

## MODELING METHODOLOGY

FROM MOODY'S KMV

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# Risk Integration: New Top-down Approaches and Correlation Calibration

### Abstract

While the sophistication and adoption of the data, models, and software systems for individual risk types has become more widespread, the tools for consistently measuring integrated risk lag. Typically, individual risk components are aggregated in ways ranging from simple summation to employing copula methods that describe the relationship between risk types. While useful, these "top-down" approaches are limited in their ability to describe the interactive effects of various risk factors that drive loss.

In this study, new top-down approaches are developed with varying degrees of sophistication and flexibility. Moreover, a bottom-up model is introduced to allow for an appropriate calibration of a correlation structure in a top-down framework that describes both interest rate dynamics as well as credit dynamics. The analysis is presented within the context of integrating two separate risk systems to arrive at an aggregated portfolio risk measure—the Fermat system, which models market risk, and Moody's Analytics RiskFrontier®, which analyzes credit risk.

The bottom-up model is designed as a two-dimensional interest rate and credit lattice that explicitly accounts for options in the valuation and risk analysis of corporate bonds and loans. In some cases—interest rate insensitive instruments in particular—the differences between the two approaches are minimal, suggesting a straightforward top-down approach is sufficient to describe integrated risk. However, the calibration becomes much more important for instruments that face both interest and credit risk. For example, the simple top-down approach can overstate Unexpected Loss (UL) by more than 25% relative to the more precise correlation-calibrated approach for a typical vanilla bond portfolio. This study provides a foundation for appropriate calibration of top-down approaches.



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# Introduction

While the data, models, and software systems for individual risk types have become more sophisticated and readily available and their adoption more widespread among financial institutions, the tools for consistently measuring the integrated risk lag behind. To meet the growing demand from regulators, senior management, and investors for the assessment of total risk, many institutions take the approach of assessing individual risk components first, and then proceed to aggregate these components up to the level of the entire institution.<sup>1</sup> Practices and techniques in risk aggregation are generally much less sophisticated than those used in measuring individual risk components. They range from simple summation with some fixed diversification percentages, to using a variance-covariance matrix between risk types, to copulas. In contrast to these “top-down” approaches, the “bottom-up” approaches take into account the impact of relevant risk types in the analysis of both individual exposures and the whole portfolio.

In this study, we propose several top-down approaches with varying levels of sophistication. Next, we compare results from top-down and bottom-up analyses of loan and interest rate sensitive bond portfolios. We then demonstrate that top-down approaches produce sufficiently accurate integrated risk measures for certain types of portfolios and over- or underestimate risks for other types. In the latter case, we show through case studies that the correlation parameters used in top-down approaches can be calibrated to significantly improve accuracy.

When considering the historical progression of risk management at traditional banks, it is not surprising that the focus has been more on the analysis of individual risk types and less on integrated risk. Historically, credit risk has represented the largest source of risk, since most corporate lending focused on floating rate instruments. However, in recent years other risk types have become more significant.<sup>2</sup> The financial organization structure has reinforced analyzing risks separately, as lending and market activities were carried out by different groups. Today, institutions typically aggregate individual risk components in ways ranging from simple summation to employing copula methods that describe the relationship between risk types in silos. This approach not only adheres to the traditional organization structure, but also has the benefit that top-down approaches are substantially simpler from a modeling and computation standpoint.

The increasing importance of credit and market derivatives and comprehensive risk analysis has led to the realized and accepted need for the practice of integrated risk assessment. In particular, the Basel II Committee prescribes total capital requirements that include credit, operational, and market risks.<sup>3</sup> In addition, many institutions recognize the importance of measuring aggregated risk, as investors and senior management increasingly want to know how much total risk the institution faces for the purpose of reporting, hedging, and active management.

The impact of the options and contingencies common in most assets and liabilities can be properly accounted for on the entire portfolio through the bottom-up approach. In this study, we focus on interest rate risk and credit risk. To arrive at some portfolio total risk measures, we develop several top-down approaches that combine the analysis results from two separate risk systems—the Fermat system, which models interest rate risk, and RiskFrontier, which analyzes credit risk. In addition, we develop a two-dimensional interest rate and credit lattice model to explicitly account for options in the valuation and risk analysis of corporate bonds and loans. We compare the exposure stand-alone risks and the co-movements between exposures from the bottom-up, two-dimensional lattice approach with those from the top-down approaches for portfolios with different characteristics, such as sensitivity to interest rates in particular. In cases where the top-down approaches produce results that are not sufficiently accurate, we illustrate how to calibrate the correlation parameters used in top-down approaches to significantly improve accuracy.

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<sup>1</sup> The recent document “Range of practices and issues in economic capital frameworks” from the Basel Committee on Banking Supervision reports methodologies, challenges, and practices in risk aggregation.

<sup>2</sup> For example, Dimitris N Chorafas, in his book, *Economic Capital Allocation with Basel II*, states that “In general, in the 1980s and through most of the 1990s economic capital tended to be allocated in the following proportions: 2/3 to credit risk, and; 1/3 to market risk. Among major money-center banks these proportions have changed, tilting towards market risk. By contrast, among conservative retail banks credit risk represents much more than 2/3 of exposure. The reason why in several big banks market risk tends to match credit risk is securitization, and particularly credit derivatives.”

<sup>3</sup> For more information, see “Range of practices and issues in economic capital frameworks,” the Basel Committee on Banking Supervision, March 2009.

