several embedded commercial real estate market forecast scenarios and also allows users to input their own views regarding specific property types and/or geographic locations. Such functionality is particularly valuable for risk managers when comparing possible outcomes from different economic outlooks. Forecast scenarios are also becoming more of a daily business necessity, given increased regulator and internal risk controller requirements for periodic stress tests. The CMM on-demand scenario analytical capabilities can significantly improve an institution's readiness to meet such continuous and rigorous demands.

#### PORTFOLIO MANAGEMENT

Portfolio management entails making numerous decisions, such as taking on additional exposures, selling or hedging existing exposures, and calculating the prices at which to do so. The Moody's Analytics CMM Canada system provides a framework that enables users to make informed decisions regarding which loans to create, under what terms, and at what price(s). In addition, risk managers can use CMM portfolio functionality to construct strategies that exploit the relative price differences between property types and local CRE markets. Additionally, CMM's EDF measures and LGD outputs can serve as inputs to calculations performed by portfolio management systems such as Moody's Analytics RiskFrontier™.

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### 3. Modeling Framework

This section first describes how commercial real estate loan credit events occur in the real world. We use an example to illustrate the importance of how collateral financials affect a loan's credit risk. We next present the conceptual framework as well as details regarding the CMM model's inner workings, including specifics on the asset process, the PD model, and the LGD model. We then describe how the components work together within the CMM Canada system. Finally, we explain how CMM implements scenario analysis and stress testing.

#### 3.1 Understanding Commercial Mortgage Credit Events

Our objective is to accurately measure the probabilities of a credit event occurring and the resulting associated losses. It is important to examine how and why credit events and losses occur. We want to make sure our model succinctly and consistently emulates real world phenomena and captures their essence.

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<sup>[1]</sup> Basel Committee on Banking Supervision, Guidelines, Guidance on accounting for expected credit losses, February 2015

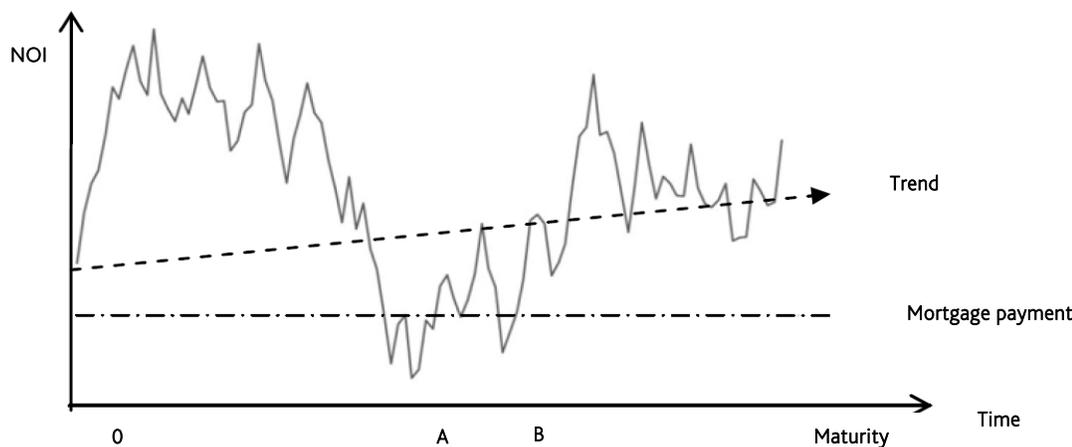
Why would any commercial real estate loan borrower default on their debt obligations? In principle, there are two primary reasons under the so-called “double-trigger” framework. The first is inadequate cash flow from the property to cover the scheduled mortgage payment; the second is that the underlying commercial properties, which serve as the secured collateral for most commercial real estate loans, are worth less than the mortgages. In other words, a commercial mortgage borrower’s ownership value, inclusive of property resale value, plus current and future incomes, less the market value of the mortgage (including current outstanding payments), becomes less than zero in the event of default. We point out that the borrower’s equity value is its economic value and takes into account embedded options, so it may differ from the book equity measure. Also, because commercial real estate is an asset class primarily focused on producing an inflation-adjusted rental income stream while preserving capital value, it is more productive to separate and focus on the income side of the ownership value. We illustrate the double-trigger framework with the following example.

#### DOUBLE-TRIGGER FRAMEWORK EXAMPLE

When a commercial mortgage is originated, the mortgage lender typically requires cushions in both leverage and debt service coverage. For example, an LTV ratio of 80% and a DSCR of 1.40 may be the threshold underwriting criteria for a particular lender. Under this threshold, the maximum loan amount is \$8,000,000, if the market value of the property is worth \$10,000,000; and the maximum annual debt service a lender would allow is \$500,000, if the property is currently generating \$700,000 in annual net operating income (NOI). In fact, since both ratios must satisfy the threshold underwriting ratios, the actual mortgage may either carry a loan amount of less than \$8,000,000 or an annual debt service less than \$500,000. Most commercial mortgages, if underwritten appropriately and absent of fraud, should, in theory, carry no (or very little) credit risk at origination. What drives the credit risk is the inherent future uncertainty, which can potentially be quantified.

For simplicity, assume that a commercial mortgage originates with an \$8,000,000 loan amount, annual debt service of \$500,000, based on a \$10,000,000 property generating \$700,000 NOI a year. The realization of future NOI is unknown and can follow an infinite number of possible paths. In the particular NOI path illustrated in Figure 3, there are periods around points A and B where the collateral’s NOI is insufficient to cover the mortgage payments.

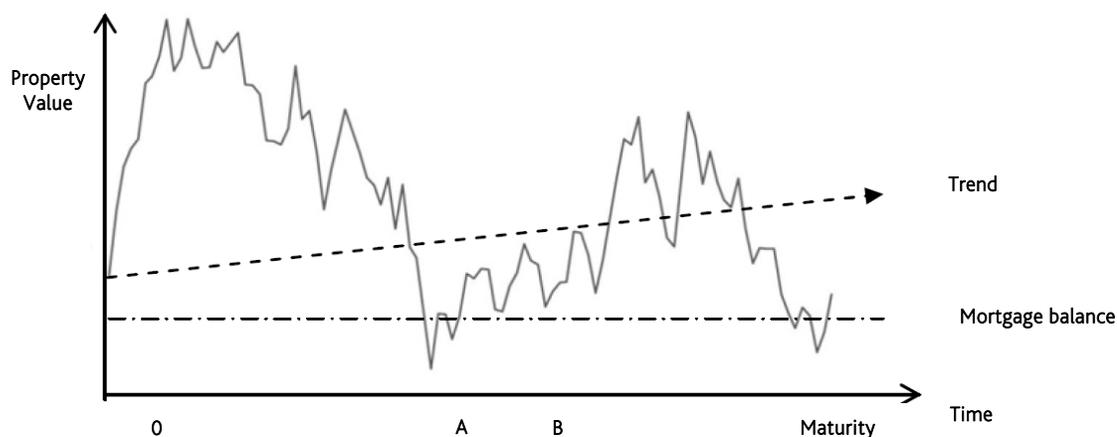
Figure 3 Evolution of a collateral property’s NOI.



Whenever a property does not generate enough NOI to cover the periodic mortgage payment, a borrower must weigh different options, as follows.

- » Cover the payment shortfall out of pocket, if the shortfall is deemed temporary and will be cured.
- » Sell the underlying property and pay back the entire remaining mortgage balance, including outstanding interest payments, if the market value of the underlying property is enough to cover all debt obligation plus a non-trivial transaction cost.
- » As a last resort, miss the mortgage payments and wait for the lender’s decision to either foreclose or restructure debt.

Figure 4 Evolution of a collateral property's value.



Obviously, what is also very important in this situation is the market value of the property, pledged as secured collateral, which the lender can take possession of in the event of default. While the property value is usually correlated with NOI, its evolution is also affected by the general conditions in both the capital and space markets, in addition to the property-specific NOI. In our example, with the particular NOI realization as in Figure 3, the property value does not necessarily follow the NOI movement in lockstep.

