Linking Stress Testing and Portfolio Credit Risk
Agenda

1. Stress testing and portfolio credit risk are related
2. Estimating portfolio loss distribution under a scenario
3. Reverse stress testing
4. CCAR style multi-period stress testing
5. Conclusion
Stress testing and portfolio credit risk are related
Stress testing and portfolio credit risk could be considered separately...

**Stress testing exercise**
- Economic scenario
- Estimated econometric model

\[ \text{Stressed PD} \times \text{Stressed LGD} = \text{Stressed EL} \]

**Portfolio level credit risk model**
- Correlations
- Instrument inputs
- Concentrations
- Portfolio loss distribution

99.9\textsuperscript{th} Percentile: Economic Capital
...however, there exist some natural links

**Stress testing exercise**

**Portfolio level credit risk model**

\[ \text{Stressed PD} \times \text{Stressed LGD} = \text{Stressed EL} \]

Output: Stressed EL
- Can the loss under the scenario differ from the stressed EL? By how much?
- Comparing stressed EL and Economic Capital

Output: Loss distribution
- Monte Carlo simulation is used to generates draws of factors to estimate the loss distribution.
- Can the simulation output (draws/losses) be linked to economic scenarios?
How can we model the links between stress testing and portfolio credit risk?

- **Draws of borrower specific credit risk factors**
  - \( \Phi_{CR1}, \Phi_{CR2}, \ldots \)
  - Joint distribution with correlation matrix \( CM \)

- **Draws of systematic credit risk factors**
  - \( 1 - RSQ \)

- **Draws of asset returns (credit quality changes)**
  - \( PD, LGD, EAD, \) Credit Migration

- **Correlations of GCorr systematic factors and standard normal macroeconomic factors (\( \Phi_{MV} \))**

- **Mapping between \( \Phi_{MV} \) and macroeconomic variables (\( MV \))**

- **Credit portfolio loss distribution on a horizon**
  - EL given a macroeconomic shock
  - Range of losses given a macroeconomic shock

- **GCorr Macro**
Correlating credit risk factors and macroeconomic variables allows for various analyses...

» **Portfolio loss distribution under a scenario**
  - For a macroeconomic scenario, we can determine the conditional distribution of the portfolio losses over one period (also called the stressed distribution).
  - Knowing the distribution allows us to estimate not only the stressed expected loss, but also percentiles. What is the probability that the losses will exceed EC under the scenario?
  - How to interpret location and shape of the conditional distribution?

» **Reverse stress testing**
  - What kind of scenarios and which factors are associated with large losses?
  - Identifying hidden risks within the credit portfolio.

» Framework can be used for a **multi-period stress testing**
  - Analytical calculations can be used to estimate multi-period stressed expected losses. No need for Monte Carlo simulation.
  - Can be applied to CCAR style stress testing.
Estimating portfolio loss distribution under a scenario
How to estimate the portfolio loss distribution under a scenario?

1. Specify macroeconomic scenario
2. Standard normal macroeconomic shocks under the scenario
3. Calculate conditional distribution of systematic credit risk factors
4. Simulate draws of credit risk factors from conditional distribution
5. Stressed portfolio loss distribution
   - Stressed EL, percentiles of the distribution, $P(\text{Stressed L} > \text{EC})$

Correlations of GCorr systematic factors and standard normal macroeconomic factors ($\phi_{MV}$):

- $\phi_{CR}$
- $CM$
- $\phi_{MV}$
## Consider four portfolios...

