

EDF™ 8.0 MODEL ENHANCEMENTS

MODELING METHODOLOGY

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ABSTRACT

The Moody's KMV EDF (Expected Default Frequency) model of public firms is the pioneering implementation of a structural model that gives investors the ability to monitor credit risk across a broad range of firms. The release of the EDF 8.0 model represents a major recalibration of the model, which incorporates both a larger default dataset and improved estimation techniques that derive the EDF term structure from credit migration.

The EDF 8.0 model refines the mapping of the Distance-to-Default (DD) to the EDF level. The resulting EDF value is a superior measure of both absolute and relative risk. The EDF 8.0 model also provides improved granularity with a wider range of EDF credit measures so that fewer credits hit the model cap and floor.

These refinements meet the increased demands that market participants are placing on quantitative credit risk models. Market participants demand models that provide more granular measure of credit risk. They also want models they can use to determine the fair value spread on a given exposure. Models need to be transparent and validated. The validity of the EDF level needs to be demonstrated given its importance in computing both required economic and regulatory capital.

In this document, we describe the process of mapping the DD to an EDF credit measure. Further, we show the impact of the mapping changes on the EDF level, required economic capital, required regulatory capital, and the EDF Credit Categories, as well as the implications for the EDF-implied spreads for both Bonds and CDS contracts. Finally, we provide more details regarding the inner workings of the model.

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1 INTRODUCTION

The EDF™ (Expected Default Frequency) model from Moody's KMV measures a specific aspect of credit risk—the probability of default. In contrast to the spread on a bond or a credit derivative, an EDF credit measure does not contain information on the loss given default or the extent of systematic risk in the underlying exposure. Further, it is not impacted by the liquidity of the bond market or the CDS market. It is an absolute measure of default risk. The ratings published by Moody's Investors Service, in contrast, are designed to provide a measure of “relative creditworthiness” (page 2, Moody's Investors Service, 2004). In aggregate, the EDF credit measures change markedly over the course of the credit cycle thereby reflecting changes in the level of default risk.

At its core, the EDF credit measure is a mapping that converts Distance-to-Default (DD), a volatility adjusted measure of market leverage, into an actual probability of default. Market leverage is based on the market value of a company's assets rather than the book value of assets. The market value of assets is calculated through an option theoretic framework that utilizes the market value of equity as an input. Consequently, an EDF credit measure allows investors to triangulate between equity and debt markets. An EDF credit measure informs market participants of the implications of changes in various inputs for credit risk. For example, if a company's future cash flow prospects diminish then its stock price will decline, therefore indicating an increase in credit risk (holding all other factors constant). Nevertheless, the sensitivity of the firm's credit risk to its stock price depends on the magnitude of its leverage and the volatility of its assets. The more debt and the more volatility a firm has, the more sensitive the firm's credit risk will be to a decline in the value of its assets. This framework, as implemented by Moody's KMV, has repeatedly been found to be robust and predictive in assessing default risk.¹ It can also be used to explain variation in spreads across credits.²

When the EDF model was first developed there was very little transparency into prices in the debt market. Further, the credit risk management systems in financial institutions were largely based on qualitative analysis that lacked benchmarks to ensure consistency across the institution, and regulators required financial institutions to hold a fixed percentage of capital against a loan without regard to its risk.

Over the past decade, there have been substantial improvements in the transparency of the credit and fixed income markets. Further, financial institutions made substantial advancements in their credit risk management systems and regulators are moving toward risk-adjusted capital requirements as a part of Basel II. Consequently, market participants rely more heavily on quantitative risk models than ever before, and market participants demand more of the models on which they rely. Market participants want granularity from their credit risk measures: models should provide meaningful differentiation in risk across the full spectrum of credit quality. Participants want the models to be based on the most current information available: liability information in the model should be kept current and up to date. They want to be able to triangulate between markets: Is the EDF value coming out of the equity market consistent with the spreads observed in the debt markets? If not, why do they differ? They want to be able to use quantitative risk models to value the loans and credit derivatives in their portfolio: Given the EDF credit measure what is an appropriate risk-adjusted return for the exposure? They want empirical validation of the model: Can they demonstrate to themselves and their regulators that the model works as intended. And finally, they want transparency into the inner workings of the model: Why did the model give the risk assessment that it did?

Over the years, we enhanced the model to meet these demands, and the EDF 8.0 model enhancements represent our latest efforts in this regard. We updated the calibration of the model to reflect a much more broad set of default data (Figure 1), which increased the granularity of the model—the model produces a larger range of meaningful EDF credit measures.³ This results in fewer exposures reaching the EDF cap and floor. We employed credit migration techniques to improve the estimated EDF term structure. Consequently, the new EDF term structure reflects more accurately the migration risk of the firm and the EDF-implied credit curves will be more useful in establishing the fair market value of future cash flows that carry default risk. We validate the EDF level as a more accurate measure of the probability of

¹ See for example: Miller, 1998; Kealhofer and Kurbatt, 2002; Kealhofer, 2003a; Arora, Bohn and Zhu, 2005; Arora, Bohn and Korablev 2005; Korablev, 2005; Stein, 2005; and Cantor and Mann, 2006.

² See for example: Bohn 2000a and b; Kealhofer, 2003b; Berndt, Douglas, Duffie, Ferguson and Schranz, 2005; and Berndt, Lookman and Obreja, 2006.

³ The previous calibration of the model was developed with data available in 1995. We have more than doubled the number of defaults in our sample since then.

default. In addition, we provide a description of how the model works and how it is calibrated, thereby providing increased transparency into the inner workings of the model.

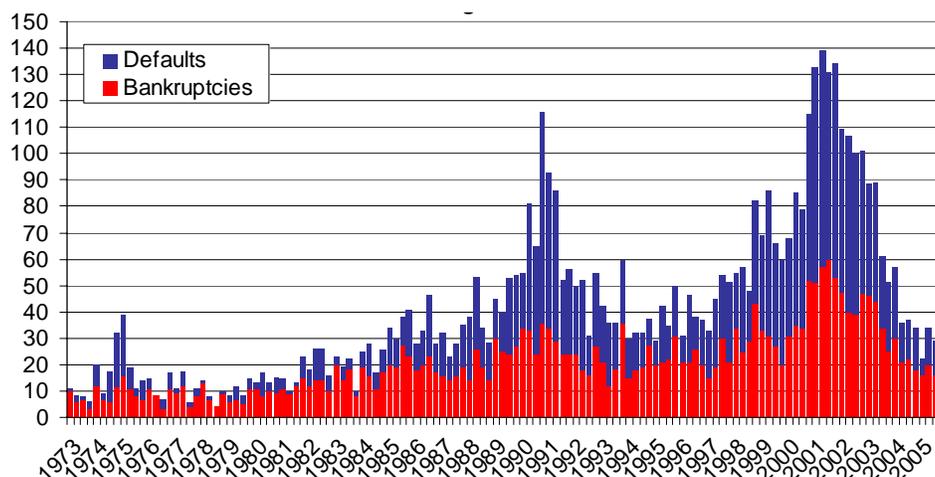


FIGURE 1 Quarterly Defaults and Bankruptcies, North American Public Companies, 1973–2006

Improving the model requires that we make changes to it. These changes have ramifications across the many applications of the model. Practitioners need to understand and prepare for these changes. In Section II, we analyze the impact of changing EDF credit measures on (i) the measures themselves, (ii) EDF-implied spreads, (iii) required economic capital, (iv) required regulatory capital, and (v) EDF Credit Categories.

In Section 2.1, we show that the new mapping increases the meaningful range of the EDF credit measures produced by the model. We improved our ability to differentiate among highly distressed firms. Therefore, we can increase the upper EDF threshold from 20% to 35%. We are also able to better differentiate risk at the high-quality end of the credit spectrum, allowing us to reduce the minimum EDF level to 1 basis point. We show that during a good stage of the credit cycle, the EDF measure is less likely to bottom out at the EDF floor, while during a poor stage of the credit cycle, the EDF measure is less likely to max out at the EDF cap. Therefore, the new EDF mapping provides the user with increased granularity.

In Section 2.2, we examine the implications of the new term structure on the EDF-implied spreads for both bonds and CDS contracts. The new EDF term structure is steeper for the high-quality credits and less downward sloping for the low-quality credits. The change is shown to be more consistent with future EDF credit measures—the forward EDF levels line up with the realized EDF levels as observed in future time periods. Accurate forward EDF levels are required to better assess the migration risks associated with mid to long term maturities. The change in the slope of the EDF term structure changes the slope of the EDF-implied spreads—it is steeper for the high-quality credits and not as downward sloping for the low-quality credits. Consequently, the model is now more accurate for pricing purposes—it will in general increase the EDF-implied spreads for low-quality longer term debt, and reduce the EDF-implied spreads for high-quality shorter-term debt. The implications for EDF-implied CDS spreads are shown as well.

The new EDF levels change both required economic capital and required regulatory capital when used as default probabilities in these calculations. We analyze the impact of the new EDF levels on both in Sections 2.3 and 2.4. We examine the impact of the changes across two large portfolios—high- and low-quality bond portfolios—on both required economic and required regulatory capital. For the high-quality portfolio (evaluated at the end of 2005) the new EDF levels have little impact on either economic capital or regulatory capital, while for the low-quality portfolio (evaluated at the end of 2005) both regulatory and economic capital fall about 10%. It is important to stress that these results differ across portfolios and time periods, and we encourage users to perform a comparable analysis on their own portfolio.

EDF values are designed to provide an absolute measure of risk. For some applications, users may prefer a relative measure of risk. EDF Credit Categories (also known as EDF-implied ratings) provide such a measure. In section 2.5, we show that the increased granularity in EDF levels make the EDF Credit Categories a more useful measure of relative risk.

We show that the increased EDF granularity results in improved granularity of EDF Credit Categories—across different stages of the credit cycle, EDF Credit Categories are less likely to bunch into one category.

The third section of the document describes the methodology and implementation decisions that were involved in mapping the Distance-to-Default (DD) to the 1-year EDF credit measure. In mapping a 1-year DD to a 1-year EDF credit measure, there are many possible choices that one could make for defining default, choosing the calibration sample, and determining the minimum and maximum EDF credit measure produced by the model. We discuss the choices that we made and the reasoning behind them. Finally, we discuss the derivation of the EDF term structure using credit migration as measured by DD dynamics.

In Section 4, we further improve the transparency of the model by providing a description of how the model, which we call the Vasicek Kealhofer (VK) model, works.⁴ We begin by discussing how current liabilities, long-term debt, convertible securities, and preferred stock are explicitly modeled. We then describe the methodology for computing volatility as the combination of *empirical volatility* and *modeled volatility*. Finally, we provide the definition of DD as used in our model. This section is intended to resolve some ambiguity that exists in both the academic literature and the marketplace around how the VK model actually works, and the differences between it and other implementations of structural models.

⁴ For an introduction to the VK model and its application, please see *Modeling Default Risk* available at www.mkmv.com.

