

MODELING METHODOLOGY

An Overview of Modeling Credit Portfolios

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Abstract

This document provides a high-level overview of the modeling methodologies implemented in Moody's Analytics RiskFrontier™. To address the challenges faced by credit risk or credit portfolio managers, RiskFrontier models a credit investment's value at the analysis date, its value distribution at some investment horizon, as well as the portfolio-referent risk of every instrument in the portfolio. The approach is designed to explicitly analyze a wide range of credit investments and contingencies, including term loans with prepayment options and grid pricing, dynamic utilization in revolving lines of credit, bonds with put and call options, equities, credit default swaps, retail instruments, commercial real estate loans, and structured instruments.

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1 Introduction

All credit portfolios, whether global, regional, corporate, small and midsize enterprises (SME), commercial real estate (CRE) or retail, face uncertainty in losses. For example, Figure 1 demonstrates the high corporate default and credit card delinquency rates during economic downturns (around the recessions of 1991 and 2001). Looking at the figure, it is evident that variation in credit quality is substantial and default risk can increase quickly and in correlation with the macro-environment. Even in the aggregate, loss variability cannot be completely eliminated by diversification. Coupled with tight lending margins, even small miscalculations in risk and pricing can undermine profitability.¹

With this in mind, credit risk can be substantially reduced through managed diversification. As investors adopt a diversification measurement, credit risk, and the rewards for bearing it, will ultimately be owned by those who can diversify it best. The challenge for every risk or portfolio manager is to measure and understand the economic risks in their portfolio, and ensure they are properly compensated. An active portfolio manager cannot do this without practical and conceptually sound methods for both measuring diversification, and determining portfolio holdings to minimize concentrations and maximize return in credit portfolios.

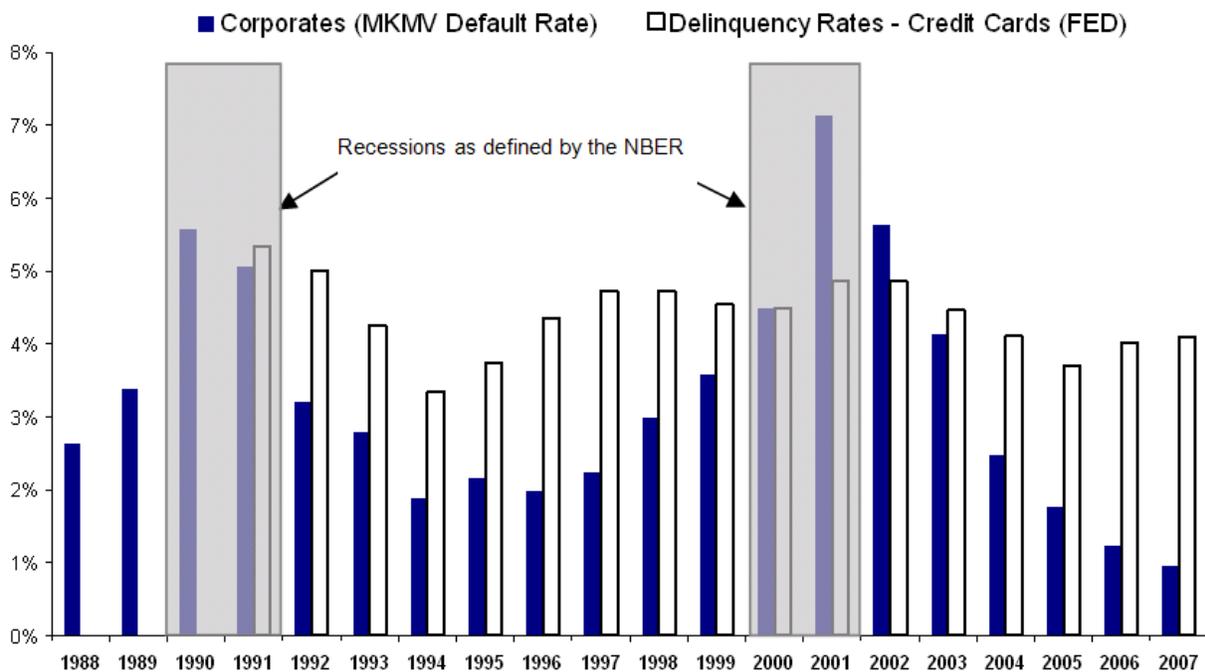


Figure 1 US corporate default and credit card delinquency rates

To address the challenges faced by credit risk or credit portfolio managers, RiskFrontier models each credit investment's value at the analysis date, its distribution of returns over an investment horizon, and the joint credit risk with all other instruments in the portfolio. The approach is designed to explicitly analyze a wide range of credit investments and contingencies, including term loans with prepayment options and grid pricing, dynamic utilization in revolving lines of credit, bonds with put and call options, equities, credit default swaps, and structured instruments. Moreover, the open

¹ For an additional discussion, see Kealhofer and Bohn (2001).

framework provides users with tremendous flexibility in specifying the valuation approach, migration model, and correlations. As such, RiskFrontier produces a quantitative set of actionable goals for portfolio management based on a granular model that can be tailored to the specific needs of an organization.

The portfolio model provides accurate economic estimates of portfolio risk measured as economic capital (i.e., Value-at-Risk, or VaR), Unexpected Loss (i.e., standard deviation, or UL), or expected shortfall (i.e., Conditional Value-at-Risk, or CVaR). It also provides accurate estimates of portfolio-referent risk for each instrument in the portfolio, measured as risk contribution (i.e., contribution to portfolio UL risk) and Tail Risk Contribution (i.e., contribution to portfolio loss). The portfolio and instrument measures account for diversification across dimensions such as country and industry, as well as reference name concentration. The model does so through a bottom-up approach that considers the following:

- The varying terms and conditions of each instrument (e.g., tenure, contingencies, and fees).
- Variation in the underlying reference entities. For example, the term structure of default probabilities such as the Moody's Analytics EDF™ (Expected Default Frequency) credit measure.
- Variation in correlations across reference entities through a correlation model, such as the Moody's Analytics Global Correlation Model™ (GCorr), which accounts for corporate, SME, CRE, and retail correlations.
- Measuring risk alone is insufficient when considering conceptually sound methods of active management. For example, an investment which adds a high level of risk to a portfolio may not be worth hedging; the decision should be influenced by the cost of the hedge. RiskFrontier provides an analysis that incorporates this necessary information to help the portfolio manager make practical and sound decisions.
- RiskFrontier provides a granular view of return and portfolio-referent risk for each investment in the portfolio. With this granular view, RiskFrontier supports action—pricing, approval, position size, hedging, selling, and structuring. It facilitates the comparison of compensation for portfolio-referent credit risk against the amount of capital needed to support a particular investment. It is important to note that return and portfolio-referent risk measures, such as risk-adjusted return on capital (RAROC), provide the portfolio manager with a rank ordering of the most beneficial and detrimental investments. The analysis guides the manager in improving portfolio performance by recommending where to focus additional investments, and indicating where hedging is needed.
- In addition, RiskFrontier can help regulated institutions address the Pillar 2 requirements of Basel II. Specifically, RiskFrontier is designed not only to measure required economic capital and quantify credit concentrations, but also to address requirements such as stress testing. More generally, its functionality addresses the Pillar 2 supervisory review processes.²