Delving into current industry practices, the Basel Committee outlines several typical approaches for top-down risk aggregation.<sup>4</sup> According to a recent Basel report, the majority of financial institutions use one or more of the following aggregation methods.

1. Simple summation through a fixed diversification percentage summation.
2. Aggregation on the basis of a risk variance-covariance matrix, also referenced as copula methods.
3. Full modeling of common risk drivers across all portfolios.

The first two approaches can be referred to as top-down approaches. Depending on the approach employed with the third, it can be either top-down or bottom-up. Under the top-down approach, the risk exposure is first assessed on each risk type and then aggregated to arrive at the overall risk measure. Under the bottom-up approach, the effects of different risk types are accounted for at the individual exposure level as well as the portfolio level.

This paper formalizes more sophisticated top-down approaches. In the simplest case, which we refer to as the traditional correlation approach, a single correlation parameter describes the relationship between any pair of risk types. While simple, a single parameter is typically insufficient to describe the joint behavior of the multitude of factors that typically drive credit and market systems (an important risk type pair). For example, RiskFrontier contains over 200 credit risk-related factors, and market risk systems frequently contain hundreds of factors. A more accurate description of the aggregated loss would require a description of the joint distribution of factors (i.e., a combined variance-covariance matrix that describes the covariance between market and credit factors).

Within the context of a simulation-based analysis, a sophisticated top-down system would jointly simulate all factors (market and credit), accounting for their covariance. In each trial, the realized credit factors would be used to look up the credit losses and the market factors would be used to look up the market losses. Since the joint variance-covariance matrix is used, total losses can then be added to arrive at a total loss distribution. Regarding top-down approaches, this paper demonstrates how we devise practical methods to integrated risk in this spirit without having to fully integrate the various Monte Carlo engines associated with the different systems. Instead, we can leverage existing systems to arrive at an aggregated estimate of risk.

Regardless of the level of sophistication associated with the integration of the variance-covariance matrix, the top-down approaches described above will not, in general, provide a complete description of aggregate risk for certain portfolios. To see why, consider the case of a fixed rate callable bond. In this case, the decision to exercise the call option depends on both interest rate risk and credit risk, and we need a two-dimensional system to know when the bond is called (at low interest rate, or good credit states, or a combination of the two). Two one-dimensional systems can only approximate the prepayment region.

Unfortunately, bottom-up approaches are computationally cumbersome. In the example above, a two-dimensional system is needed versus two one-dimensional systems that are needed with the top-down approach. Despite this challenge, in this study we present results from a two-dimensional lattice which jointly analyzes interest rate and credit risk. We utilize the system to benchmark a top-down approach, which is comparably parameterized. In some cases, interest rate insensitive instruments in particular, the differences between the two approaches are minimal, suggesting the top-down approach is sufficient in describing integrated risk. However, results can be substantively different for instruments that face both interest rate and credit risk. For example, the simple top-down approach can overstate Unexpected Loss (UL) by more than 25% relative to the more precise bottom-up approach for a typical vanilla bond portfolio. This study provides a reference point for when the top-down approach requires refinement, as well as appropriate calibration of the top-down approach in situations where distortions are substantive.

The remainder of the paper is organized in the following way.

- “Top-down Approaches to Risk Integration” introduces top-down approaches for risk measurement.
- “Bottom-up Approaches to Risk Integration” describes the bottom-up methodology used in this study.
- “The Correlation between Interest Rates and Credit Risk” discusses and motivates the interest rates and credit correlation parameterization used in this study.

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<sup>4</sup> “Range of practices and issues in economic capital frameworks,” the Basel Committee on Banking Supervision, March 2009.

- “Calibrating Correlation Parameters in Top-down Approaches” illustrates how to calibrate the correlation parameters used in top-down approaches.
- “Conclusion” provides concluding remarks.

## Top-down Approaches to Risk Integration

As discussed in the introduction, financial organizations are typically required to assess overall risk, and the organization units often have sophisticated risk systems in place. For example, a unit responsible for the trading book is likely to have a good idea of its market risk exposure, but their system will not typically take into account the credit risk of the banking book. The top-down approaches offer a solution where each risk source is analyzed separately and combined in order to arrive at the overall risk picture. While the top-down approach is convenient in that it utilizes existing systems and faces a relatively low computation burden, it faces shortcomings when it comes to accuracy.

In this section, we discuss several top-down approaches to risk aggregation. Some of these approaches are common and discussed in the Basel working paper, and some, to the extent of our knowledge, are new.<sup>5</sup> We omit the straightforward summations methods from the discussion, given its extreme simplicity. We focus on the following approaches: Traditional correlation, multifactor model through conditional simulation, and multifactor model through calibrated loss distribution. Note that combinations of these approaches are possible as well.