2 IMPACT OF MODEL CHANGES

The model changes impact all of the different uses of EDF credit measures, and practitioners will want to understand the impact. First, the EDF values for individual firms will change. Second, the EDF-implied spreads will change along with the implied credit curves. Third, changing EDF values will change both required economic and required regulatory capital. If the changes in the EDF values were to result in large changes in either required economic capital or required regulatory capital, then practitioners and their regulators would need to understand the impact and plan accordingly. We assess the impact on two large bond portfolios. Finally, we show that improved granularity of EDF credit measures result in more useful EDF Credit Categories, which provide a relative measure of default risk (in addition to the absolute level the EDF credit measures themselves provide). We discuss each of these in turn.

2.1 Increased Granularity in EDF Credit Measures

The overall change in the one-year DD-to-EDF mapping function is shown in Figure 2. The black line plots the 1-year EDF values computed under the original mapping against the 1-year EDF values computed under the new mapping on a log-log graph.⁵ The grey line plots the 45 degree line as a reference—when the black line crosses the grey line, the new EDF value equals the old EDF value. We see that the lowest quality companies that had the highest EDF values of 20%, are now better differentiated under the new mapping. At the other end, the group of companies that had EDF values truncated at 0.02% under the previous mapping, can now be differentiated under the new mapping. This group of companies now has EDF values that range from 0.01% to 0.04%. In the middle of the distribution, the new EDF values are generally lower than the old values—an EDF level of 0.50% under the original mapping becomes an EDF value of approximately 0.33% under the new mapping. In November 2006, the boundary between investment grade and speculative grade credits was approximately 0.06%. For these credits, the EDF values are approximately unchanged.

⁵ Graphed in this fashion, the “major tick marks” represent 0.01%, 0.10%, and 1.00%. The first “minor tick marks” represent 0.02%, 0.20%, and 2.00%, respectively. The second “minor tick marks” represent 0.03%, 0.30%, and 3.00%.

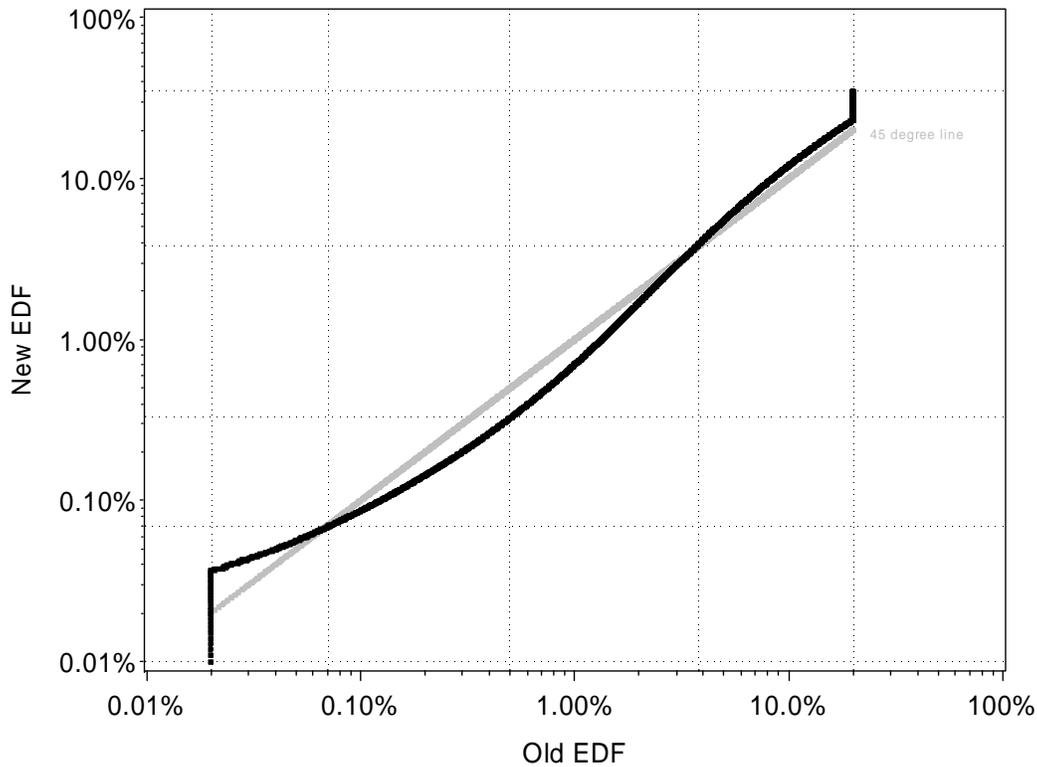


FIGURE 2 Overall Changes in One-Year EDF Credit Measures

2.1.1. Impact in Different Credit Environments

Just as the EDF credit measures anticipate deterioration and improvements for individual firms, aggregate EDF credit measures anticipate changing business climates for an industry, region, or country. At the end of 1998, EDF levels rose rapidly across the board even with a rising equity market because volatility increased as well. Subsequently, the actual default rate soon increased. During the 2004–2006 period, EDF values fell across the board reflecting decreased volatility and liabilities. Both credit spreads and actual default rates were also very low during this time period. The original mapping had a tendency to “max out” during a recession and “bottom out” during an expansion. These tendencies lead to a loss in granularity during extreme periods in the credit cycle.

The new mapping reduces the number of firms that hit the boundaries. At the end of 2001, a time of credit turmoil, the old mapping assigns an EDF level of 20% to about 14% of the population, while the new mapping reduced the capped population to about 7% (right-hand panel of Figure 3). In the benign credit environment at the end of 2005, the number of firms at the minimum EDF credit measure reduced from 11% under the previous mapping to 5% under the updated mapping because the floor lowered from 0.02% to 0.01% (left-hand panel of Figure 3).

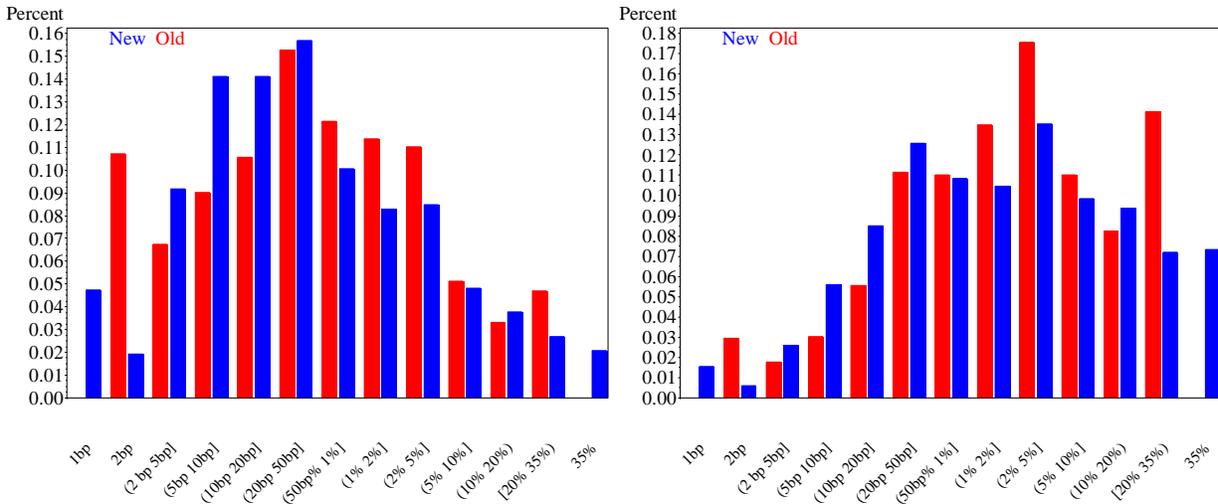


FIGURE 3 Increased Granularity in Both Risky and Benign Credit Environments

The left-hand panel represents the distribution of the new and old EDF credit measures during the good stage of the credit cycle (end of 2005), while the right-hand panel represents a poor stage of the credit cycle (end of 2001).

2.1.2. Level Validation under the New Mapping

A group of firms with a particular EDF measure, by construction, has the same expected number of defaults irrespective of the particular stage of the business cycle. Using the Moody’s KMV public firm default database, we can compare the predicted distribution of default rates versus the realized default rate. Making such a comparison requires a somewhat involved analysis due to the fact that common risk factors shared by individual firms cause the distribution of defaults to be highly skewed. A large negative shock has a disproportionate impact on default rates relative to a large positive shock. Consequently, the predicted mean default rate will be greater than the predicted median default rate. To deal with this issue, we simulate the distribution of predicted defaults using the EDF credit measures and a single factor Gaussian model for asset returns following Korabev and Kurbat (2002).

In Figure 4, we compare the observed default rates with the distribution of predicted default rates. The actual default rate is plotted against the median, the 10th percentile and the 90th percentile of the distribution. In every year, the realized default rate is close to the predicted and within the 10th to 90th percentile of the distribution. The largest departure is in 2003 when the model substantially over-predicted default rates. The over prediction of defaults in 2003 implies that there was a positive and unanticipated shock to the economy during this time period. Note that there is no clear tendency for the model to overstate defaults during a recession and understate them during an expansion or vice versa. This contrasts with agency ratings. When a similar methodology is applied to measure the level of risk associated with agency ratings, default rates within a rating group are clearly elevated during recessions.⁶

⁶ cf., Section 2.7 of Bluhm, Overbeck, Wagner, 2003; or Appendix D of Dwyer, 2006.

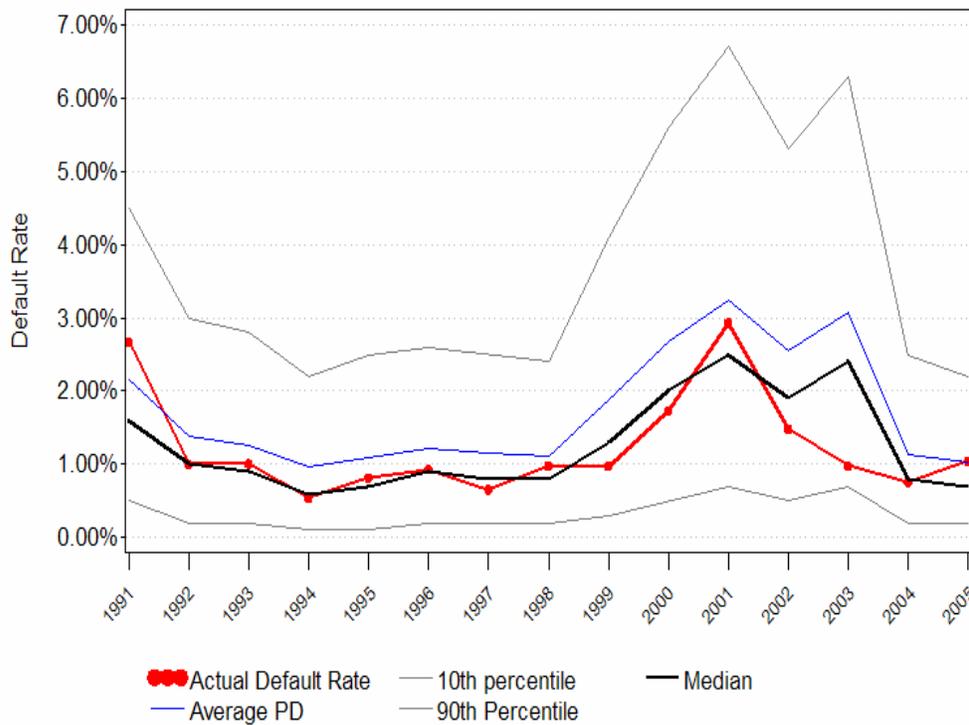


FIGURE 4 Predicted versus Actual Defaults

Figure plots the predicted distribution of the default rate versus actual default rates for the population of U.S. Non-Financial Corporations with more than \$300 million in Sales.