As illustrated in Figure 4, the property value drops below the mortgage balance around point A, but not point B. Toward the end, the property value declines again around loan maturity even though the property's NOI remains well above the scheduled mortgage payment amount, shown in Figure 3. It is straightforward to make the following observations with this particular example, as shown in Table 1.

Table 1

### Decision Analysis on Default Probabilities

DECISION POINT	DSCR	LTV	DECISION ANALYSIS	CREDIT RISK MEASUREMENT
A	< 1.0	> 100%	High probability of default	EDF <sub>A</sub>
B	< 1.0	< 100%	Not a clear-cut default choice	EDF <sub>B</sub>
Maturity	> 1.0	≈ 100%	High refinancing risk	EDF <sub>Maturity</sub>

While the above discussions illustrate the financial aspects of CRE borrower default drivers, we should note that decades of actual experience with commercial mortgage defaults also clearly teach us that a borrower's decision to default is not purely a financial matter. For a CRE asset that is illiquid and difficult to value and to sell easily, the borrower's decision to default is influenced by both financial facts and subjective assessment of the situation, leading to the so-called "sub-optimal" (non-ruthless) default behaviors observed at the aggregate level.<sup>4</sup> We emphasize here that a vast majority of borrowers do make very rational and near-optimal decisions regarding defaults; it is the inability to observe and to record many loan, property, and borrower-specific decision factors that lead to empirically-observed, "sub-optimal" default rates in aggregate. Furthermore, even perfectly explainable and rational behavior on the individual level can still appear to be "sub-optimal" using aggregate data alone.

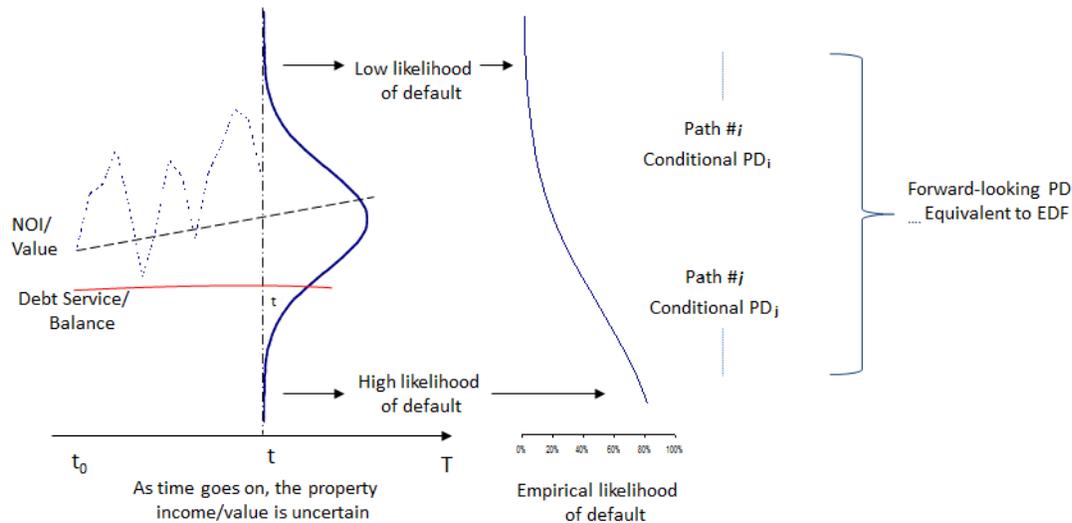
To create a credit risk model that is relevant for business users, we must anchor the analysis on empirical data. While theoretical thinking is very useful as a starting point in disentangling the causes and consequences from a rational economic-reasoning perspective, a model will not be accurate and useful for business applications if it is simply an intellectual exercise without the benefit of actually-observed historical data. Thus, our modeling approach combines rational economic reasoning with insights gleaned from careful analysis of the empirical data.

<sup>4</sup> Note, "sub-optimal" is viewed from the borrower's perspective. In fact, since the lender takes the opposite position, the "sub-optimal" borrower's behavior actually adds value to the lenders' CRE portfolio. Because, in aggregate, CRE borrowers do not exercise their default options ruthlessly, most lenders' CRE operations have been able to live through the cyclical troughs of CRE market downturns without being completely destroyed.

### 3.2 Model Setup and Details

Because commercial mortgage credit events fundamentally depend on the realized financial conditions of the underlying collateral and the market environment it operates within, our modeling process starts by understanding and quantifying the dynamic processes and uncertainties surrounding CRE assets. In addition, at the time of financial distress, because commercial mortgage borrowers make default decisions based beyond purely financial considerations, we employ empirical data and statistical analysis to measure the “sub-optimal” or “behavioral” aspect of mortgage defaults. In other words, our modeling framework can be thought of as consisting of two major parts: the underlying commercial properties' stochastic dynamics following a structural approach; and empirically calibrated default probability measurements, conditional upon realized property financials.

Figure 5 The CMM modeling framework consists of two major parts.



Mathematically, a commercial real estate loan's EDF credit measure at a particular point in time  $t$  is:

$$EDF_t = \sum_{i=1}^N Prob(X_{i,t}) \cdot Prob(Default|X_{i,t}) \quad (1)$$

where  $X_{i,t}$  denotes all the relevant financial variables at the loan, property, and market levels measured at time  $t$  for a particular path  $i$ ,

$Prob(X_{i,t})$  measures the probability of a particular realization of  $X_{i,t}$ ,

and

$Prob(Default|X_{i,t})$  is a conditional default probability function given the realized  $X_{i,t}$ .

$EDF_t$  (i.e. the unconditional  $EDF$ ), is simply a summation of the probabilities of all possible realizations of  $X_{i,t}$  multiplied by corresponding conditional default rates for those realizations.

We find that this composite modeling approach leads to the most effective credit risk model by combining the best of both worlds. On the one hand, the dynamics of variable  $X_{i,t}$  follow a structural stochastic process that is fully parameterized based on extensive historical observations of the commercial property financials. On the other hand, the calibration of the conditional default rate function  $Prob(Default|X_{i,t})$ , derived from rigorous statistical analysis, captures the “sub-optimal” exercise of the default options, and therefore, produces accurate and realistic EDF measures. This composite approach is similar in spirit to Moody's Analytics EDF credit measure model approach for publicly traded firms,<sup>5</sup> as that model is also a structural approach with a robust implementation grounded in empirical data.

<sup>5</sup> Nazeran, Pooya and Douglas Dwyer, 2015, “Credit Risk Modeling of Public Firms: EDF9.” Moody's Analytics White Paper, 2015.

Another benefit of our approach is that it naturally leads to LGD measures that are economically and structurally correlated to PD, because  $LGD_t$  can be estimated as another conditional function that draws  $X_{i,t}$  as the dependent variables. This method offers significant improvement over an *ad hoc* approach for approximating the relationship between PD and LGD.

In essence, our CMM model consists of three key elements:

- » **Parameterizations of Asset Dynamics and Volatility (the Asset dynamics model).** In this step, the CRE collateral's NOI and value processes in the future are parameterized, based on property type and geographic location, in conjunction with the known financial and leasing information at the starting point.
- » **Calculation of EDF Measure (the EDF or PD model).** At a future time  $t$ , given realized NOI and values generated by Monte Carlo simulations, the model first calculates the default drivers, including DSCR and LTV, and then estimates conditional PD via the conditional default rate function  $Prob(Default|X_{i,t})$ . Provided by the known distribution characteristics of NOI and value from the first step, the model then estimates unconditional EDF measures as well as Stressed PD measures as point estimates from the full range of conditional PD distributions.
- » **Calculation of LGD (the LGD model).** Given a simulated market value of the collateral at a future time  $t$ , the model estimates conditional LGD via an empirically-determined loss function. An unconditional LGD is the weighted sum of conditional LGD values, with weights being the corresponding conditional PD values.

We can also easily calculate other risk measures such as EL, UL, and yield degradation since the model performs a full Monte Carlo simulation from which EDF measures and LGD have been calculated. The remainder of this section offers a brief introduction to these key model elements.