<table>
<thead>
<tr>
<th></th>
<th>U.S. Corporates</th>
<th>Global Corporates</th>
<th>U.S. CRE</th>
<th>U.S. Retail</th>
<th>Aggregate Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Commitment</strong></td>
<td>35 billion USD</td>
<td>26 billion USD</td>
<td>13 billion USD</td>
<td>13 billion USD</td>
<td>87 billion USD</td>
</tr>
<tr>
<td><strong># exposures</strong></td>
<td>2,266</td>
<td>3,734</td>
<td>130</td>
<td>250 (homogenous pools)</td>
<td>6,380</td>
</tr>
<tr>
<td><strong># counterparties</strong></td>
<td>1,133</td>
<td>1,867</td>
<td>130</td>
<td>250 (homogenous pools)</td>
<td>3,380</td>
</tr>
<tr>
<td><strong>Weighted Avg. PD</strong></td>
<td>1.40%</td>
<td>1.58%</td>
<td>1.29%</td>
<td>0.79%</td>
<td>1.35%</td>
</tr>
<tr>
<td><strong>Weighted Avg. LGD</strong></td>
<td>40%</td>
<td>40%</td>
<td>25%</td>
<td>40%</td>
<td>38%</td>
</tr>
<tr>
<td><strong>Weighted Avg. RSQ</strong></td>
<td>36%</td>
<td>39%</td>
<td>35%</td>
<td>7%</td>
<td>32%</td>
</tr>
<tr>
<td><strong>Locations and Types of Exposures</strong></td>
<td>Diversified across U.S. industries (Automotive, Oil Refining, Telephone, Paper, Banks, etc.).</td>
<td>Exposures from Japan, Europe, Australia. Diversified across industries, similarly to the U.S. portfolio.</td>
<td>U.S. Commercial Real Estate Exposures</td>
<td>U.S. Residential Mortgage Exposures</td>
<td>Combination of the 4 portfolios</td>
</tr>
<tr>
<td><strong>Concentration</strong></td>
<td>The 10 largest exposure account for 14% of the total commitment.</td>
<td>The 10 largest exposure account for 8% of the total commitment.</td>
<td>The 10 largest exposure account for 66% of the total commitment.</td>
<td>The 10 largest pools account for 37% of the total commitment.</td>
<td></td>
</tr>
<tr>
<td><strong>EL</strong></td>
<td>0.69%</td>
<td>0.83%</td>
<td>0.47%</td>
<td>0.38%</td>
<td>0.65%</td>
</tr>
<tr>
<td><strong>Capital wrt EL, 10bps</strong></td>
<td>12.49%</td>
<td>11.31%</td>
<td>16.35%</td>
<td>8.38%</td>
<td>9.80%</td>
</tr>
</tbody>
</table>
CCAR 2013 Severely Adverse scenario over one year

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Return over period 2012 Q3 – 2013 Q3 given by the scenario</td>
<td>-0.05</td>
<td>0.35</td>
<td>0.48</td>
<td>-0.72</td>
<td>-0.11</td>
<td>1.35</td>
</tr>
<tr>
<td>Corresponding standard normal shock – $\phi_{MY}$ under the scenario</td>
<td>-2.07</td>
<td>1.6</td>
<td>2.44</td>
<td>-2.55</td>
<td>-1.63</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Putting the CCAR 2013 Severely Adverse scenario in a perspective...

Time series of annual log changes on Dow Jones Total Stock Market Index

Comparison: The worst annual return occurred during the crisis and had a magnitude of about -0.5.
Selecting a set of macroeconomic variables for each of the portfolios…

- Objective: determine a set of macroeconomic variables with as much explanatory power as possible, while preserving economically intuitive relationship between losses and the macroeconomic variables.

\[
N^{-1}(Loss) = \sum_{k} \beta_k \phi_{MV,k} + \alpha + \varepsilon
\]