2.2 EDF Credit Measures at Horizons Greater than One Year

As we will discuss in section 3.2, the EDF 8.0 model uses Distance-to-Default Dynamics to estimate EDF levels beyond one year. In this section, we show the implications of using DD Dynamics on the EDF term structure and the EDF-implied credit curves.

The price of a promise to pay by a specified date in the future will depend on how far in the future the date is—what is the term of the loan. It will also depend on how likely it is that the promise will be met as well as what happens when the promise is not met. To manage credit risk, and mark to market bonds, loans or credit derivatives, the practitioner needs a credit curve and a risk-free term structure to determine the appropriate price for a given exposure.

When investing in risk-free assets, the expected return on a strategy of buying a bond with a one-year term and then reinvesting in another risk-free one-year bond each year may not equal the return on buying longer maturity bonds. A whole taxonomy of terminology developed to allow market participants to compare these two strategies. In securities with default risk, the same issues apply as well as some new ones: what is paid in the event of default and how much additional return is required for taking on the default risk. Consequently, some additional terminology is necessary.

Like the term structure of interest rates, the term structure of the EDF measure can be expressed in many ways. In interest rates, one can speak of the spot rate, the forward rate, and the yield to maturity. Similarly, we define the Forward, Annualized, and Cumulative EDF credit measures. These terms are defined in the box “EDF Term Structure Terminology” directly following.

EDF credit measure Term Structure Terminology:

- The cumulative t -year EDF value ($CEDF_t$) is the probability of default between now and date t .
- The forward EDF value between times $t-1$ and t ($FEDF_{t-1,t}$) is the probability of default between times $t-1$ and t conditional upon survival until $t-1$. It can be calculated as $(CEDF_t - CEDF_{t-1}) / (1 - CEDF_{t-1})$. This concept is analogous to a forward interest rate.
- The annualized t -year EDF value (EDF_t) is the 1-year EDF level that would generate ($CEDF_t$) given constant Forward EDF credit measures. It can be calculated as $1 - (1 - CEDF_t)^{1/t}$. This concept is analogous to the internal rate of return.
- Another useful concept is that of the hazard rate, which can be thought of as the Forward EDF credit measure per unit time over a very short time period. This concept is analogous to a continuously compounded interest rate.

An upward (downward) sloping term structure means that the Annualized EDF credit measures are increasing (decreasing) with maturity or equivalently that the Forward EDF credit measures are increasing (decreasing) with t .

There are several desirable properties for an EDF term structure. The most basic is that forward EDF values should be positive. The second is that the term structure of the worst quality credits should be downward sloping—both the Annualized and Forward EDF credit measures should decrease with term. Conditional upon survival, the risk of low-quality credits should decline over time on average, so their Forward EDF credit measures will decline. Third, the term structure of the highest quality credits should be upward sloping. The risk of the highest quality credits can not decline, so on average their Forward EDF credit measure will increase.

Figure 5 presents the Forward EDF credit measures for six different buckets of companies, for both the original and the new mapping.⁷ The buckets have been determined by their one-year EDF value. This figure demonstrates that for the original EDF term structure, Forward EDF values were always positive and they were upward sloping for the highest quality credits. For the lowest quality credits, the Forward EDF values were constant at 20% for the first three years due to the cap and then decreased.

One desirable property of the EDF term structure is that it should be broadly consistent with the so-called expectations hypothesis. In the literature on interest rates, the expectations hypothesis states that the expected spot interest rate should equal the forward interest rate. A mathematically equivalent statement is that the yield to maturity of an n -period bond should equal the expected return associated with a strategy of rolling over a one-period bond for n -periods. While the data may not completely support the expectations hypothesis, it provides a useful framework for interpreting the data: an upward-sloping term structure of interest rates implies that interest rates are expected to rise and vice-versa (cf, Cochrane, 1999).

⁷ Note that in this graph the vertical axis is on a log scale. Consequently, the solid grey lines represent an EDF credit measure level of 0.10%, 1.00%, and 10.00% respectively. The next dotted line above the grey line represents 0.20%, 2.00%, and 20.00%, respectively and so forth.

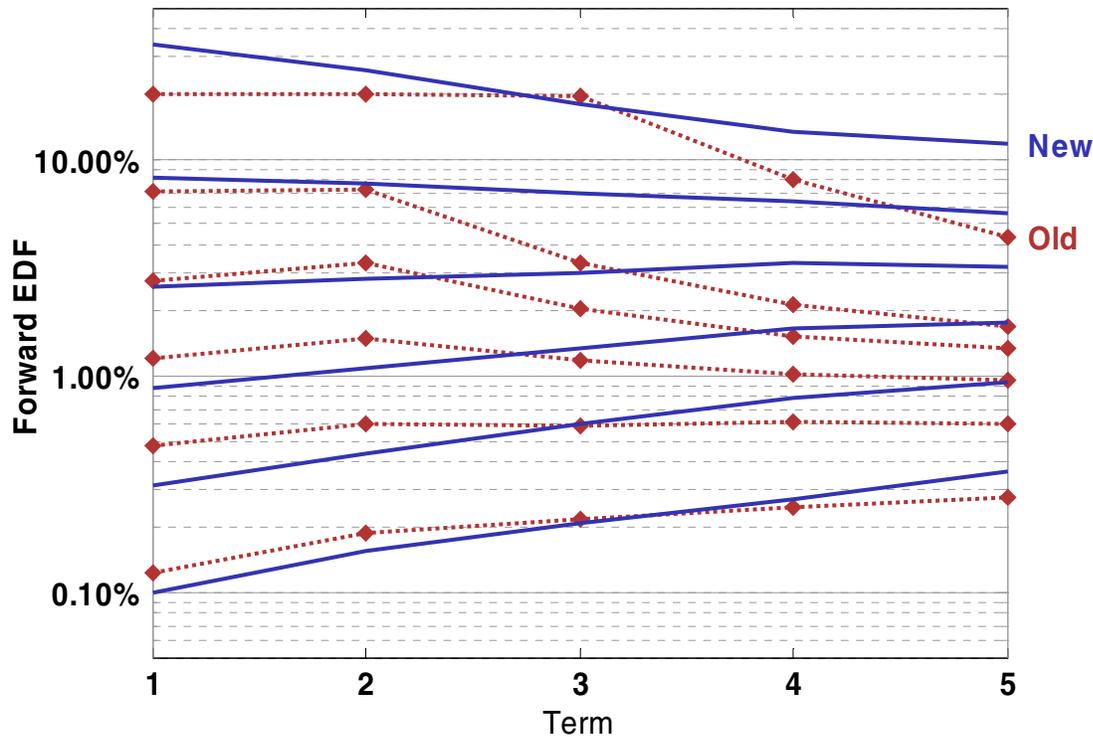


FIGURE 5 Forward EDF Credit Measures under the New and Old Mapping

The expectations hypothesis provides a useful framework for evaluating the consistency of the EDF term structure. Over any given time period the forward EDF values may be greater than or less than the realized EDF values in the future due to aggregate shocks in the economy that were not anticipated by the stock market. Nevertheless, when the forwards EDF credit measures are consistently either greater than or less than the realized future EDF credit measures, the realism of the model's term structure is drawn into question.

When testing the original mapping, we discovered that the Forward EDF credit measures did not obey the expectations hypothesis. The Forward EDF levels were too low when compared to realized spot EDF levels later in time for low-quality companies in both good and bad stages of the credit cycle. The left-hand panel of Figure 6 shows that for high-risk companies over an 11-year time period the forward 3- to 4-year EDF credit measure were less than the realized one-year EDF credit measure three years later. In other words, the term structure was too downward sloping for risky companies.

The right-hand panel of Figure 6 shows that over the long-run the new term structure is more in line with the expectations hypothesis. The 3- to 4-year forward EDF values exceed the 1-year EDF values three years later in time periods when the later realized EDF values are low and vice-versa. Over the full-time period, the average of the 3- to 4-year EDF credit measure approximately matches the 1-year EDF credit measure three years later. This improvement derives from using DD Dynamics to compute the EDF term structure. More details on this methodology are provided in section 3.2.

The impact of an accurate forward EDF credit measure is largest in contexts where one would use this measure directly. For example, suppose that one had a five-year bond that was hedged with a four-year CDS contract. With such a hedge one is still exposed to the risk of default between years four and five. The probability of such a default occurring is the probability of surviving until year four ($1 - \text{CEDF}_4$) multiplied by the probability of default between years four and five ($\text{FEDF}_{4,5}$). Suppose that this exposure had a risk profile comparable to the riskiest firms in Figure 5, then under the new mapping the default risk associated with the uncovered portion of the position is 3.9% versus 2.1% under the old mapping. The new mapping gives nearly double the risk assessment of the uncovered position relative to the old mapping.

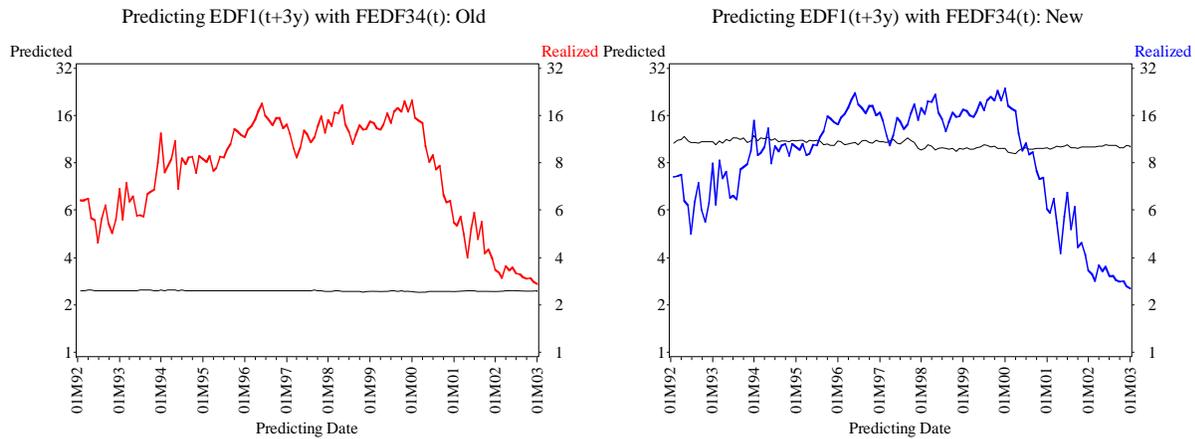


FIGURE 6 3-yr Forward EDF credit measure vs 1-yr EDF credit measure in three years

2.2.1. Implications for Pricing

In pricing credit risk, the metric that the market currently focuses on is the spread—the difference between the yield and the risk-free rate for a bond. Consider a bond that pays N coupons of c at dates (t_1, t_2, \dots, t_N) and then pays a principal of \$1 at the terminal time, T . The spread, s , solves the equation below:

$$P = \sum_{i=1}^N c\beta(t_i) \exp(-st_i) + \beta(T) \exp(-sT) \quad (1)$$

where P is the price of the bond and $\beta(t) = \exp(-r(t)t)$ is a function representing the risk-free term structure.