### 3.2.1 Asset Dynamics and Volatility

Asset dynamics refers to the inner workings and quantifiable causal relationships of the commercial properties' financial performance, and asset volatility refers to the uncertainty around the financial performance. While conceptually one can measure all sorts of financial-related variables, including rents, occupancy rates, revenue, expense, NOI, capital expenditure, and market value, etc., the direct drivers behind commercial mortgage defaults are primarily DSCR and LTV, which are, in turn, driven by NOI and property market value.

Modern financial theory views any asset, including commercial real estate, as having two independent risk driver sources:

- » One related to the overall market movement — market or systematic risk
- » Another related to the specifics of individual assets — non-systematic or idiosyncratic risk

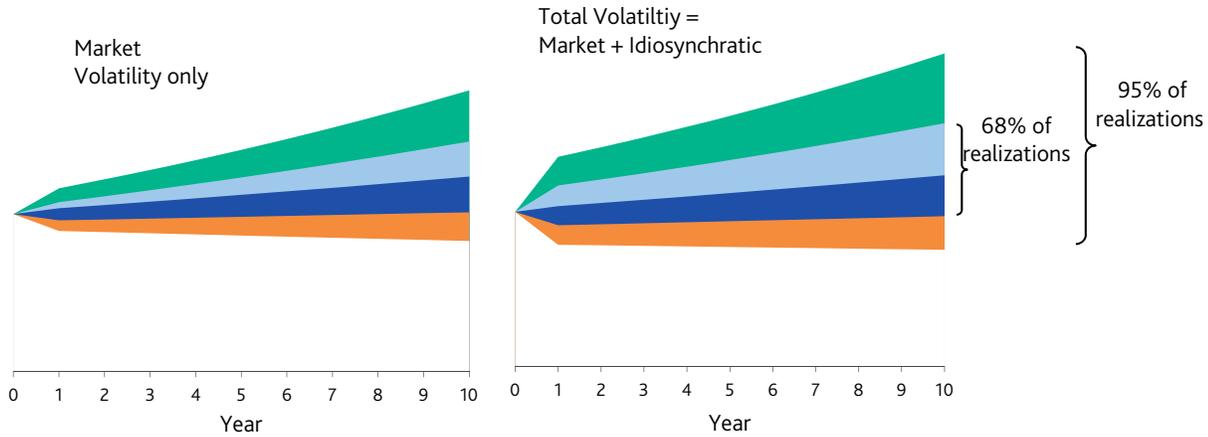
Intuitively, this separation is no different than the commonly practiced attribution analysis, where both performance (returns) and risks (volatility) can be traced back to either market-wide trends or asset-specific conditions. A commercial property's income or value can be approximated as follows:

$$E_{i,t} = E_{m,t} + \varepsilon_{i,t} \quad (2)$$

$E_{i,t}$  represents the realization of NOI or value changes in log form for the  $i$ th property at time  $t$ , and  $E_{m,t}$  represents the changes of market-wide index at the same time  $t$ , and  $\varepsilon_{i,t}$  is the idiosyncratic component of the  $E_{i,t}$  movement that remains after stripping out the market-driven component.

Figure 6 illustrates the importance of a risk model when considering both systematic and idiosyncratic risks. The chart shows a typical CRE asset that displays substantially higher total risk than market risk alone.

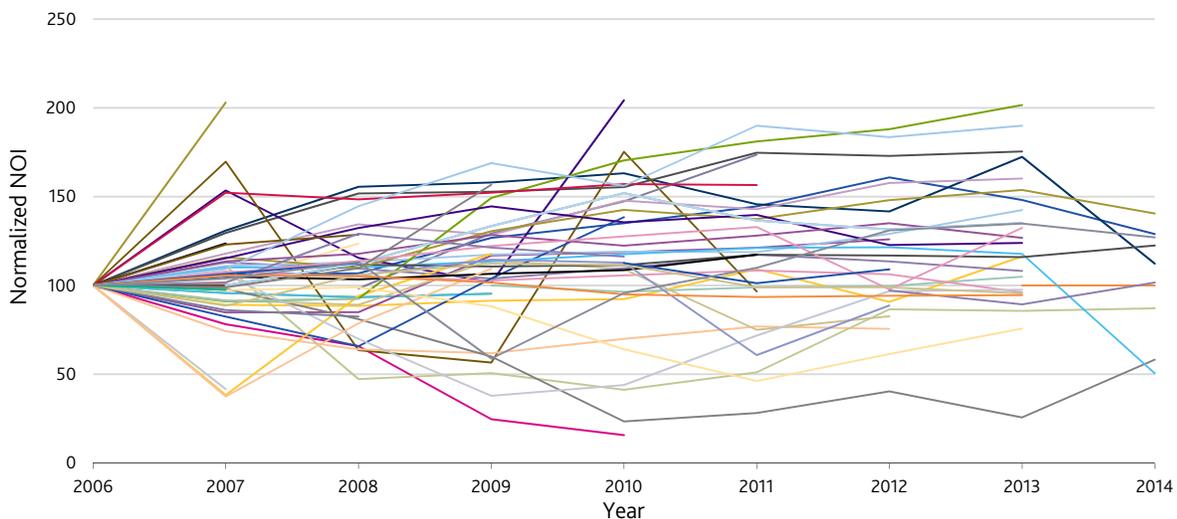
Figure 6 Distributions of NOI or values.



Because real estate is a very location- and property type-specific business, the market here is defined by property type and location. In other words, the Toronto office market is considered a distinct market from the Toronto apartment market or the Montréal office market, and so on. Drawing from a large historical time-series database, which includes both market-wide performance statistics and property-specific operating financials covering a substantial portion of Canada, we can estimate and parameterize the asset dynamics and volatilities as specified in Equation (2) for most of the active CRE markets in Canada.

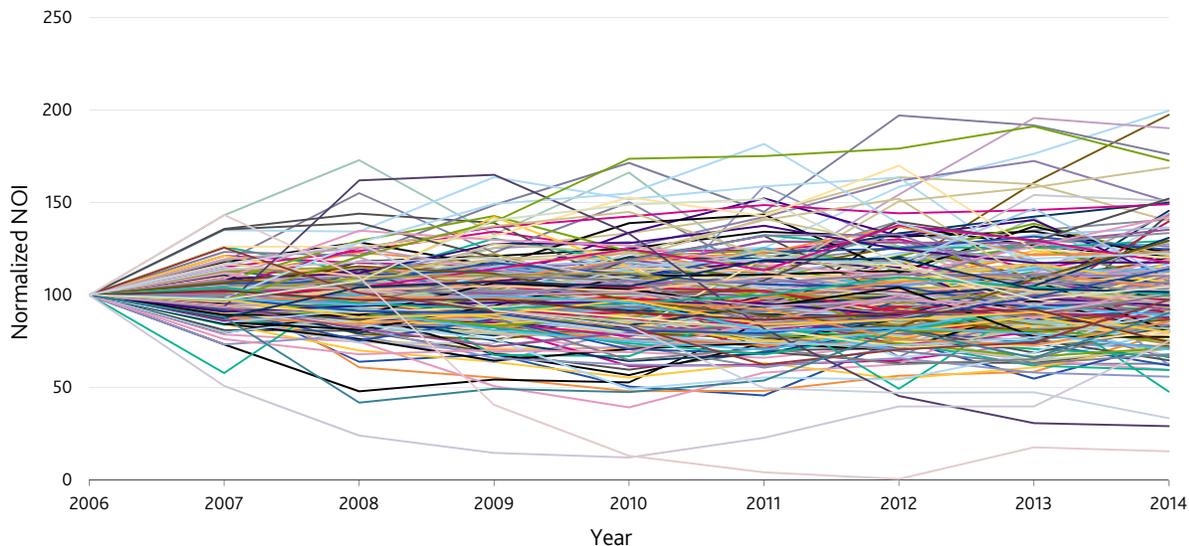
To help understand how CMM implements the Monte Carlo simulations of the collateral NOI, Figure 7 provides an example of the actually observed property-level NOI values of office properties in the Toronto office market. Since DSCR is determined to be the most important empirical variable, in CMM Monte Carlo simulations, we focus on the NOI simulations based on the observed historical patterns by property type and location, such as the one shown in Figure 7. In particular, we simultaneously simulate the random realization of independent factors: the market-wide and the property idiosyncratic factors.