We focus on the aggregation of loss distributions as opposed to value distributions, since it is more robust to possible double-counting of instrument values that might be considered under different risk types. Moreover, we utilize the expected value of the distribution as the loss reference point (i.e., we measure losses in excess of Expected Loss) since it results in a loss distribution with the convenient property that the mean equals zero. This allows for the distributions to be aggregated without worrying about double-counting means; the mean of the sum will be equal zero as well. More formally, the loss ( $L$ ) and value ( $V$ ) distributions of risk type  $k$  face the following relationship:

$$\widetilde{V}_k = E[\widetilde{V}_k] - \widetilde{L}_k \quad (1)$$

If two value distributions,  $i$  and  $j$ , are aggregated we get  $\widetilde{V}_i + \widetilde{V}_j$ . The mean of this distribution,  $E[\widetilde{V}_i] + E[\widetilde{V}_j]$  will not equal the correct mean,  $E[\widetilde{V}_{i \oplus j}]$ , where  $i \oplus j$  denotes the combined distribution of risk types  $i$  and  $j$ . The difference comes from risk types that contain overlapping assets. For example, if counterparty risk associated with a market instrument is accounted for in the credit portfolio, the instrument will show up in both the market portfolio and credit portfolio. Naïvely combining the values of the portfolios will double count the value of this instrument. Instead, if the analysis is conducted in loss in excess of Expected Loss space, the mean is always zero.

## Traditional Correlation

Under this approach, which is referenced as copulas in the Basel working paper, the loss due to each risk source is defined by its marginal loss distribution. These marginal distributions might be constructed independently, using different risk systems. For example, one loss distribution might represent market risk in the bank’s trading book, while two others might represent credit risk for two bank units. To combine these loss distributions, a correlation structure is specified that consists of a copula function and the corresponding variance-covariance matrix. This structure describes the relationship between various risk sources. A joint normal distribution is typically specified, but a variety of copula functions are available in practice.

Depending on the chosen copula function and the desired integrated risk statistics, analytic or simulation calculation methods can be employed. For example, if a simulation is used, then during each trial losses from different risk systems are aggregated to construct the total loss distribution. As a practical matter, uniform random variables can be simulated

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<sup>5</sup> See “Range of practices and issues in economic capital frameworks,” the Basel Committee on Banking Supervision, March 2009.

and mapped to the respective inverse cumulative loss distribution so that their joint distribution adheres to the chosen copula and covariance matrix. The resulting losses are aggregated to arrive at the integrated loss for that trial.

Formulaically, the joint loss distribution function can be represented by

$$P[\tilde{L}_1 \leq l_1, \dots, \tilde{L}_N \leq l_N] = F_{Total}(F_1^{-1}(l_1), \dots, F_N^{-1}(l_N), \Sigma), \quad (2)$$

where  $F_k^{-1}(\cdot)$  is the inverse cumulative distribution function for the risk type  $k$ ,  $F_{Total}(\cdot, \dots, \cdot, \Sigma)$  is the chosen copula function, and  $\Sigma$  is the underlying covariance matrix that describes the covariances between loss distributions for different risk types. The total loss is just the sum of the losses due to different risk types. Therefore, its distribution can be calculated analytically or via simulation based on the joint distribution above.

## Multifactor Model through Simulation

The traditional approach requires the description of correlation between the loss distributions and not the underlying factors. Frequently, the correlation is described through a single parameter. In general, this single correlation parameterization is difficult to obtain directly and is typically insufficient to describe the correlation structure. After all, each risk source is often driven by multiple factors and so the correlation between risk sources is likely multifaceted and not describable through a simple structure. To address this shortcoming of the traditional approach, we introduce approaches that are based on the correlations between the underlying factors. At the same time, we maintain the practicality of the traditional approach and allow the use of the existing risk systems for the construction of loss distributions for different risk types and sources.

Even with a comprehensive description of the correlation structure across all factors for all risk sources, the joint simulation of all factors is not straightforward from an implementation standpoint. Each risk system will require the input of the relevant factor realizations for each trial. Depending on the infrastructure of the systems, this can make the procedure time-consuming and, in some cases, impossible. There are several ways to circumvent that problem.

The first approach is based on the conditional simulation of the underlying factors. We start by describing the joint covariance structure for the underlying factors of all risk systems. The knowledge of the joint distribution structure allows for the construction of conditional distribution for a subset of the factors given the realization of the rest of the factors. Knowing the conditional distributions provides a means for the conditional simulation. For a particular realization these operations can be done, for example, in a sequential manner.

First, one would simulate a factor set for the first loss distribution,  $f_1$  and calculate the corresponding loss value for the loss distribution 1 using, for example, a bottom-up approach. Then, using the assumed global factor model, construct the conditional distribution for the factor set  $f_2$  or distribution that describes  $f_2 | \bar{f}_1$ .

After simulating the factor realizations from that conditional distribution it will be possible to calculate the loss value for the second loss distribution. To obtain the final value this operation is then repeated with each additional factor set, i.e., the last distribution in the sequence will be simulated using the conditional distribution for the factor set  $N$ ,  $f_N | f_1, \dots, f_{N-1}$ . Finally, for each trial, losses from different risk systems are aggregated to construct the total loss distribution and the total loss function is

$$L^{Total} = L^1(f_1) + L^2(f_2 | f_1) + \dots + L^N(f_N | f_1, \dots, f_{N-1}), \quad (3)$$

where  $f_k | f_1, \dots, f_{k-1}$  denotes the conditionally simulated factor realizations, i.e., the set of factors  $f_k$  is simulated from the conditional distribution for that set if factors and conditions are set by the values of realizations for factor sets  $f_1, \dots, f_{k-1}$ .

## Multifactor Model through Calibrated Loss Distribution

The approach described in the previous subsection is very flexible with regard to the valuation part of the risk systems used for the construction of loss distribution; it does not impose any restriction on the value and loss calculations for a given set of factor realizations. Depending on the correlation structure, it might be analytically difficult to construct or implement conditional distributions for the factor sets under the analysis. In these cases, the risk can be integrated using a calibrated version of the loss distributions along with unconditional joint simulation of underlying joint factor set.