Equation (1) shows how to compute the spread on a bond from the bond's price and the risk-free rate. The spread is a concept that isolates the credit risk component of a bond's price. Now the task is to relate the spread to the probability of default or the EDF credit measure. In going between an EDF credit measure and a spread, we require four additional concepts: a market price of risk, a measure of the systematic risk of the exposure, an LGD, and a zero-EDF curve.⁸ For a zero-coupon bond with default risk, the spread is given by:

$$s = -\log(1 - LGD * CQDF(T)) / T \quad (2)$$

where LGD is loss given default and CQDF is the cumulative risk-neutral default probability (which we refer to as the quasi-default frequency). To move between the physical and risk-neutral measure, we require a market price of risk. Systematic risk in a security will lead to a risk premium being built into the observed credit spreads. Consequently, the observed credit spread will exceed the level that compensates for expected losses under the so-called physical measure. The framework of risk-neutral pricing allows one to capture this difference by converting the physical probability of default into a risk-neutral probability of default. The structural framework allows for this to be done by assuming that the asset value process is Brownian under both the physical and risk-neutral measure.⁹ This framework implies the following conversion of the EDF credit measure from the physical to the risk-neutral measure:

⁸ For small firms, we also make an adjustment to reflect that small firms trade at greater spreads than larger firms with comparable risk.

⁹ Note that the Brownian motion assumption is only made to convert the physical probability of default to the risk-neutral probability of default. We use an empirical mapping to convert the DD into the physical probability of default (the EDF credit measure), which accounts for the skewed nature of asset returns.

$$CQDF(T) = \Phi\left(\Phi^{-1}(CEDF(T)) + \rho\lambda\sqrt{T}\right) \quad (3)$$

where ρ is the correlation between the firm's asset returns and the market returns, λ is the market price of risk and Φ and Φ^{-1} are the standard cumulative normal distribution and its inverse, respectively.¹⁰ CreditEdge Plus™, CreditMark™, and Portfolio Manager™ use a variety of techniques for estimating these parameters.

Reasonable representative values for LGD, ρ , and λ are 0.5, 0.47, and 0.6, respectively. We can use these to convert a FEDF value into a FQDF as well as to compute the spread associated with a zero-coupon bond for different terms. Figure 7 presents the zero-coupon spreads for the new and old term structure. The term structure of spreads implied by the new mapping are steeper for the highest quality credits and more downward sloping for the worst quality credits relative to the original mapping.

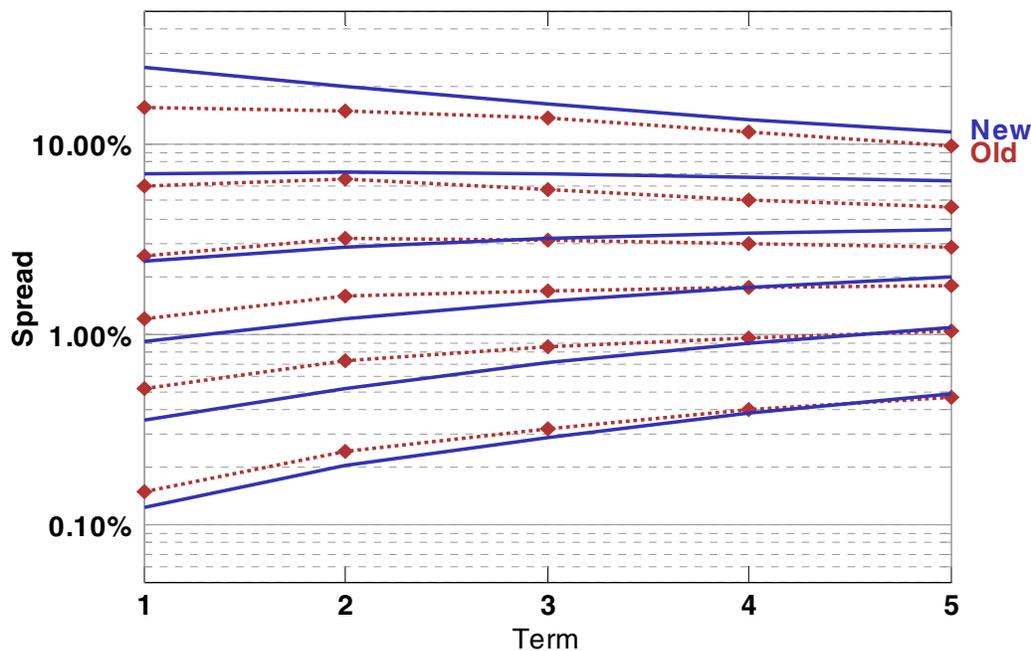


FIGURE 7 Zero-Coupon Spreads under the New versus the Old Mapping

Moving from Zero Coupon Bonds to CDS Pricing

A CDS contract is an agreement between two parties (a buyer and seller of default protection). The buyer of protection pays a fixed spread on a given notional amount at a specified interval until default occurs or the contract ends at a specified maturity. When and if default occurs, the seller of protection pays the notional amount, but receives the defaulted bond associated with the underlying credit with a face value equal to the notional amount of the contract. Given the term structure of risk-neutral EDF credit measures and an LGD assumption, one can solve for the EDF-implied spread, which is the spread that sets the value of the *premium leg* of the contract equal to the value of the *protection leg* of the contract.

¹⁰ The interested reader can find several variations of the derivation of this conversion in our literature (cf., Bohn, 2000, Kealhofer, 2003, as well as Agrawal, Arora, and Bohn 2004).

Figure 8 compares the EDF-implied CDS spreads for the new versus the old mapping. For low EDF values, the term structures of CDS spreads are steeper with the new mapping. For the most risky group, the term structure is more inverted and the spreads are considerably higher.

Such term structures are valuable in marking to market existing CDS contracts. Unless the CDS contract involves an up-front fee, a CDS contract that has just been signed has a mark-to-market value of zero. If spreads increase after the contract is signed, however, the buyer of protection can now demand a cash settlement from the seller of protection to cancel the contract. If spreads decline, the buyer of protection will have to pay the seller to cancel the contract.

To value such contracts, one needs a credit curve—a term structure of spreads. In the CDS market, on many names one may only see liquidity on a five-year CDS. One could construct a credit curve from the five-year CDS, by assuming a constant hazard rate to value the contract. The assumption of a constant Forward EDF (or a constant hazard rate) results in a flat credit curve. Such a credit curve typically overstates the market spread of CDS contracts on high-quality credits at maturities of less than five years. Consequently, one tends to place a higher value on CDS contracts when *owning* protection on high-quality credits as these contracts mature relative (relative to Figure 8), and one would undervalue CDS contracts when *owning* protection on high-quality credits as these contracts mature (relative to Figure 8).

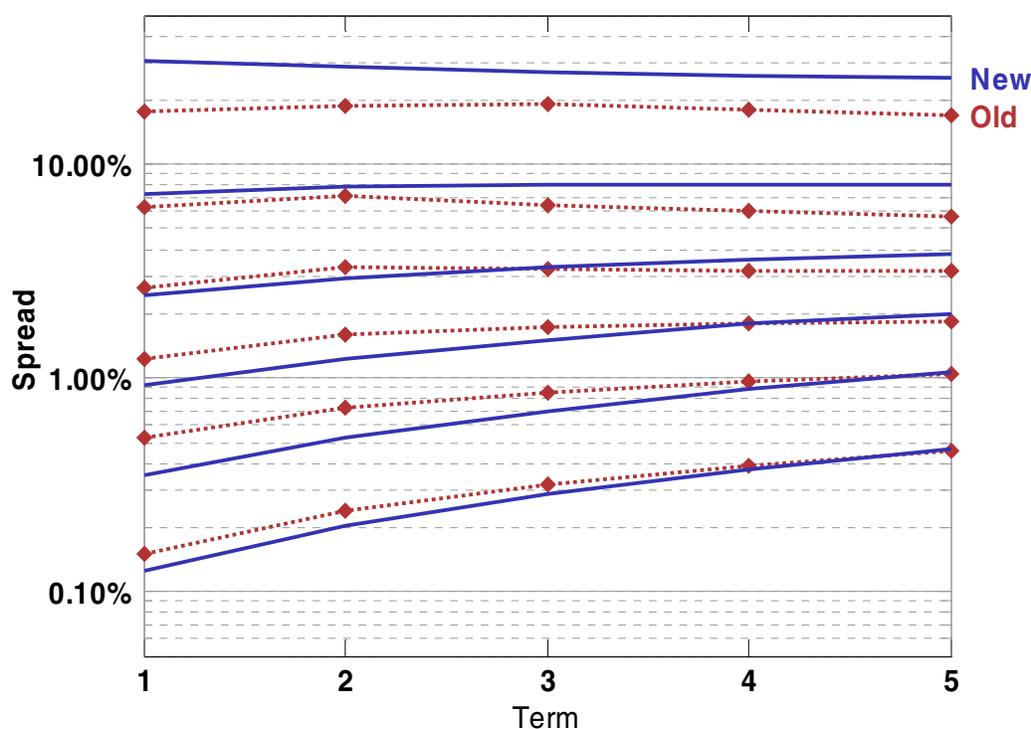


FIGURE 8 EDF-Implied CDS Spreads, New versus Original Mapping

2.2.2. Improved Power in Explaining Market Spreads

To compare the quality of EDF term structures under different mapping functions, we look into the cross-sectional correlation between five-year EDF values and five-year CDS spreads. If the new mapping function improves the term structure, we expect to see a higher explanatory power of EDF5 to observed spreads. With the old and the new EDF term structures we separately run the following cross-sectional regressions:

$$\log(CDS_{it}) = \alpha_i + \beta_i \log(EDF5_{it}) + \varepsilon_{it} \quad (4)$$

where $CDS_{i,t}$ and $EDF_{5,i,t}$ are the observed five-year CDS spread and five-year EDF measure for company i in month t . We compare the explanatory powers of the two term structures, as measured by regression R-squares, and find the new term structure generally outperforms. Figure 9 reports the comparison for the sample of investment grade companies.