To summarize, the portfolio model behind RiskFrontier helps the risk manager or portfolio manager overcome the high-dimensional problem associated with understanding portfolio-referent risk and return on the underlying investments in the portfolio. The remainder of this document provides an overview of how RiskFrontier addresses these challenges. This paper is organized in the following way:

- Section 2 discusses the valuation methodologies.
- Section 3 explains how risk and return of each of the individual investments are modeled within the portfolio.
- Section 4 provides an overview of the Global Correlation model (GCorr), as well as a discussion of other correlation models that can be utilized in RiskFrontier.
- Section 5 explains the Monte Carlo methods that amalgamate instrument-level valuation, risk-and-return analysis with correlations to build up the distribution for the portfolio, and provide actionable portfolio-referent risk measures for each instrument. This section also reviews several features associated with Monte Carlo analysis,

² For detailed discussions, see Gibbon (2007).

including DealAnalyzer®, which allows for portfolio-referent risk-and-return calculations to be conducted rapidly for a new investment or hedge position without reanalyzing the entire credit portfolio.

2 Valuation

Valuation is fundamental to credit portfolio analysis. Given the lack of market prices for most credit instruments, an accurate model is essential. The valuation model used in marking a portfolio to market can have dramatic effects on the perceived portfolio values, as well as the rank-ordering of instruments' return and portfolio-referent risk. A portfolio or risk manager must ensure an economically consistent valuation approach that correctly handles specifics of each instrument across the entire portfolio.

Within the context of RiskFrontier, there are two classes of valuation models: the valuation models used to analyze single-name instruments (e.g., loans or bonds), and models used for structured instruments (e.g., CDOs). We discuss horizon value distribution in Section 3.

2.1 Single-Name Credit Instruments

In the spirit of the open framework, RiskFrontier provides substantial modeling flexibility on the valuation front. Broadly speaking, users can value instruments using book-style methodologies, market-based measures (e.g., prices), or the Moody's Analytics lattice valuation method.³ The approach that best fits an institution depends on its needs. In practice, institutions utilize differing approaches across their sub-portfolios if, for example, market prices are not consistently available for some sub-portfolio. Moreover, many institutions analyze their portfolios under multiple settings to get different views of their portfolio value, such as book value versus mark-to-market.

Under book value, the value of an instrument is par less amortized upfront fees. While book value provides a sense for the instrument value at origination (value is typically near par at origination), it does not adjust in any way for changes in the credit quality of the underlying reference entity. Book valuation can be very useful for evaluating the performance of an instrument's contractual pricing. It is also useful in that it can easily align with an institution's other systems that recognize only value changes resulting from cash losses.

If data are available, users can utilize market prices or credit curves (through spreads or matrix pricing) to value instruments. This option is useful for instruments that are either the same as, or similar to instruments that are traded regularly. Although the application of prices is straightforward (value is equal to price), valuation using credit curves entails employing a reference rate to discount contractually promised cash flows. It is important to understand the caveat that applying credit curves from one market or instrument to another can lead to erroneous analysis. This is relevant when instruments have differing terms and conditions which impact their respective spreads. As will be demonstrated below, this issue is addressed in the lattice structure, which explicitly models terms and conditions.

The lattice structure values an instrument using a bottom-up approach. Cash flows are modeled as a function of the contract as well as the state of a reference entity's credit quality. The structure lends itself to the explicit modeling of credit contingencies. Figure 2 provides a simplified visual illustration of the lattice structure. Credit states for the reference entity are represented on the vertical axis, and time is represented on the horizontal axis. A time-credit state combination is referred to as a "node." The probability of migrating from any one node to another at a subsequent time point is captured by the transition probabilities, which can be based on the Moody's Analytics Empirical Credit Migration model, or on a user-provided transition matrix (e.g., rating-based migration). The default state is an absorbing state, so all transition probabilities from the default state to non-default states are zero.

³ For a detailed discussion of the non-lattice valuation methodologies, see "Modeling Credit Portfolios." For additional details regarding the lattice models, see "Modeling Credit Portfolios," and Levy, Hu, and Li, (2007).

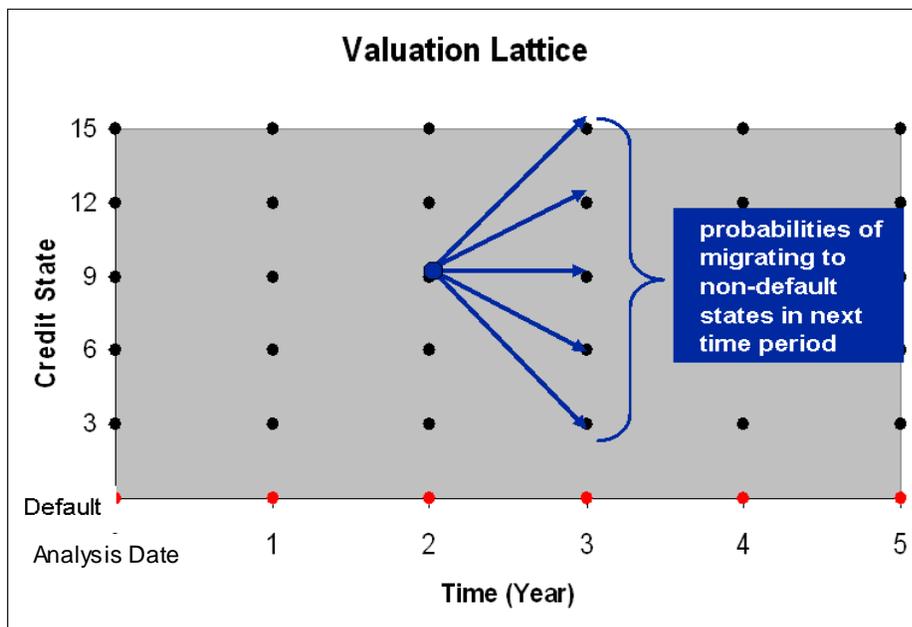


Figure 2 The lattice

As the figure demonstrates, the lattice allows for the modeling of credit qualities over time, credit migration probabilities, and credit-contingent cash flows. Given these, one can model contingencies that are a function of the reference entity's credit state. For example, the nodes for which a prepayment option on a loan is exercised can be determined by comparing the value of the instrument as an ongoing concern with the value to the borrower associated with prepayment.

