MOODY'S
ANALYTICS

Joint Modeling of Conditional Credit Migration and Default: New Answers to Old Problems

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Agenda

1. Overview
2. Converting a TTC Transition Matrix to a Conditional Transition Matrix
3. PD Based Approach to Modeling Credit Transition and Default
4. Comparing the Two Approaches
5. Summary

1 Overview

Questions

Suppose a company is rated Ba (PD=0.3%) as of Q3 2013, how would you describe its credit quality in Q4 2013?

- A rating transition matrix
- A distribution of PD
- Very likely the transition matrix and the PD distribution is dependent upon the macro environment

» Does your institution use a through-the-cycle (TTC) internal rating/PD framework?

» Does your institution have a through-the-cycle transition matrix?

» Does your institution need a point-in-time (PIT) internal rating/PD framework, conditional upon the macroeconomic scenario?

» Does your institution need a point-in-time (PIT) transition matrix, conditional upon the macroeconomic scenario?

How to Model Credit Migration and Default Conditioning on Macro Variables?

- Many institutions need to model credit/rating migration for various applications:
  - Stress testing and other regulatory purposes
  - Provisioning and ALLL
  - Business planning
- But it is challenging to relate an average through-the-cycle (TTC) migration matrix, or PD distribution, to macro conditions, for a number of reasons:
  - Number of parameters to be estimated for an *n*-state matrix: *n**(*n*-1) (90 for n=10)
  - Lack of data of sufficient length and breadth
    - Requires data over at least one economic cycle
    - Most internal rating system are TTC, so credit transitions are sparse
    - Difficult to customize the transition matrix by sector or geography
- "**Sound rating transition models require two fundamental building blocks: a robust time series of data and well-calibrated, granular-risk rating systems**" — *Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice (Federal Reserve System, August 2013)*

We Present Two Solutions

- Approach #1: Translating a TTC migration matrix to a PIT migration matrix framework by constructing a “credit state” factor and relating it to macroeconomic variables

- Approach #2: Modeling the distribution of the PD changes over time and relating the changes to macroeconomic variables

- We illustrate and compare the two approaches using our RiskCalc CCA EDF and comment on how they can be applied using a bank’s internal data

2

Converting Through-the-Cycle (TTC) Migration Matrix to Conditional Migration Matrix

Relating a TTC Migration Matrix to a PIT Conditioning on Macro Variables

» A typical process usually involves the following steps*:

- Representing the rating transition matrix by a single summary measure;
- Estimating a time-series model linking the summary measure to scenario variables;
- Projecting the summary measure over the nine-quarter planning horizon, using the parameter estimates from the time-series model;
- Converting the projected summary measure into a full set of quarterly transition matrices.

--**Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice (Federal Reserve System, August 2013), p24.*

» Following the above mentioned steps, we show a generic approach** to calibrating a single credit cycle factor *Z,* to capture the observed variation of transition matrices in different economic conditions.

— ***Rating Migrations*, Malte Kleindiek, 2005.

Estimating Transition Matrix Using EDF Data

- Define Rating Buckets:
  - US RiskCalc 4.0 CCA EDF in CRD data
  - Define bin cutoff such that each bin has the same quintile, 7 non-default buckets, with 1 being the highest rating; rating 8 is the default bucket.

- To estimate the transition matrix for a specific quarter, first select the data for this quarter and the next quarter, then calculate the average EDF for each Customer ID within the quarter, and assign the ratings according to its average EDF.

- The last column of a transition matrix is the average default probability.

- Count the frequency of migration for each initial grade and normalize by the number of firms and 1 minus average default probability.

Empirical Average, Peak, and Trough Migration Matrices

1994 to 2012

88.46%	8.87%	1.24%	0.64%	0.39%	0.25%	0.10%	0.05%
11.59%	73.90%	11.31%	1.63%	0.76%	0.51%	0.22%	0.08%
1.13%	14.93%	68.81%	12.15%	1.64%	0.86%	0.34%	0.13%
0.34%	1.70%	16.19%	67.14%	12.19%	1.64%	0.58%	0.23%
0.14%	0.50%	1.88%	16.15%	68.24%	11.62%	1.08%	0.39%
0.05%	0.22%	0.57%	1.72%	14.65%	72.62%	9.40%	0.78%
0.02%	0.05%	0.13%	0.32%	0.88%	11.42%	84.92%	2.27%

2011 Q1

93.97%	3.85%	0.97%	0.54%	0.34%	0.19%	0.09%	0.05%
32.27%	60.76%	4.41%	1.28%	0.71%	0.35%	0.15%	0.08%
4.10%	37.69%	51.55%	4.43%	1.18%	0.69%	0.23%	0.13%
1.05%	5.30%	40.42%	47.25%	4.08%	1.33%	0.34%	0.23%
0.41%	1.39%	5.42%	39.69%	48.11%	3.83%	0.76%	0.39%
0.20%	0.68%	1.45%	4.74%	35.13%	53.18%	3.84%	0.79%
0.08%	0.11%	0.38%	0.96%	2.41%	26.25%	67.54%	2.26%

2008 Q3

61.95%	**36.19%**	0.95%	0.43%	0.20%	0.17%	0.06%	0.05%
0.65%	50.97%	46.24%	1.21%	0.46%	0.35%	0.04%	0.08%
0.10%	1.14%	43.86%	52.05%	1.81%	0.69%	0.21%	0.13%
0.02%	0.14%	1.14%	43.65%	53.18%	1.41%	0.23%	0.23%
0.00%	0.06%	0.26%	1.13%	47.97%	49.60%	0.59%	0.39%
0.00%	0.03%	0.08%	0.26%	1.16%	60.64%	37.04%	0.79%
0.00%	0.00%	0.00%	0.01%	0.05%	0.68%	96.77%	2.48%

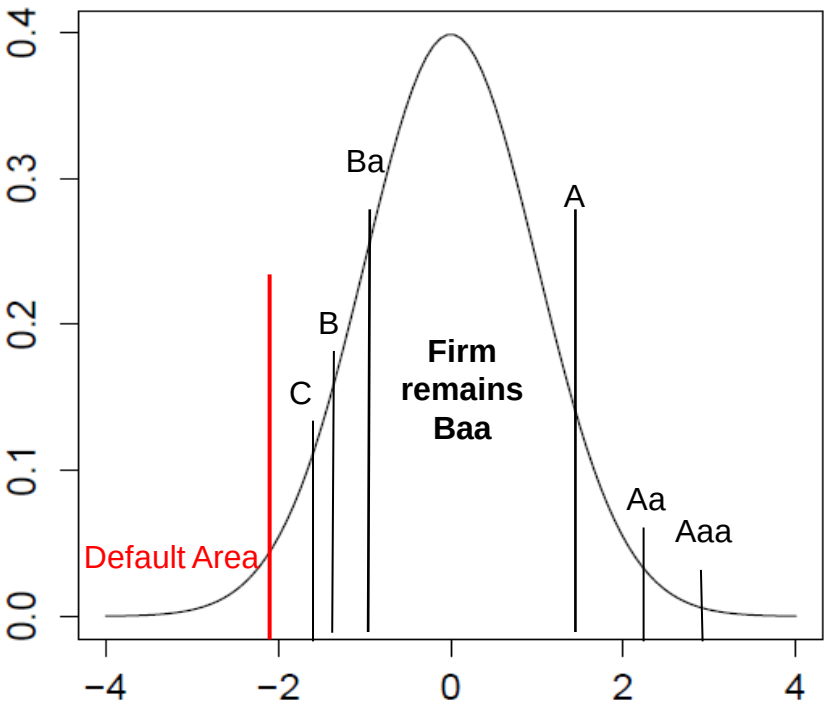
Representing TTC and Conditional Migration Matrix