As with the approach described in the previous subsection, a description of the joint covariance structure for the underlying factors is needed. Using these jointly simulated factor realizations, the loss values for each risk type can be constructed separately using the actual risk system or its calibration. To arrive at the total loss distributions, losses from each risk system are aggregated.

For example, suppose that for risk type 1 a bottom-up analysis is conducted while for the rest we have to use the calibrated loss functions. Then the total loss function is represented by

$$L^{Total} = L^1(f_1) + \widehat{L}^2(f_2) + \dots + \widehat{L}^N(f_N), \quad (4)$$

where  $\widehat{L}^k(f_k)$  are the calibrated loss functions and the factor sets  $f_1, \dots, f_N$  are simulated jointly and unconditionally.

To calibrate a particular loss distribution  $k$ , one can use regression techniques to estimate parameters for a polynomial representation, such as Figure 1 and the following equation:

$$L_i^k = \beta_k^0 + \sum_{j=1}^{NF_k} \sum_{l=1}^{N_P} \beta_k^{j,l} (f_{k,i}^j)^l + \sum_{j=1}^{NF_k} \sum_{l=1}^{NF_k} \beta_{k,cross}^{j,l} (f_{k,i}^j f_{k,i}^l) + \sqrt{1 - R_k^2} \sigma_k \varepsilon_{k,i}, \quad (5)$$

where  $f_{k,i}^j$  is the realization of factor  $j$  used in construction of loss distribution  $k$  during trial  $i$ ,  $R_k^2$  is the R-squared for the polynomial calibration of loss function  $k$ ,  $\sigma_k$  is the sample standard deviation of loss distribution  $k$ , and  $\varepsilon_{k,i}$  is the idiosyncratic portion of the loss distribution  $k$  for trial  $i$ . The betas, R-squared, and sample standard deviation calculated during the polynomial calibration will be used for the calculation of  $\widehat{L}^k(f_k)$  for a given set of realizations of factors and idiosyncratic draws.

A benefit of this calibration approach is its ability to quickly provide losses for a particular loss distribution for a random set of underlying factors realizations. The shortcoming is the loss in precision coming from using a calibrated function as opposed to the actual loss distribution. Loss of precision can be offset by choosing a more appropriate calibration function, such as a higher-degree polynomial.

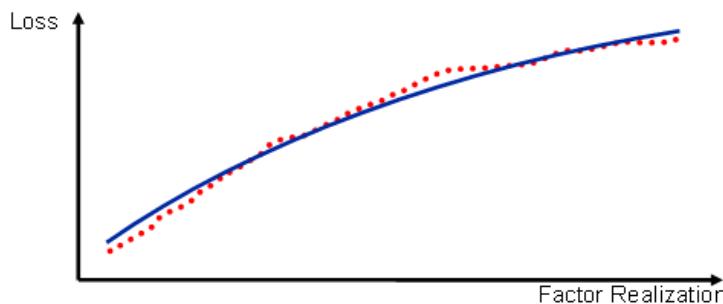


Figure 1 Calibrated loss distribution

## Bottom-up Approaches to Risk Integration

In this section, we describe a bottom-up approach to consistently evaluate instrument losses accounting for different risk sources and then aggregate. In particular, we model both the credit and interest rate risks in a two-dimensional lattice. We first build credit and interest rate lattices separately, then we construct the two dimensional lattice. This framework allows us to model both the credit risk and interest rate risk and their interactions in a consistent way at instrument level. For example, the issuer of a callable bond would exercise the call option when its credit quality improves, or the risk-free interest rate drops, or some combination of these two scenarios. A top-down approach would have difficulty modeling this type of behavior in a consistent way.

### Credit Lattice

Figure 2 shows the credit lattice structure used in RiskFrontier.

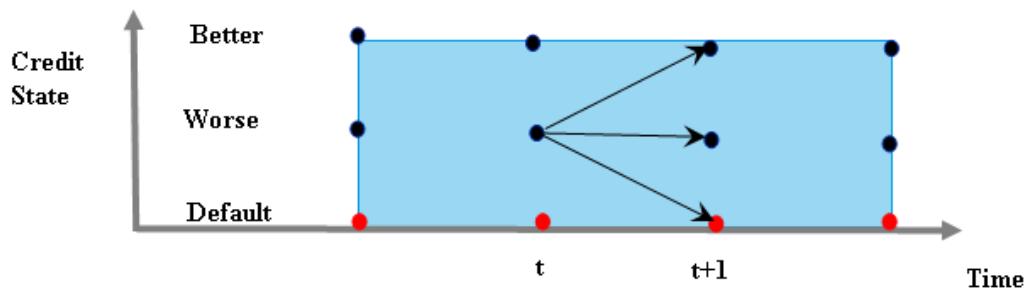


Figure 2 RiskFrontier credit lattice structure

Each node on the lattice represents the credit quality of the reference entity at the corresponding time. The probabilities of credit migration are based on the Moody's Analytics Distance-to-Default (DD) dynamics model or on a user-provided migration model (such as a rating-based migration). A backward valuation procedure is used to calculate the value of an instrument at the analysis date, using the risk-neutral valuation framework. Moreover, the lattice model provides an estimate of the distribution of the value of an instrument at horizon, which is used to conduct risk analysis of the portfolio. One advantage of the lattice model is its ability to explicitly model and account for options and credit-contingencies such as prepayment options of bank loans, call and put options of corporate bonds, and dynamic usage schedules for revolvers. The credit-only lattice assumes a deterministic interest rate structure.