Valuation within the lattice structure is conducted recursively backward in time. Starting at the second to last period, expected cash flows from the last period are discounted back to arrive at a continuation value at each node; for a vanilla loan there is no uncertainty in value of the non-default nodes in the last period, so no calculation is necessary. Note that the default probability depends on the node; starting at a high node (i.e., a high credit quality) will result in a lower probability of default and higher value. The value will typically be lower for lower credit quality nodes. After adding any accumulated cash flows (e.g., interest payments, amortized principal or prepayment penalties), the value for nodes in the preceding period are computed, with any contingencies taken into account.

A few subtleties are worth pointing out. First, a separate lattice is constructed for each instrument. After all, contingencies and terms and conditions are instrument-specific, and so cash flows at each node are instrument-specific. Second, the probability of reaching a node is specific to the reference entity. This follows from the fact that migration probabilities depend on the default probability term structure for the reference entity; the default probability term structure can be different for each reference entity in the portfolio. Third, valuation requires discounting of cash flows that face risks, such as default risk. Similar to the capital asset pricing model (CAPM), the lattice accounts for the systematic portion of risk in cash flows. However, instead of discounting cash flows at a beta-adjusted rate, the lattice discounts cash flows using risk-neutral valuation techniques.

The motivation for using risk-neutral valuation techniques is driven by their computational ease when applied to the problem at hand. These techniques were pioneered by Black and Scholes (1973), and Merton (1973) in their seminal work on option pricing. Merton (1974) applied the same analysis to the pricing of corporate liabilities, and created a structural framework that established the theoretical underpinning of subsequent developments in academic research and industrial application pioneered by Moody's Analytics. It is important to note that similar to the CAPM framework, each asset's covariance with the market determines its own systematic risk adjustment; the risk-neutral valuation method includes an adjustment for each reference entity's systematic risk.

With the lattice in hand, one can analyze a wide range of dynamics associated with credit contingencies, since an arbitrary cash flow can be associated with each node. For example, one can model the common observation that deterioration in credit quality is associated with a drawdown of credit lines, as demonstrated in Figure 3. The lattice accurately accounts for differing usage and non-usage fees, and the impact of the payoff or drawdown on cash flows. In the same spirit, the RiskFrontier lattice models loans with prepayment options or grid pricing, callable or puttable bonds, CDSs (explicitly accounting for counterparty risk), custom instruments with described arbitrary cash flows, and exposure profiles that capture the credit component of instruments facing risks other than credit risk, such as derivatives. The lattice also models equity, both public and private.

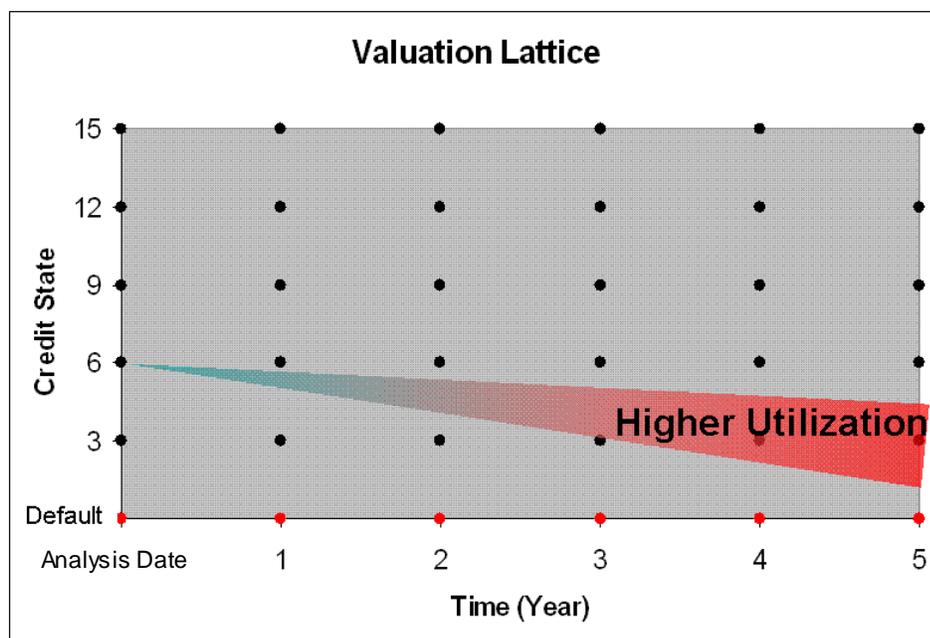


Figure 3 The lattice structure applied to modeling the dynamic usage pattern in a line of credit

As mentioned above, RiskFrontier allows for modeling of migration using the Moody’s Analytics Empirical Credit Migration model, or through a user-specified credit migration matrix (e.g., a ratings-based migration matrix). The Moody’s Analytics Empirical Credit Migration model uses information from the Moody’s Analytics EDF model to determine the probability of migrating to different credit states. As demonstrated in Dwyer and Qu (2007), the EDF model provides the most accurate point-in-time measure of default probability. As such, an EDF-based migration model provides the most accurate description of how actual point-in-time credit quality (i.e., default probability) evolves over time. Alternatively, users can specify a Moody’s transition matrix if the interest is to model migration using a through-the-cycle measure of credit quality.

To get a sense for the dynamics associated with different migration models, Figure 4 presents a Moody’s one-year migration matrix (top left), as well as a Moody’s Analytics EDF-based one-year migration matrix (bottom right). Although the Moody’s Analytics matrix is not derived directly from the Moody’s Analytics Empirical Credit Migration model, it does represent the kind of dynamics observed in the Moody’s Analytics Empirical Credit Migration model. In particular, one can see that EDF credit measures are substantially more dynamic than agency ratings; the probability of remaining within a Baa category is much higher under a ratings-based approach than the probability of remaining within

an EDF-range that corresponds to a Baa rating.⁴ This follows from the fact that EDF values measure credit quality at a point-in-time, while Moody's ratings are through-the-cycle measures, and do not change as frequently.⁵