- The rating transition probability can be represented by a normal random variable *x*. Each initial credit rating has a corresponding x^G.
- *x* consists of two components:
  - The idiosyncratic component, a standard normal variable *Y*.
  - The systematic component, a standard normal variable *Z*.
  - $x^G = \sqrt{1-\varphi}Y^G + \sqrt{\varphi}Z$ The correlation between *Z* and *x* is φ.
- The bins $(x_g^G, x_{g+1}^G]$ are estimated from the empirical historical average transition matrix, so that the average (TTC) quarterly migration probability from rating *G* to *g* is given by $P(G,g) = \Phi(x_{g+1}^G) - \Phi(x_g^G)$ and conditional migration probability by (fix *Z*, solve probability for *Y*)

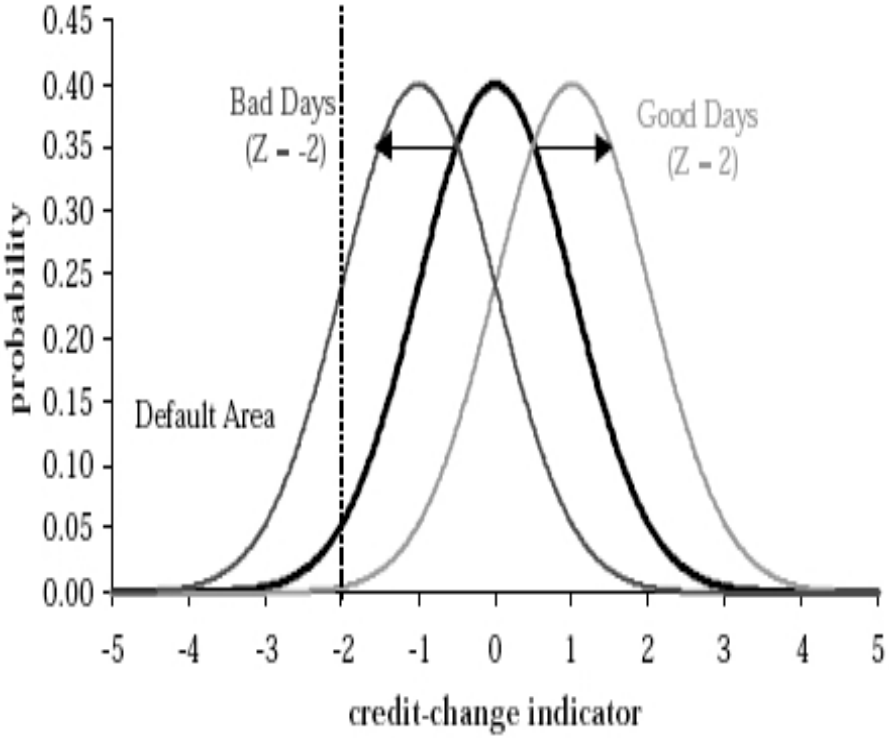
$$\pi\left(x_g^G, x_{g+1}^G, Z_t\right) = \Phi\left(\frac{x_{g+1}^G - \sqrt{\varphi}Z_t}{\sqrt{1-\varphi}}\right) - \Phi\left(\frac{x_g^G - \sqrt{\varphi}Z_t}{\sqrt{1-\varphi}}\right)$$

Representing TTC and Conditional Migration Matrix

Historical Average Migration Matrix



Conditional Rating Transitions



Estimating *Z*

» Given φ, we want to find a Z_t that is a solution to the following minimization program:

$$\min_{Z_t} \sum_G \sum_g \frac{n_{t,G}\left(P_t(G,g) - \pi\left(x_g^G, x_{g+1}^G, Z_t\right)\right)^2}{\pi\left(x_g^G, x_{g+1}^G, Z_t\right)\left(1 - \pi\left(x_g^G, x_{g+1}^G, Z_t\right)\right)}$$

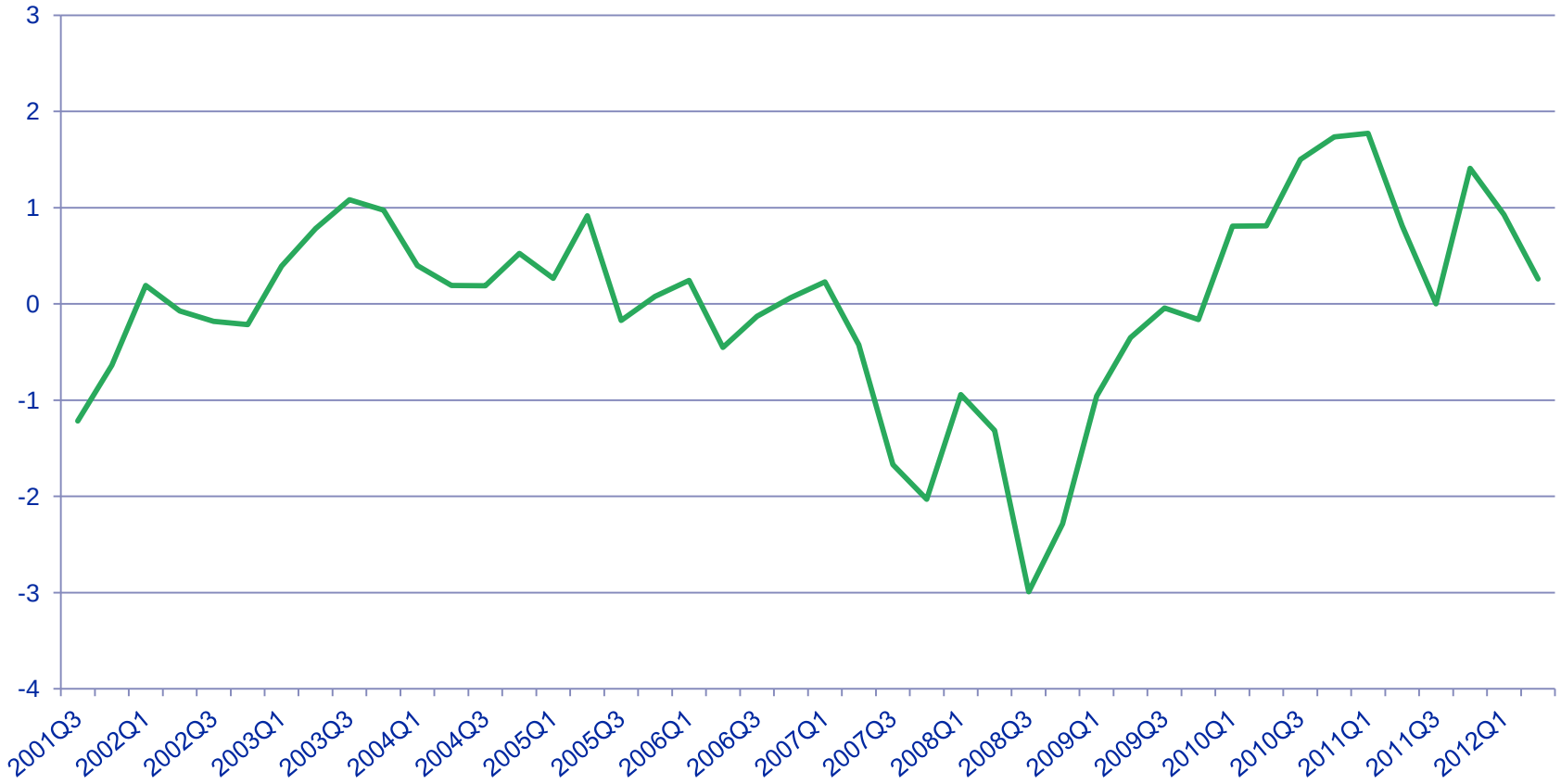
$n_{t,G}$ denotes the number of firms in rating G, and $P_t(G,g)$ is the empirical estimation of transition matrix at quarter *t*. We put higher weights on those rating buckets that have more observations and lower uncertainties on the conditional migration, i.e. $\pi\left(x_g^G, x_{g+1}^G, Z_t\right)$ is close to 0 or 1.