Figure 7 Property-level observations of normalized NOI for Toronto office properties.



Meanwhile, a particular realization of the collateral's NOI can also deviate from the market factor by idiosyncratic variations. Given a particular realization of the market path, the final realized NOI could be better than, the same as, or worse than the market-wide growth rates. We have observed in the actual model implementation that the end results of CMM's NOI simulations for the Toronto office market shown in Figure 8 are indeed quite similar to that shown in Figure 7, which confirms the validity and robustness of our collateral models.

Figure 8 CMM's NOI simulations for Toronto Office.



### 3.2.2 Modeling Conditional Default Probability

Once we fully specify the asset processes, the next problem is to solve the probability of borrowers choosing to default, based on a particular realization of NOI and market value of the underlying collateral. Fundamentally, this is a question about the conditional default probability. Note, in an abstract structural default modeling approach, such as the Merton model, a loan automatically defaults once the asset's market value falls below the mortgage value, since the amount of debt serves as the absorbing boundary. While still incorporating this powerful notion, our modeling framework expands to include the following important practical considerations, making the model truly relevant in the real business world:

- » **The market value of a specific asset is actually unknown and is a somewhat subjective measure.** As such, in an empirical model, we must resort to other directly-observable measures to complement imprecise measures of asset value. It is also important to recognize that a periodic income stream should be explicitly factored into the valuation equation of a CRE asset.
- » **Net operating income (NOI)**, directly observable and ubiquitously measured and recorded, is of predominant financial and decision-making importance to borrowers. Notice, a particular CRE asset is fundamentally an income-producing asset, and, unlike a corporation, it has no potential to grow its business base, since its physical size and location are fixed once built. As we find throughout our empirical work, the debt-service-coverage ratio (DSCR), which measures the level of NOI relative to the periodic mortgage payments, explains a large portion of historical defaults.
- » **Defaulting (or not) is a borrower's choice, rather than a strict rule that must be followed.** While financial factors such as DSCR and LTV are of critical importance, choices are behavioral in nature and subjective, incorporating both quantifiable and non-quantifiable factors. Note, we consider that the conditional default rate is not only a default probability conditional upon known financial and operating ratios, but it is also the probability of a borrower choosing to default. To date, it appears that the best way to model this kind of behavioral issue is via extensive statistical analysis using large panel datasets, which is exactly what we incorporate.

Effectively, our conditional default probability model is a multi-factor empirical function that links empirical default rates to key explanatory variables at both the property/loan and the market levels. Our empirical research identifies the following key statistically-significant variables in explaining historical defaults:

- » Asset level financial ratios: DSCR and LTV
- » Market cycle factors: vacancy rate, market-wide price changes, and market condition at origination
- » Other: recourse vs non-recourse, core vs. non-core property type, and loan seasoning, etc.

The functional model form:

$$Prob(Defaul\text{t}|X_{i,t}) = f[T_1(DSCR_{i,t}), T_2(LTV_{i,t}), \dots] \quad (3)$$

where  $X_{i,t}$  denotes all the explanatory variables listed above,

$T_1$  and  $T_2$  are transformations of DSCR and LTV,

and  $f$  is an empirical-based function to capture the non-linear relationship between PD and explanatory variables.

Table 2

### Explanatory Variables in the Conditional Default Probability Model

NO.	CATEGORY	MEASUREMENT	RELATIONSHIP
1	Debt Coverage	DSCR (debt-service-coverage ratio)	Lower DSCR leads to higher PD
2	Leverage	LTV (loan-to-value ratio)	Higher LTV leads to higher PD
3	Systematic Factor (Space Market)	Market-wide vacancy rate, specific to each property type and location	Weak space market (higher vacancy rate) leads to higher PD
4	Systematic Factor (Capital Market)	Market-wide price change	Weak capital market (slower and negative price appreciation) leads to higher PD
5	Origination Quality	Market-wide vacancy rate at loan origination	Loans originated in stronger CRE market environment tends to have higher PD due to loosening of the underwriting criteria
6	Seasoning	Loan age	PD peaks around years 3-7
7	Property Type	Dummy for non-core property types	Non-core property types have higher PD due to higher operating business characteristics
8	Guarantee	Indicator for guarantee	Loans which have guarantee have lower PD due to its recourse nature

As Table 2 shows, in addition to loan and property level factors, market-wide factors play a significant role. We use the market vacancy rate as a proxy for the contemporaneous space market condition, the market-wide price change as a proxy for the commercial real estate capital market condition, and the market condition at origination to approximate the average underwriting quality (similar to the vintage effect discussed in some industry literature). That is, when the commercial property market is tight and experiences low vacancy rates, myopic lenders tend to loosen underwriting criteria and admit more lower-quality loans than when the property market experiences high vacancy rates.

The other explanation is that these market factors serve as proxies for the option value of borrower equity positions. Given the same DSCR and LTV, when the market is good, the option values of borrower equities tend to be higher (at least from a regular borrower's viewpoint) than when the market is bad. So, if the DSCR is 0.8 and LTV is 100%, the troubled borrower is more likely to hold onto the property without defaulting than when the general market condition is favorable, and the same borrower is more likely to default given the same DSCR and LTV than when the prevailing market condition is deteriorating.

In general, for the conditional default probability model, we find that the variables we identify lead to accuracy ratios (AR) of 50% to 60% or more (equivalent to ROC ratios of 75% to 80% or more), both in- and out-of-sample.

### 3.2.3 Modeling Loss Given Default

Loss given default (LGD) measures loss severity if the loan is already in default. When a commercial mortgage is in default, if the lender decides to first take possession of the collateral property and then dispose of it to recover mortgage principal, then the LGD is simply a function of the disposition value of the collateral in relation to the unpaid loan amount plus transaction and administrative cost. The lender may decide to work out and restructure the loan if the expected LGD is too high by taking the foreclosure route, but the decision to restructure largely depends upon the perceived collateral liquidation value. Our LGD model is a sum of two components:

- » **Loss from principal.** Loss due to the difference between a collateral property's liquidation value and the face unpaid principal balance of the commercial mortgage.
- » **Loss due to costs and expenses,** including, but not limited to, lost interest, transaction costs, legal and administrative expenses, and property maintenance and renovation costs, etc.

In other words, a commercial mortgage's LGD can be expressed as:

$$LGD_t = g[T_3(LTV_{i,t}), Y] \quad (4)$$

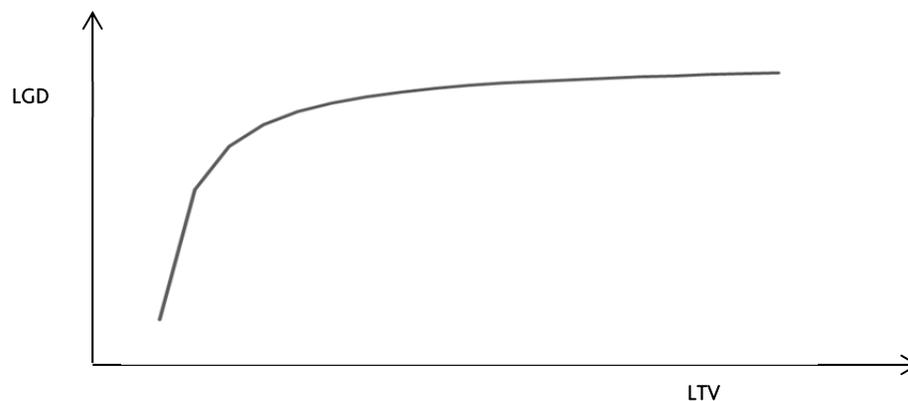
where  $T_3$  is a transformation function of LTV to make it linear to  $LGD_t$ ,

$Y$  denotes the empirical variables proxy for disposition costs,

and  $g$  is a linear specification.

