A Unified Decision Measure Incorporating Both Regulatory Capital and Economic Capital

Abstract

Required economic capital (EC) and regulatory capital (RegC) are two measures frequently used in loan origination and other decisions related to portfolio construction. EC accounts for economic risks such as diversification and concentration effects. When used in measures such as return on risk-adjusted capital (RORAC) or Economic Value Added (EVA™), EC can provide useful insights that allow institutions to optimize risk-return profiles, facilitate strategic planning and limit setting, as well as quantify risk appetite. Meanwhile, when RegC is binding, an institution faces a tangible cost, in that additional capital is needed for new investments that face a positive risk weight. Given these observations, both EC and RegC should influence decision making. After all, a deal with lower RegC but the same EC is favorable, and a deal with lower EC but the same RegC is favorable.

In this paper, we formalize RORAC and EVA measures that incorporate both RegC and EC. The new measures allow institutions to rank-order their portfolios and potential deals in a way that accounts for economic risks and regulatory charges. We find that in a traditional one-year horizon analysis, we can compute RORAC by considering EC as the risk measure, and RegC as a cost or tax that decreases the return on the investment.

To demonstrate the effects of this approach, we use the IACPM/ISDA portfolio to quantify the impact of the RegC constraint. In this exercise, 14.55 percent of the instruments that were otherwise viewed as "favorable" become "unfavorable" from a portfolio-referent investment perspective when we consider both RegC and EC.
# Table of Contents

1 Introduction .................................................................................................................................................. 4

2 Unified Measures ...................................................................................................................................... 5

3 Impact of the RegC Constraint .................................................................................................................. 9

4 Conclusion .................................................................................................................................................. 13

Appendix A Formalized Framework .......................................................................................................... 14

Appendix B Test Portfolio Description .................................................................................................... 18

References .................................................................................................................................................... 19
1 Introduction

With the advent of Basel III and increased regulatory pressure stemming from the financial crisis, financial institutions face regulatory capital (RegC) constraints that increasingly impact strategic investment plans. As a byproduct of the increased focus on RegC, some financial institutions have reduced their attention to required economic capital (EC). Some argue that with a binding regulatory capital constraint, EC is irrelevant. Unfortunately, with such focus on RegC, information content in EC is lost. RegC and EC are actually two distinct and very relevant measures. EC accounts for diversification, concentration effects, and other economic risks. When used in measures such as return on risk-adjusted capital (RORAC) or Economic Value Added (EVA™), EC can provide useful insights that allow institutions to optimize risk-return profiles, facilitate strategic planning and limit setting, and quantify risk appetite. That said, when RegC is binding, an institution faces a tangible cost, in that additional capital is needed for new investments that face a positive risk weight. Intuitively, RegC and EC should both influence investment decisions; given two otherwise identical deals (including EC), the deal with lower RegC is preferable. Alternatively, given two otherwise identical deals (including RegC), the deal with lower EC is preferable. In this paper, we formalize RORAC and EVA measures that incorporate both RegC and EC. These measures allow institutions to rank-order their portfolios and potential deals to account for economic risks and regulatory charges.

While it is clear that firms focusing only on RegC or EC will have a reduced ability to manage risk and performance, it is less clear how to formalize a decision-making variable that incorporates both regulatory and economic considerations. To address this challenge we consider the fundamental problem faced by a financial institution that must choose a combination of investments to maximize value for its stakeholders. We borrow from traditional settings such as those used to derive the capital asset pricing model (CAPM), or those used to derive economic decision-making rules such as RORAC and EVA. The difference is that we conduct the analysis in which the financial institution faces a regulatory constraint. The constraint limits the institution’s ability to borrow, and forces it to hold a level of book equity to cover the RegC associated with its asset base.

Upon analysis, we find that the unified decision variables have similarities with their traditional counterparts. The difference is that accounting for RegC constraint results in an effective RegC cost or tax, which decreases the return on the investment. The unified measures are intuitive and have the following appealing properties:

- They account for the economic risks coming from concentration and correlation effects. An asset’s risk measure will be higher if, all else being equal, it is more correlated with the portfolio or if it is more likely to be in distress.
- They account for cross-sectional variation in regulatory charges, so that investments with higher regulatory risk weights are less attractive, all else being equal.
- They go beyond common approaches used to bring together RegC and EC (for example, taking the maximum of RegC and EC as the risk measure), which invariably lose important information. The measure incorporates both RegC and EC in a unified fashion, so that both ultimately influence decision making.
- As with traditional measures, the institution can utilize a single unified decision variable to rank-order deals and portfolios in a way that accounts for economic risks and regulatory charges. Thus, the measures can be easily integrated in an institution’s investment decision process, facilitate strategic planning and limit setting, and help quantify risk appetite.
- Accounting for the RegC charge has economic significance. We use the IACPM/ISDA portfolio to quantify the impact of the RegC constraint. In this exercise, 14.55 percent of the instruments that were otherwise viewed as "favorable" become "unfavorable" from a portfolio-referent investment perspective when we consider both RegC and EC.