### Interest Rate Lattice

Before describing the two-dimensional lattice, we next explain how we model a stochastic interest rate process in a lattice framework. The interest rate lattice is similar to the credit lattice in Figure 1, except that the nodes represent different short rate levels at different time points. The interest rate lattice tracks the stochastic interest rate movement through time. In this study, the interest rate lattice is constructed based on the well known Hull-White model for the short rate process:

$$dr_t = (\theta(t) - \kappa r_t) dt + \sigma dW_t, \quad (6)$$

where  $\sigma$  is the instantaneous volatility of the short rate,  $\kappa$  reflects mean-reverting speed. A large  $\sigma$  or small  $\kappa$  indicates a more volatile interest rate environment. The time varying parameter  $\theta(t)$  is related to the initial interest rate term structure:

$$\theta(t) = \frac{\partial f}{\partial T}(0, T)|_{T=t} + \kappa f(0, t) + \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t}), \quad (7)$$

where  $f(0, t)$  is the instantaneous forward rate that applies to time  $t$  observed at time 0. It can be computed as  $f(0, t) = -\frac{\partial \log(P(0, T))}{\partial T}|_{T=t}$ , where  $P(0, T)$  is the price at time 0 of a zero coupon bond with maturity  $T$ .

One of the main advantages of the Hull-White model is its analytical tractability. Bonds and European options on interest rate can be valued analytically; standing at time  $t$ , the distribution of future short rate can be determined analytically in terms of the short rate at time  $t$ . The main disadvantage of the Hull-White model is that it does not exclude negative interest rates. Our interest rate lattice framework is flexible enough to potentially accommodate other interest rate models, such as those by Cox, Ingersoll, and Ross (1985), Black, Derman, and Toy (1990), and Black and Karasinski (1991), etc.

One key issue in constructing an interest rate lattice is its calibration. Even though we used the analytically tractable Hull-White model, we still need to calibrate the lattice due to the discretization over time and over interest rate level, and the truncation of interest rate range to be non-negative. We choose the interest rate level of each node and calculate the transition probabilities between nodes at consecutive time points so that the zero coupon bond prices of all maturities calculated via the risk-neutral backward induction on the lattice match the ones implied by the initial term structure. Additional calibration can be performed on the volatility of interest rates, which can be achieved by using the market prices of interest rate derivatives such as swaptions.

## Two-dimensional Lattice

Figure 3 shows a two-dimensional lattice.

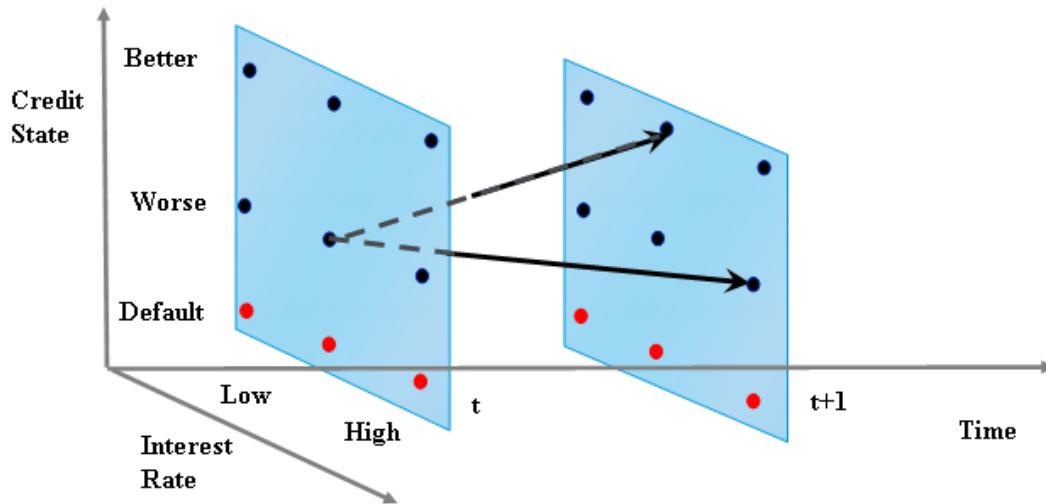


Figure 3 Two-dimensional lattice

Once we have the two separate one-dimensional lattices for credit and interest rate, we construct a two-dimensional lattice as their Cartesian product. At a given time  $t$ , the lattice node is represented by  $S_t = (R_t, r_t)$ , where  $R_t$  and  $r_t$  correspond to the credit and interest rate lattice nodes at time  $t$ . We also need to find the one period transition

probability  $P(S_t, S_{t+1})$ . Given the marginal transition probabilities along the credit dimension and along the interest rate dimension, and correlation between credit and interest rate movements, the joint transition probabilities can be calculated on the two-dimensional lattice through a copula. Empirically, we have found that correlation between changes in interest rates and default events are not statistically significant after conditioning on the Moody's Analytics EDF<sup>TM</sup> (Expected Default Frequency) credit quality measure and the term structure of interest rates. Therefore it is reasonable to set the correlation between credit and interest-rate risk to zero, which significantly simplifies the two-dimensional lattice construction. In this case,  $P(S_t, S_{t+1}) = P(R_t, R_{t+1}) \cdot P(r_t, r_{t+1})$ .