Loan-to-value (LTV) ratio is the most important factor affecting LGD for CRE loans. LTV's explanatory power is mainly driven by the denominator ( $V$ ), the market value of the collateral. Figure 9 illustrates the relationship between LGD and LTV.

**Figure 9** Relationship between LGD and LTV.



Because the CRE market is not efficient and carries significant transaction costs, the liquidation price of distressed commercial properties tends to trade below the market price of comparable non-distressed assets. We find that the degree of distress in the actual loss severities data could be partially proxied by another observable variable: time to liquidation. Time to liquidation is directly correlated with lost interest, transaction, and property maintenance costs. Collateral size captures the percentage cost of some relatively fixed expenses on the legal and administrative sides. For example, \$1 million legal and administrative costs would add 2% LGD to a \$50 million loan, while the same cost would add 10% LGD to a \$10 million loan.

Table 3

#### Explanatory Variables in the Conditional LGD Model

NO.	CATEGORY	MEASUREMENT	RELATIONSHIP
1	Loss on Principal	1 – Recovery rate from collateral liquidation	Higher LTV leads to higher LGD
2	Loss on Interest and property maintenance costs	Estimated time between the default event and resolution	Longer time to resolution leads to higher LGD
3	Administrative and legal costs	Relative size of the property	Larger collateral corresponds to slightly lower LGD due to some fixed costs on asset liquidation

As implementing the CMM model uses Monte Carlo simulation techniques, we can simulate numerous future asset income and value paths. These simulated asset values provide a direct measure of loss from principal, thus enabling an LGD measure that is also structurally and causally correlated to the EDF measure.

### 3.2.4 Putting It All Together

In summary, our CMM Canada model applies an asset-based composite approach that combines a structural asset evolution process and empirically calibrated functions for both EDF measures and LGD. The steps for calculating EDF and LGD measures for a given commercial real estate loan include:

1. Simulate a large number of asset income paths and values by considering both systematic and idiosyncratic uncertainties.
2. Calculate a series of DSCR, LTV, and other explanatory variables along each simulation path.
3. On each simulation path  $i$ , estimate its conditional PD and LGD measures at time  $t$ ,  $PD_{i,t}$  and  $LGD_{i,t}$ , given the simulated realizations of  $DSCR_{i,t}$ ,  $LTV_{i,t}$  and other contemporaneous explanatory variables using Equation 3 and Equation 4.
4. Calculate unconditional EDF measure at time  $t$ ,  $EDF_t$ , using Equation (1).
5. Calculate unconditional EL at time  $t$ ,  $EL_t$ , using the formula:

$$EL_t = \frac{\sum_{i=1}^N (\text{Prob}(\text{Default}|X_{i,t}) \times LGD_{i,t})}{N} \quad (5)$$

where  $i$  refers to individual simulation path that has equal probability of asset realization, and  $N$  is the total number of simulation trials. Unconditional LGD at time  $t$ ,  $LGD_t$ , follows by  $LGD_t = EL_t / EDF_t$ .

One key benefit of implementing the Monte Carlo simulation is the resulting full-range distribution of conditional PD and conditional loss rates via a large number of random draws. Very naturally, from there we obtain all the point estimates regarding UL and various stressed PD measures and stressed loss rates at any user-specified confidence levels.

### 3.2.5 Maturity EDF

As discussed earlier, theoretically, CRE loans default either when NOI falls below the scheduled mortgage payment or when the property value falls below the outstanding loan balance. Additionally, CRE loans might also default if the loan has a balloon payment at maturity and the borrower cannot secure new financing for the payment. Even if a borrower is in good standing during mortgage term, he still may be unable to refinance at the maturity due to existing market conditions, such as higher interest rates or tighter underwriting standards, among other factors. In this situation, the borrower may have to choose default, called a maturity default, even if the underlying collateral may still generate enough cash flows to cover the mortgage payment.

CMM Canada estimates maturity EDF measures based on DSCR and LTV under the new loan underwriting criteria when a loan matures. We find the separation of maturity risk from term risk provides a more accurate measure of the total credit risks a lender faces, in particular in situations where an interest-only (IO) loan or a below market-rate loan reaches its maturity.

### 3.2.6 Construction Loans

Compared to permanent loans, construction loans have different kinds of risks associated with them, such as the borrower's ability to complete the proposed project on time and within budget. Evaluating construction loan performance can be challenging, because the assumption of the property's "as completed" market value and implied rental and operation income at stabilization are all highly uncertain. Additionally, market conditions may also play a substantial role in determining credit risk. For example, due to a general economic slowdown or due to increased competition, the demand from prospective tenants or clients may decrease, increasing the default risk of construction loans. Another risk is that, if the actual rental rates during lease-up are lower than the projected rents or stabilized occupancy rate turns out to be substantially lower than Pro Forma assumptions, the property will be unable to generate sufficient income to cover the debt service. These risks can be quantitatively modeled, and modeling these types of uncertainties fits naturally within the scope of our framework.

Compared to a permanent loan that finances existing properties, a new construction does not have rental income or NOI to begin with. Therefore, our model begins with assumptions on LTV and "shadow" NOI.<sup>6</sup> In addition, our Monte Carlo simulations derive the probabilities of market rent and value decline during the construction loan term that lead to a credit loss.

<sup>6</sup> It is permissible to take the projected NOI number directly from a Pro Forma, if available. We only need best estimates of NOI and value, assuming the new construction is successfully completed.

Our modeling framework for construction loans is fundamentally the same as that of permanent loans. The model begins with an option-based approach, where a construction loan is viewed as a derivative of the collateral property, and the default risk of construction loans is fundamentally determined by the underlying collateral process. Similar to permanent loans, we model the asset process by first decomposing asset drivers into local market risk factors and idiosyncratic factors. The key difference here is that construction properties typically exhibit much wider variance in realized rents and NOIs compared to stabilized properties with long-term existing leases. The stochastic nature of these risk factors naturally defines the financial risks of new construction. Conditional upon realizations of the asset processes, including property income and value, we estimate default probabilities and loss severities using parameters calibrated to the model portfolio.

In the absence of interim operating income and minimal risk of term default/prepayment, we find that our modeling approach works very well.

At loan maturity, when a construction loan must be paid off in its entirety (taken out by a permanent lender), construction loan default risk is mainly a function of LTV and DSCR at loan maturity, in addition to local real estate market factors.

### 3.3 “What-if” Questions and Scenario Analysis

As our model adopts a strong, economically sensible causative specification during each step, it provides users with a powerful and reliable framework in which they can ask “what-if” questions regarding the model’s various inputs and then examine the effects on credit risks of any input changes. For example, a user may be interested in comparing the EDF measures between the baseline economic forecast scenario ( $S_0$ ) and a stressed economic forecast scenario ( $S_1$ ). Furthermore, the user may also want to test the EDF measures differences if the future CRE market volatility differs from the past, which is a legitimate exercise, given the vast amount of literature pointing to the existence of time-varying volatilities. It would be of tremendous business value to complete the report shown in Table 4 by leveraging CMM as one of the main tools.

Table 4

#### Scenario Analysis Over the Holding Period: An Example of Model Flows

SCENARIO	MACROECONOMIC ASSUMPTIONS	CRE MARKET ASSUMPTIONS	CRE ASSET VOLATILITY	CRE PORTFOLIO EDF	CRE PORTFOLIO EL
S: Base case	GDP Unemployment ... ..	NOI growth Value growth ... ..	1.0 times historical market vol	EDF	EL
S: Stressed case	GDP Unemployment ... ..	NOI growth Value growth ... ..	x times historical market vol	EDF	EL



Users can change one or all three categories of forward-looking assumptions to conduct scenario analysis. In addition, both macroeconomic and CRE market outlook scenarios can be either direct user inputs or sourced to third-party vendors within the CMM application. In fact, there are a number of third-party vendors who provide forward-looking macroeconomic and CRE market scenarios as fee-based forecasting services. Users may find it cost-effective and informative to take advantage of the available location-specific forecasts from those forecasters. For example, CMM Canada users can employ the CRE markets forecasts based on CBRE Econometric Advisors (EA) and/or macroeconomic forecasts provided by Moody’s Analytics.