To summarize, RegC and EC provide distinct measures, each of which is relevant in its own way. The challenge is that two variables cannot be used to rank-order investments at the same time, so a single decision-making statistic is necessary. In

1See for example Laurie Carver’s article, “Bye, Robot” in Risk, July 2012.
this paper, we address this challenge by formalizing a decision-making variable that incorporates both regulatory and economic considerations. This approach follows traditional portfolio theory and formalizes the notion of a regulatory constraint to derive a decision-making variable akin to RORAC or EVA that incorporates both regulatory and economic considerations.

We organize the remainder of the paper as follows:

» Section 2 introduces a formal metric for estimating RORAC and EVA within the context of an organization that cares about economic risk while facing a RegC constraint.

» Section 3 demonstrates the impact of the RegC constraint through an analysis of the IACPM/ISDA portfolio on asset origination and strategic planning.

» Section 4 offers concluding remarks.

» Appendix A provides a formal derivation of the expression introduced in Section 2.

» Appendix B details the IACPM/ISDA portfolio characteristics.

2 Unified Measures

This section introduces formal decision variables that incorporate economic risk, as well as the equity capital constraint described in Section 1. We begin with a brief review of common variables used in the context of EC and RegC. We then discuss unified measures that can be used when a regulatory capital constraint is introduced. Appendix A also provides formal derivations.

EC is defined as the amount of capital set aside at the analysis date, sufficient enough to absorb losses at horizon, $1 - \alpha$ percent of the time. More formally, portfolio capital at $t$ ($\text{Cap}_{P,\alpha,t}$) is defined implicitly through the following equation:

$$\Pr_t\left[\text{Loss}_{P,t} \geq \text{Cap}_{P,\alpha,t} (1 + r_{D,t})\right] = \alpha$$

Here, $r_{D,t}$ represents the borrowing rate and $\text{Loss}_{P,t}$ represents the distribution of portfolio loss at time $t+1$. Allocation of risk to each asset $j$ is typically measured as Risk Contribution ($\text{RC}_{j,t}$), which is the contribution to portfolio unexpected loss ($\text{UL}_{P,t}$), or tail risk contribution ($\text{TRC}_{j,t}$), which is the contribution to a tail loss event. Formally, each is computed as:

$$\text{RC}_{j,t} = \frac{\partial \text{UL}_{P,t}}{\partial N_{j,t}} = \rho_{j,P} \text{UL}_{j,t}$$

$$\text{TRC}_{j,t} = \frac{\partial \text{Cap}_{P,\alpha,t}}{\partial N_{j,t}} = E \left[ \frac{\text{Loss}_{j,t}}{P_{j,t}(1 + r_{D,t})} \right] = \text{Cap}_{P,\alpha,t}$$

2 In the interest of exposition, we abstract from the definition of Loss in this paper, recognizing that it is equal to a loss reference point less the horizon portfolio value. Depending upon the analysis, the definition can include cash losses as well as credit migration effects. For a detailed discussion, see Levy (2008).

3 For a formal discussion, see Levy (2008) and references therein.
Where \( N_{j,t} \) represents the notional of asset \( j \) invested at time \( t \), and \( P_{j,t} \) represents the price of \( j \) at time \( t \) per unit of \( N_{j,t} \). Capitalization rate (CR) is another variable that is frequently used to allocate \( \text{Cap} \) across the instruments in the portfolio. CR can be defined using either \( RC_{j,t} \) or \( TRC_{a,j,t} \) as the risk basis:

\[
CR_{RC_{j,t}} = \frac{P_{j,t}N_{j,t}RC_{j,t}}{\sum_t P_{j,t}N_{j,t}RC_{j,t}} \frac{\text{Cap}_{p,a,t}}{P_{p,t}}
\]

or

\[
CR_{TRC_{a,j,t}} = \frac{P_{j,t}N_{j,t}TRC_{a,j,t}}{\sum_t P_{j,t}N_{j,t}TRC_{a,j,t}} \frac{\text{Cap}_{p,a,t}}{P_{p,t}}
\]

Traditional strategic planning and asset origination systems that consider economic measures of portfolio-referent risk target a measure of risk-adjusted return (such as RORAC) when determining whether, and how much, to invest in an asset. More formally, one can define a decision variable such as RC- or TRC-based RORAC for asset \( j \) as:

\[
RORAC_{j,t} = \frac{ES_{j,t}}{CR_{j,t}} + r_{D,j}
\]

Here, \( ES_{j,t} = P_{j,t} - r_{D,j} \) represents the excess expected return, or expected spread, where

\[
r_{j,t} = E_t \left[ \tilde{r}_{j,t} \right] = \frac{E_t \left[ \frac{P_{j,t+1} + CF_{j,t+1}}{P_{j,t}} \right]}{P_{j,t}} - 1
\]

is the expected return on asset \( j \) between time \( t \) and \( t+1 \), and \( CF_{j,t+1} \) represents any cash flow per unit of notional at time \( t+1 \). The general idea is that an organization can improve its risk-return profile by investing in projects for which RC- or TRC-based RORAC is greater than that of the portfolio:

\[
RORAC_{j,t} \geq RORAC_{p,t}
\]

For RC-based RORAC, the decision is equivalent to a Sharpe ratio \( \left( SR_{j,t} = ES_{j,t}/RC_{j,t} \right) \) or EVA-based decision rule, where the organization can improve its risk-return profile by investing in projects that satisfy:

\[
SR_{j,t} \geq SR_{p,t} = \frac{ES_{p,t}}{\sigma_{p,t}}
\]