» For a given φ, we solve for time series Z_t. Then we choose φ such that $Var(Z_t) = 1$ In this case, φ is about 10%, inline with our GCorr RSQ for SME and small public firms.

» A positive *Z* represents the time when each initial credit rating has a lower probability to downgrade and default.

Time Series of Z

Interpretation of Z: a standardized index of credit quality change. Large positive (negative) Z implies more upgrade (downgrade).



Estimating *Z* and Migrations Conditional on Macro Variables

» We can fit Z as a function of a number of macro variables, Z=function(intercept, Dow Jones return, credit spreads, mortgage rate, etc)+error term

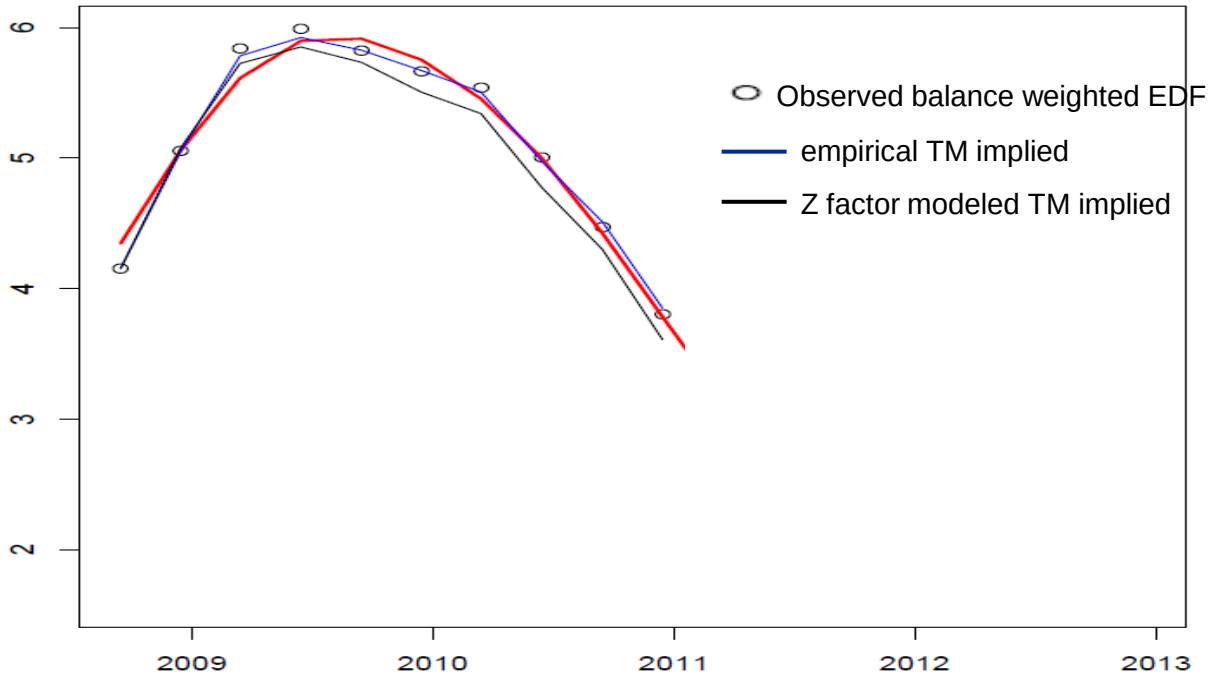
– The fitted line is depicted in red

» The modeled credit Transition matrix is given by:

$$\hat{\pi}(x_g^G, x_{g+1}^G, \hat{Z}_t) = \Phi\left(\frac{x_{g+1}^G - \sqrt{\rho}\hat{Z}_t}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_g^G - \sqrt{\rho}\hat{Z}_t}{\sqrt{1-\rho}}\right)$$

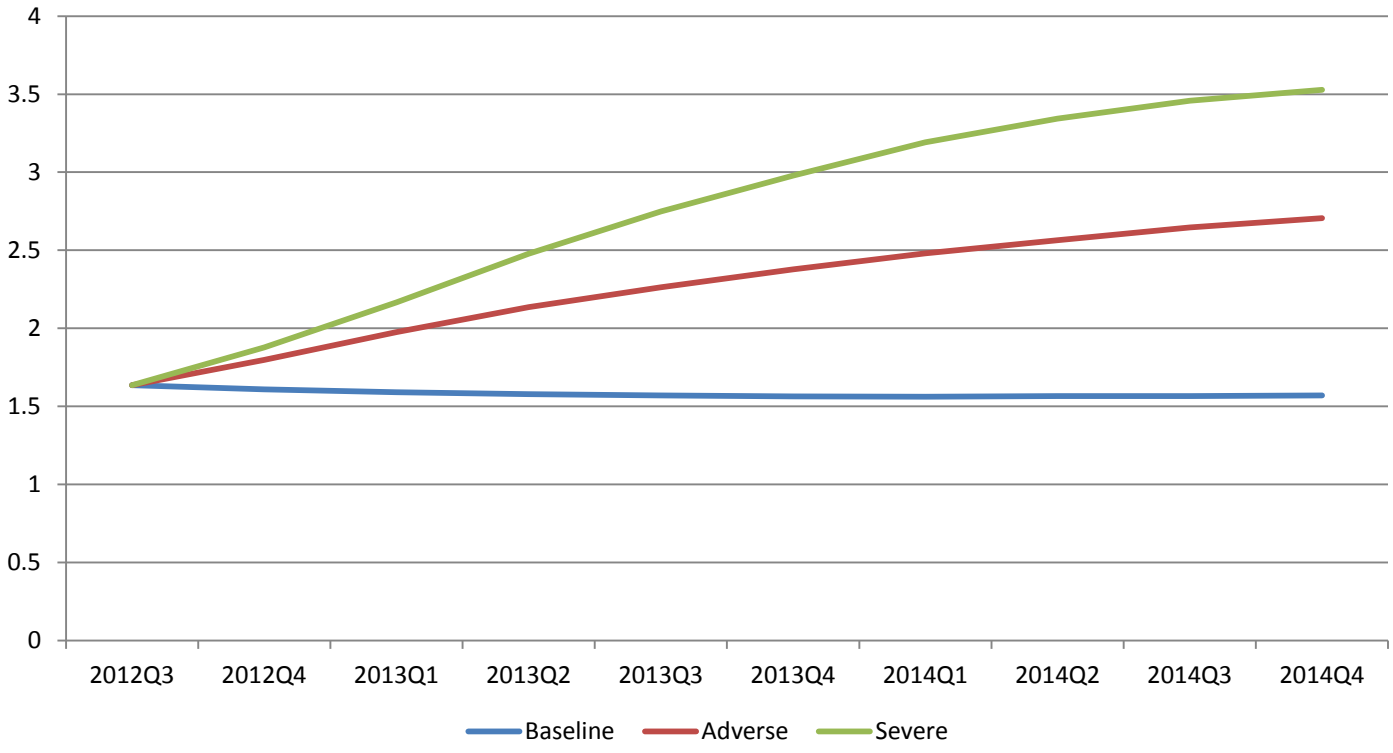
Validation: Matching Weighted Average EDF

» The cohort is fixed at the beginning of the analysis period, which we set to be 2008Q3. From the initial firm balance, apply the transition matrix consecutively and calculate the weighted average EDF for the next 9 quarters. Compare the results with the empirical estimation of weighted average EDF.