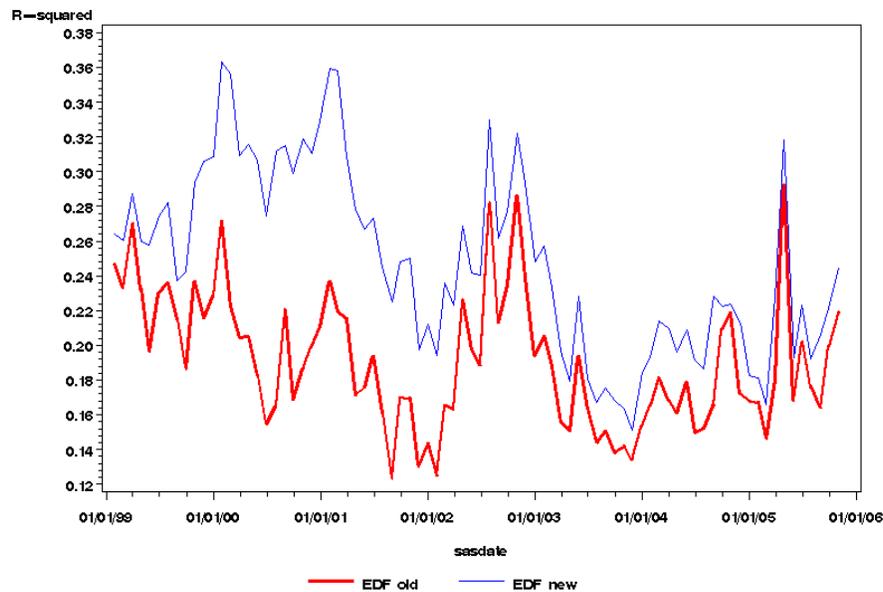


FIGURE 9 R² between CDS Spread and EDF for Investment Grade Credits

2.3 Required Economic Capital

Changing the EDF credit measures, of course, change the required economic capital of a credit portfolio if EDF credit measures are used in calculating it. Required economic capital is a concept that financial institutions use to manage their portfolio risk. It can be thought of as the stock of money that the financial institution needs to set aside to ensure that it can withstand losses within a given time horizon, typically one year, consistent with a given solvency probability, typically derived from the institutions senior debt rating. In the Moody's KMV Portfolio Manager framework, this is computed using a multi-factor model. Monte-Carlo methods are employed to explicitly account for both concentration risk and migration risk. Required economic capital captures the risk implications of a financial institution having substantial exposures that are undiversified because of concentrations in either a limited number of firms, a limited number of industries, a limited number of geographic regions, or a combination of the three. Further, because of migration risk, an investment-grade portfolio can suffer substantial losses without suffering any defaults.

Required economic capital is not additive across portfolios: the required economic capital of a portfolio of one bond is larger than the required economic capital of a portfolio of ten bonds of comparable risk (when economic capital is measured as a percentage of portfolio value). Consequently, the impact of the new mapping on required economic capital varies from portfolio to portfolio. As an illustration, we can check the implications of the new mapping on two large bond portfolios. One is an investment-grade portfolio while the other is a speculative-grade portfolio.¹¹ For each portfolio, we compute required economic capital with EDF credit measures from both the original and the new mapping. The high-quality portfolio has 3,113 bonds, while the low-quality portfolio has 1,886 bonds. Table 1 provides a description of each portfolio by EDF Credit Category.

The required economic is measured with respect to expected loss, and the target probability is set as 10 bps. The comparisons are reported in Table 2. For the investment-grade portfolio, the change in required economic capital is minimal. This change reflects that at the end of 2005, most investment-grade exposures had EDF levels below 0.10%—a 0.06% EDF credit measure remains at 0.06% under the new mapping. For the speculative-grade portfolio, required

¹¹ These portfolios were designed to replicate an investment-grade index and a speculative-grade index at the end of 2005.

economic capital falls by about 10%, which reflects that most of the EDF credit measures will be lower for speculative-grade bonds under the new mapping. It is important to note that these implications differ from portfolio to portfolio. Further, they depend on the stage of the credit cycle. Users should run a similar exercise on their own portfolios.

TABLE 1 Benchmark Portfolios

EDF Credit Category	High-Quality Portfolio				Low-Quality Portfolio			
	Bonds	Value (Billions)	Median EDF Value		Bonds	Value (Billions)	Median EDF Value	
			Old	New			Old	New
Aa	407	266.9	0.02%	0.01%	1	0.2	0.02%	0.01%
A	374	224.9	0.02%	0.02%	6	1.4	0.02%	0.02%
Baa	1,853	1,158.8	0.04%	0.04%	152	61.7	0.04%	0.05%
Ba	420	207.6	0.10%	0.09%	574	241.0	0.17%	0.13%
B	51	27.2	0.39%	0.25%	555	223.8	0.52%	0.34%
Caa	8	2.6	2.39%	2.14%	571	169.3	3.15%	3.07%
Ca					27	5.1	17.38%	28.76%
Total	3,113	1,888.0			1,886	702.5		

TABLE 2 Portfolio Analysis for Required Economic Capital

Portfolio	MTM exposure (U.S.\$ trillion)	Required Economic Capital	Unexpected Loss
Investment Grade with Old EDF levels	1.891	5.1%	0.75%
Investment Grade with New EDF levels	1.888	5.1%	0.76%
Speculative Grade with Old EDF levels	0.707	11.8%	2.2%
Speculative Grade with New EDF level	0.700	10.6%	2.0%

2.4 Impact on Required Regulatory Capital

Required regulatory capital is a straightforward calculation that has its roots in a limiting distribution derived in Vasicek (1987).¹² Like required economic capital, it is intended to compute how much capital a bank needs to hold in order to be protected against a one in one thousand worst-case scenario. Banks are required to hold such capital in reserves. It is also designed to be “*portfolio invariant*, i.e., the capital required for any given loan should only depend on the risk of that loan and must not depend on the portfolio it is added to” (Basel Committee, 2005). While this leads to a tractable formula, it is important to note that regulatory capital does not measure that which typically causes bank failures—excessive concentrations of capital in a small number of exposures with related risks. Consequently, regulators caution against relying on it too heavily.¹³ Nevertheless, it can act as a binding constraint for some financial institutions, so it is important to compare the impact of the new mapping and the old mapping on regulatory capital due its prominence in the Basel II regulatory framework.

Under a single factor Gaussian model (i.e., the Vasicek Model), the probability of a firm defaulting conditional upon the aggregate shock is given by:

¹² The Basel Committee (2005) explains the motivation behind the regulatory capital formula. Lopez (2002) provides some further background.

¹³ The Basel Committee (2005) notes that if a bank does not meet the ideal of a well-diversified portfolio, then the bank is “expected to address this under Pillar 2 of the framework.” Pillar 2 refers to the supervisory review process that is intended to ensure both that banks maintain adequate capital to support all business risks as well as engage in continual improvement of risk management techniques (cf., Paragraph 720 of “A Revised Framework”).

$$\Pi(r_M) = \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}r_M}{\sqrt{1-\rho}}\right) \quad (5)$$

Where r_m is the aggregate shock (the market return), PD is the unconditional probability of default, ρ is the correlation factor and Φ and Φ^{-1} are the standard cumulative normal and its inverse, respectively. The aggregate shock is assumed to be drawn from a standard normal distribution. Therefore, large negative aggregate shocks (negative market returns) increase the probability of default. As the number of exposures goes to infinity, the default rate becomes arbitrarily close to (5) conditional upon the aggregate shock. One out of a thousand times, there will be an aggregate shock of -3.09 or worse.¹⁴ Therefore, one out of a thousand times, the default rate for the portfolio will be at least:

$$\frac{\Phi^{-1}(PD) - \sqrt{\rho} \times \Phi^{-1}(.001)}{\sqrt{1-\rho}} = \frac{\Phi^{-1}(PD) + \sqrt{\rho} \times 3.09}{\sqrt{1-\rho}} \quad (6)$$

To get from this number to the value at risk for an actual portfolio, one simply multiplies each exposure by Exposure At Default and Loss-Given-Default and then sums across all the exposures. To get to capital requirements, one subtracts expected loss from Value at Risk.¹⁵

As required regulatory capital is a non-linear function of PD, maturity and LGD, the impact of the change in the calibration on required regulatory capital for a given portfolio depends on the distribution of the PDs across the portfolio. To give a sense of how required regulatory capital will change at the portfolio level, we compute required regulatory capital on the high- and low-quality portfolios from the previous section. The results are reported in Table 3. For the high-quality portfolio, required regulatory capital is largely unchanged. For the low-quality portfolio, required regulatory capital falls from 10.4% to 9.6%. Figure 10 presents regulatory capital for both the high and low-quality portfolio by EDF Credit Category. Note that the impact on regulatory capital is small and the largest impact is on the B credits. Required regulatory capital for this group is lowered from 9.7% to 8.4%. It is important to note that these implications differ from portfolio to portfolio. Further, they depend on the stage of the credit cycle. Users should run a similar exercise on their own portfolios.

TABLE 3 Portfolio Analysis for Required Regulatory Capital

Portfolio	MTM exposure (U.S.\$ trillion)	Required Regulatory Capital
High-Quality with Old EDF levels	1.887	4.71%
High-Quality with New levels	1.884	4.77%
Low-Quality with Old levels	0.703	10.4%
Low-Quality with New levels	0.696	9.6%

¹⁴ A standard normal has a mean of zero and a standard deviation of 1. Therefore, a property of this distribution is that 1 out of a 1,000 times the outcome is more than -3.09 (standard deviations) below zero (the mean).

¹⁵ The actual calculation is somewhat more involved. There is a maturity adjustment. Further, the correlation parameter is taken as a function of PD and the size of the firm. There are different correlation assumptions for residential mortgages and qualifying revolving retail exposures. The explanatory note by the Basel Committee (2005) gives the complete calculation.

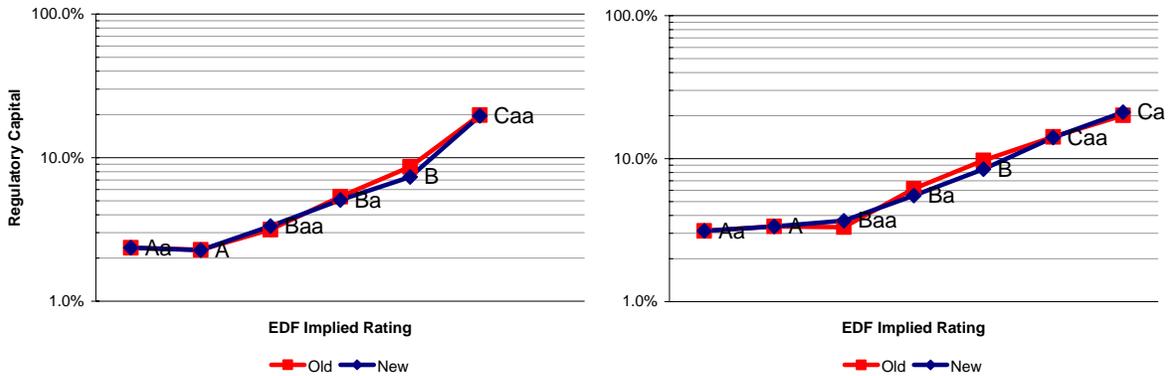


FIGURE 10 Required Regulatory Capital by EDF Credit Category for the High and Low Quality Portfolios

2.5 Impact on EDF Credit Categories

The EDF credit measures provide an absolute measure of risk, but they can also be used to derive the EDF Credit Categories, which are a relative measure of risk. For some applications, users may prefer a relative measure of risk. The EDF Credit Categories provides such a measure. The increased granularity in EDF levels makes the EDF Categories a more useful measure of relative risk.

When the EDF model first came out, users did not have an intuitive sense as to whether an EDF value of 0.56% was a good or a bad credit. They were comfortable with interpreting the scales of the ratings agencies: a Baa3 or a BBB- was a borderline investment-grade company. Consequently, we developed a mapping to convert EDF credit measures into EDF Credit Categories. Prior to Moody's acquisition of KMV, we used only the S&P scale. Shortly following the acquisition, we adopted the Moody's rating scale as well. An EDF Credit Category of a Baa3 implies that the company has an EDF value comparable to the typical EDF values of Baa3 companies.