Moody’s Analytics does not endorse any third-party forecast services nor validates their accuracy. Rather, in absence of a large, publicly traded market for the CRE asset class, we find that the forward-looking views given by industry experts provide useful perspectives, although the eventual market movement may occasionally deviate from their actual forecasts.

### 3.4 Forward-looking Views: Forecasts

Once formalized and adopted as the uniform basis for enterprise-wide analysis, scenarios become known as “forecasts” in many settings. Macroeconomic scenario-based stress-testing approach has becoming a business necessity in modeling credit risks of

commercial real estate loans driven by explicit regulatory mandates and internal risk management needs. In the commercial real estate world, no publicly traded marketplace exists where a collective forward-looking view can be inferred clearly, as is the case for the stock market. Market participants must rely upon CRE domain experts to help form their views (forecasts) or arbitrarily set up their own views based on limited publicly-available information.

CMM Canada uses a baseline forecast grounded on expected trend/ market views. The major modeling components can be divided into a two-step process:

- » In the first step, the model translates forward-looking macroeconomic assumptions into CRE market factors including vacancy, rent, NOI, and cap rate/price.
- » The second step involves running credit risk models by utilizing the outputs from Step 1, coupled with institution-specific CRE portfolio information and forward-looking volatility measures.

We already discussed Step two in Section 3.2. Our main focus here is describing the first step of translating macroeconomic assumptions to CRE market factors. The model utilizes data sources from CBRE-EA and Moody's Analytics to estimate the forward-looking CRE market variables. We developed individual models that quantify how macroeconomic performance affects CRE market performance by each property type. To explore and understand the empirical relationship and to screen candidate variables, we conducted univariate analysis and correlation analysis. Real estate markets are well known for their stickiness and, to incorporate this effect, the model often uses the lagged macroeconomic drivers as opposed to concurrent, or contemporaneous explanatory variables. For example, lagged household income and lagged GDP were among the candidate variables for multifamily vacancy model. Similarly, separate models were developed for each property type for CRE market variables vacancy, rent, NOI and value.

These forecasts or hypothetical scenarios are available in CMM Canada at the local market level by different property types wherever data permits. Users can leverage these local market forecasts to facilitate granular credit risk analysis across multiple locations.

## 4. Empirical Data

A model is only as good as the empirical data allows. The shortage of long-term, detailed, reliable data poses a major challenge for developing a credit risk model for Canadian commercial real estate loans. In the process of developing our model, we assembled a collection of datasets covering different aspects of the CRE asset and loan markets. Collectively, these datasets serve as the foundation for both model development and model validation.

### 4.1 Data Sources

The datasets used in CMM model development and validation provide both aggregate market statistics and property- and loan-level information. Data sources include:

- » **MSCI's IPD real estate index:** IPD index tracks performance of commercial properties with a total capital value of \$129.0 billion as at June 2015. Provides national, provincial, metro, and submarket (if applicable) level aggregate market statistics, mainly market value, rent, vacancies, and cap rates for major property types beginning in 1999. Coverage for cap rate and market value at national level dates back to 1985.
- » **CB Richard Ellis Econometric Advisors (CBRE EA):** The CBRE Econometric Advisors market coverage data is more comprehensive, as it also includes very detailed submarket-level statistics, mainly market rents and vacancies, for office and industry property types. The raw data from CBRE EA contains proprietary individual leasing information generally not available in the public domain, so this data has been very accurate historically in measuring vacancy rates and net effective rents for office and industrial properties. The earliest data from CBRE EA goes back to 1981.
- » **Canada Mortgage and Housing Corporation (CMHC):** Provides the most detailed aggregate market statistics, mainly for multifamily rent and vacancies at CMA level. CMHC conducts biannual rental market survey that covers all urban areas with populations of 10,000 and more. CMHC covers approximately 190 markets across Canada, beginning in 1987.
- » **Trepp's CMBS Deal Library:** One of the largest commercially available databases in the CMBS universe, the Deal Library contains comprehensive information and history on both individual loans and properties that serve as collateral within the CMBS transactions. The database contains Canadian deals beginning in 1997.

- » **Moody's Analytics' Economic and Consumer Credit Analytics Group (formerly known as Moody's Economy.com):** Provides historical time-series data for a myriad of macroeconomic variables of interest, including GDP, unemployment rate, inflation, interest rate, and home prices.
- » Various published studies and reports, including the series of studies conducted by Cynthia Holmes, et al. The unique importance of the Cynthia Holmes studies lies in analyzing the statistical relationship between guarantees and probability of default in the Canadian commercial mortgage lending landscape.

While each of the above datasets is indispensable for our model development, none is completely sufficient on its own. We take a mosaic approach that pieces all of the information and empirical analysis together within the coherent overarching framework explained in Section 3. The final CMM model is the result of exhaustive empirical analyses and careful triangulation from complementing insights garnered from multiple data sources.

## 4.2 Market Coverage

CMM Canada covers major property types: apartment, office, retail, industrial, and hotel. Table 5 shows the metropolitan areas covered by CMM Canada. Each CMA is covered by at least one property type. When CMA-level data is not available, CMM Canada uses market data at both national and provincial level for all the major property types. To facilitate more refined location-specific analysis, CMM Canada also uses submarket-level data and forecasts. For example, Table 6 shows a snapshot for submarkets covered for Calgary Office.

Table 5

### CMA's Covered

NO.	CMA	NO.	CMA
1	Abbotsford-Mission (B.C.)	18	Ottawa-Gatineau (Ont. Part) <sup>7</sup>
2	Barrie (Ont.)	19	Peterborough (Ont.)
3	Brantford (Ont.)	20	Québec (Que.)
4	Calgary (Alta.)	21	Regina (Sask.)
5	Edmonton (Alta.)	22	Saguenay (Que.)
6	Ottawa-Gatineau (Que. Part) <sup>7</sup>	23	Saint John (N.B.)
7	Greater Sudbury (Ont.)	24	Saskatoon (Sask.)
8	Guelph (Ont.)	25	Sherbrooke (Que.)
9	Halifax (N.S.)	26	St. Catharines-Niagara (Ont.)
10	Hamilton (Ont.)	27	St. John's (N.L.)
11	Kelowna (B.C.)	28	Thunder Bay (Ont.)
12	Kingston (Ont.)	29	Toronto (Ont.)
13	Kitchener-Cambridge-Waterloo (Ont.)	30	Trois-Rivières (Que.)
14	London (Ont.)	31	Vancouver (B.C.)
15	Moncton (N.B.)	32	Victoria (B.C.)
16	Montréal (Que.)	33	Windsor (Ont.)
17	Oshawa (Ont.)	34	Winnipeg (Man.)

<sup>7</sup> CMA Ottawa-Gatineau has been further categorized into two parts – Quebec part (Gatineau) and Ontario part (Ottawa).

Table 6

**Coverage for Calgary, Office at Sub-market Level**

NO.	SUBMARKET	NO.	SUBMARKET
1	Central Core	6	Northeast
2	East End	7	Northwest
3	Mid-West Core	8	South
4	West End	9	South Central
5	Beltline		

## 5. Model Validation

Validating a quantitative risk model is both a theoretical and an empirical problem. By theoretical, we refer to the general guidelines of model development, which involve not only academic rigor, but also sound and time-proven business experience. Validation refers to whether or not the model makes econometric sense as well as common business sense, and also whether or not the model specifies an economically-sensible and long-lasting causative relationship that links credit risks with key risk drivers.