Projection Under CCAR 2013 Scenarios

- Scenarios: 2013CCAR Baseline, Adverse and Severe.
- Calculate the Z from the model from 2012Q4 to 2014Q4 using the given macro scenarios. Calculate the transition matrix, and the firm accounts weighted average EDF



Some Comments

» Federal Reserve expects BHCs that use rating transition models to have robust time series of data that include a sufficient number of transitions, which allows BHCs to establish a statistically significant relationship between the transition behavior and macroeconomic variables. Data availability has been a widespread constraint inhibiting the development of granular transition models because a sufficient number of upgrades and downgrades are necessary to preclude sparse matrices. In order to overcome these data limitations, BHCs have often relied on third-party data to develop rating transition models.

Source: Capital Planning at Large Bank Holding Companies: Supervisory Expectations and Range of Current Practice (Federal Reserve System, August 2013), p24.

» The results of approach 1 are based on RiskCalc CCA EDF, which is generally considered as PIT PD. Many banks do not have PIT ratings/PDs, thus it is difficult to construct quarterly transition matrix, which makes estimating Z factor challenging

» **How do we solve the problem without PIT quarterly transition matrix?**

» **How do we construct sector specific conditional transition matrix?**

3

PD Based Approach to Modeling Credit Migration and Default

Fed CCAR C&I PD Model

"The first stage of the modeling process is the estimation of a series of equations relating historical changes in the median probability of default for 12 different borrower industries, six credit quality categories, and countries of incorporation to macroeconomic variables, including changes in stock price volatility and the spread on BBB-rated corporate bonds. Default probability data are derived from expected default frequency estimates."

"These equations are used to project quarterly changes in PD at the borrower industry-credit quality-country level over the stress scenario horizon ..."

"The next stage is to use detailed, loan-level information submitted by the 19 BHCs to calculate expected losses as of September 30, 2011 for every loan. Probability of default for each loan is estimated by mapping its internal credit rating assigned by the BHC to a standardized rating scale and then linking these standardized ratings to default probabilities ..."

Source: Comprehensive Capital Analysis and Review 2012: Methodology for Stress Scenario Projections, March 12, 2012

How Would You Improve Upon the "Fed Model"?

Macro Variables		Credit Migration
❑ GDP	⟷	❑ Shift of EDF distribution
❑ Unemployment		❑ Sector Effect
❑ S&P 500		❑ Rating Effect

1. Divide the population between large corporate and middle-market

2. More granular and industry buckets

- Dependent variable: the shift of EDF distribution for each sector/rating bucket.
  - » Change of Median EDF: $mEDF_t - mEDF_{t-1}$
  - » Change of mean Log EDF: $m(\ln(EDF_t)) - m(\ln(EDF_{t-1}))$
  - » Change of Pseudo DD: $N^{-1}(mEDF_t) - N^{-1}(mEDF_{t-1})$
  - » Change of Logit-median EDF: $\ln(\frac{mEDF_t}{1-mEDF_t}) - \ln(\frac{mEDF_{t-1}}{1-mEDF_{t-1}})$

- Main Model: Two-way Fixed Effect Model

$$y_{s,r,t} = \rho y_{s,r,t-1} + \alpha_s + \alpha_r + \sum_{i=1}^{M} (\beta_{i,s} + \beta_{i,r}) X_{i,t} + \varepsilon_{s,r,t},$$

  - » *s* denotes sectors, *r* denotes ratings, *t* denotes time points
  - » *Xs* are macroeconomic variables
  - » Different variations of the main model: sector- or rating-effect only
  - » 1st lag of dependent variable: high persistence of dependent variable

The Resulting Stressed PD Model for Middle Market

- Use the sample deciles of RiskCalc EDF data to evenly divide the entire sample into 10 rating buckets and 13 sectors.
- Dependent variable: the change of mean log-EDF from current quarter to the next quarter for each sector/rating bucket.
- Perform transformations and stationarity checks on the CCAR macroeconomic variables, and start the variable selection process.
- Use the full, two-way fixed effect model with the 1st order autoregressive term to run both univariate and multivariate regressions of dependent variables on macroeconomic variables. Select one set of macroeconomic variables that fit the data well, make economic sense, and are robust.
- Compare the reduced form models with the full model for the selected macroeconomic variables. Finalize the model form.
- Run the rating buckets bootstrapping to generate the model estimators for each PD value for each sector/rating bucket.

Granularity Issue of Rating Buckets

Issues

- Different clients define rating/PD range differently, and it may change over time
- Low observations count for certain buckets
- Biased estimation or no available data for certain buckets