Or, equivalently,

\[
EVA_{j,t} = E_t \left[ \frac{P_{j,t+1}N_{j,t} + CF_{j,t+1}N_{j,t}}{1 + SR_{p,t}RC_{j,t} + r_{D,j}} - P_{j,t}N_{j,t} \right] \geq 0
\]

The use of these measures for decision making can be justified through the same fundamental economic model of a utility maximizing agent that justifies the capital asset pricing model (CAPM). While typically used to describe investment

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\(^4\) In practice, the numerator is frequently measured using a version of net income. The denominator and the numerator are frequently augmented to account for non-credit related returns and risk (for example, operational risk charges). Given the purpose of this paper, we abstract from these issues.
choices for individuals, the models are also used to describe a firm’s optimal choice of investments given the firm’s objective to maximize stakeholders’ utility (for example, shareholders)\(^5\).

That said, the justification of the decision variables outlined above assumes perfect markets with no (regulatory) constraints on borrowing. We now explore which decision variables should be used when facing regulatory constraints. We begin this discussion by defining RegC as the minimum level of book equity to be held for a certain level of assets as defined by regulatory mandates such as the Basel Accord. More formally, RegC is represented in this paper as follows:\(^6\)

\[
\text{BookEquity}_t = \sum_i N_{ij} - D_i \geq \sum_i N_{ij} \cdot RWC_{ij}
\]

In other words, an organization must have sufficient book equity to cover risk-weighted assets multiplied by the minimum capital ratio, or risk-weighted capital (RWC), equivalent to the requirement that the difference between the total portfolio notional of \( \sum_i N_{ij} \) and leverage \( D_i \) be greater than \( RWC_{ij} \).

Notice, RegC is a linear requirement; \( RWC_{ij} \) is independent of name concentration \( N_{ij} \), and independent of correlation effects \( RWC_{ij} \) is not a function of \( N_{ij} \) for \( i \neq j \). And so, RegC is not a risk measure that accounts for correlation and concentration effects.

In the context of RC-based risk measures, the resulting decision variables, augmented with a hat (\(^\wedge\)), reflect the RegC charge:

\[
\wedge SR_{j,t} = \frac{\wedge ES_{j,t}}{RC_{j,t}}
\]

\[
\wedge SR_{P,t} = \frac{\wedge ES_{P,t}}{\sigma_p}
\]

Where

\[
\wedge ES_{j,t} = r_{j,t} - r_{D,t} \cdot f_{RWC_{j,t} - \text{adjustment}}
\]

and

\[
\wedge ES_{P,t} = r_{P,t} - r_{D,t} \cdot f_{RWC_{P,t} - \text{adjustment}}
\]

Where

\[
f_{RWC_{j,t} - \text{adjustment}} = \frac{1 - RWC_{P,t} \left( 1 - RWC_{j,t} \right) / P_{j,t}}{1 - RWC_{P,t}}
\]

\(^5\)See for example Feldstein and Green (1983).

\(^6\)It is worth highlighting that regulatory requirements are more complex than what is described in this stylized setting. For example, multiple classes of liabilities enter into the definitions of Tier capital, or more complex balance sheets recognizing regulatory capital offsets. For the purpose of this discussion, we focus on the simple setting where there are two classes of liabilities, Equity and Debt, and a fully funded long vanilla portfolio.
We also compute a RegC-adjusted EVA, another measure frequently used in decision making:

$$\text{EVA}_{jt} = \frac{E_j \left[ P_{jt} N_{jt} + CF_{jt} N_{jt} \right]}{1 + \text{SR}_{jt} \cdot RC_{jt} + r_{dj} \cdot f_{\text{RWC}_{jt}} \cdot \text{adjustment}} - P_{jt} N_{jt}$$

The relationships have similarities to the classical relationships in the equations above, but have the additional adjustment to the cost of debt that reflects the impact of a binding RegC constraint. Focusing on the Sharpe ratio equation, it is interesting that the RegC constraint affects only in the numerator through $$\text{ES}_{jt}$$, with the RC in the denominator remaining identical to the value obtained in the unconstrained setting. Focusing now on $$\text{ES}_{jt}$$, we see it is similar to the traditional measure of expected spread, with the cost of debt being adjusted by $$f_{\text{RWC}_{jt}} \cdot \text{adjustment}$$.

$$f_{\text{RWC}_{jt}} \cdot \text{adjustment}$$ is a function of $$RWC_{P,j}$$, $$RWC_{jt}$$, and $$P_{jt}$$. Ultimately, the RegC-based Sharpe ratios have very intuitive properties that are worth highlighting:

- SR_{jt} and EVA_{jt} decrease as RWC_{jt} increases.

- SR_{P,j} \leq SR_{jt} under relevant parameters.\(^7\)

- SR_{P,j} decreases with RWC_{P,j} under relevant parameters.\(^8\)

- $$f_{\text{RWC}_{jt}} \cdot \text{adjustment}$$ increases as $$P_{jt}$$ increases. This is an interesting byproduct of the RegC being measured as a proportion of notional rather than price. As a result above (below) par instruments become less (more) attractive, all else equal. Intuitively, an above (below) par asset faces a tighter (looser) borrowing constraint as a proportion of price when compared to notional.