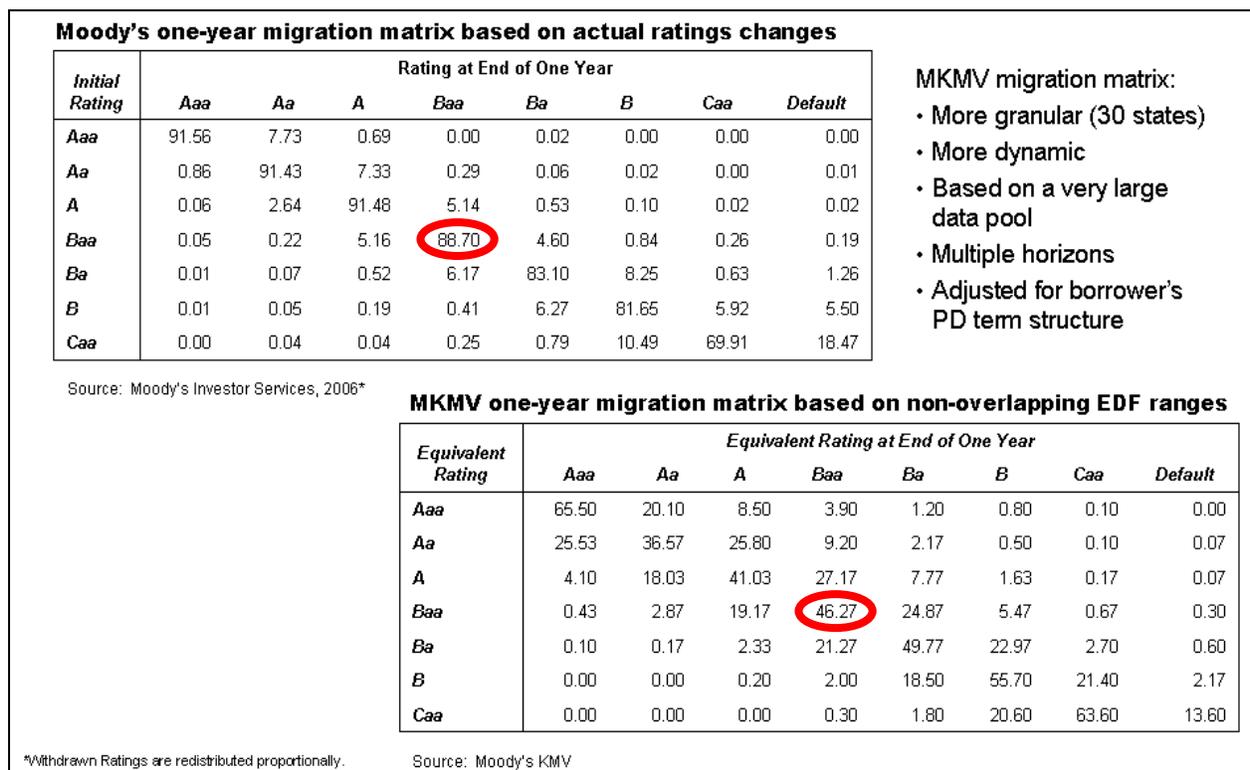


Figure 4 A comparison of a Moody's rating transition matrix and a Moody's Analytics EDF-based transition matrix

2.2 Structured Instruments

The challenges of modeling structured instruments are numerous. Much of the complexity stems from the dependence of a structure's cash flows on a collateral pool whose credit qualities are heterogeneous. Understanding the combination of events and associated probabilities that lead to possible cash flow realizations can be computationally costly. For example, using a lattice structure to value a CDO with 100 reference entities in the collateral pool would require constructing a grid with 100 dimensions (i.e., one for each reference entity). This would be impractical. To avoid these costs, RiskFrontier does not utilize the lattice structure to value structured instruments. Instead, RiskFrontier makes simplifying assumptions and applies an analytic valuation method.

The analytic valuation approach is similar to the approaches presented in Gregory and Laurent (2003), and Hull and White (2004). The methodology relies on two fundamental modeling assumptions. First, the collateral terms and conditions, and correlations are assumed to be uniform across the collateral pool.⁶ Second, losses can be described through a pass-through waterfall structure. More specifically, losses impact a subordinated class (e.g., a tranche) when

⁴ It is worth mentioning that the relationship between EDF measures and ratings can be non-monotonic. As such, this exercise is supposed to provide a sense for the dynamics; a formal comparison is much more difficult.

⁵ This implies that default rates associated with an agency rating varies over time. For example, Baa-rated firms experienced a 10-fold increase in default rates from 1998 to 2001.

⁶ It is worth noting that the simplified correlation structure is only applied to valuation. As we will discuss shortly, risk and return incorporate the rich correlation structure that is available in RiskFrontier.

losses on the collateral pool exceed the attachment point. The tranche will be worthless after losses on the collateral pool exceed the detachment point. However, the structured instrument can be parameterized to model CDOs, BDSs, CLOs, and asset-backed securities of various types, with different waterfall structures.

Figure 5 provides a simplified visual representation of the analytic approach. The first step is to build up the loss distribution for the underlying collateral pool at each point in time, as depicted at the top of the figure. Cash flows to the tranche can be constructed by mapping the distribution of losses on the collateral pool to losses on the tranche, which are based on the attachment point (a) and detachment point (d), as depicted in the lower right-hand part of the figure. Tranche value is computed by discounting risk-neutral expected tranche cash flows back to the analysis date.