Once we have the two-dimensional lattice, principal and coupon cash flows of an instrument can be calculated at each lattice node based on the credit quality and interest rate level of the node. We then use backward induction to calculate the instrument value at the analysis date and evaluate the embedded options at each step. The value distribution of the instrument at horizon can also be calculated on the two-dimensional lattice by combining the cash flows received before horizon and future cash flows valued at horizon. This distribution can be expressed as  $\{V(S_H), P(S_H)\}$ , where  $V(S_H)$  is the expected value of the instrument at state  $S_H = (R_H, r_H)$  at horizon  $H$ , and  $P(S_H)$  is the probability of reaching  $S_H$  at  $H$ .

## Calculating Portfolio Risk Measures

In order to analyze the risk and return of a portfolio of instruments that are sensitive to both credit and interest rate risks, we first perform a bottom-up analysis on each instrument on the two-dimensional lattice and obtain the associated value distribution at horizon. We then simulate the realized credit and interest rate state at horizon, and determine the instrument values at horizon. The sum of these simulated instrument values is the portfolio value for a simulation trial. More specifically, the interest rate state is simulated based on the transition probability from the analysis date to different interest rate levels at horizon. The credit states are then simulated using a normal copula approach, where the asset returns have a joint Normal distribution. With the interest rate and credit states simulated for all instruments, the corresponding instrument values are determined based on each instrument's value distribution at horizon.

Repeated simulation trials produce a portfolio value distribution and loss distribution. Based on these distributions, further portfolio risk and return measures such as expected spread, Unexpected Loss, and economic capital are calculated.

## The Correlation between Interest Rates and Credit Risk

Whether we take a top-down or bottom-up approach in integrating risk, we need correlation estimates to link the various sources of risk sources. For the remainder of this paper, we focus on interest rate and credit risk integration, and now turn our attention to the co-movement of these two risk sources.

Generally speaking, lead and lag dynamics between interest rates and the credit cycle are complicated. An example is the relationship between default and interest rate when the economy is in recession and the default rate is high. Interest rates are often relatively low through the traditional central bank monetary policy of lowering rates in the hope of stimulating the economy. When the economy improves, the central bank tends to raise rates. Given that government rates often form the basis of the cost of capital faced by the companies, when the interest rates increase, a firm must generate a higher rate of return on its assets to stay in business. If the cost of capital is higher than the rate of return for a particular company, that firm will run into financial insolvency or bankruptcy. In other words, the central bank is raising rates in an effort to slow the economy. Therefore, we may conjecture that the relationship between default risk and interest rates is sensitive to some measure of where the economy is in the business cycle and/or other macroeconomic factors. Moreover, co-movements between interest rates and default risk may exhibit different behavior whether analyzed contemporaneously or in a causal or predictive setting.

The Moody's Analytics working paper titled "The Relationship between Default Risk and Interest Rates: An Empirical Study" analyzes, among other things, the contemporaneous correlation between interest rates and defaults at the quarterly frequency. Specifically, following question is addressed: Conditional on the current term structure of interest rates as well as on EDF credit measures, are distributions of future interest rates and defaults correlated? The study finds that correlation between unexpected changes in interest rates and default are not statistically significant, suggesting that within the context of an integrated economic capital model one can set the correlation between credit and interest rate risk to zero. It is worth discussing this point further, given the discussion above, in which we describe common patterns between movements in interest rates and defaults.

The empirical study does not claim that there is no relationship between interest rates and default rates. The claim is that, on average, an increase in interest rate can be equally associated with an increase or decrease in default rates, depending on the sequence of events that lead to the change in environment. Motivated by this study, the correlation between credit and interest rates is set to zero for the remainder of the paper. Let us stress that the qualitative results of this paper relating the top-down and bottom-up approaches will not change if the correlation is non-zero, and so we should not take this assumption too literally when interpreting the final results.

## Calibrating Correlation Parameters in Top-down Approaches

In this section, we compare aggregated portfolio results using a top-down approach with those using a bottom-up approach. Moreover, we introduce a correlation calibration methodology, which allows us to better approximate results from a bottom-up system using a top-down approach. We choose the setting to represent a possible institutional structure where risks are aggregated across two separate systems using a top-down approach—market losses coming from interest rate risk are analyzed in the Fermat system, while credit losses are calculated with RiskFrontier.

In this section, we use portfolios comprised of pools of large numbers of homogeneous instruments. Depending on the exercise, instruments are either floating or fixed rate, and may or may not have a prepayment/call option. For the market risk component, the uncertainty comes from the interest rates with dynamics described by a Hull-White model. Additionally, reference entities face a flat annualized default probability term structure of 1% and R-squared of 20%. All instruments mature in five years and have a loss given default (LGD) value of 50%. The analysis focuses on comparison of Unexpected Loss and correlation between instruments within and across the four types.

The remainder of the section is organized as follows.

- “Portfolios of Vanilla Instruments” compares portfolio UL from top-down and bottom-up analysis for vanilla fixed and floating rate instruments.
- “Portfolios of Instruments with Options” compares portfolio UL from top-down and bottom-up analysis for prepayable and callable floating and fixed rate instruments.
- “Impact of Stochastic Interest Rate on Cross-instrument Loss Correlations” analyzes the correlation structure across instruments in different portfolios of fixed, floating, vanilla, and prepayable/callable instruments.
- The analysis in the first three sections is leveraged in “Calibration of Correlation Parameters Used in a Top-down Approach” when calibrating correlation in the top-down approach so that the results match those produced from the bottom-up system.