We explore the quantitative impact of these properties in the next section when we introduce the IACPM/ISDA portfolio.

But first, we extend the analysis to capital-based decision rules, and obtain the following conditions that must be met in order for an investment to make sense:

$$\text{RORAC}_{jt, \text{RC}} = \text{SR}_{jt} \sum_j P_{jt} N_{jt} R_{C_{jt}} \geq \frac{\text{Cap}_{P,j}}{\text{Cap}_{P,a,j}} + r_{dj} \text{RORAC}_{P,j} = \frac{\text{ES}_{P,j}}{\text{Cap}_{P,a,j}} + r_{dj}$$

and

\(^7\)The relevant condition is that $$-r_{dj} + r_{dj} RWC_{P,j} N_{P,j} < 0$$ equivalent to $$\frac{N_{P,j}}{P_{P,j}} RWC_{P,j} (1 - RWC_{P,j}) < 1$$ which will be the case given that $$RWC_{P,j}$$ is relatively small.

\(^8\)First taking derivative with respect to RWC,

$$- \frac{-r_{dj} + r_{dj} RWC_{P,j} N_{P,j}}{(1 - RWC_{P,j})^2} + r_{dj} \frac{N_{P,j}}{P_{P,j}} = r_{dj} \left( \frac{-1}{(1 - RWC_{P,j})^2} + \frac{N_{P,j}}{P_{P,j}} \right).$$

Focusing on the last equality, we point out that the value of the portfolio $$\{P_{P,j}\}$$ is unlikely to fall below the capital threshold $$1 - RWC_{P,j}$$ and even less likely to fall below the square of the threshold, while having the financial institution survive as an ongoing concern. Thus, it is safe to say that the derivative is negative under relevant parameters.
Notice that $\hat{\text{Cap}}_{p,t,j}$ and $\hat{\text{TRC}}_{a,t,j}$ are risk measures and are associated with an adjustment for RegC, while above we highlight that RC remains at its original value. The reason is that Loss, which enters into the calculation of capital and TRC, is measured in some cases in excess of expected spread. Expected spread is now adjusted and represented as $\hat{\text{ES}}_{j,t}$.

A subtlety exists in the derivation of the framework as presented Appendix A that is worth highlighting. The derivation of $\hat{\text{ES}}_{j,t}$ produced a more general expression than what we present above:

$$\hat{\text{ES}}_{j,t} = r_{j,t} - r_{\xi,t} + \left( r_{\xi,t} - r_{D,t} \right) \left( 1 - RWC_{C,j,t} \right) / P_{j,t}$$

Here, $r_{\xi,t}$ can be interpreted to represent the sum of the return on a risk-free asset, if the institution faces no borrowing constraints and has a positive premium associated with the borrowing constraint. In other words, if the institution does not face a borrowing constraint, the $r_{\xi,t} = r_{D,t}$. While the functional form of $r_{\xi,t} = \frac{r_{D,t}}{\left( A - B \cdot RWC_{C,j,t} \right)^{C}}$, with open parameters $A$, $B$, and $C$, was chosen to arrive at the equation above, other functional forms can be considered. The appropriateness of one form over another depends upon an institution’s subjective association with the costs associated with the regulatory constraint. The problem is similar in spirit to an organization choosing RC versus TRC in risk allocation, or the appropriate target probability by which to measure capital.

## 3 Impact of the RegC Constraint

This section illustrates the RegC constraint impact on asset selection. We conduct an analysis for a portfolio whose characteristics were designed by the IACPM/ISDA and has been used in their study on the comparison of credit capital models. The portfolio represents a reasonably diversified global loan portfolio that consists of 6,000 term loans associated with 3,000 counterparties across seven countries and different industries. Appendix B provides more details regarding the composition of the portfolio.

We use Moody’s Analytics RiskFrontier™ to estimate unadulterated expected returns as well as portfolio-referent risk (for example, RC, TRC). Required RegC is calculated using the Basel II AIRB capital formula.

Table 1 shows an overview of the analysis results.

<table>
<thead>
<tr>
<th>Table 1 Analysis Results</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Exposure Amount</td>
<td>100 billion USD</td>
</tr>
<tr>
<td>MTM Value</td>
<td>96.4 billion USD</td>
</tr>
<tr>
<td>Expected Spread</td>
<td>1.24 billion USD</td>
</tr>
<tr>
<td>Unexpected Loss</td>
<td>2.21 billion USD</td>
</tr>
<tr>
<td>EC</td>
<td>8.2 billion USD</td>
</tr>
<tr>
<td>RegC</td>
<td>8.32 billion USD</td>
</tr>
<tr>
<td>Traditional Sharpe Ratio</td>
<td>0.562</td>
</tr>
<tr>
<td>RegC-Adjusted Sharpe Ratio</td>
<td>0.553</td>
</tr>
</tbody>
</table>
As depicted Figure 1, a comparison between exposure-level RegC and TRC-based EC shows that they are correlated. However, it is worth noting that the relationship is not linear. RegC tends to be higher than EC for the less risky instruments, and tends to be lower than EC for riskier instruments. This relationship is largely due to the fact that the calculation of RegC and the analysis in RiskFrontier are conducted using different correlation models. RegC is obtained using a single-factor model, and the correlation parameter is calculated as a decreasing function of probability of default (PD) using the following formula specified by the Basel Committee:

$$R = 0.12 \times \frac{1 - e^{-50 \times PD}}{1 - e^{-50}} + 0.24 \times \left(1 - \frac{1 - e^{-50 \times PD}}{1 - e^{-50}}\right)$$