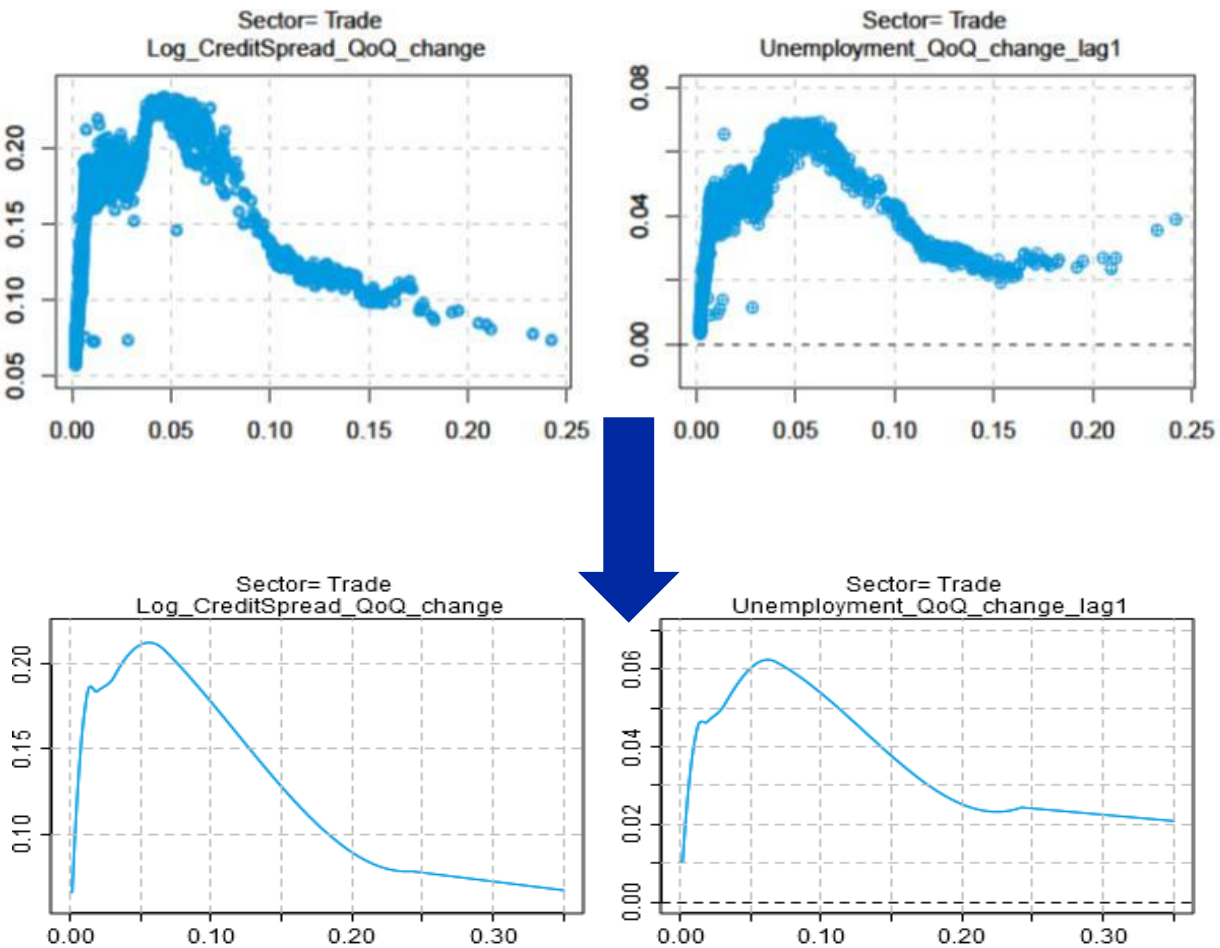
Solution

- Bootstrap random rating buckets
  - generate many pairs of bucket mean PD and model estimates
- Non-parametric approach to fit the curve of model estimates vs. PD values

Resulting model

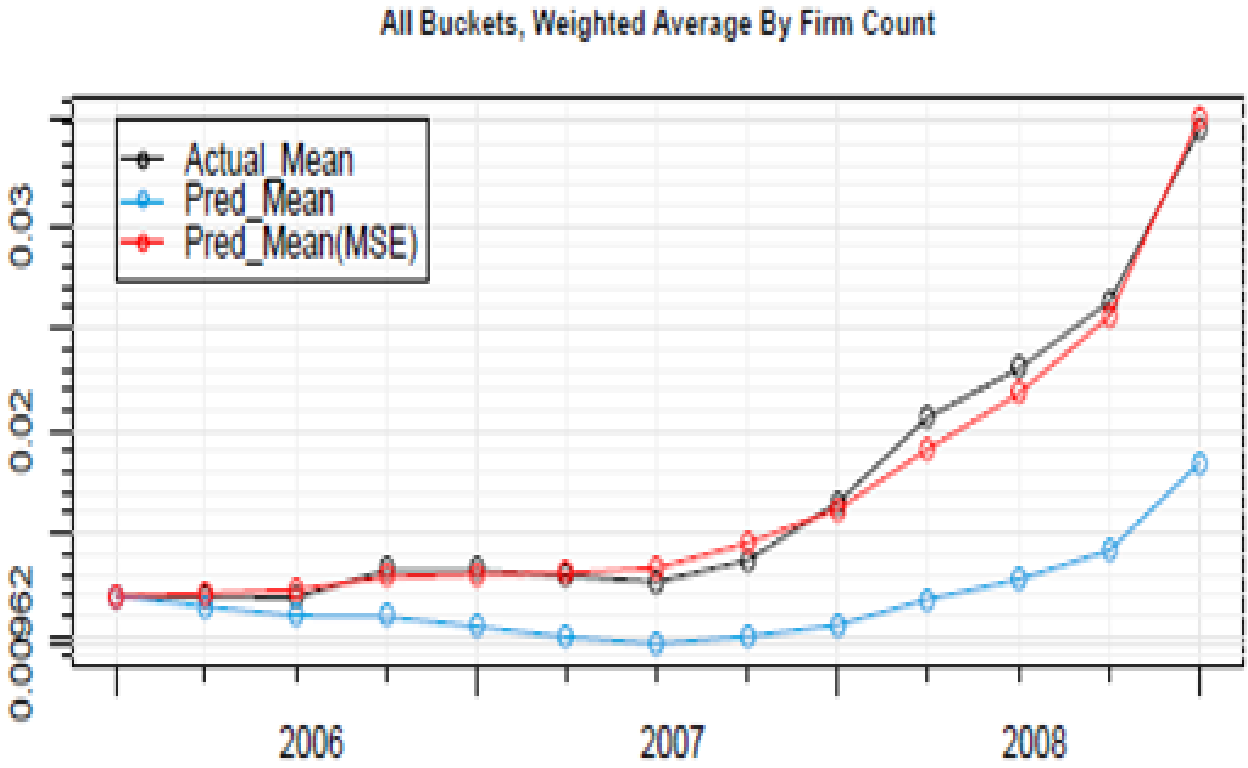
- The final model is granular and is not dependent on the rating bucket definition

Bootstrapping: Coefficients vs. PD



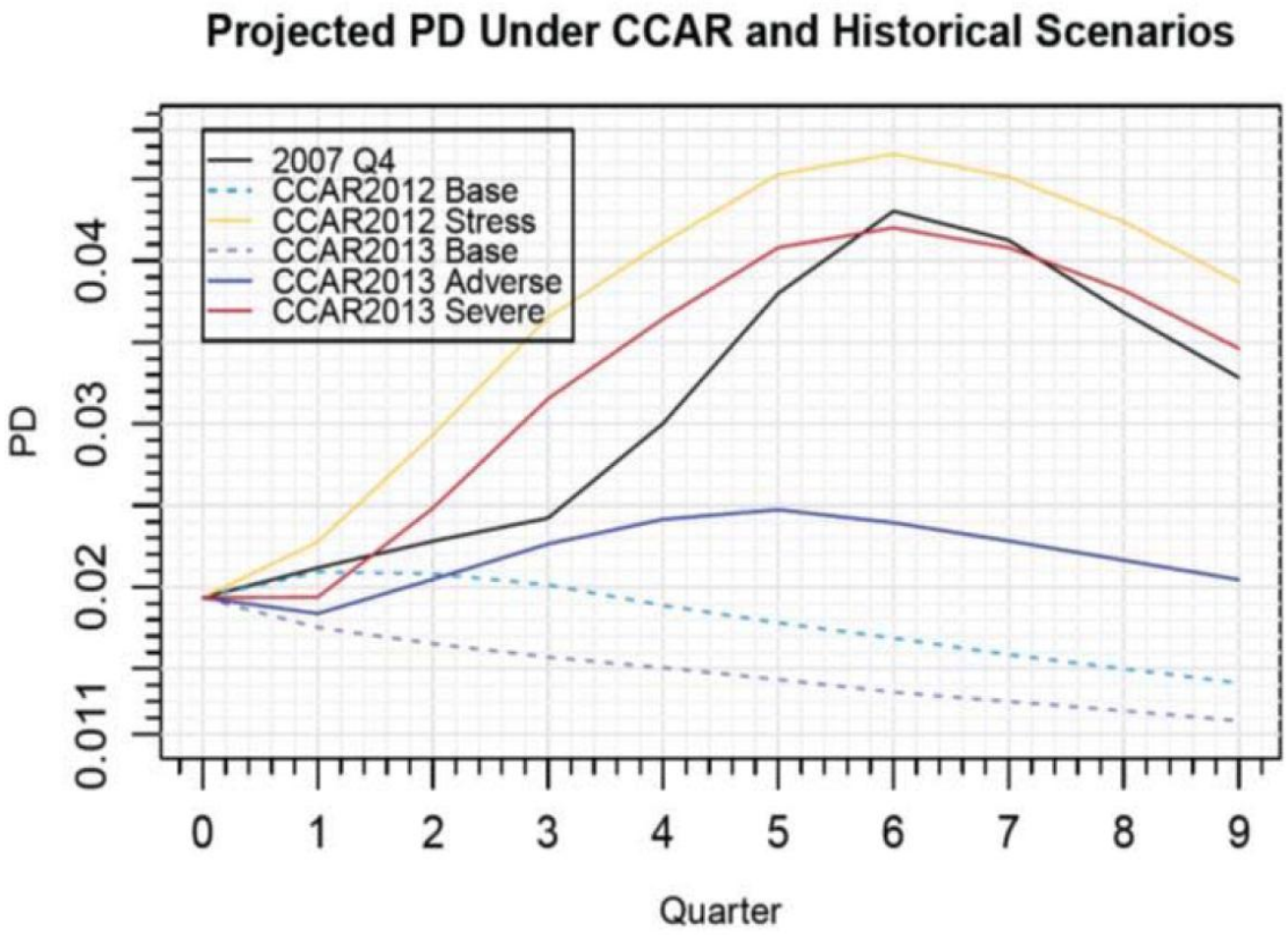
- Simulate many different ways to define the rating bucket
- Estimate the model for each simulation
- Associate the coefficients with the starting mean PD of each bucket
- Use firm-level fitting errors to estimate mean squared errors (MSE)
- Local regression to fit a continuous curve of coefficients for all starting PDs
- Final Model Output: A table of coefficients vs. PD