### Portfolios of Vanilla Instruments

In the first exercise we compare the result of top-down and bottom-up approaches for the portfolios of vanilla instruments, i.e., instruments without any options. As shown in Table 1 there is almost no difference for the floating rate instruments, and considerably higher UL for fixed rate instruments. It is not surprising to see the almost equivalent numbers for the floating rates because almost all uncertainty in interest rates is already accounted for. The exaggerated UL for the top-down approach results from inappropriate accounting for interest rate risk in the default state—under the bottom-up approach the default event to some extent hedges the interest rate risk.

**Table 1** Results for top-down and bottom-up approaches: Portfolios with vanilla instruments

Unexpected Loss					
	Market Only	Credit Only	Integrated (Top-down)	Integrated (Bottom-up)	Relative Difference
Floating Rate (Vanilla)	0.3	3.2	3.2	3.2	~0%
Fixed Rate (Vanilla)	6.5	3.1	7.2	5.7	27%

## Portfolios of Instruments with Options

For this case study we compare the results for the portfolios with instruments that have embedded optionalities. Table 2 shows that we see no difference in UL for floating rate instruments and a considerably lower number for the fixed rate instruments. Once again, it is not surprising to see the almost equivalent numbers for the floating rates because almost all uncertainty in interest rates is already accounted for. The muted UL for the top-down approach is driven by the double counting of states where embedded options are exercised.

**Table 2** Results for portfolios of instruments with embedded optionalities

	Unexpected Loss				
	Market Only	Credit Only	Integrated (Top-down)	Integrated (Bottom-up)	Relative Difference
Floating Rate (Prepayable)	0.5	2.2	2.3	2.3	~0%
Fixed Rate (Callable)	1.4	2.6	2.9	3.9	-26%

## Impact of Stochastic Interest Rate on Cross-instrument Loss Correlations

In this subsection, we analyze cross-instrument loss correlations for fixed, floating, vanilla, and prepayable/callable instruments. Table 3 presents pairwise correlations for these instruments when interest rates are deterministic, while Table 4 presents pairwise correlations when interest rates are stochastic. Not surprisingly, correlations are immune to interest rate dynamics for floating rate instruments. On the other hand, fixed rate instruments are affected significantly.

The pairwise loss correlations between two fixed rate instruments jumps from around 13% to about 30%–40% when we switch the interest rate from deterministic to stochastic. The reason is that high interest rate decreases values of all fixed rate instruments, and low interest rate increases values of all fixed rate instruments. Therefore, introducing additional interest rate risk would stretch the loss distributions of all the fixed rate instruments in the same direction, and hence increase the cross-correlation.

**Table 3** Pairwise correlations: Deterministic interest rates

Deterministic Interest Rate	Floating Rate (Vanilla)	Floating Rate (Prepayable)	Fixed Rate (Vanilla)	Fixed Rate (Callable)
Floating Rate (Vanilla)	15.1%	13.1%	15.2%	13.7%
Floating Rate (Prepayable)	13.1%	11.1%	12.9%	12.2%
Fixed Rate (Vanilla)	15.2%	12.9%	12.9%	13.7%
Fixed Rate (Callable)	13.7%	12.2%	13.7%	13.0%

**Table 4** Pairwise correlations: Stochastic interest rates

Stochastic Interest Rate	Floating Rate (Vanilla)	Floating Rate (Prepayable)	Fixed Rate (Vanilla)	Fixed Rate (Callable)
Floating Rate (Vanilla)	15.3%	13.2%	9.6%	9.4%
Floating Rate (Prepayable)	13.2%	11.7%	6.5%	6.6%
Fixed Rate (Vanilla)	9.6%	6.5%	38.8%	33.1%
Fixed Rate (Callable)	9.4%	6.6%	33.1%	29.7%

The results presented in this section suggest that the comprehensive risk aggregation might have significant impact on the allocation of capital across the instruments in the aggregated portfolio.

## Calibration of Correlation Parameters Used in a Top-down Approach

As demonstrated previously, the top-down approach may produce substantially different  $UL$  for portfolios that are sensitive to interest risk. In this subsection, we introduce a methodology in which results from the previous subsection, “Impact of Stochastic Interest Rate on Cross-instrument Loss Correlations,” can help reduce the  $UL$  gap between the top-down and the bottom-up approaches. Specifically, correlation is calibrated between interest and credit factors in the top-down aggregation.