An implication of the Basel correlation formula is that a borrower with higher PD will always have lower correlation. This forced relationship makes it hard to account for the possibility that both PD and correlation of an obligor may increase at the same time, as observed in Cai, Levy, and Patel (2009), Zhang, Zhi and Lee (2008), as well as in several cases, such as the default of Lehman Brothers in 2008. Meanwhile, RiskFrontier utilizes a multi-factor correlation model, and the R-squared value is specified by IACPM/ISDA. Some of the obligors have both high PD and high R-squared.

As described later in this section, the difference in the correlation model is an important factor that drives the RegC constraint. Finally, it is worth highlighting that the RegC does not account for name concentration (EC does), as we measure it in this paper. Name concentration is limited in the IACPM/ISDA portfolio as compared with typical bank portfolios, which frequently have concentrations that vary in orders of magnitude. Thus, we expect the correlation between RegC and EC to be even lower in many cases.

Figure 1  The relationship between TRC-based EC and RegC for the IACPM/ISDA portfolio.

Next, we explore the impact of $f_{RWC_{ij}\text{-adjustment}}$ on expected spread. Figure 2 demonstrates that as RegC increases, the difference between $ES_{i,j}$ and $\overline{ES}_{i,j}$ increases. This is as expected given the properties of the RegC-adjustment outlined in

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9The relationship between RegC and RC-based capital is similar and excluded for brevity.


11For details, see IACPM/ISDA (2006).
the section above. With the median expected spread in this portfolio at 35bps, the impact is non-trivial. We explore the impact of the RegC-adjustment on portfolio choice in greater detail below.

Before we proceed, notice that the relationship between RegC and the impact on expected spread is not strictly monotonic. This is, in part, a result of $P_{jt}$ entering into $R_{Wj} - \text{adjustment}$. As discussed in Section 2, this is driven by RegC being measured as a proportion of notional rather than price.

Figure 2  The relationship between RegC and the difference between the traditional expected spread ($ES_{jt}$) and RegC-adjusted expected spread ($\bar{ES}_{jt}$).

Now, focusing on RegC-adjusted Sharpe ratio, it is only slightly lower when compared with the traditional Sharpe ratio measure. However, we will see a wide range of cross-sectional variation in the impact of the RegC-adjusted Sharpe ratio at the exposure level. In general, the exposures with higher traditional Sharpe ratios tend to also have higher RegC-adjusted Sharpe ratios, as shown in Figure 3.

For some exposures, the Sharpe ratio changes significantly after the RegC constraint is imposed. For instance, the traditional Sharpe ratio of the circled exposure in Figure 3 is approximately 0.7, much higher than the portfolio’s traditional Sharpe ratio. This implies that one should increase the weight of this instrument in order to improve the return-to-risk ratio of the entire portfolio. However, with the constraint, the Sharpe ratio of this exposure decreases to less than 0.3, indicating that one should reduce its weight in the portfolio. As reported in the graph, the correlation between the RegC-adjusted Sharpe ratio and the traditional Sharpe ratio for this portfolio is 87%.
To find out what exposure characteristics may affect the impact of the RegC constraint upon the investment decision(s), we break down the overall portfolio into three categories. The exposures in the first category have lower-than-portfolio traditional Sharpe ratios but higher-than-portfolio RegC-adjusted Sharpe ratios. The second category represents the opposite case: exposures have higher-than-portfolio traditional Sharpe ratios but lower-than-portfolio RegC-adjusted Sharpe ratios. All other exposures belong to the third category, meaning that either both traditional and RegC-adjusted Sharpe ratios are higher than portfolio, or both are lower than portfolio.

Table 2  Portfolio Categories and Exposure Characteristics

<table>
<thead>
<tr>
<th>Category</th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Exposures</td>
<td>30</td>
<td>873</td>
<td>5,097</td>
</tr>
<tr>
<td>Average PD</td>
<td>0.0685</td>
<td>0.00477</td>
<td>0.0297</td>
</tr>
<tr>
<td>Average LGD</td>
<td>0.548</td>
<td>0.568</td>
<td>0.561</td>
</tr>
<tr>
<td>Average Maturity</td>
<td>5.099</td>
<td>2.089</td>
<td>2.511</td>
</tr>
<tr>
<td>Average RSQ</td>
<td>0.242</td>
<td>0.158</td>
<td>0.201</td>
</tr>
<tr>
<td>Average Basel RSQ</td>
<td>0.145</td>
<td>0.222</td>
<td>0.190</td>
</tr>
<tr>
<td>Average RWC</td>
<td>0.173</td>
<td>0.0422</td>
<td>0.0916</td>
</tr>
<tr>
<td>Average EC</td>
<td>0.232</td>
<td>0.0125</td>
<td>0.0678</td>
</tr>
</tbody>
</table>
We observe some patterns from the table:

- The number of instruments in Category 2 is higher than that in Category 1. In other words, more instruments will have lower-than-portfolio Sharpe ratios with the RegC constraint.