Predicted Multi-Quarter Bucket Mean PD and Actual Bucket Mean PD



Note: Given we estimate a regression model on bucket mean of log EDF, a convexity adjustment is important in matching the observed bucket mean of EDF.

Projection Under CCAR and Historical Scenarios



How Do We Create Transition Using This Approach?

» With starting PD in the range from 0 bps to 35%, this granular approach can create average transition in the scale of PD, with incremental unit of 1bps. In other words, the transition is a special one, 3500X3500, with only 1 non-zero transition per row.

» With starting PD given by an internal rating framework, how do we construct a transition matrix conditioning on macroeconomic varaibles?

Converting Granular PD Model to Conditional Migration Model

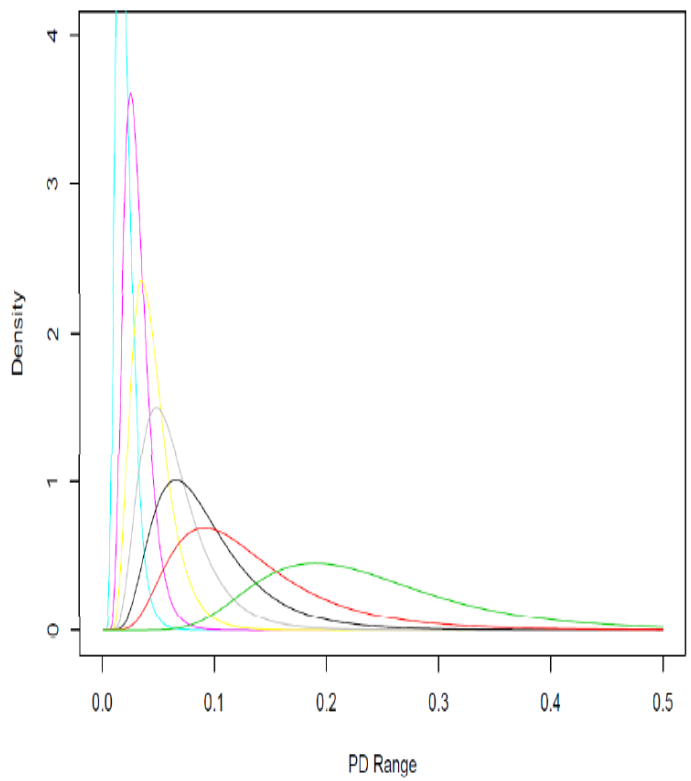
- The granular PD model effectively describes the first two moments (mean and variance) of log(PD) at quarter end for firms with any starting PD in a given sector, conditional upon macro variables up to the current quarter.
- The conditional distribution is assumed to be log normal:

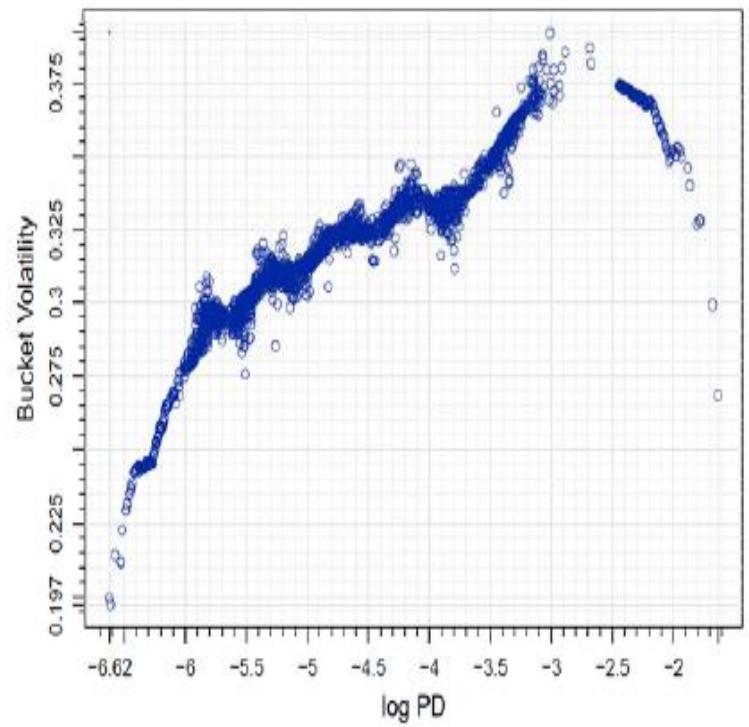
$$\log(PD_{t+1}) - \log(PD_t) \mid X_{t+1} \sim N\left(\mu(PD_t, X_{t+1}), \sigma(PD_t)^2\right)$$

- The above model provides a conditional credit migration in a continuous fashion if we can estimate the conditional variance term

- For any discrete rating bucket definition, we can produce conditional credit migrations matrix by “discretize” the continuous migration distribution

Conditional (log Normal) Distributions for Different Starting PDs





Note: the conditional mean (first moment) of logPD is given by the granular approach; the graph on the right shows the relationship between bucket volatility (second moment) with starting (log)PDs.