First, we calculate an approximate aggregated portfolio  $UL$  based on the bottom-up approach. Next, we choose a calibrated top-down correlation to match the bottom-up  $UL$ . We then use the calibrated correlation between credit and interest risks in the construction of the joint loss distribution and calculation of the relevant risk statistics such as Economic Capital and Expected Shortfall. Analytically, the calibration is based on the formula that relates the  $UL$  for the portfolios:

$$UL_{bottom-up, credit+market}^2 = UL_{credit}^2 + UL_{market}^2 + 2\rho_{top-down, calibrated} UL_{credit} UL_{market} \quad (8)$$

Or, after the rearrangement,

$$\rho_{top-down, calibrated} = \frac{UL_{bottom-up, credit+market}^2 - (UL_{credit}^2 + UL_{market}^2)}{2UL_{credit} UL_{market}} \quad (9)$$

In Equation (9) the portfolio  $UL$  for market risk only and for credit risk only are typically available from the respective risk systems. The  $UL$  for the combined market and credit risks is either obtained from analyzing the portfolio through a bottom-up approach or calculated or approximated if the  $UL$  for the instruments in the combined portfolio and the pairwise correlations between them are known:

$$UL_{bottom-up, portfolio}^2 = \sum_i UL_{bottom-up, i}^2 + \sum_{i \neq j} \rho_{i,j} UL_{bottom-up, i} UL_{bottom-up, j} \quad (10)$$

Next, we illustrate through an example how the bottom-up portfolio  $UL$  is estimated first using Equation (10), and then the correlation between credit and market risks is calibrated using Equation (9). Suppose we have a portfolio that contains four pools of homogeneous instruments with the pool sizes shown in the last row of Table 5. Each pool is of a different instrument type with instrument notional amount all equal to \$100K. In the notation of Equation (10),  $UL_{bottom-up, i}$  is the  $UL$  for instrument  $i$  in currency units and the numerical values are reported in the first row of

Table 5;  $\rho_{i,j}$  is the pairwise instrument loss correlation between instruments  $i$  and  $j$ , whose numerical values are reported in Table 4. In particular, the diagonal elements in the table represent the correlations of the two homogeneous instruments of the same type. Off diagonal values are the correlations two instruments of two different types.

We then calculate the credit risk-only  $UL$  for our sample portfolio described in the last row of Table 5 using the credit risk system. Similarly, we calculate the market-only portfolio  $UL$  using the market risk system. Finally, we apply Equation (9) to find the implied correlation between the market and credit risk systems for the top-down approach to match the bottom-up portfolio  $UL$ .

**Table 5** Sample portfolio statistics (Currency Units are in thousands)

	Floating Rate (Vanilla)	Floating Rate (Prepayable)	Fixed Rate (Vanilla)	Fixed Rate (Callable)
Instrument UL in Currency Units	8.0	6.4	9.1	7.1
Instrument Analysis-date Value in Currency Units	98.22	96.25	96.93	94.76
Number of Instruments in the Sample Portfolio	100	200	300	400

The bottom-up portfolio  $UL$  is 3.58M according to the formula Equation (10). More precisely, in our example there are 1000 terms for instrument  $ULs$  in currency units in the first sum in Equation (10). The pairwise instrument loss correlation  $\rho_{i,j}$  in the second sum is one of the 16 entries in Table 4. For example, in the second sum of Equation (10), there are 2 by 100 by 200 terms accounting for pairs between vanilla floating rate and prepayable floating rate instruments, and  $400 \cdot 400 - 400$  terms accounting for pairs between callable fixed rate instruments. We calculate the credit risk-only portfolio  $UL$  to be 2.60M, and the market risk-only portfolio  $UL$  to be 1.88M. Using Equation (9), the calibrated top-down correlation is

$$\begin{aligned}\rho_{top-down,calibrated} &= \frac{UL_{bottom-up,credit+market}^2 - (UL_{credit}^2 + UL_{market}^2)}{2UL_{credit}UL_{market}} \\ &= \frac{3.58^2 - (2.60^2 + 1.88^2)}{2 \cdot 2.60 \cdot 1.88} = 25.8\%\end{aligned}\tag{11}$$

Thus, for the construction of the integrated loss distribution under top-down approach for our example portfolio we would use the correlation of 25.8%.

Note that our case studies and the illustration of correlation calibration are based on four instrument types whose risk drivers—probability of default (PD) values, LGD values, RSQ values, etc.—are chosen to represent the average levels. As a result, the correlation calibration will achieve more accuracy if the instruments in the user's portfolio are closer to those in the case studies. When this is not the case, the calibration can be performed at a more granular level than discussed above. Specifically, one can analyze a larger number of pools of homogeneous instruments using the bottom-up approach and obtain instrument level correlation coefficients similar to those in Table 4, but for different groupings of PD, LGD, RSQ, etc.

## Conclusion

Bottom-up approaches in integrated risk management, while more accurate, are much costlier computationally. Meanwhile, top-down approaches take advantage of existing systems for different risk sources and avoid the computational burden. In this study, we discuss and formalize several top-down approaches that range in sophistication, flexibility, and computational efficiency. Additionally, we compare the top-down approaches with a comprehensive bottom-up approach that accounts for both interest rate risk and credit risk at the instrument level. We find that for portfolios of instruments that are not very sensitive to interest risks (such as floating rate loans), a top-down approach is sufficient for a practical application. For portfolios of instruments that are sensitive to interest rate dynamics in addition to credit risks (such as fixed rate bonds), the risk measures produced by top-down approaches could be substantially different from those produced using a bottom-up approach. For these portfolios where a straightforward top-down approach does not produce accurate results, we illustrate, analytically and through case studies and examples, how analysis results from a bottom-up approach could be used to calibrate the correlation parameters used in the top-down risk integration. The correlation calibration approach is particularly useful and practical for institutions that demand high accuracy in risk integration on a regular basis where a bottom-up analysis at each time becomes computationally impractical. Such institutions can take advantage of the computational efficiency and flexibility of top-down approaches and perform a bottom-up analysis and correlation calibration less frequently.

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