- Exposures in Category 1 have higher RegC. This finding may seem to be counterintuitive, since it means that an exposure that requires more RegC will become more attractive with the constraint. However, if we look at the difference between RegC and EC, we can see that, relatively speaking, capital calculated using the Basel formula is lower than that obtained from the EC model for Category 1.

- Exposures in Category 1 have higher PD and higher R-squared, but lower Basel correlation. As discussed earlier, correlation is decreasing in PD in the Basel formula while using the IACPM/ISDA-provided R-squared, and exposures with high PD can also have high correlation. These exposures will appear to be less risky using the Basel correlation formula.

- Exposures in Category 1 have longer maturity. The implication is that the migration model used in EC analysis has a more pronounced impact on risk than the Basel maturity adjustment.

- All three categories have about the same average loss given default (LGD), implying that LGD does not affect the impact of the RegC constraint on Sharpe ratio. This finding is not surprising since the value and treatment of LGD are similar in RegC calculation and EC analysis. However, it is worth mentioning that this may not be the case if PD-LGD correlation is accounted for in the EC model.

To summarize our findings, accounting for RegC along with EC can make a substantial difference in the rank-ordering of instruments. In particular, when RegC is high relative to allocated EC, an instrument is more likely to be reclassified as "unfavorable" once accounting for the regulatory constraint.

4 Conclusion

RegC and EC are both relevant in investment decisions and strategic planning. In this paper, we design a model that considers both in a single RORAC- or EVA-style decision variable. The variable is intuitive, in that economic risk is still the relevant risk measure, with RegC entering into the decision as a cost or tax that decreases the return on investment.

The RegC-adjusted measure has appealing properties: the RegC-adjusted portfolio Sharpe ratio is lower than the traditional Sharpe ratio, and the RegC-adjusted portfolio Sharpe ratio for both the portfolio and instrument decrease with their respective RWC. We find that the impact on strategic planning and decision making when accounting for RegC and EC in the proposed fashion can be substantial. When analyzing the IACPM/ISDA portfolio, 14.55 percent of the instruments that were otherwise viewed as favorable become unfavorable when we consider both RegC and EC, with instruments that have higher RegC relative to allocated EC being impacted the most.
Appendix A Formalized Framework

This appendix formalizes the framework introduced in Section 2. We begin with a utility maximizing agent and derive the agent’s optimal investment strategy in a setting where the agent faces no RegC constraint. We then derive the optimal investment strategy in a setting where the agent faces a binding RegC constraint. As discussed in the main text, while this framework is typically used to describe investment choices for individuals, the models are also used to describe a firm’s optimal choice of investments given the firm’s objective to maximize stakeholders’ utility (for example, shareholder’s).¹²

The unconstrained optimization problem:

It is well known that complete markets and a representative agent with quadratic utility results in a conditional CAPM relationship. Specifically, a representative agent maximizes lifetime utility by trading off consumption and investment at each period:

\[
U(C_t) = \max \sum_{t} \gamma^t (C_t - bC^2_t)
\]

Subject to:

\[
C_t = \sum_j (P_{jt} + CF_{jt})N_{jt-1} - D_{jt}(1 + r_{jt}) - \sum_j P_{jt}N_{jt} + D_t
\]

The first order conditions, along with the existence of a risk-free asset imply that the agent will want to hold assets to the point where:

\[
\frac{ES_{jt}}{RC_{jt}} = \frac{ES_{P,t}}{\sigma_{P,t}} \quad \forall j
\]

In other words, at the optimal portfolio, the Sharpe ratio (or ratio of expected excess return to risk) is equal to the market Sharpe ratio for all instruments.

If there is an opportunity to invest in a new asset \( j \), the institution can compare the price of the asset with the present value of future expected cash flows. Discounting will be relative to the amount of risk the asset adds to the portfolio. The asset should be purchased as long as:

\[
\frac{ES_{jt}}{RC_{jt}} \geq \frac{ES_{P,t}}{\sigma_{P,t}}
\]

Or, equivalently:

\[
P_{jt} \leq \frac{E[ P_{jt+1} + CF_{jt+1} ]}{1 + RC_{jt} \frac{ES_{jt}}{\sigma_{jt}} + r_{jt}}
\]

Within the context of a general equilibrium setting, the framework conforms to the CAPM structure where all asset returns adhere to the following relationship:

¹²See for example Feldstein and Green (1983).
\[ E_t \left[ r_{jt} - r_{D,t} \right] = \beta_{jt} E_t \left[ r_{\text{market},t} - r_{D,t} \right] \]

Where

\[ \beta_{jt} = \frac{\text{Cov}(r_{\text{market},t}, r_{jt})}{\sigma_{\text{market},t}^2}. \]

Optimizing with a binding RegC constraint:

\[ \max \; U(C) = E_0 \left[ \sum_t \gamma^t \left( C_t - bC_t^2 \right) \right] \]

s.t.

\[ C_t = \sum_j \left( P_{jt} + CF_{jt} \right) N_{jt-1} - D_t (1 + r_{D,t}) - \sum_j P_{jt} N_{jt} + D_t \]