The methodology is based on computing the median EDF level of a Aaa, Aa, A, Baa, Ba, B, Caa, Ca, and C credits. The actual cutoffs are computed to ensure that the mid-point of a Baa2 credit is the median value for a Baa credit. This mapping is updated on a monthly basis. This procedure works well because of the reasonable relationship between EDF credit measures and ratings: the median EDF level by broad rating category is increasing as ratings decline. It also is, in principal, invariant to the calibration of the EDF credit measures—if you triple the EDF level then EDF Credit Categories will not change. Nevertheless, in the original mapping EDF credit measures had a tendency to hit the floor or the cap depending on the stage of the credit cycle. For example, in 2005 the median EDF credit measures of Aaa and Aa were both 0.02%. Further, the median EDF credit measure of an A credit was very close to 0.02%. Consequently, the distribution of EDF Credit Categories had a “lump” at Aaa–Aa3, and there were very few A1, A2, A3 and Baa1 credits.

Figure 11 shows the improvements by comparing distribution of EDF Credit Categories for three different stages of the recent credit cycle, from the risky period of 2001 until the benign period of 2005. With the old mapping, we see a large percentage of companies that are mapped to the Aaa and Aa classes at the end of 2005. In 2001, many firms were classified as Caa1 or worse. Under the new mapping, the distribution of EDF Credit Categories is much more stable due to the greater granularity in the EDF credit measure.

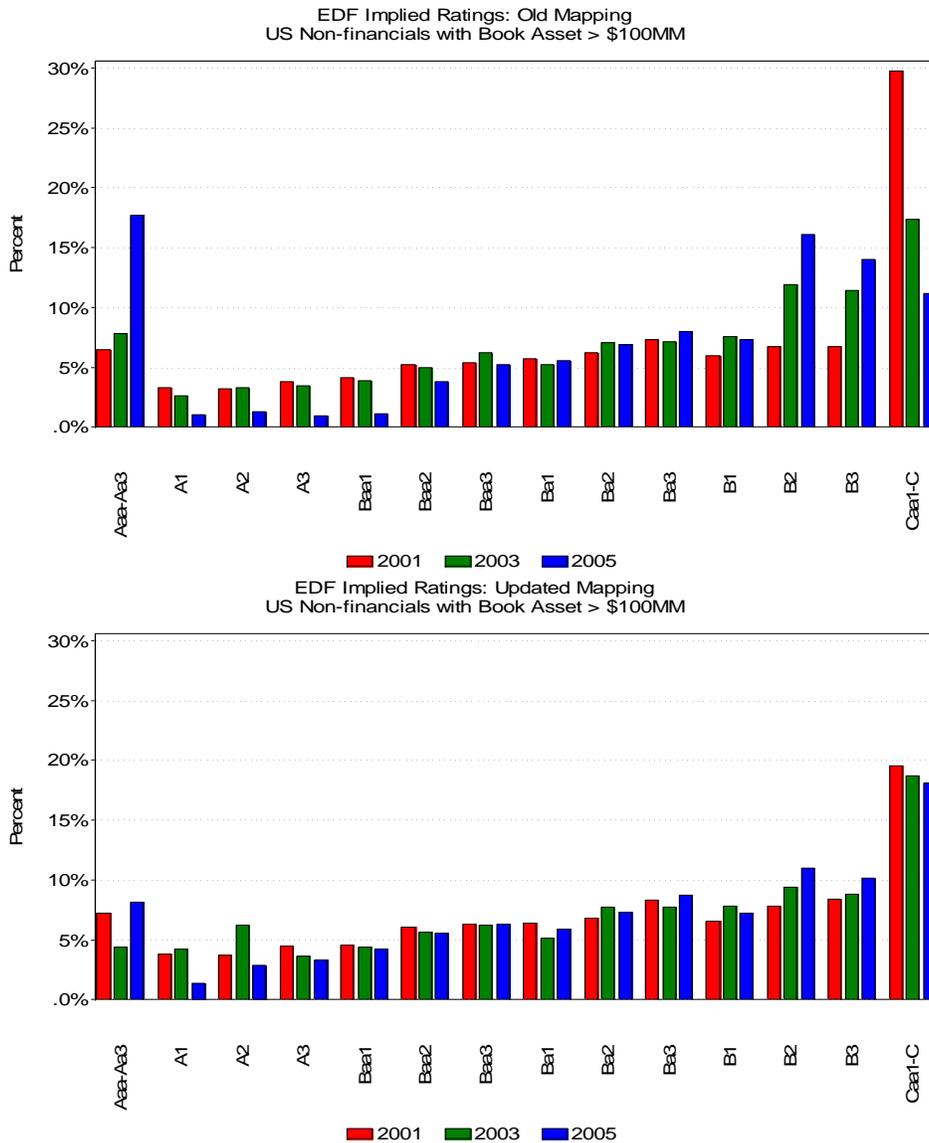


FIGURE 11 The Distribution of EDF Credit Categories under the Old Model (above) and the New Model (below)

The EDF Credit Category can be used as a relative measure of risk. The distribution of EDF Credit Categories is derived from the distribution of actual ratings in the market. As the distribution of actual ratings is generally stable over time, the distribution of EDF Credit Categories will be stable over time as well. Consequently, the percentage of exposures with an investment grade EDF Credit Categories will be stable over time even though the absolute risk of such exposures is changing as measured by the EDF value. Alternatively, one could use the quartiles of EDF credit measures to facilitate a relative analysis as well: one can compare the EDF level of an exposure to the first, median, and third quartile EDF levels for comparable firms. This method allows one to make a relative comparison within specific groups of exposures (e.g., the North American automobile industry).

3 MAPPING METHODOLOGY

We divide the mapping methodology discussion into two sections. The first is the one-year DD-to-EDF mapping, and the second is the process used to estimate the EDF term structure. In the EDF 8.0 model, we follow the approach used in prior EDF model versions to construct the mapping function for the one-year horizon. As we are now using a much larger database, this leads to a new DD1-to-EDF1 mapping function. For longer horizons, we adopt a new approach that is based on credit migration measured by the dynamics of DD.

3.1 Mapping of the 1-yr Distance-to-Default to the 1-yr EDF Credit Measure

Constructing a one-year empirical mapping from DD1 to EDF1 is conceptually straightforward. The mapping is designed to answer the following question: For a given risk level as measured by DD1, what is the likelihood of default happening in the coming 12 months? We obtain the answer from a long history of empirical default experience.

We group company-month observations in buckets with similar DD1s. We then track each observation for 12 months to flag all the defaults we observe. The empirical default rate is the ratio between the total number of defaults and the total number of DD1 observations. We do this exercise for all DD1 buckets, and build a look up table that maps the DD1 to default rates. We then fit a smooth function through each bucket that determines the Expected Default Frequency as a function of DD1.

In mapping the DD to an EDF value at a one-year horizon, there are several implementation decisions that are important for the user to recognize. We describe them below.

3.1.1. Definition of Default

In building a public firm model, it is critical to develop a definition of default that is consistent with an actual economic loss suffered on the part of creditors.¹⁶ Further, the definition needs to be such that it can be widely and consistently applied across many countries. While we track many default types, it is useful to divide them into three groups:

- Missed payments
- Bankruptcy
- Distressed exchange

A missed payment is the failure to make a contractually required payment by its due date. This can include a bond payment, payment on a bank loan, as well as payments on trade debt. Such an event is considered a default without regard to how long the payment is delayed. A missed dividend on a preferred stock is not considered a default. While a preferred stock has priority over common stock in receiving dividends, a company is not contractually obligated to pay such dividends. Therefore, we follow the standard practice of not considering such an event to be a default.

A bankruptcy should be thought of as a filing for legal protection from creditors due to financial distress. In the U.S., this would include either Chapter 11 or Chapter 7 filings. In Canada, it includes the filing under the Creditors Arrangement Act. In the U.K., it would include an Administrative order. In Japan, it includes both bankruptcy and rehabilitation. In Japan, the specific application differs by the type and the size of the company.

The most ambiguous concept of a default is a distressed exchange. Companies restructure debt for many reasons that are unrelated to financial distress. A restructuring becomes a distressed exchange when the restructuring is explicitly done to avoid bankruptcy proceedings.

Users of the model are often interested in how this definition of default compares with the one set forth in Basel II. The key difference is that Basel II only considers a missed payment a default when it becomes 90 days past due.

¹⁶ The definition of default for a public firm model differs from the definition used in private firm models (cf., Dwyer and Kocagil, 2004) reflecting the differences in the data sources.

Consequently, our definition is conservative relative to the Basel definition in this aspect. Basel II also allows a financial institution to classify a company as a default if “it is determined that the obligor is unlikely to pay its debt obligations (principal, interest, or fees) in full.”¹⁷ It is not clear how one could include such a default type in a model of public defaults that is intended to be used across multiple financial institutions. Nevertheless, if a company is highly unlikely to pay its debt obligations, then it should eventually miss a payment, become bankrupt, or restructure its debt through a distressed exchange. If so, it would be included in our definition of default.

3.1.2. Missing Defaults

We calibrate the model to a population where we are confident that we have comprehensive coverage of default events over a long time period. We calibrate the model to a population where we are confident that we have comprehensive coverage of default events over a long time period. This population is U.S. non-financial public firms with more than \$300 million in sales from 1980–2004.¹⁸ Another reason for choosing this sample is that the vast majority of debt outstanding to public companies is owed by companies with more than \$300 million in sales. For example, in the corporate sample, over 98.4% of outstanding debt is taken by companies in this group. Finally, as discussed below, there is less ambiguity around what constitutes a default event for larger public companies relative to small public companies. We test the performance of the model on many other samples, and validate the model outside of the region it was developed on a regular basis.¹⁹

For larger firms, we are confident that we are collecting a comprehensive database of default events. We compared our database with clients on several occasions and repeatedly found our coverage to be more comprehensive and thorough. Following the integration of KMV with Moody’s Risk Management Services, we compared the coverage in the legacy KMV default database with the legacy Moody’s Risk Management Services database, which resulted in a handful of defaults being added to our database (Dwyer and Stein, 2006). For the smallest public firms, however, there is the problem of hidden defaults. For such firms, either the founders of the company or a venture capital firm may have lent the company money in the forms of both subordinated debt and equity. Further, the terms of the debt are often involved. Contractually, a company may be allowed to delay payments with an automatic increase in interest rates.²⁰ They can also allow for *payment in kind*—the delivery of company stock in lieu of cash. Consequently, the distinction between a covenant default and a default that causes an actual economic loss is often unclear.

On an ongoing basis, Moody’s KMV researches companies that have very high EDF credit measures and then disappear from our data. For small companies we often find evidence of financial distress that would not necessarily be considered a default. For example, Storage Computer Corp, a company with \$5mm in assets, was delisted on February 1, 2005. It had the maximum possible EDF credit measure, which was 20%. It filed a 10-Q in November 2004 that noted the use of stock based compensation in lieu of salary to conserve cash. Another example is Cirilium Holding Inc, which reported positive net income due to debt forgiveness by a key shareholder in March 2004. Similarly, OrbitTravel.com paid debt in equity on an ongoing basis when note payments became due. The value of the equity may have exceeded the payment due, in which case it would not have been a default. Nevertheless, OrbitTravel.com eventually filed for bankruptcy in 2004 long after it had become delisted.