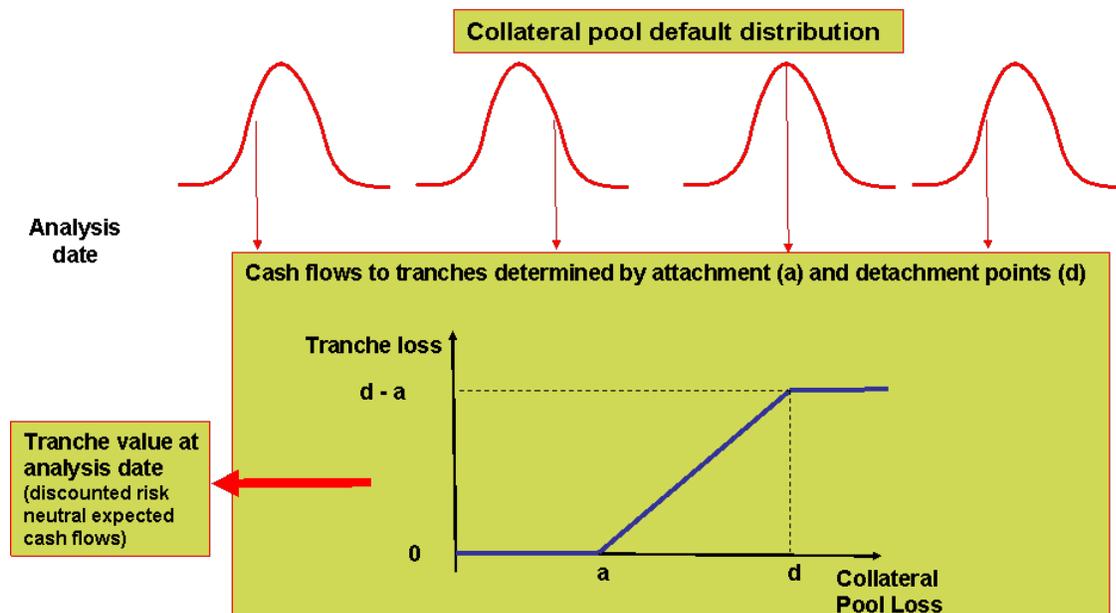


Figure 5 The analytic valuation approach to modeling structured instruments

RiskFrontier allows users to offset distortions associated with the two fundamental modeling assumptions related to the analytic valuation method. First, distortions associated with collateral pool homogeneity are offset using information specific to the deal, as well as available market information. For example, the correlation parameter that is used for pricing can be calibrated to those implied by market prices, or an external model.⁷ It is important to note that the semi-analytic approach used for analyzing risk and return does not rely on homogeneity. For example, it leverages the chosen correlation model (e.g., MKMV GCorr) along with Monte Carlo techniques and does not have the same shortcomings. Second, users can also offset distortions associated with a pass-through waterfall structure. The attachment point can be calibrated so that the distress probability of the structured instrument corresponds to that produced by a model which accounts for the more complex waterfall structure. Similarly, the detachment point can be calibrated to the loss given distress on the structured instrument. This approach to setting attachment and detachment points allows users to leverage a richer stand-alone valuation tool in a computationally more efficient framework that accounts for portfolio concentration effects.

⁷ For a more detailed discussion, see “Modeling Credit Portfolios.”

3 Measuring the Stand-Alone Instrument Risk and Return

We now discuss how the horizon value distribution is constructed and used, along with the valuation model, to measure an instrument's stand-alone risk and return. Similar to valuation, the model that defines the horizon value distribution is fundamental to credit portfolio management. The instrument's horizon distribution, when combined with the analysis-date value, defines the risk and return on the instrument. As in Section 2, "Valuation," the horizon valuation models used to analyze single-name instruments (e.g., loans or bonds) are different from those that are used for structured instruments (e.g., CDOs). As such, this section is divided into two parts.

3.1 Single-name Credit Instruments

RiskFrontier provides substantial flexibility when modeling the value distribution at horizon. As with analysis-date value, users can model the distribution incorporating book-style methodologies, default-no-default settings, market-based measures (e.g., prices or spreads), or utilizing the Moody's Analytics lattice structure.

Similar in spirit to book value, RiskFrontier allows users to define the expected horizon value of an instrument as an ongoing concern (i.e., conditional on not having defaulted) using a linear or exponential pull-to-par approach. As with book valuation, horizon linear and exponential valuation do not account for changes in the credit quality of the underlying reference entity.

Like with analysis-date valuation, market prices or credit curves (e.g., spreads or matrix pricing) can be used to describe the horizon value of an instrument as an ongoing concern. In the case of credit curves, the user also specifies the distribution of spreads at horizon. In other words, the Moody's Analytics Empirical Credit Migration model would not be used in the analysis. Instead, each horizon credit state is associated with a forward credit curve which, along with the reference rate, is used to discount the remaining promised cash flows. Here, the practitioner must be aware of the same caveats as in analysis-date valuation: applying the distribution of spreads from one market to another can lead to erroneous analysis when instruments have differing terms and conditions.

Finally, at horizon the user can compute the expected value and value distribution of an instrument using the lattice methodology. Figure 6 provides a simplified visual representation of how the horizon value distribution is constructed. In the example, the analysis is conducted to a horizon of two years. The value of the instrument at each of the non-default nodes at horizon is a combination of post- and pre-horizon cash flows. These cash flows include not only coupons or fees, but also any cash flow-associated contingencies, such as prepayment.

Using the lattice, valuation of cash flows after horizon is conducted using risk-neutral valuation techniques as discussed in Section 2.1, "Single-name Credit Instruments," (i.e., backward induction). In this example, a higher credit state will typically be associated with a higher value, because the probability of default is lower. Cash flows before horizon are assumed to be reinvested and aggregated at horizon (i.e., forward). It is worth pointing out that the different possible paths to a node are considered when aggregating these cash flows.⁸ The single *default* node at horizon actually represents a set of states. That is, in the event of default, recovery value is uncertain and is modeled through a beta distribution with parameters from Moody's Analytics LossCalc™, or a user-specified mean and variance.⁹

To arrive at the instrument value distribution in the non-default states, it is necessary to associate a value with each horizon node, as well as a probability of arriving at that node. The probability of arriving at each horizon node is computed by concatenating the probability distribution at each time-step before horizon. In Figure 6, there are two time steps before horizon. As such, the probability of arriving at any horizon node is computed by considering the different possible paths that lead to this node. Of course, the default node is an absorbing state. As discussed in Section 2.1, the

⁸ It is assumed that all cash flows realized before horizon are reinvested to horizon at the credit default risk-free rate.

⁹ In fact, users can specify an entire term structure for the expected recovery and variance in RiskFrontier. This can be particularly useful when modeling project finance where recovery is typically lower at the initial phases of the project.

probability of arriving at a node is determined by the chosen credit migration model (e.g., the Moody's Analytics Empirical Credit Migration model).

It is important to stress that the relevant distribution of states at horizon is the distribution observed in the physical world (i.e., the distribution under the physical measure). For example, for the purpose of risk management, it is relevant to focus on actual probabilities of default and probabilities of the counterparty improving or deteriorating in credit quality. This physical distribution is distinctly different from the distribution under the risk-neutral measure, which is relevant when valuing risky future cash flows. In other words, the relevant probability of defaulting or migrating to a credit state at horizon is the physical probability. Once that credit state is realized, the value of the instrument can be computed using risk-neutral valuation techniques that combine future physical probabilities and the preference of investors to avoid risk, as expressed in market prices.

With the horizon value distribution in the non-default and default states, and the analysis-date value in hand, the computation of risk and return is straightforward. In particular, unexpected loss is computed as the standard deviation of the return, and the expected spread, which measures compensation for risk, is computed as the expected return less the credit-risk-free rate.