Equity_t = \sum_j N_{jt} - D_t \geq \sum_j N_{jt}RWC_{jt}\]

When the RegC constraint is binding, the second constraint becomes:

\[ D_t = \sum_j (1 - RWC_{jt}) N_{jt} \]

and can be rolled into the consumption/wealth constraint:

\[ C_t = \sum_j \left( P_{jt} + CF_{jt} \right) N_{jt-1} - \sum_j (1 - RWC_{jt-1}) N_{jt-1} (1 + r_{D,t}) - \sum_j P_{jt} N_{jt} + \sum_j (1 - RWC_{jt}) N_{jt} \]

The first order conditions of \( U(C) \) with respect to \( N_{jt} \) yields:

\[ \frac{\partial U}{\partial N_{jt}} = \frac{\partial}{\partial N_{jt}} E_0 \left[ u(C_t) + u(C_{t+1}) \right] \]

\[ = E_0 \left[ u'(C_t) \cdot (-P_{jt} + (1 - RWC_{jt})) + u'(C_{t+1}) \cdot (P_{jt+1} + CF_{jt+1} - (1 - RWC_{jt}) \cdot (1 + r_{D,t})) \right] \]

\[ = 0 \]

Recognizing that \( u'(C_t) = \gamma t \cdot (1 - 2bC_t^2) \) yields:

\[ E_0 \left[ \gamma t \cdot (1 - 2bC_{t+1}) \cdot (P_{jt+1} - (1 - RWC_{jt})) \right] = E_0 \left[ \gamma t+1 \cdot (1 - 2bC_{t+1}) \cdot (P_{jt+1} + CF_{jt+1} - (1 - RWC_{jt}) \cdot (1 + r_{D,t})) \right] \]

Taking expectations at time \( t \) and rearranging yields:

\[ 1 = E_t \left[ g \cdot \frac{1 - 2bC_{t+1}}{1 - 2bC_j} \cdot \frac{P_{jt+1} + CF_{jt+1} - (1 - RWC_{jt}) \cdot (1 + r_{D,t})}{P_{jt} - (1 - RWC_{jt})} \right] \]

Define the risk charge adjusted return to be:
\[ \hat{r}_{j,t} = E_t \left[ \frac{P_{j,t+1} + CF_{j,t+1} - (1 - RWC_{j,t}) \cdot (1 + r_{D,t})}{P_{j,t} - (1 - RWC_{j,t})} - 1 \right] \]

And we can represent the equation as follows:

\[ 1 = E_t \left[ \gamma \frac{1 - 2bC_{t+1}}{1 - 2bC_t} \cdot (1 + \tilde{r}_{j,t}) \right] = Cov_t \left[ \gamma \frac{1 - 2bC_{t+1}}{1 - 2bC_t} \cdot (1 + \tilde{r}_{j,t}) \right] + E_t \left[ \gamma \frac{1 - 2bC_{t+1}}{1 - 2bC_t} \cdot (1 + \tilde{r}_{j,t}) \right] \]

Rewriting

\[ \frac{1 - 2bC_t}{\gamma E_t [1 - 2bC_{t+1}]} + 1 + \hat{r}_{j,t} = \frac{-Cov_t \left[ 1 - 2bC_{t+1}, \tilde{r}_{j,t} \right]}{E_t [1 - 2bC_{t+1}]} \]

This equation also holds for the portfolio:

\[ \frac{1 - 2bC_t}{\gamma E_t [1 - 2bC_{t+1}]} + 1 + \hat{r}_{P,t} = \frac{-Cov_t \left[ 1 - 2bC_{t+1}, \tilde{r}_{P,t} \right]}{E_t [1 - 2bC_{t+1}]} \]

Subtracting the equation for \( j \) from the portfolio yields:

\[ \hat{r}_{P,t} - \hat{r}_{j,t} = \frac{-Cov_t \left[ 1 - 2bC_{t+1}, \tilde{r}_{P,t} \right]}{E_t [1 - 2bC_{t+1}]} - \frac{-Cov_t \left[ 1 - 2bC_{t+1}, \tilde{r}_{j,t} \right]}{E_t [1 - 2bC_{t+1}]} \]

With \( C_t = \tilde{r}_{P,t} \) we have

\[ \hat{r}_{P,t} - \hat{r}_{j,t} = \frac{2b\sigma_{P,t}}{1 - 2b\delta_{P,t}} - \frac{2b\delta_{P,t}}{1 - 2b\delta_{P,t}} \]

Rewriting:

\[ \frac{\hat{r}_{P,t} - \hat{r}_{j,t}}{\delta_{P,t} - \hat{r}_{P,t}} = \frac{2b\delta_{P,t}}{1 - 2b\delta_{P,t}} \equiv \hat{\lambda}_{P,t} \]

In equilibrium, this equation holds for all assets. Rewriting yields:

\[ \frac{\hat{r}_{j,t} + \hat{\lambda}_{P,t} \sigma_{P,t} - \hat{r}_{P,t}}{RC_{j,t}} = \hat{\lambda}_{P,t} \]