Model theoretical validation also involves imposing a parsimonious structure and designing the model so that it is intuitive and simple to understand. A parsimonious structure allows the model to focus on the most important factors while leaving out "accidental" variables that are either redundant or have a one-time transient effect only. Meanwhile, an intuitive model is more traceable and more likely to be used appropriately by regular business users with limited statistical backgrounds. Moreover, a model should be theoretically validated by examining the directional response of PD/LGD to explanatory variables. For example, one expects PD to increase with declining DSCR. This type of directional relationship can be first validated via a theoretical angle. Finally, common sense and business experience also apply when checking the theoretical reasonableness of model results. For example, since we will not be able to predict *ex ante* which loans will default and which will not default, a model is probably poorly constructed if it consistently outputs PDs in the 95% – 100% range for an input DSCR of 1.0.

Beyond theoretical considerations, a model must be empirically validated, most importantly, via the out-of-sample data. Moody's Analytics has pioneered and refined the use of empirical validation in commercial credit models, and we validate the CMM Canada model using those proven testing processes.

It is useful to consider the model's empirical validation from two separate yet related metrics.

- » **Model power**— the ability of the model to rank-order individual loans from more to less risk. Power describes how well a model discriminates between defaulting ("bad") and non-defaulting ("good") loans.
- » **Model calibration** — the consistency of the aggregate-level EDF and LGD measures when compared to the actual realization for a portfolio of loans. It also measures relative risk magnitudes between subsets of a portfolio. For example, whether or not a model can predict correctly if group A has twice the default rates as group B.

These two dimensions are indispensable for a good model, as model power ensures its ability to rank score individual risks, and model calibration ensures the EDF level closely matches the actual default rates throughout portfolios and through time.

For model power, we apply walk-forward, *k*-fold, and bootstrapping techniques. These tests include out-of-sample testing (using defaults and non-defaults not used in model development, such as "hold-out" sample) and extending the inference regarding the population from a sample.

### 5.1 Walk-forward Tests

The walk-forward testing approach allows users to test models and modeling methodologies while controlling for sample and time dependence.<sup>8</sup> The test proceeds as follows. We estimate the model up to a certain year and score the observations in the next year. These model scores are out-of-time and out-of-sample. We then re-estimate the model including one more year of data and repeat the analysis for the next year and continue until the end of the sample. For each version of the model, we calculate the accuracy ratio of next year's scores by comparing them to actual realized default rates. Under this approach, we select the

<sup>8</sup> See Dwyer and Kocagil (2005)

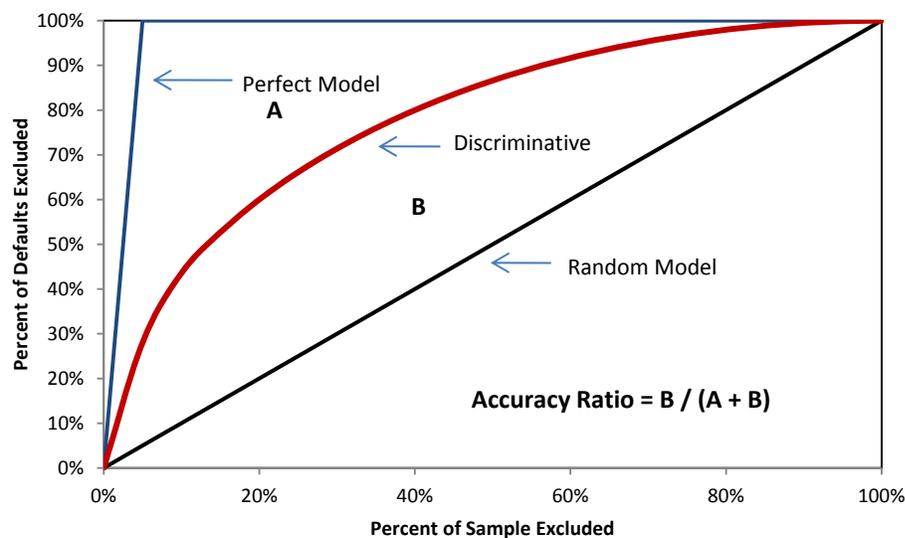
development dataset prior to a certain cutoff date, and the validation dataset begins after that date. Multiple tests can be conducted as one moves the cutoff dates backward and forward. This technique reduces the chances of “over-fitted” models, since the testing process never uses data used to fit model parameters. At the same time, the approach allows modelers to take greater advantage of the data by using as much of the information as possible to fit and to test the models. The results from our out-of-sample tests show a high degree of discriminatory power for the CMM Canada model, shown in Table 7. A model is considered to be better than a random guess if the Area Under the Receiving Operating Curve (AUROC) is above 50% and the accuracy ratio is above 0%. A perfect model achieves the maximum possible 100% for both AUROC and accuracy ratios. While sample dependent, accuracy ratios of 50% or above are often associated with good predictive power.

Table 7

### Out-of-Sample Model Power from the Walk-forward Method

DEVELOPMENT DATA	OUT-OF-SAMPLE YEAR	AUROC	ACCURACY RATIO
Up to 2010	2011	76.4%	52.7%
Up to 2011	2012	80.2%	60.5%
Up to 2012	2013–2014	84.3%	68.5%

Figure 10 Cumulative Accuracy Profiles (CAP) and Accuracy Ratio.



## 5.2 *k*-fold Tests

Similar results on the model's power can be obtained using alternative validation techniques such as *k*-fold and cross-validation analysis. The *k*-fold analysis tests model stability vis-à-vis different data segments. In this analysis, we divide commercial mortgages into *k* sub-samples. We then estimate the model on the sample while excluding the observations in the set  $\{k=1\}$ . This model is used to score the observations in the set  $\{k=1\}$ . Such scores represent true out-of-sample estimates. We repeat this process for each of the *k* sub-samples. Afterward, we average the out-of-sample scores for each of the *k* sub-samples and calculate the accuracy ratio and the power curve. We then compare these results with the corresponding in-sample accuracy ratios and power curve. In addition, we check to see whether the parameter estimates for each explanatory variable are stable across the different samples. We find that model performance is well-maintained both in- and out-of-sample in the *k*-fold analysis.

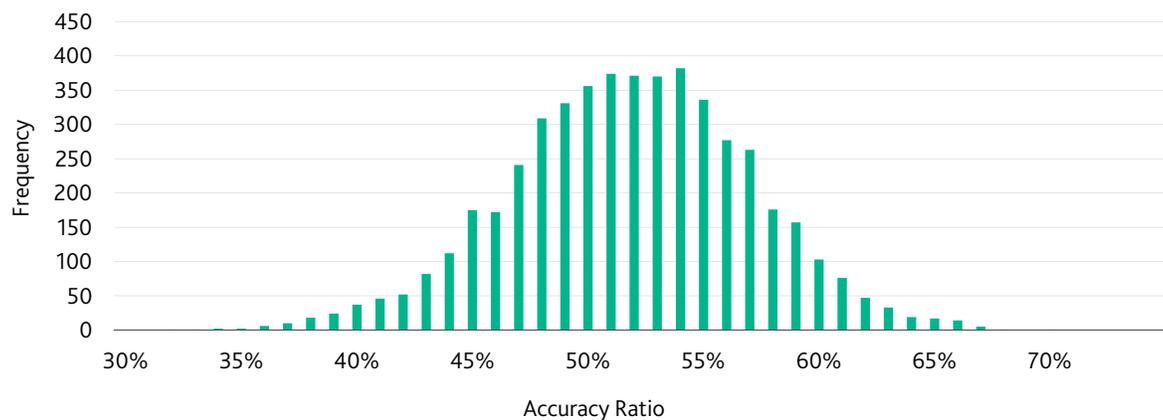
Table 8

**Out-of-Sample ROC and Accuracy Ratios Using *k*-fold Method**

SAMPLE	AUROC	ACCURACY RATIO
Subsample 1	75.1%	50.2%
Subsample 2	75.3%	50.6%
Subsample 3	84.5%	69.1%
<i>k</i> -fold Overall	78.3%	56.6%

**5.3 Bootstrapped Accuracy Ratio**

As we have limited loan-level default data, we can use the dataset at hand as a “proxy population” and perform the sampling with replacement on the available sample. We repeat this process 5,000 times to create 5,000 bootstrap samples. Figure 11 shows the results at various confidence intervals. The mean accuracy ratio is 51.3%.