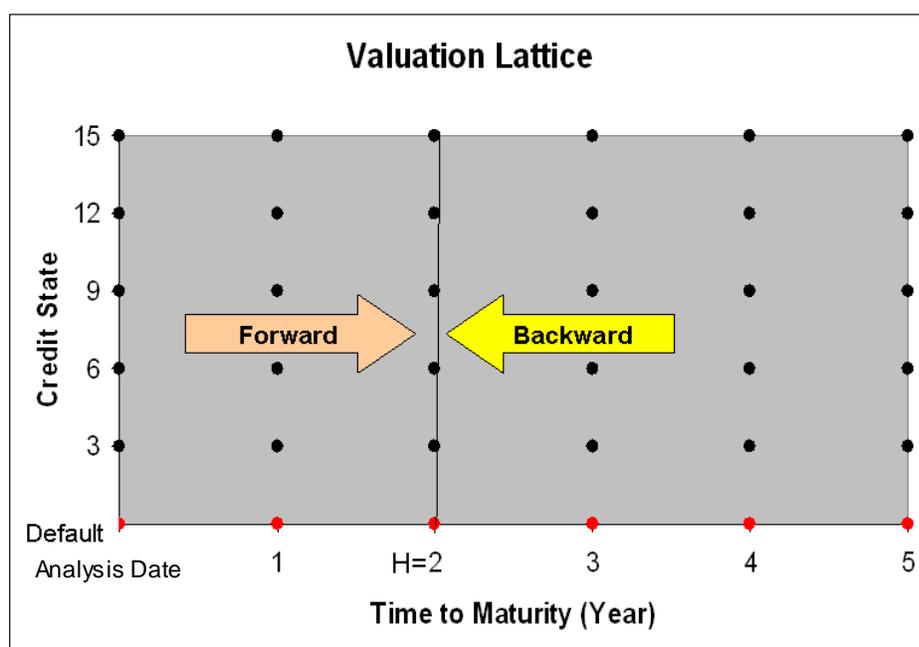


Figure 6 How an instrument's value distribution at horizon is generated

3.2 Structured Instruments

As discussed in Section 2.1, "Single-name Credit Instruments," *valuation* of a structured instrument is complex and time-consuming. When compared with single-name instruments, the complexity associated with modeling the *horizon value distribution* is much higher for a structured instrument. One challenge is to keep track of the horizon credit states of all reference entities in the collateral pool, as they play a role in the horizon value of the structured instrument. Another challenge is to associate a probability to each combination of reference entity credit states. This is coupled with the desired property that the model properly account for credit events associated with reference entities, which overlap the collateral pool of the structured instrument and other parts of the portfolio.

To overcome these challenges, RiskFrontier uses a semi-analytic approach. This approach combines a Monte Carlo simulation, which accurately represents cash losses prior to horizon, with the analytic valuation method (described in

Section 2.2, “Structured Instruments”), which represents the horizon value of the structured instrument as a going concern. The analytic horizon value is pre-computed for each point of a two dimensional grid: one dimension is the cumulative loss on the collateral to horizon; the other dimension is the average credit quality of the remaining principal (average forward 1-year default probability). The structured instrument’s horizon value distribution is constructed through simulating each reference entity’s horizon credit state in accordance with other parts of the portfolio. The value from cash flows prior to horizon is then combined with the forward-analytic value from the grid to arrive at a value at horizon.

It is important to stress that within the context of the horizon value distribution, homogeneity is only assumed when computing the analytic value of the structured instrument as a going concern at each point on the grid. Meanwhile, the simulation accurately accounts for heterogeneity in the default probability, the likelihood of multiple defaults as implied from the specified correlation model, loss given default, time to maturity, and notional. Thus, risk and return metrics incorporate the heterogeneous characteristics of the collateral pool. In addition RiskFrontier is uniquely able to identify the concentration risk of reference entities that span the collateral pool and other parts of the portfolio.

The last step of the analysis accounts for inconsistencies between assumptions (e.g., correlation) used in constructing the horizon value distribution, and assumptions used in valuation at the analysis date. To rectify this discrepancy, the simulated horizon values are shifted by a constant. This constant is set to guarantee that the discounted expected value (using risk-neutral valuation techniques) is equal to the value at the analysis date.

Finally, it is worth pointing out that unlike single-name instruments, stand-alone risk and return statistics for structured instruments are simulation-based. Unexpected loss is computed as the standard deviation of the return, and the expected spread is computed as the expected return less the credit risk-free rate.

4 Correlation

A credit portfolio or structured product is composed of multiple instruments. Clearly, the overall risk of a portfolio depends not only on the risks of the individual instruments, but also how the future values of these investments are correlated. Given the importance of correlation in determining the risk and return on a portfolio, Moody’s Analytics has developed a rich global factor model (GCorr Corporate) that provides pairwise asset (not equity) correlations for approximately 33,000 publicly traded firms, the Private Firm R-squared Calculator that provides mid-market correlations, as well as correlation parameterizations for a wide range of asset classes, such as retail instruments, CRE and asset-backed securities. In addition to the Moody’s Analytics models and data, RiskFrontier provides substantial flexibility by allowing specification of an arbitrary factor model or pairwise correlation matrix for reference entities in the portfolio, where the joint distribution can be described through a Normal copula. To facilitate in parameterization, the following three structures are available:^{10,11}

- **Flexible Factor Model.** It is an expansion of the Moody’s Analytics factor model (GCorr), in which additional factors (e.g., retail, SME or CRE factors) are included. This allows users to refine the correlation model associated with the extended asset classes, while leveraging the richness of GCorr in a single unified correlation framework.
- **General Factor Model.** It allows users to specify an arbitrary linear factor model. The factors can be economic in nature, or represent latent factors. The general factor model framework requires users to specify the covariance matrix between the factors.
- **Pairwise Correlation Model.** It provides the most flexibility, because users specify the correlation between every pair of reference entities in the portfolio.

¹⁰ For a detailed discussion of the correlation module, see “Modeling Credit Portfolios.”

¹¹ The Private Firm R-squared Calculator can be used to estimate asset correlation between private firms and between private firms and other assets found in the credit portfolio.

Regardless of the specified correlation structure, RiskFrontier utilizes a bottom-up approach where correlations describe the co-movement in credit states, not the co-movement instrument values that are part of the output. Parameterized correlations can represent the correlation of credit states to be used to describe the co-movement in forward default probabilities as represented in the lattice structure and the Moody's Analytics Empirical Credit Migration model, or alternatively the co-movement in credit curves. The details of how the joint movement of credit states is combined with the instrument horizon value distributions to describe the joint distribution of instrument values can be found in the Monte Carlo section that follows.

The remainder of this section provides an overview of the GCorr Corporate model, which uses a factor model rather than direct historical observations to measure asset return correlations between firms. The factor model approach leverages information in areas where data are plentiful and allows for inference in areas where data are scarce and noisy. Zeng and Zhang (2001) use extensive empirical data to show that historical correlations are subject to a large amount of sampling error, thereby limiting their usefulness in predicting future correlations. The predictive power of a factor model, in contrast, stems largely from its control over these errors. A good follow-up paper is Zhang, Zhu, and Lee (2008) who provide a link between the predicted default correlations from the GCorr Corporate model and correlations implied directly from realized corporate defaults.