By calibrating the level of our model on the population of firms with the fewest missing defaults (those with sales greater than \$300 million), we are focusing on the population where we have the most comprehensive coverage of default events and the population that has the most outstanding debt. If we were to calibrate to the population of smaller firms, an upward adjustment would have been required in order to avoid the significant downward bias that the hidden defaults issue would have caused.

¹⁷ Paragraph 452 of *A Revised Framework*.

¹⁸ The research behind the EDF 8.0 model enhancements goes back several years. As 2005 and 2006 were years with minimal default activity, the inclusion of these years was found to have little impact on the DD-to-EDF mapping. Nevertheless, we included these years in our validation work.

¹⁹ See for example: Arora, Bohn and Korablev, 2005, and Korablev, 2005.

²⁰ For example, a health care company with less than a \$100 million in sales appears to have deferred payments on a note which had the terms: “Company was required to make principal debt payments of \$900 on each of March 24, June 24, and September 24, 2004, together with accrued interest thereon. If the Company did not fulfill its contractual payment obligations, the annual interest rate on the unpaid principal and interest increased to 15.00% from 8.00% until such default was cured.”

3.1.3. Practical Considerations: EDF Floor and Cap

A practical issue that must be addressed in the mapping is the maximum and minimum EDF levels produced by the model. The original DD-to-EDF mapping capped the EDF credit measure at 20% and had a minimum EDF credit measure of 0.02%. The justification for the cap was that above a certain threshold a lower DD did not empirically correspond into higher default rates. The justification for the floor was that below about 0.10% we stopped observing defaults. To extrapolate the EDF credit measure below 0.10% we validated the low EDF levels against spreads: A company with an EDF value of 0.02% is expected to have a lower spread than a company with an EDF value of 0.06%.

At different stages of the credit cycle, users of the model found that both the floors and the caps to be frustrating. During the recession of 2001, a substantial number of firms had an EDF value of 20%, while in 2006 a significant number of firms had an EDF value of 0.02%. Users often want more granularity in EDF credit measures. They want to be able to say which companies are the riskiest within a set of companies that all have an EDF measure of 20% (or a set of companies that all have an EDF measure of 0.02%). In fact, some users prefer EDF credit measures with neither a floor nor a cap.

The new calibration continues to have both a floor and a cap, but the cap rose to 35% while the floor reduced to 0.01%. The new cap and floor are considerably less likely to bind—the sets of exposures with the same EDF credit measures will be much smaller. The reasons for having a floor and a cap with the new calibration are the same as with the old calibration, but with more data we are able to increase the dynamic range of the model with confidence.

Cap of the mapping function

In empirically mapping DDs to EDF credit measures, we expect the empirical default rate to be a decreasing function of DD. We do in fact see such a relationship, except for exposures with extremely low DDs. For those firms, we do not actually observe default rates continuing to rise so we cannot build a reliable mapping. We set a maximum EDF value for all companies with DD measures below a certain level DD_1^A . Because we have more than doubled the number of observed defaults for the EDF 8.0 model mapping, we have been able to increase the maximum EDF credit measure from 20% to 35%.

Floor of the mapping function

Within the almost 30,000 companies covered by the EDF model, there is a large cross-section in terms of default risk. For very high-quality companies, after the DD exceeds a certain level of DD_1^B , we have never empirically observed a default. For even higher quality companies, after DD exceeds a certain level of $DD_1^C > DD_1^B$, we cannot discern a relationship between the CDS spreads and the DD. To assign EDF credit measures to these high-quality names, we map all companies where DD1 exceeds DD_1^C to a floor EDF value, and extrapolate the mapping function for DD1 values between DD_1^B and DD_1^C . Because of improvements in default data and CDS spreads data, we can now lower the EDF floor from 0.02% to 0.01%.

3.2 Building an EDF Term Structure from Distance-to-Default Dynamics

To manage credit risk, the risk measure needs to match the term of the exposure. One complication in estimating a term structure of default probabilities is the issue of missing data. Whether it be a sample of rated firms, public firms, or private firms, over long time periods a certain portion of the firms will disappear from the sample and it is not clear whether they actually defaulted.²¹ In the context of our public firm model, one firm that has an EDF credit measure in one period may drop from the sample for a variety of reasons that include: (i) the firm is acquired or merges with another firm; (ii) the firm becomes privately held through either a leveraged buyout or some other mechanism and is therefore delisted from the exchange; (iii) the firm goes out of business but is able to meet all of its obligations; (iv) the firm goes

²¹ In the duration modeling literature, this phenomenon is referred to as data censorship.

into default. When tracking a cohort of firms over long time periods, the number of censored observations—firms that no longer have an EDF credit measure—can be substantial.²²

When implementing a term-structure, the long-term cumulative EDF credit measure needs to be adjusted to correct for the censorship issue. If one were to build a five-year cumulative EDF credit measure by computing the percentage of observations that are observed defaulting within five years, one would understate the CEDF credit measure due to the issue of data censorship.²³

To alleviate this issue, the EDF 8.0 model employs a credit migration based approach (that we refer to as Distance-to-Default dynamics) to build a term structure.²⁴ The spirit of this approach is as follows. Suppose we have the one-year mapping function and have a company's current credit state as measured by its DD. The current DD implies the current one-year EDF level of the company through the DD-to-EDF one-year mapping. If we know the company's DD in one year, we could use the same mapping function to get its second-year default rate. Thus the cumulative two-year default rate can be derived from the first-year default rate and the distribution of the second-year DD, which in turn can be derived from empirical DD migration.

Mathematically, let $EDF_1 = m_1(DD_1)$ be the one-year mapping function. Further, let DD_2^0 be the two-year DD at the current time and DD_1^1 be one-year DD in one year, where superscripts 0 and 1 denote the time period when the DD is observed while the subscript denotes the term of the DD, i.e., DD_i^j is the i term DD observed when $t=j$. Let $\pi(DD_2^0, DD_1^1)$ be the probability density function of one-year DD migration conditional upon the current value of two-year DD (and that the company has a one-year DD in the following year), i.e.,

$$\pi(DD_2^0, DD_1^1) = \Pr(DD_1^1 | DD_2^0) \quad (7)$$

Let $FEDF_{12}$ be the forward EDF credit measure between the end of year one and end of year two. It is the current assessment of the second-year default rate. Given the one-year mapping function and credit migration distribution, we can derive

$$FEDF_{12}(DD_2^0) = \int_{-\infty}^{+\infty} \pi(DD_2^0, DD_1^1) m_1(DD_1^1) dDD_1^1 \quad (8)$$

Therefore, the forward EDF credit measure from year 1 to year 2 is simply the expectation of the one-year EDF value in one year. The two-year cumulative default rate $CEDF_2$ can be derived from

$$1 - CEDF_2 = (1 - EDF_1)(1 - FEDF_{12}) \quad (9)$$

²² For the sample of all U.S. non-financials companies during 1990 to 2006, the percentage of firms that are censored at a five-year horizon ranges from 20% to 60% depending on the initial Distance-to-Default value. Another example is Duffie, Eckner, Horel, and Saita (2006). They noted that twice as many firms exit their sample due to mergers and acquisitions than exit due to bankruptcy and other defaults (see Table 1). Their sample is a subset of the U.S. industrial firms from 1980 to 2005.

²³ The rating agencies confront a similar issue when computing historical default rates by rating category. Moody's Investor Service makes an adjustment for rating withdrawals when computing a long-term cumulative default rate in their default studies (cf., Hamilton and Varma, 2006). Other rating agencies may publish lower historical default rates by rating category because they do not make such an adjustment.

²⁴ We do not make an adjustment at the one-year horizon, because *censorship* is a relatively minor issue at short horizons and has been dealt with using the dataset with sizes greater than \$300 million.

This approach is easily extended to longer horizons. Note that this procedure ensures that the forward EDF credit measure is always positive.

3.2.1. Calibration of the Empirical Distribution of DD1 Migration

The empirical distribution of DD migration, $\pi(DD_2^0, DD_1^1)$, is calibrated on almost two decades of monthly DD data on all U.S. companies. Specifically, we pool all monthly DD2 observations—for convenience let's denote them as DD_2^0 —and bucket them into small DD_2^0 groups. Within each DD_2^0 group, for each observation we track the company to find their realized DD1 one year later, denoted as DD_1^1 . The distribution of DD_1^1 is fitted using a parametric distribution. These distributions across DD_2^0 groups, determine the function $\pi(DD_2^0, DD_1^1)$ in equation (7) and (8). The distribution thus obtained provides the historical average behavior of credit migration.

3.2.2. Extrapolating EDF Term Structure Beyond Five Years

The current model does not estimate DD measures for horizons beyond year five. However, long-horizon default probabilities are frequently needed in valuing long-term credit instruments. In the past, two approaches evolved at Moody's KMV, which we describe below, that extrapolate the first five years of EDF term structures into longer horizons. We also present a new approach, which is computationally efficient as well as consistent with observed characteristics of the term structure of credit risk.

The first approach was developed in the early 1990s when extrapolation was needed for our first generation Portfolio Manager. This approach assumes a company has constant forward EDF credit measure beyond year five, and the constant is its five-year annualized EDF credit measure, i.e.,²⁵

$$FEDF_{t-1,t} = EDF_5, \text{ for } t > 5 \quad (10)$$

This is equivalent to

$$EDF_t = EDF_5, \text{ for } t > 5 \quad (11)$$

This approach is attractive to portfolio analysis because of its computational efficiency in Monte Carlo simulation. However, there is an undesired feature of this extrapolation. Usually PD term structures are believed to have a mean-reverting behavior, in that the annualized EDF values should to be upward sloping for high-quality companies, and downward sloping for low-quality companies. When we set longer horizon PD to a constant, this mean reverting behavior abruptly stops at year five—the annualized EDF value is a constant after year five.

The second approach for EDF term structure extrapolation was developed in the early 2000s to facilitate the valuation of long-term credit instruments in CreditMark. This approach is motivated by the mean reverting behavior of the PD term structure, and assumes a common asymptote for all companies in cross section. The term structures of companies extend to this common asymptote at different speeds. The extrapolation was calibrated to obtain the daily common asymptote. The shape of the term structure after year 5 under this approach is more intuitive than the first approach, but due to computational efficiency, it is difficult to implement in simulation based portfolio analysis.

The EDF 8.0 model adopts a third approach. It assumes that after year four the term structure extends with a constant forward rate, i.e.,

²⁵ This is because, for example, $(1 - EDF_6)^6 = (1 - EDF_5)(1 - FEDF_{56}) = (1 - EDF_5)^6$.

$$FEDF_{t-1,t} = FEDF_{45} = 1 - \frac{(1 - EDF_5)^5}{(1 - EDF_4)^4}, \text{ for } t > 5 \quad (12)$$

The apparent advantages of this approach are twofold. First it is computationally convenient and is easy to adopt in portfolio analysis. Second, it results in upward-sloping term structures for high-quality companies and downward-sloping term structures for low-quality companies (with respect to annualized EDF credit measures). It therefore represents a compromise of the first two approaches.