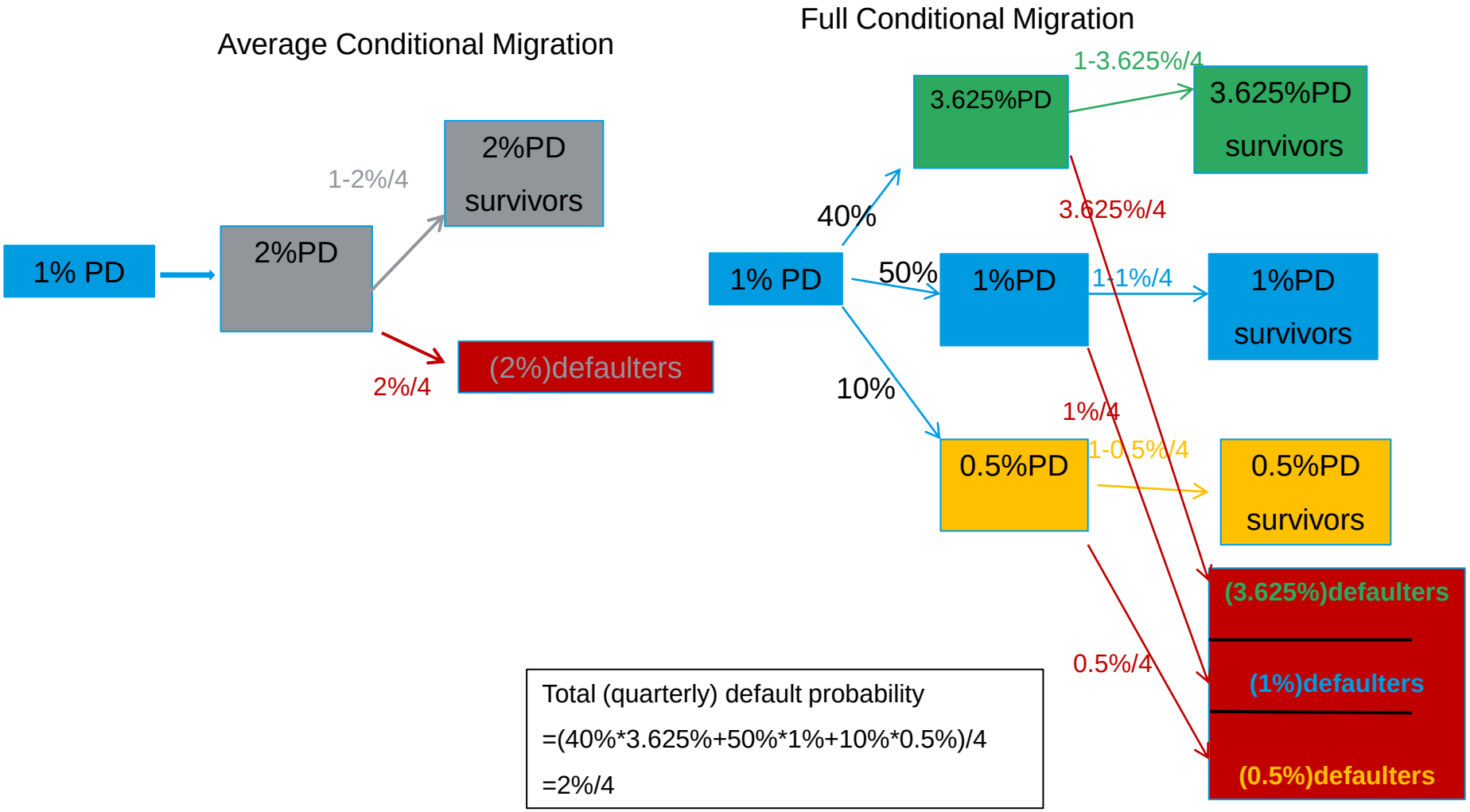
Constructing the Conditional Transition Matrices

- Basics idea: a pool of firms all start with 1% PD, after a negative economic shock, according to the granular model, on average they become 2% PD firm. At the end of the quarter, 2%/4= 0.5% of them default, the remaining firms have 2% PD on average, without considering the distribution of migration around the mean. Now considering possible migrations around the mean, we can have 40% chance down grade to 3.625% PD firms, 50% chance remain 1% PD and 10% chance upgrade to 0.5% PD firms before defaulting according the new probabilities. So on average we still have annualized PD of

40%*3.625%+50%*1%+10%*0.5%=2%!

The migration probabilities 40%, 50% and 10% as well as PD possibilities of 3.625%, 1% and 0.5% are determined by the conditional first and second moments of the log normal distribution assumption.

Calculating the Conditional Transition Matrices



Total (quarterly) default probability

=(40%*3.625%+50%*1%+10%*0.5%)/4

=2%/4

Calculating the Conditional Transition Matrices

- Define rating bucket cutoff points: $0 = x_0, x_1, \ldots, x_{N-1}, x_N = 100\%-, x_{N+1} = default.$
- Quarter *t* bucket *i* starting PD: $PD_{i,t}, i = 1,2, \ldots, N$. Projection of next quarter mean log-PD: $\mu_{i,t+1} = f_{PD}\left(\log\left(PD_{i,t}\right), Z_{t+1}\right)$, f_{PD} function is the "granular PD model." We assume the distribution of PD in quarter *t+1* is log normal: $LN\left(\mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)$.
- The conditional probability of transition to bucket *j =1,2,..N* (before default) is $TP_{i,j,t}(Z_{t+1}) = \int_{x_{j-1}}^{x_j} f\left(x, \mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)dx$, which has an average (annualized) PD of

$$PD_{i,j,t+1}(Z_{t+1}) = \frac{\int_{x_{j-1}}^{x_j} x \cdot f\left(x, \mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)dx}{\int_{x_{j-1}}^{x_j} f\left(x, \mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)dx}.$$

- The total probability of default (annualized) is

$$TP_{i,1,t}(Z_{t+1}) \cdot PD_{i,1,t+1}(Z_{t+1}) + TP_{i,2,t}(Z_{t+1}) \cdot PD_{i,2,t+1}(Z_{t+1}) + \ldots + TP_{i,N,t}(Z_{t+1}) \cdot PD_{i,N,t+1}(Z_{t+1}) + \int_1^\infty \min(x,1) \cdot f\left(x, \mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)dx \approx \int_0^\infty x \cdot f\left(x, \mu_{i,t+1}, {\sigma_{i,t+1}}^2\right)dx \approx \mathrm{E}\left(PD_{i,t+1} \middle| Z_{t+1}\right)$$
$$= e^{\mu_{i,t+1} + 0.5 \cdot {\sigma_{i,t+1}}^2}$$

which is the average conditional PD in the next quarter as in the "granular PD model"

- The final conditional non-default transition probabilities:

$$TP_{i,j,t}(Z_{t+1}) \cdot (1 - PD_{i,j,t+1}(Z_{t+1})), j = 1,2, \ldots, N$$

- At quarter *t+1*, for any bucket, the new starting PD is the balance-weighted average of all firms (regardless of the buckets they come from last quarter) that fall in that bucket.

4

Comparing the Two Approaches

Comparing the Two Approaches

Benefits of Approach #1:

» The concept of transition matrix is familiar to many

» Many banks have various TTC transition matrices and various processes built upon them

» Transforming a TTC matrix to a conditional matrix can be readily achieved through a single factor "Z"

Challenges:

» Estimation of the Z factor requires well behaved quarterly transition matrix

» It is difficult to make the approach "granular", e.g., make the sector specific conditional transition matrix

Benefits of Approach #2:

» The approach can easily produce granular conditional transition matrix

» It is not dependent on the definition of rating bucket

» Only a parsimonious set of parameters to estimate

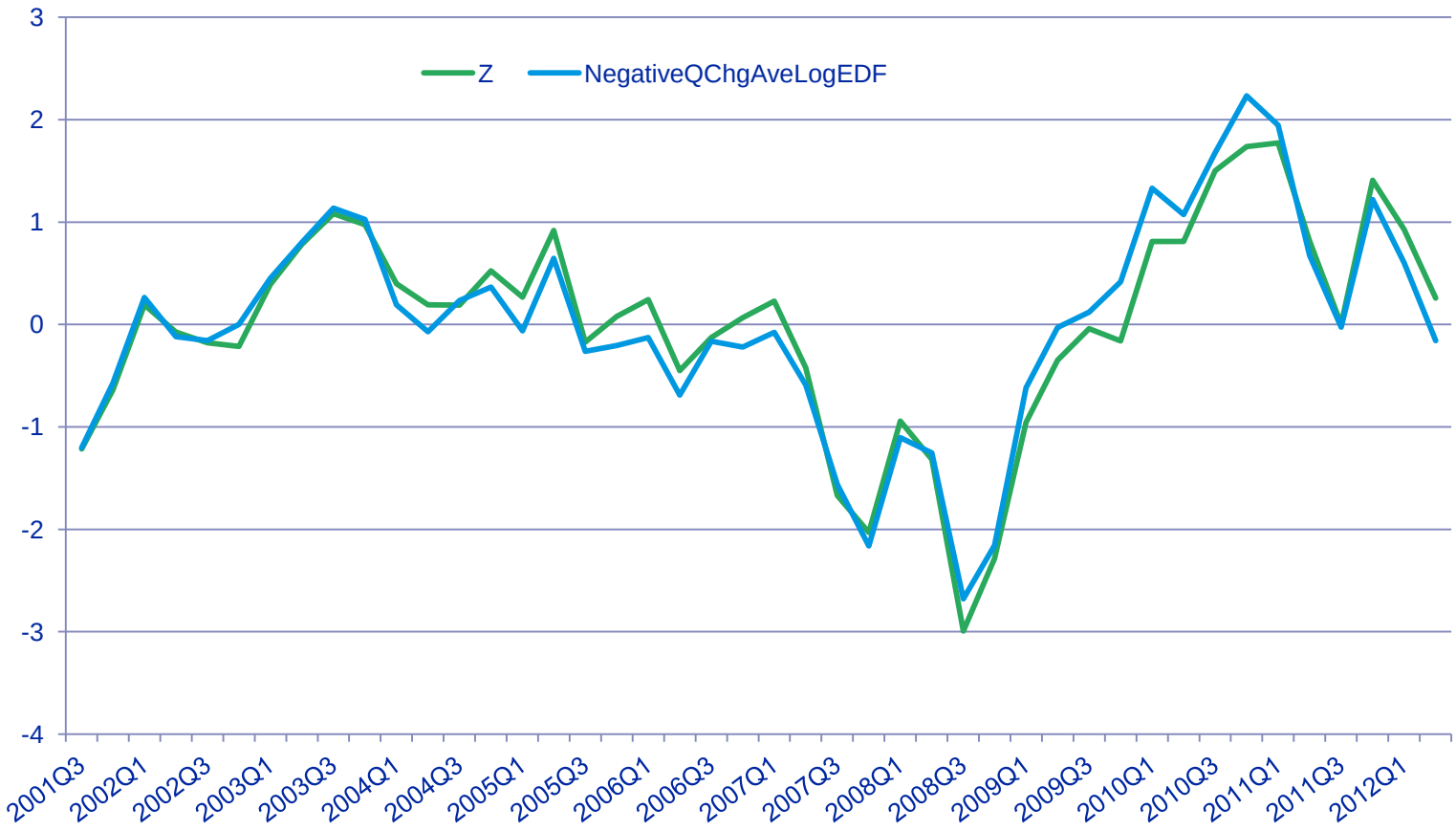
Challenges:

» Requires a PIT PD framework to start with

» The concept of continuous PD distribution can be a "foreign concept" to start with

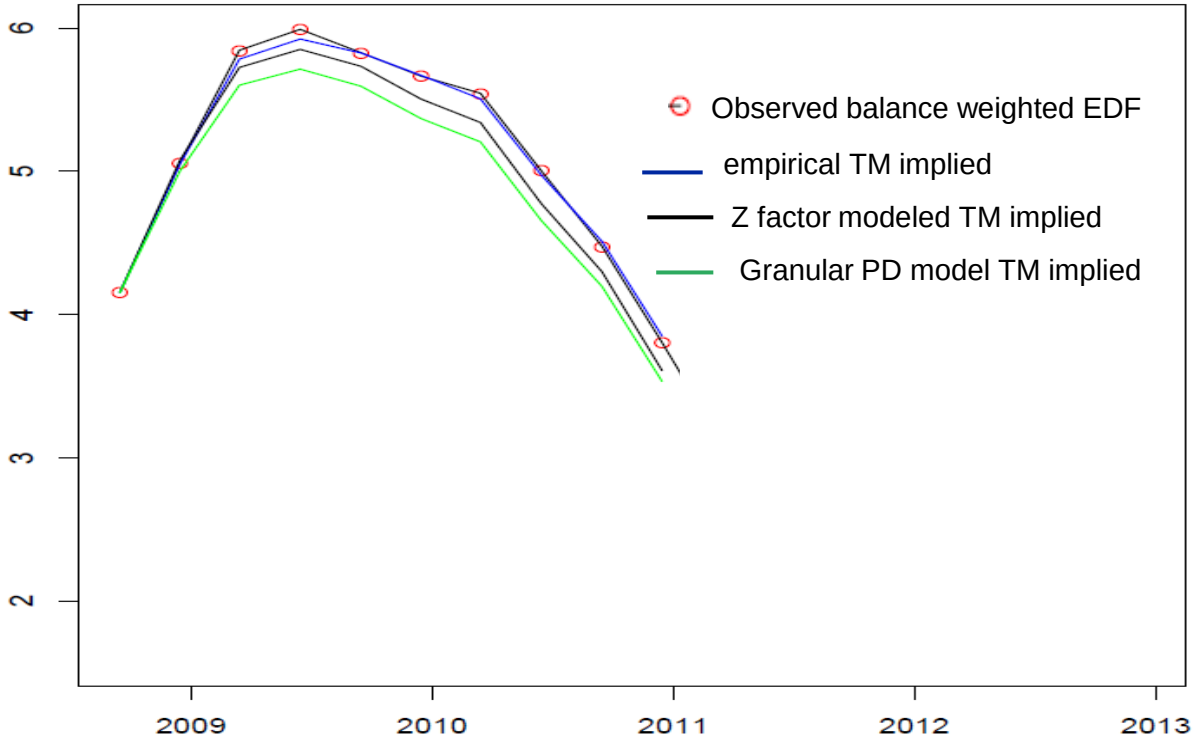
What is the "Z" Factor in Approach 2

Answer: Normalized negative quarterly change of average log EDF!



Two Approaches Produce Similar Results

» The cohort is fixed at the beginning of the analysis period, which we set to be 2008Q3. We apply the transition matrix consecutively and calculate the weighted average EDF for the next 9 quarters. Compare the results with the empirical estimation of weighted average EDF



Alternative Ways to Construct the "Z" Factor

Instead of estimating the Z factor from minimizing the distance between the TTC and PIT transition matrices, we can construct the "Z" factor directly:

1. By using PIT PD such as EDF
   - » This approach would allow one to construct sector specific Z factors

2. By using other credit quality index such as historical charge-off rate and credit spreads
   - » They needs to be transformed to standardized measure of credit quality change

3. Z can be linked to macro economic variables

4. The condition transition matrix (given Z) is given by:

$$\hat{\pi}(x_g^G, x_{g+1}^G, \hat{Z}_t) = \Phi\left(\frac{x_{g+1}^G - \sqrt{\rho}\hat{Z}_t}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_g^G - \sqrt{\rho}\hat{Z}_t}{\sqrt{1-\rho}}\right)$$

5

Summary

Summary

- Many institutions need to model conditional credit transition and default for stress testing and other application----a challenging task because of data and other technical constraint

- "**Sound rating transition models require two fundamental building blocks: a robust time series of data and well-calibrated, granular-risk rating systems**"

- We present two approaches to modeling conditional credit transition and default
  - They have parsimonious representations are relatively simple to implement and yield robust results
  - They can be used to construct sector specific transition matrix
  - They can be used to supplement banks' internal data/approach

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