Some practitioners have expressed valid concerns about measuring default correlations since they are difficult to estimate; defaults are infrequent and typically occur once, at most, for any entity. The approach described below cleverly addresses this dynamic aspect of default correlations by decomposing default (and value) correlations into two component pieces: asset (business value) correlation, and credit quality, which is enormously dynamic and requires forward-looking measures and constant monitoring. The description below is about the estimation of firm-level asset correlations. The practitioner in RiskFrontier can then put these together with highly predictive credit quality measures and get a useful and predictive view of portfolio risk.

A factor model approach imposes a structure on the correlation of asset returns. The correlation between the asset returns of any pair of firms can be explained by the firms' relationships to a set of common factors. In particular, GCorr Corporate can be thought of as having a factor model within a factor model. To begin, a firm's credit risk can be decomposed into a systematic component (i.e., the firm's composite factor) and a firm-specific (i.e., idiosyncratic) component; this can be thought of as the first factor model. The systematic component can be described by a second factor structure. In particular, a firm's composite factor is defined by the firm's industry and country composition. The dynamics are driven by industry- and country-specific loadings on a set of global, regional, and industrial sector factors that are common to all countries and industries. Furthermore, each industry and country has a specific risk component that is unrelated, or idiosyncratic, to other industry- and country-specific risks. Thus, the GCorr Corporate asset correlation between two firms is defined by the covariance between the two respective composite factors, along with the correlation between each firm and its own composite factor. More formally, the variables that define the GCorr Corporate correlation structure are the following:

- The relation between a firm and its composite factor (this is referred to as R-squared or RSQ).
- A firm's industry and country composition.
- The loadings on the common global, regional, and industrial sectors that define the correlation between industry-country combinations.

Figure 7 provides a visual representation of the Moody's Analytics GCorr Corporate factor structure. Starting from the top, a firm's risk is decomposed into systematic and firm-specific risk; this decomposition is determined by the firm's R-squared. The systematic risk is defined by the firm's industry and country composition (i.e., a composite factor). As demonstrated in the bottom row, dynamics for each industry and country are defined by loadings on the global, regional, and industrial sectors. The remaining industry and country risks are specific to the respective industry and country.

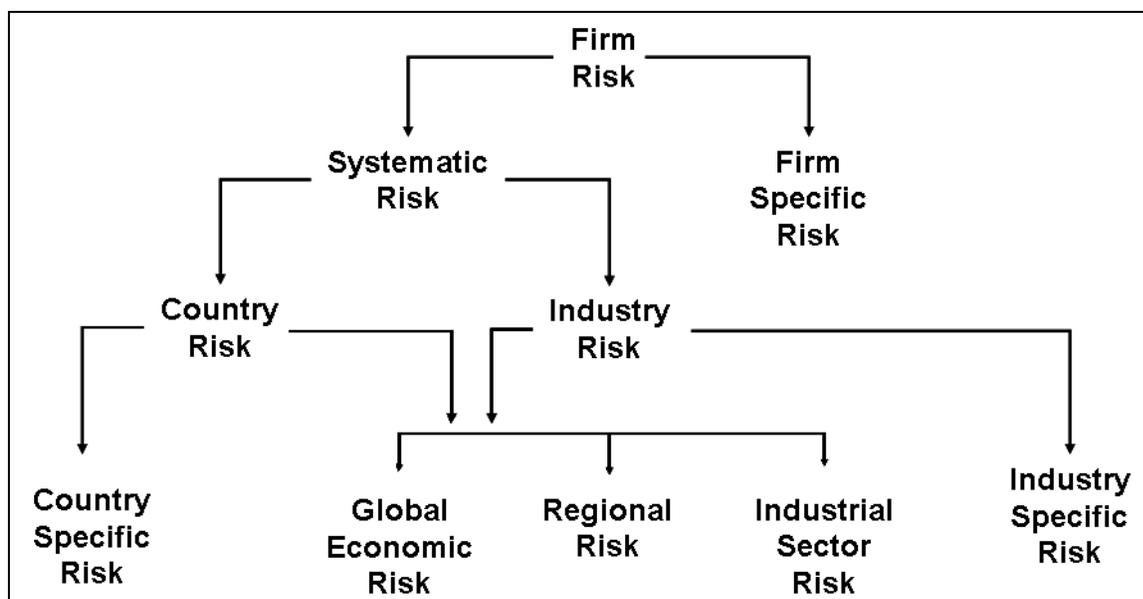


Figure 7 The GCorr Corporate Factor Structure

5 Bringing it All Together with Monte Carlo Simulation

So far, this document has provided an overview of valuation models, models describing horizon value distribution at the instrument level, and the correlation module (along with GCorr). This section describes how these components are used in the Monte Carlo engine to construct the portfolio value distribution, as well as portfolio-referent risk statistics.

As discussed above, the problem of analytically computing the joint distribution of returns for a credit portfolio is a daunting task. Instead of describing the joint distribution analytically, the Monte Carlo engine samples from the joint distribution of credit states as defined by the correlation model. The engine then utilizes the mapping between credit state and instrument value (available through the instrument horizon value distribution) to sample from the joint distribution of instrument values. Said another way, the Monte Carlo engine simulates the credit states of the world for the underlying reference entities in the portfolio, then maps credit states to a value for each instrument. By simulating a large number of trials, where each trial represents one combination of credit states for all the reference entities, one can obtain a description of the joint distribution of instrument values.

Figure 8des a visual representation of the simulation for two firms. Focusing on the graph in the top right, the x-axis represents the credit state for reference entity X, and the y-axis represents the credit state for reference entity Y. Ovals on the graph represent iso-probability curves for the joint distribution of credit states. Curves in the center have a higher probability mass and those further out have less, indicating that extreme credit states are less likely. The fact that the ovals are squeezed toward the top right and bottom left indicate that the credit states are correlated; reference entity X is more likely to realize a bad horizon credit state if reference entity Y realizes a bad credit state.

To aid with the visual representation, the marginal distribution of credit states for reference entities X and Y are plotted along the x and y-axis. The shaded portion of left x-axis (bottom y-axis) represents the default states for reference entities X and Y. Meanwhile, the value of each instrument, as a function of credit state, is depicted on the top left and bottom right. In each trial, the Monte Carlo engine samples from the joint distribution of credit states. This is done by simulating the underlying factors as defined by the correlation model, associating the factor loadings for each reference entity to arrive at the systematic portion of the reference entity's risk. In addition an idiosyncratic shock is simulated for each firm to arrive at a credit state for each firm. If the realized credit state falls below the default threshold, a random recovery amount is simulated. If the reference entity is not in default, the credit state is mapped onto a going-concern instrument value.