**Figure 11** Accuracy ratios using bootstrapping technique.**5.4 Calibrating PD Levels for the Overall Canadian Commercial Mortgage Universe**

As mentioned above, we have limited loan-level default data. The data we can access may not necessarily reflect the average loan quality for the entire commercial mortgage industry. In other words, the default rate in our model development sample may differ from the true default rates for the entire population, which we define as the overall commercial mortgage market in Canada. An example is limited Canadian CMBS data availability for loans originated before 2000. Therefore, we must ensure that our model is robust and consistent with default rates observed over multiple commercial real estate market cycles.

**CONSTRUCTING HISTORICAL PORTFOLIOS**

While we do not have details on every single loan throughout history, we do have enough loan origination statistics at useful aggregate levels to construct a pseudo-historical portfolio or a model portfolio. We can therefore compare the performance of this portfolio with historical and out-of-sample default rates. Commencing in 1990, the portfolio contains both permanent loans and construction loans that have typical characteristics in terms of DSCR, LTV, maturity, amortization term, etc. as observed in the mortgage market. The loans cover the major Canadian markets and all five major property types. The resulting model portfolio is dynamic in nature, as new loans are added each year and matured loans are deleted. We model construction loans as interest-only floating rate loans, and we model permanent loans as combinations of fixed and floating, amortizing and interest-only loans.

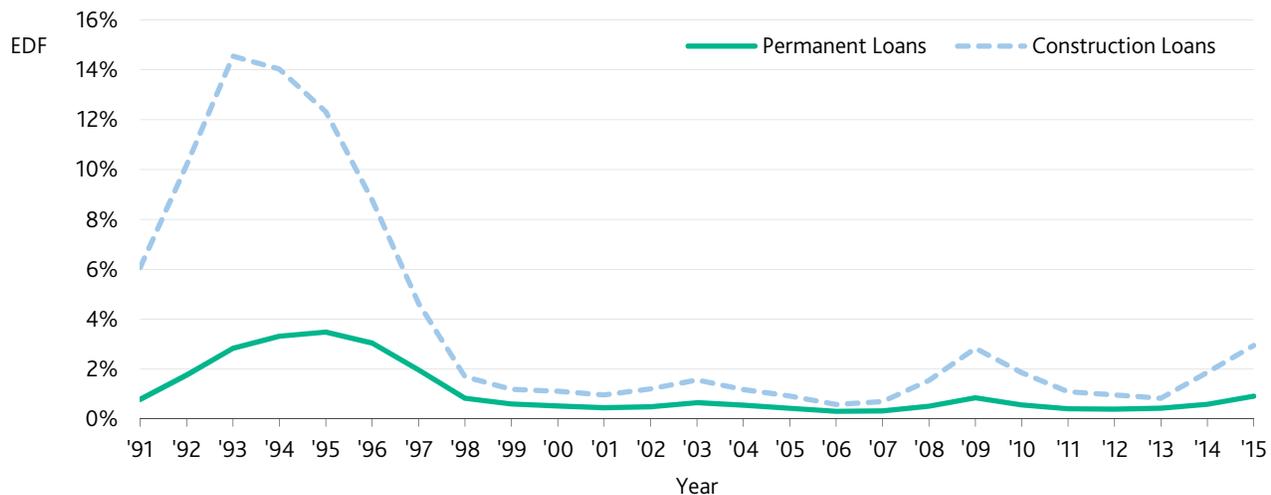
**RUN THE MODEL THROUGH HISTORY**

After constructing a model portfolio that we believe is a good proxy for the actual historical portfolio, we run the model portfolio through our model. The model uses historical commercial real estate market data to dynamically update a loan's contemporaneous DSCR, LTV, and general market cycle variables. For example, for a loan originated in 2001 with DSCR of 1.3 and LTV of 70% at origination, due to continuously deteriorating real estate market conditions following its origination, its

contemporaneous DSCR dropped to 1.28 and an LTV of 75% in 2002, and it further dropped to 1.26 and LTV of 82% in 2003, and so on.

The local real estate market data we use to update DSCR and LTV are imputed market-wide NOI indices by property type and location from CBRE Econometric Advisors historical rent and vacancy data, in conjunction with IPD indices and CMHC data in the corresponding markets. We then aggregate the predicted PD for each quarter in Figure 12, showing credit cycle with EDF peaking in the mid-1990s, followed by minor peaks in the early 2000s and around 2008–2009. The level and timing of the EDF level peaks coincides with observed default rates in Canada, verifying our model's calibration giving accurate out-of-sample, predicted PD. Also, we see a clear distinction of the risk levels between permanent and construction loans. As expected, construction loans have much higher EDF levels during downturns as compared to during good times. For example, the difference between EDF levels in construction and permanent loans during 2008–2009 is much higher than during the 2005–2006 boom period.

Figure 12 CMM Canada estimated default rates for Model Portfolios.



Because model calibration also involves relative risk measurements between subsets of a portfolio, we conduct another validation exercise to compare out-of-sample PD level accuracy by buckets. In this exercise, we bucket out-of-sample observations into 20 buckets based on our estimated PD and then compare the average PDs with actual realized default rates for each group, shown in Figure 13. The overall PD levels appear to fit quite well across rating groups based on estimated PD.

Figure 13 Fitting between CMM Canada estimated PD and actual realized default rates.



Using a variety of validation techniques, we find that the CMM Canada model estimates credit risks reasonably well, conditional upon accurate inputs. We believe that the model produces satisfactory risk measurements over the past several commercial real estate cycles, given the empirical evidence we can access. We also believe that the model is applicable to all major market segments: commercial bank, credit unions, and insurance companies.

The CMM Canada application is flexible enough to allow users to conduct historical analyses. As a result, users can perform back-testing using an institution's historical portfolios to compare model results to see if they match realized default and loss rates when using the validation metrics from the power, as well as the calibration perspectives. Before users implement the CMM Canada application within their organizations, we encourage them to thoroughly understand and evaluate the model to ensure appropriate use.

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## 6. Summary

The commercial real estate and commercial mortgage markets continue to evolve. We have seen numerous changes on many important fronts: demand for commercial real estate space driven by macroeconomic forces, local supplies driven by each CMA's physical layout and zonings, and capital sources increasingly fluid and global in nature. Furthermore, the recent credit crisis has heightened regulatory scrutiny of commercial mortgage lending.

Despite all the changing factors, we find that a commercial property's financials, transpired through DSCR and LTV, continue to play a dominant role in affecting mortgage defaults and losses. Other market-wide factors, such as the prevailing commercial real estate market condition and underwriting pressures, continue to influence the magnitude of point-in-time default rates via their impact on default option values and borrower behaviors. This little-changed relationship between a loan's financial performance and its credit risk makes a quantitative model not only possible, but extremely useful for any serious risk management practice.

After extensive research on a very rich collection of both public and private data sets, we have developed unprecedented insights into commercial mortgage's credit risk drivers. The result is our CMM Canada modeling framework, which balances the need for a highly predictive model with a robust, intuitive, transparent, and highly-flexible model.

In the CMM Canada application, we also source extensive commercial real estate market data and forecasts at the property type and metropolitan area/submarket levels where data exists. This enables clear differentiation of the credit risks of otherwise similar CRE loans but located in different areas and experiencing different economic and real estate market cycles. We also perform exhaustive validation using a variety of innovative approaches to confirm the model is stable and predictive as designed.

Based on all the evidence to date, we firmly believe that our CMM Canada framework will prove indispensable for financial institutions when implementing quantitative credit risk tools in loan origination, portfolio management, scenario analysis, stress testing, and for meeting regulatory requirements.

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