<table>
<thead>
<tr>
<th></th>
<th>U.S. Corporates</th>
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<th>U.S. CRE</th>
<th>U.S. Retail</th>
<th>Aggregate Portfolio</th>
<th>Global Corporates</th>
<th>Aggregate Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. - Real GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. – Unempl. Rate</td>
<td>0.060</td>
<td></td>
<td>0.040</td>
<td>0.069</td>
<td>0.087</td>
<td>0.082</td>
<td></td>
</tr>
<tr>
<td>U.S. – BBB Yield</td>
<td>0.068</td>
<td></td>
<td></td>
<td>0.064</td>
<td>0.050</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>U.S. – Dow Jones</td>
<td>-0.078</td>
<td>-0.051</td>
<td>-0.040</td>
<td>-0.013</td>
<td>-0.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. – House Price Index</td>
<td>-0.054</td>
<td>-0.054</td>
<td>-0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. - CRE Index (NCREIF)</td>
<td>-0.043</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. – VIX</td>
<td>0.055</td>
<td></td>
<td></td>
<td>0.040</td>
<td>0.050</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>Japan – Real GDP</td>
<td>-0.031</td>
<td></td>
<td></td>
<td></td>
<td>-0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K. – Real GDP</td>
<td>-0.035</td>
<td></td>
<td></td>
<td></td>
<td>-0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe – STOXX 50 Stck. Mkt.</td>
<td>-0.008</td>
<td></td>
<td></td>
<td></td>
<td>-0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan – Nikkei Stock Market</td>
<td>-0.093</td>
<td></td>
<td></td>
<td></td>
<td>-0.027</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K. – FTSE Stock Market</td>
<td>-0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.911</td>
<td>-1.732</td>
<td>-1.738</td>
<td>-2.015</td>
<td>-1.867</td>
<td>-1.732</td>
<td>-1.867</td>
</tr>
<tr>
<td>R-squared</td>
<td>54%</td>
<td>47%</td>
<td>26%</td>
<td>24%</td>
<td>59%</td>
<td>38%</td>
<td>53%</td>
</tr>
</tbody>
</table>
Distribution of the U.S. Corporates portfolio losses under the CCAR Severely Adverse scenario...

**Macroeconomic variables:** Unemployment Rate, BBB Corporate Yield, Dow Jones Total Stock Market Index, and VIX

Density of losses over one year

- **Unconditional distribution**
- **Stressed distribution = distribution under the scenario**

### Statistics of the stressed loss distribution

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>8.42%</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>3.15%</td>
</tr>
<tr>
<td><strong>P25</strong></td>
<td>6.22%</td>
</tr>
<tr>
<td><strong>P50 (Median)</strong></td>
<td>8.08%</td>
</tr>
<tr>
<td><strong>P75</strong></td>
<td>10.27%</td>
</tr>
<tr>
<td><strong>P99</strong></td>
<td>17.39%</td>
</tr>
<tr>
<td><strong>P(Stressed L &gt; EC)</strong></td>
<td>7.13%</td>
</tr>
</tbody>
</table>

- **Stressed EL = 8.42%**. Percentile of the unconditional distr. = 99.45%
- **Capital (EL + Capital wrt EL) = 13.18%**. Percentile of the unconditional distr. = 99.9% (10bps target probability)
Stressed distribution of the aggregate portfolio losses...

Macroeconomic variables: Unemployment Rate, BBB Corporate Yield, Dow Jones Total Stock Market Index, and VIX

Stressed distribution depends on the joint distribution of individual loss distributions. This in turns depends on the correlations among portfolios, which is determined by the factor structure.

Note: Capital = EL + Capital wrt EL
What determines the stressed distribution?

The stressed distribution is a combination of two effects: severity of the scenario and correlation of the portfolio and macroeconomic variables.

Single-factor, default/no-default example: PD=2% and RSQ=10%.

- $\phi_{MV}$ – shock
- $\rho_{MV,CR}$ – correlation of risk factor and macroeconomic shock

The stressed distribution is a combination of two effects: severity of the scenario and correlation of the portfolio and macroeconomic variables.

$\phi_{MV} = -3$ and $\rho_{MV,CR} = 100\%$

Stressed EL = 12.2%

EC = 12.8%

$\phi_{MV} = -3$ and $\rho_{MV,CR} = 60\%$

Stressed EL = 6.5%

EC = 12.8%

$\phi_{MV} = -2$ and $\rho_{MV,CR} = 100\%$

Stressed EL = 6.7%

EC = 12.8%

$\phi_{MV} = -2$ and $\rho_{MV,CR} = 60\%$

Stressed EL = 4.4%

EC = 12.8%
How to interpret the stressed loss distribution?

General patterns:
- The *more adverse scenario*, the more the stressed distribution is shifted to the area of *large losses*.
- The *more the macroeconomic variables explain systematic risk* of the portfolio, the *less dispersion* in the stressed distribution and the *higher the stressed expected loss*.