Computing portfolio and portfolio-referent risk statistics is straightforward after a large sample of instrument values is simulated. For portfolio level statistics, instrument values can be aggregated for each trial to arrive at a sample of portfolio values. Unexpected Loss (i.e., standard deviation or UL) for the portfolio can be estimated by taking the standard deviation of the sample. Similarly, portfolio capital can be computed by looking at the loss threshold associated with a particular target probability. Meanwhile, portfolio-referent risk statistics can be computed by relating the instrument value with the portfolio value. The covariance between the instrument value and the portfolio value can be estimated for the purpose of computing risk contribution (i.e., contribution to UL risk) at the instrument-level. Meanwhile Tail-Risk Contribution (i.e., contribution to the event of an extreme portfolio loss) is computed by keeping track of the loss on each instrument when the portfolio falls within the specified loss interval.

Having laid out the mechanics of how the portfolio and portfolio-referent risk statistics are computed, it is worth considering the economics of when it is relevant to focus on each of the measures. In particular, an institution's preferences should determine which statistics are most relevant. For example, an institution whose primary focus is to minimize the year-to-year changes in portfolio value should probably focus on portfolio UL and each instrument's contribution to UL (i.e., risk contribution). If, instead, an institution's primary concern is with minimizing the likelihood of incurring extreme losses (e.g., to protect their Aaa rating), then the focus should probably be on capital measured at a low target probability, and focus on each instrument's contribution to those extreme outcomes (i.e., Tail-Risk Contribution). In some cases, multiple stakeholders may have different preferences. For example, an institution's shareholders may care about UL risk, while regulators may be concerned with the likelihood of extreme loss. One approach is for the institution to focus on a low target probability capital measure at the portfolio level, and risk contribution when allocating risk at the instrument level. Loosely speaking, this approach allows the institution to rank order and optimize its portfolio relative to its own preferences, while adhering to the constraints placed upon it by regulators.

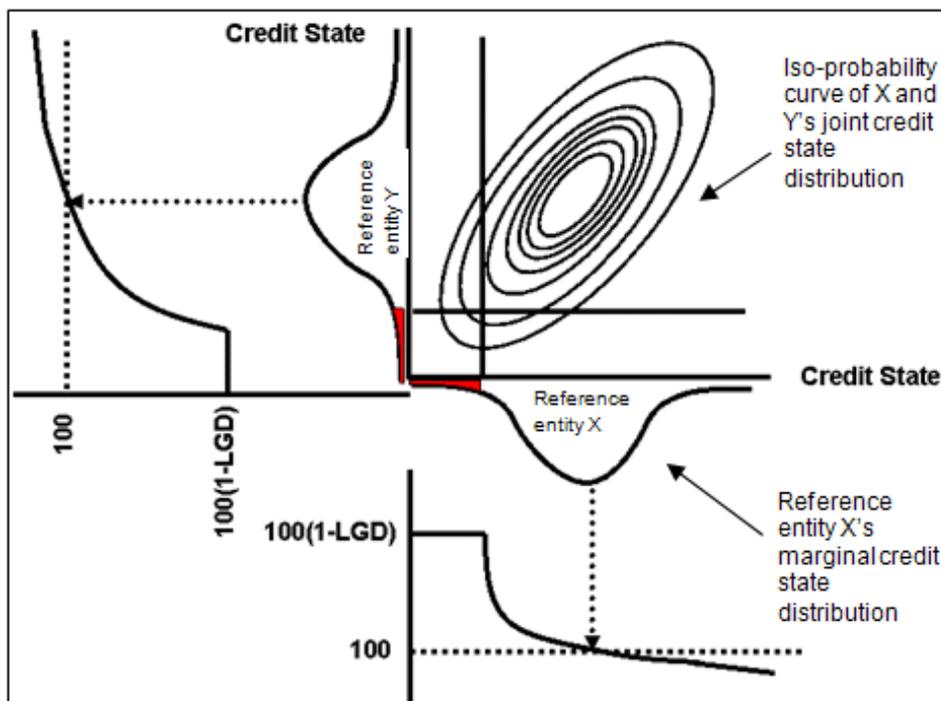


Figure 8 The Monte Carlo engine

The following four features related to the Monte Carlo engine are worth highlighting:

- Simulation Services distributes the Monte Carlo calculations across multiple processors and machines. This allows for an improvement in computation speed, as well as an ability to analyze portfolios with a large number of instruments. The engine utilizes a Multi-Stream Monte Carlo approach that controls the sequence of pseudo-random numbers in such way that ensures that results are not affected by whether the calculations are performed in a distributed fashion.
- An importance-sampling approach to simulation is available in RiskFrontier. The methodology improves upon the simulation speed without loss of accuracy. The basic idea behind importance sampling is to change the distribution from which the random samples are taken in a way that preserves the statistic; asymptotically, as the number of trials approaches infinity, importance sampling will produce the same simulated statistics as standard Monte Carlo. A greater concentration of the sample is chosen from the region with the greatest impact on the calculation. For computing risk statistics, this means sampling those scenarios that lead to large losses. Although performance is portfolio-dependant, the approach frequently reduces the number of simulations by a factor of ten or more (i.e., one-tenth the number of trials are needed to achieve the same level of accuracy).
- RiskFrontier facilitates the analysis of large homogeneous pools of instruments. The functionality allows for accurate analysis of a homogeneous pool of arbitrary size in a fraction of the time it would otherwise take to analyze each instrument individually; it takes about as much time to analyze the pool as it does a stand-alone instrument. This functionality is useful when, for example, modeling retail pools (e.g., a pool of credit card loans), where the benefits of granularly analyzing characteristics of each borrower and loan may not be sufficient to offset the increased computation time and data requirements. When modeling instruments using the homogeneous pool functionality, the model is designed to compute simulated statistics with approximately the same level of accuracy as when the instruments were modeled individually.
- DealAnalyzer is a calculation engine that brings portfolio analytics to the desktops of loan originators and active managers. DealAnalyzer provides the ability to structure, value, and price transactions accurately in light of existing credit portfolio risk. DealAnalyzer leverages the Multi-Stream Monte Carlo methodology to analyze the return and portfolio-referent risk characteristics of a deal without reanalyzing the entire portfolio. Intuitively, Multi-Stream Monte Carlo allows for the timely regeneration of random sequences for the existing portfolio, so that simulated values from the deal can be appropriately associated with values from the existing portfolio's value distribution, allowing for the computation of risk statistics.

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