We define \( r_{\xi,t} = \hat{r}_{P,t} - \hat{\lambda}_{P,t} \sigma_{P,t} \), which under complete markets results in \( r_{\xi,t} = r_{D,t} \) with all of the equations collapsing to the ones presented in the unconstrained problem. Expanding and renormalizing the numerator and denominator by \( P_{j,t} \) rather than \( P_{j,t} - \left( 1 - RWC_{j,t} \right) \) we obtain our RegC-adjusted Sharpe ratio equation:

\[ \boxed{SR_{j,t} = \frac{r_{j,t} - r_{\xi,t} + (r_{\xi,t} - r_{D,t}) \left( 1 - RWC_{j,t} \right)}{RC_{j,t} / P_{j,t}}} \]
The final step is parameterization of \( r_{\xi,j} \). One option is to parameterize the utility function parameter \( b \) using the relationship above,

\[
\frac{\hat{\nu}_{P,j} - \bar{\nu}_{j,t}}{\hat{\sigma}_{P,j} - RC_{P,j}} = \frac{2b\hat{\nu}_{P,j}}{1 - 2b\hat{\nu}_{P,j}},
\]

under complete markets. We can then use that utility parameter to solve for \( r_{\xi,j} \). Unfortunately quadratic utility implies that absolute and relative risk aversion are either increasing or constant, as wealth \( (W) \) increases, and the parameter can result in odd dynamics for various levels of RWC. Instead, we choose a parametric form of

\[
r_{\xi,j} = \frac{r_{D,j}}{(A - B \cdot RWC_P)^C}
\]

where parameters \( A, B, \) and \( C \) can be calibrated to best reflect portfolio and organization dynamics. The functional form is chosen so that \( r_{\xi,j} \geq r_{D,j} \) and \( r_{\xi,j} \) increases with \( RWC_{P,j} \). The condition that \( r_{\xi,j} \geq r_{D,j} \) is necessary in order for the RegC constraint to be binding, and the condition that \( r_{\xi,j} \) increases with \( RWC_{P,j} \) intuitively follows from the fact that the impact of the constraint will be more severe as \( RWC_{P,j} \) increases.

In addition, one can further refine the parameters by setting boundary conditions at \( RWC_{P,j} = 1 \) where the \( r_{\xi,j} = \infty \) implying \( A = B \). In addition, the boundary condition \( RWC_{P,j} = 0 \) where the constraint should not be binding, and \( r_{\xi,j} = r_{D,j} \) implies that \( A = 1 \). Finally, we set \( C = 1 \) when reporting results in Section 3. The final relationship implies:

\[
\bar{ES}_{j,t} = r_{j,t} - \frac{r_{D,j}}{1 - RWC_{P,j}} + \frac{r_{D,j} RWC_{P,j}}{1 - RWC_{P,j}} \left( 1 - RWC_{j,t} \right) / P_{j,t}
\]

Or, alternatively

\[
\bar{ES}_{j,t} = r_{j,t} - r_{D,j} \cdot f_{RWC_{j,t} - adjustment}
\]

Where

\[
f_{RWC_{j,t} - adjustment} = \frac{1 - RWC_{P,j} \left( 1 - RWC_{j,t} \right) / P_{j,t}}{1 - RWC_{P,j}}
\]
Appendix B  Test Portfolio Description

This appendix provides a detailed description of the test portfolio used in the numerical example described in Section 3. The following description is found in the IACPM/ISDA publication *Convergence of Credit Capital Models*.

The $100 billion test portfolio is comprised of two term loans, to each of 3,000 obligors, across a diverse set of industries (643 NAICS codes), and seven countries dispersed along eight whole-grade rating buckets and varying LGD values. Exposure amounts vary from $1MM to $1.250MM, and tenors ranged from six months to seven years. “R-squares” (the degree to which obligors exhibit systematic vs. idiosyncratic risk) varied from 10% to 65%. Contractual spreads over a risk-free rate are chosen so that the mark-to-market value of the exposures at time zero, relative to specified required market spreads, would be approximately par. The characteristics of the test portfolio are provided as follows:

**Portfolio Size**
- Exposures: 6,000
- Portfolio Size: $100 Billion

**Obligors**
- Number of Obligors: 3,000
- Rating Scheme: eight ratings buckets
- Credit rating: Average = BBB
- Industry Classifications: 61 Moody’s Analytics industries, 643 NAICS Codes (6 digit)
- Countries: seven countries

**Facilities**
- Facility Type: 100% Term Loans
- Fixed vs. Floating: 100% Floating Rate

**Exposure Distribution by Facility**
- Mean: $16.7 million
- Standard Deviation: $101.7 million
- Minimum: $1 million
- Maximum: $1.250 million

**Tenor Distribution by Facility**
- Mean: 2.5 years
- Standard Deviation: 1.7 years
- Minimum: six months
- Maximum: seven years

We analyze the portfolio with the average LGD set at 56.8% and PD values based upon the following PD-rating mapping table.

**Table 3  PD-Rating Mapping**

<table>
<thead>
<tr>
<th>Rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
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</thead>
<tbody>
<tr>
<td>PD</td>
<td>0.01%</td>
<td>0.063%</td>
<td>0.063%</td>
<td>0.534%</td>
<td>3.81%</td>
<td>9.96%</td>
<td>25.5%</td>
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References