Why should be there any dispersion in the stressed distribution at all?
- Assume that macroeconomic variables do not completely explain systematic risk of a large diversified credit portfolio.
- In this case, conditioning on macroeconomic variables does not completely determine portfolio loss, and hence the loss can differ from the conditional expected loss.
- What conditioning on macroeconomic variables does is shifting and changing the shape of the portfolio loss distribution.

Example: a real-estate portfolio stressed using a national house-price index.
- Specifying a national-house price index does not completely determine situations in house market in individual MSAs. The residual movements in these individual markets drive the dispersion in the stressed distribution.
Reverse stress testing
What is reverse stress testing analysis?

» The general question is – what are the scenarios that can lead to large losses?

   – For example FSA presents stress testing as “…a new requirement for a firm explicitly to identify and assess the scenarios most likely to cause its current business plan to become unviable…”

   …The intention behind the introduction of this new requirement is to encourage firms: first, to explore more fully the vulnerabilities of their current business plan (including ‘tail risks’ as well as milder adverse scenarios); second, to make decisions that better integrate business and capital planning; and third, to improve their contingency planning…”

   Source: Stress and scenario testing by Financial Services Authority, December 2008

» More specifically:

   – Which business lines cause most vulnerability of the bank? In other words, if there is a tail event, which of its business lines have likely contributed to it most?

   – What kind of macroeconomic scenarios are associated with a tail event?
How to conduct a reverse stress testing analysis?

**Monte Carlo simulation:**
In each trial generate draws of the factors and calculate the portfolio loss.

**Output of Monte Carlo simulation**

<table>
<thead>
<tr>
<th>Trial</th>
<th>Simulated macroeconomic factors</th>
<th>Simulated systematic credit risk factors</th>
<th>Portfolio loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\phi_{MV1}, \phi_{MV2}, \ldots$</td>
<td>$\phi_1, \phi_2, \ldots$</td>
<td>$L_{Trial\ 1}$</td>
</tr>
<tr>
<td>2</td>
<td>$\phi_{MV1}, \phi_{MV2}, \ldots$</td>
<td>$\phi_1, \phi_2, \ldots$</td>
<td>$L_{Trial\ 2}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Possible conversion to observable macroeconomic variables $MV_1, MV_2, \ldots$ using mappings

Select the trials where losses fell within an interval:
Distribution of systematic credit risk factors across the trials?
Distribution of macroeconomic variables across the trials?
### Scenarios associated with the tail event…

#### Output of Monte Carlo simulation

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>61,523</td>
<td>0.14</td>
<td>-0.22</td>
<td>0.03</td>
<td>-3.30</td>
<td>-3.37</td>
<td>-3.17</td>
<td>-3.37</td>
</tr>
<tr>
<td>55,461</td>
<td>0.15</td>
<td>-0.56</td>
<td>0.07</td>
<td>-2.22</td>
<td>-2.30</td>
<td>-2.17</td>
<td>-2.40</td>
</tr>
<tr>
<td>31,358</td>
<td>0.56</td>
<td>-1.10</td>
<td>0.11</td>
<td>-2.83</td>
<td>-2.37</td>
<td>-2.97</td>
<td>-3.05</td>
</tr>
<tr>
<td>11,401</td>
<td>0.63</td>
<td>-0.31</td>
<td>-0.08</td>
<td>-3.26</td>
<td>-3.02</td>
<td>-3.40</td>
<td>-2.33</td>
</tr>
</tbody>
</table>

#### Portfolio Losses

<table>
<thead>
<tr>
<th></th>
<th>US Corp</th>
<th>Global Corp</th>
<th>US CRE</th>
<th>US Retail</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.7</td>
<td>26.1</td>
<td>13.0</td>
<td>13.0</td>
<td>85.9</td>
<td></td>
</tr>
</tbody>
</table>

One of the scenarios associated with extreme losses.
Reverse stress testing of corporate portfolios: which countries exhibit adverse shocks in the tail event?

U.S. Corporates
Trials where L > EC. Average of factor (custom index) draws across these trials.

Interpretation
Unconditionally:
Mean = 0, std = 1
The tail event:
Mean = -3.3, std = 0.3

Global Corporates
Trials where L > EC. Average of factor (custom index) draws across these trials.
Reverse stress testing of the aggregate portfolio: which countries exhibit adverse shocks in the tail event?

Trials where $L > EC$. Average of factor (custom index) draws across these trials.

**Aggregate Portfolio**

Emerging Markets

USA, Japan, Australia, Europe
Reverse stress testing of the CRE portfolio: which markets exhibit the biggest distress in the tail event?

Markets with the largest shocks in the tail event

The tail event is associated with severe shocks in the Hotel markets in the southwest of the U.S. and several other markets (Hotel, Multi-Family Housing, Retail) in Florida and NY-NJ-PA area

U.S. CRE Portfolio

The tail event is associated with less severe shocks in the Industrial and Office property markets in various parts of the country (Midwest and South, for example).

Markets with the smallest shocks in the tail event
Taking reverse stress testing one step further...

The tail event = Aggregate portfolio losses exceed its 99th or 99.9th percentile.

What is the distribution of the macroeconomic variables in the case of this tail event?

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Unconditional</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution Mean</td>
<td>0.02</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Std</td>
<td>0.19</td>
<td>0.20</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>1% Tail</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.33</td>
<td>-0.30</td>
<td>-0.08</td>
</tr>
<tr>
<td>Std</td>
<td>0.31</td>
<td>0.33</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>0.1% Tail</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.45</td>
<td>-0.53</td>
<td>-0.11</td>
</tr>
<tr>
<td>Std</td>
<td>0.41</td>
<td>0.44</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Distribution in the tail event (beyond the 99.9th percentile) Based on only 100 observations. To achieve a higher precision, more trials are required.
How to interpret reverse stress testing results?

» A reverse stress testing analysis answers the following question:
  – What are the scenarios, as defined by factors or macroeconomic variables, that we can observe in the tail event?

» Results of a reverse stress testing analysis can help reveal hidden vulnerabilities in a credit portfolio.
  – Results are not given merely by geographical or sector distribution of exposures. They depend on a combination of effects:
    » Stand-alone characteristics of the exposures (PD, LGD), impact of systematic risk (RSQ), correlations across the systematic factors, correlations of systematic factors with macroeconomic variables.

» There is not a single scenario corresponding to the tail event:
  – In mathematical terms, the factors and macroeconomic variables still exhibit dispersion across the trials where losses are large.
CCAR style multi-period stress testing
What is the impact of correlation between a credit risk factor and a macroeconomic variable?

> \( \phi_{US,Oil} \) = systematic credit risk factor of U.S. “Oil, Gas, and Coal Expl/Prod” industry.

> \( \phi_{\Delta OilPrice} \) = standard normal shock representing oil price changes.

> Effect of the negative two standard deviation shock to the oil price: \( \phi_{\Delta OilPrice} = -2 \)?

Unconditional distribution of \( \phi_{US,Oil} \):

- Mean = 0
- Std = 1

Correlation:

\[
\text{Corr} \left( \phi_{US,Oil}, \phi_{\Delta OilPrice} \right) = \rho = 41\%
\]

Conditional distribution of \( \phi_{US,Oil} \), given \( \phi_{\Delta OilPrice} \):

\[
\phi_{U.S.,Oil} | \phi_{\Delta OilPrice} \sim \mathcal{N} \left( \rho \phi_{\Delta OilPrice}, 1 - \rho^2 \right)
\]

A mapping links the standard normal macroeconomic shock \( \phi_{\Delta OilPrice} \) to observable changes in the oil price, \( \Delta OilPrice \).

For example: \( \phi_{\Delta OilPrice} = -2 \leftrightarrow \Delta OilPrice = -52\% \) over a quarter.
Calculation of the stressed expected losses

» Stressed expected loss on the portfolio is the sum of stressed expected losses on individual instruments:

\[
E \left[ L_t \mid Sc_{1,t}^{Cumul} \right] = \sum_{k \in \text{instruments}} E \left[ L_{k,t} \mid Sc_{1,t}^{Cumul} \right]
\]

Stressed expected loss on the portfolio for quarter \( t \)

Stressed expected loss on instrument \( k \) for quarter \( t \)

» Which instrument level parameters are stressed?
- The framework stresses PD and LGD.
- The calculation uses input values of the other parameters. → The input values of those parameters, namely UGD and CMT should reflect the scenario!

» Stressed expected loss on an instruments in the portfolio, over the first quarter after the analysis date:

\[
E \left[ L_{k,1} \mid Sc_{1,1}^{Cumul} \right] = CMT_{k,1} \times UGD_{k,1} \times PD_{k,1}^{Quarter} (Sc_1) \times LGD_{k,1} (Sc_1)
\]

Stressed quarterly PD

Stressed LGD
The first step of calculating the stressed credit risk parameters: determining the stressed distribution of the systematic credit risk factors over quarter \( t \).

\[
\begin{align*}
    r_{C,t} & \sim N\left( \Sigma_{r,MV} \times \Sigma_{MV}^{-1} \times \Phi_{MV,t}, \Sigma - \Sigma_{r,MV} \times \Sigma_{MV}^{-1} \times \Sigma_{MV,r} \right) \\
    \Phi_{CR,t} | Sc_t & \sim N \left( \sum \right)
\end{align*}
\]

Custom index = weighted sum of the country and industry factors

\( \Phi_{MV} \) = standard normal macroeconomic factors

\( f \) = mapping between standard normal macroeconomic factors and macroeconomic variables

\( \rho \) = strength of the relationship between \( \Phi_{CR} \) and macroeconomic variables
Stressed PD calculation

» Conditional LGD for an instrument over quarter $t$, given the systematic factor:

$$FDP_{k,t,cs}(\phi_{CR,k,t}) = N\left(\frac{N^{-1}(FDP_{k,t,cs}) - \sqrt{RSQ_k} \phi_{CR,k,t}}{\sqrt{1 - RSQ_k}}\right)$$

Unconditional forward PD given a credit state at the beginning of $t$

Conditional forward PD given a credit state at the beginning of $t$

Conditional PD formula, in a Gaussian single factor model

However, we do not know the value of $\phi_{CR}$, only its distribution under the scenario. Calculation of the stressed forward PD, given the scenario:

$$FDP_{k,t,cs}(Sc_t) = \int FDP_{k,t,cs}(\phi_{CR,k,t}) d(\phi_{CR,k,t} | Sc_t)$$

Stressed distribution of the custom index

$$FDP_{k,t,cs}(Sc_t) = N\left(\frac{N^{-1}(FDP_{k,t,cs}) - \sqrt{RSQ_k} \left[\sum m \beta_m f_m(MV_{m,t}^{Scenario})\right]}{\sqrt{1 - RSQ_k \rho_k^2}}\right)$$

Stressed forward PD

Note: for the first quarter, $FPD = PD^{Quarter}$
Stressed LGD calculation

» Conditional LGD for an instrument over quarter $t$, given the systematic factor:

$$LG{D}_{k,t,cs} (\phi_{CR,k,t}) = \text{The conditional LGD is implied by the Moody’s Analytics PD-LGD correlation framework, which links recovery return to systematic factors}$$

Conditional LGD given a credit state at the beginning of $t$

» However, we do not know the value of $\phi_{CR}$, only its distribution under the scenario.

Calculation of the stressed forward LGD, given the scenario:

$$LG{D}_{k,t,cs}(Sc_t) = \int LG{D}_{k,t,cs} (\phi_{CR,k,t}) \, d(\phi_{CR,k,t} | Sc_t)$$

Stressed distribution of the custom index

$$LG{D}_{k,t,cs}(Sc_t) = \int Bet a^{-1} \left(1 - N_{a,b}(z), (k_k - 1)LGD_{k,t}, (k_k - 1)(1 - LGD_{k,t}) \right) \times \varphi(z, M^{Scenario}V_t)dz$$

Converting the recovery return to a random variables with unconditionally Beta distribution. $a,b$ – implied by the PD-LGD parameters.

Stressed density of the recovery return, given default and the scenario

Note: the integral must be evaluated numerically.
Incorporating the effect of credit migration...

» Stressed expected loss on an instruments in the portfolio, over the quarter \( t \) \((t>1)\) after the analysis date:

\[
E[L_{k,t}|Sc_{1,t}^{Cumul}] = CMT_{k,t} \times UGD_{k,t} \times \sum_{cs} TP_{k,1\rightarrow t,cs_0\rightarrow cs}^{Cumul}(Sc_{1,t-1}) \times FPD_{k,t,cs}(Sc_t) \times LGD_{k,t,cs}(Sc_t)
\]

Non-default credit states \( cs \) at the beginning of quarter \( t \)

Stressed transition probability: probability of migrating from credit state \( cs_0 \) on the analysis date to credit state \( cs \) at the beginning of quarter \( t \)

Stressed forward PD and LGD over quarter \( t \), given credit state \( cs \)

Illustrating the role of credit migration
Stressed expected losses on portfolios assuming the CCAR 2013 supervisory scenarios

Analysis date: end of 2012 Q3
Scenario over 9 quarters: 2012 Q4 – 2014 Q4

U.S. Corporates

Macroeconomic variables: Unemployment Rate, BBB Corporate Yield, Dow Jones Total Stock Market Index, and VIX

Average $\rho = 75\%$.

Quarterly Stressed Expected Losses

Global Corporates

Macroeconomic variables: Eurozone Real GDP, Japanese Real GDP, UK Real GDP

Average $\rho = 40\%$.

Quarterly Stressed Expected Losses

Cumulative drop in Japanese GDP under:
- CCAR Adverse scenario = 8.4%
- CCAR Severely Adverse scenario = 7.0%

The international CCAR variables do not explain a large enough portion of the portfolio systematic risk. The dispersion of the loss around the stressed EL remains large under the scenario.
Stressed expected losses on portfolios assuming the CCAR 2013 supervisory scenarios

Analysis date: end of 2012 Q3
Scenario over 9 quarters: 2012 Q4 – 2014 Q4

U.S. CRE

Macroeconomic variables: Real GDP, Dow Jones Total Stock Market Index, House Price Index, and CRE Index

Average ρ = 51%.

In both cases, the losses are strongly linked to the real estate market indexes. The loss paths are therefore impacted by the scenario paths of House Price Index and CRE Index.
Firms with different level of systematic risks will respond differently to the scenarios....

**U.S. Corporates**

**Macroeconomic variables:** Unemployment Rate, BBB Corporate Yield, Dow Jones Total Stock Market Index, and VIX

Two instruments from the U.S. Corporates portfolio:
- Both are exposures to the Construction industry ($\rho = 77\%$)
- Both have input/unconditional PD = 18bps (annual)

The larger firm is more sensitive to systematic risk and its PD increases more under the adverse scenario.
Firms in different industries will respond differently to the scenarios....

U.S. Corporates

**Macroeconomic variables:** Unemployment Rate, BBB Corporate Yield, Dow Jones Total Stock Market Index, and VIX

Two instruments from the U.S. Corporates portfolio:
- Both have input/unconditional PD = 18bps (annual)
- Both counterparties have RSQ = 27%

Counterparty operates in **Steel & Metal Products** industry

Correlation with the macroeconomic variables $\rho = 79\%$

Counterparty operates in **Telephone** industry

Correlation with the macroeconomic variables $\rho = 68\%$

Steel & Metal Products industry is more sensitive to the state of the economy, the PD of the exposure to that industry increases more under the adverse scenario
Conclusion
Conclusion

» There exist intuitive links between stress testing and portfolio credit risk.

» The links can be modeled by correlating credit risk factors and macroeconomic variables.

» Such a framework allows for:
  - Estimation of conditional loss distribution under a scenario
  - Reverse stress testing – what are the scenarios we can observe in case of the tail event?
  - Multi-period stressed expected loss calculation.

» Results depend on a combination of effects:
  - Stand-alone characteristics of exposures
  - Portfolio concentrations across locations and sectors.
  - Factor structure and factor correlations, including correlations of credit risk factors and macroeconomic variables.

» The framework can enhance portfolio analysis by linking it to real world scenarios and by identifying factors associated with the tail event.