The term structures resulting from the third approach are consistent with dynamics of the shorter horizon EDF credit measures. In the literature on the term structure of interest rates, researchers have built the link between interest rate term structure and the dynamics of the short rate. Das, Duffie, and Kapadia (2004) applied the interest rate term structure analogue to link PD term structure with short-term PD, and we follow the same approach modeling one-year EDF dynamics with a one factor stochastic model using the estimated parameters to derive the EDF term structures. The resulting shape of term structure is very close to the simple approach we employ in equation (7). Consequently, we are using the method of “constant forwards” to extrapolate after five-years in the Moody’s KMV public firm model.

4 THE MOODY'S KMV PUBLIC FIRM MODEL

The concept of using an option theoretic framework combined with stock prices to estimate default risk was controversial when first developed. Nevertheless, thirty years after Merton wrote “On the Pricing of Corporate Debt” the usefulness of the basic idea and the framework has largely become accepted. Today, the structural approach has had many applications in risk management. Structural models are used to:

- Provide early warnings on declining credits
- Streamline the loan origination process
- Determine value-at-risk and required economic capital
- Determine required regulatory capital
- Actively manage portfolio risk
- Mark to market both loans and credit derivatives
- Measure default correlations between firms
- Assess the appropriate risk adjusted return on corporate transactions

The basic idea behind a structural model is straightforward. Consider a holding company that owns stock in another company and that the market value of these holdings is V . Further, the company has a debt payment of D due at a fixed point in time, T . Owning the equity of such a holding company is equivalent to owning an option to buy the stock at a price of D with an expiration date of T . Owning the debt is equivalent to owning a risk-free bond that pays D at time T and being short a put option on the stock with an exercise price of D and an expiration date of T . In this example, the firm defaults if the value of assets, V , is below D at the expiration date T . One can use the simple Black-Scholes option formula to determine the value of equity. The four inputs to this equation are the debt payment, D which we refer to as the default point, the market value of the firm's assets, V , the volatility of assets, σ_A , and the risk-free interest rate, r . The probability that the obligations will not be met is a function of the firm's DD, which represents the number of standard deviations that the firm is away from the default point. DD can also be viewed as a volatility-adjusted market-based measure of leverage.²⁶

This simple idea allows practitioners today to use information from the highly liquid equity market to obtain an estimate of what credit risk should be in the fixed income market—a market that *even today* is much less liquid. There are many steps involved in converting this idea into a model that can be used to support decision making by investors. Currently, the Moody's KMV Public Firm model produces EDF credit measures for approximately 30,000 firms worldwide on a daily basis. These EDF values are used by hundreds of institutions to actively manage credit risk.

For a structural model to be widely implemented, it first needs to be able to incorporate several different types of contingent claims. Second, the model requires an estimate of asset volatility even when a time series of equity prices is either unavailable or not relevant. Such cases occur for newly public companies and for companies that undergo a merger, acquisition or spin-off that represents a substantial change in their business. Third, one needs a consistent and credible measure of the default point. We discuss each of these in turn. Finally, one needs to map the DD to an actual term structure of EDF values, which we discussed in Section 3.

4.1 Determining the Value of a Firm's Assets and the Default Point

The standard Merton model assumes two types of claims to the cash flow generated by a firm—debt with no coupons and equity with no dividends. This model posits that the underlying assets can be represented by a geometric Brownian motion process that is parameterized by volatility and a drift term. The debt is a one-time payment at a specified point in time. One can then obtain an analytic expression for the value of equity by constructing a risk-free portfolio and solve the resulting partial differential equation using two boundary conditions relating to the value of the option at expiration

²⁶ For a more thorough description, see *Modeling Default Risk*.

and the value of the option when the underlying assets become worthless. In this context, the actual solution is the Black-Scholes option pricing formula.²⁷

The structural model that forms the foundation for the Moody's KMV public firm model is the Vasicek-Kealhofer (VK) model.²⁸ The VK framework assumes five types of claims on the firm's cash flow:

- Short-term liabilities
- Long-term liabilities
- Convertible securities
- Preferred stock
- Common stock

Incorporating these different types of contingent claims into the model changes both the asset value process and the boundary conditions.

Dividends, Coupons, and Interest Expense

Cash leakages in the forms of dividends on stock, coupons on bonds, and interest expense on loans impact both default probabilities and the value of debt. For example, consider two firms with identical assets and debt but one pays a larger dividend. Obviously, the firm paying the larger dividend has a higher default probability: even though the dividend may be cut in the event of distress, any higher dividend payments made until distress became apparent would reduce the cash flow available to service debt. The VK model incorporates cash outflows directly into the different types of claims on a firm's assets.

Convertible Securities

Companies may issue securities that can be converted into equity at a specified conversion rate. Such securities are often preferred stock, but bonds can be convertible as well. By issuing such securities, the firm is effectively selling a portion of the upside return that otherwise belongs to the holders of common stock. Consider two firms: A and B. Both have the same assets and debt. Company B, has a convertible security outstanding, so the fully-diluted shares outstanding exceeds the common shares outstanding. Under this scenario, Company B has a lower market value of equity than Company A even though the default probability is the same. Holders of common stock in company B sold a portion of the upside return to the holders of the convertible security. This difference becomes reflected in one of the boundary conditions for the VK model—that as the asset value of the firm becomes arbitrarily large the derivative of equity value with respect to asset value becomes equal to the ratio of the shares outstanding divided by the number of fully diluted shares outstanding. The dilution effect of convertible securities reduces the sensitivity of the value of equity to the value of assets. If this dilution effect is ignored, when observing the equity value of a company that has a large amount of convertible securities, one will underestimate the market value of assets and overstate the probability of default.

Current and Long-term Liabilities

In structural models that use an absorbing default barrier, two approaches have been taken to defining the barrier. The default event can be driven by creditors forcing the company into default when the asset value falls below the barrier. Alternatively, if there are no protective covenants, the company can choose to default when the value of equity falls to zero—if the value of assets fall below a certain threshold, then the holders of equity chose to stop making payments on

²⁷ See Section III of Merton 1974. In the same paper, Merton develops several useful extensions to the “standard Merton model.”

²⁸ Oldrich Vasicek developed the analytic model. Stephen Kealhofer led the research efforts that determined many of the empirical implementation details.

the debt and thereby turn over the firm's assets to debt holders.²⁹ In the VK model, the barrier is exogenous, in the sense that creditors put the debt back to the firm as soon the value of assets hits the value of the default point. The barrier is implemented as current liabilities plus a portion of long-term debt. We refer to this barrier as the default point. This barrier forms the second boundary condition of the model—the value of equity is equal to zero when the value of the firm's assets is equal to the default point.

Preferred Stock

Preferred stock has both equity and debt characteristics, and the model takes both of these aspects into consideration. The VK model is able to incorporate various types of preferred stocks, including tradable preferred stocks and convertible preferred stocks. This completes the description of the different types of claims on the cash flows covered by the VK model.

Vasicek's model is an extension of other models that treat equity as a down-and-out perpetual option. Such extensions are necessary to credibly implement the structural modeling framework for actual firms. Actual firms have convertible securities, pay dividends, coupons and interest payments. These liabilities need to be explicitly modeled to generate a reasonable measure of default risk—particularly in the cases where these payments are unusually large.

In implementing the model, the default point is updated for every firm on a monthly basis based on publicly available information. For non-financial firms, the inputs are current liabilities and long-term liabilities less minority interest and deferred taxes.³⁰ There is often some ambiguity as to how to classify various items on a firm's balance sheet into long-term debt, long-term liabilities and current liabilities. Care is taken in making these distinctions. We refer back to original company reports and filings in addition to cross referencing the data against other sources when necessary. Given the default point, asset volatility and the risk-free interest rate, one can solve for the value of assets that sets the modeled value of equity equal to the observed value of equity.

4.2 Determining the Volatility of Assets

The Black-Scholes pricing formula was originally derived to value equity options with the volatility of equity being an important input. There is vast literature on different ways to model the volatility of equity. Extensions to the Brownian motion process include instantaneous jumps in equity value and time-varying stochastic volatility. One could implement these approaches directly from the observed time series of equity returns.

In structural models, one writes down an asset process and solves for the equity process as implied by the model. If the asset process has a constant volatility, volatility in the implied equity process is time varying. One implication of constant asset volatility is that as the value of assets falls close to the default barrier, equity volatility increases because of increased leverage. Consequently, de-levering a measure of equity volatility tends to understate the volatility for the firm's assets as asset value approaches the default barrier. Therefore, a direct measure of asset volatility is required, which is one of the key challenges in implementing a structural model: one needs to estimate the volatility of an unobserved variable. The VK model iteratively estimates both the value of a company's assets and its volatility.

Any estimate of asset volatility will be just that—an estimate. There are two sources of information available to estimate this volatility: information that is specific to the given firm (such as its equity price and liabilities history), and information for the population of comparable firms (their equity prices and liabilities history). We use a firm's specific information to estimate what we call empirical volatility, and we utilize the information from the population of comparable firms to estimate what we call modeled volatility. We combine the two. The weight on empirical volatility (relative to modeled volatility) is determined by the length of the time series of equity prices that is used in estimating empirical volatility.

The intellectual origins of combining empirical and modeled volatility can be found in Vasicek (1973). This paper develops a Bayesian approach to estimating β in the context of a Capital Asset Pricing Model. In this paper, rather than

²⁹ For early implementations of so-called exogenous and endogenous default barriers, see Black Cox (1975) and Leland (1994), respectively.

³⁰ For financial firms, these numbers are calculated by Moody's KMV from the balance sheet, see Sellers and Arora, 2004 for details.

estimating a firm's β through simply the ordinary least squares (OLS) regression of the firm's returns on the market returns, the paper starts with a prior distribution of the firm's beta and then uses Bayesian methods to compute the posterior. The prior distribution of a firm's β is the distribution of β of comparable firms. Vasicek argues that the optimal estimate of β is the mean of the posterior distribution. Under this method, one combines the OLS estimate of β with the prior expectation of β to achieve an optimal estimation of β (in the Bayesian sense). When combining the two estimates, the weight on the OLS regression estimate increases with the number of observations used in the OLS estimate of β .

4.2.1. Empirical Volatility

Given volatility and the default point, one can solve for the asset value that sets the value of equity implied by the VK model equal to the observed value. Thus for a given firm, one can construct a time-series of asset values. One can then compute what we call *empirical volatility* through an iterative method mentioned in *Crosbie and Bohn (2003)*: Using the VK model we compute a time series of asset values and hedge ratios from which we *de-lever* equity returns into asset returns. We compute the resulting volatility of asset returns, and then iterate until convergence. Remarkably, Duan, Gauthier, and Simonato (2004) showed that an iterative approach could be viewed as an application of the EM algorithm: an iterative procedure for estimating empirical volatility turns out to be the maximum likelihood estimate of asset volatility.

The iterative approach is superior to “solving for two equations and two unknowns” that was found in some academic and commercial implementations of structural models. As discussed in Ericsson and Reneby (2005), the issue with the two equations and two unknowns approach is that it assumes that equity volatility is a constant which is inconsistent with a structural model in which asset volatility is a constant. For companies with rapidly falling asset values, equity volatility is increasing rapidly due to changes in leverage. Consequently, asset volatility becomes understated for companies with rapidly increasing EDF credit measures. At KMV, the iterative approach grew out of poor results associated with attempts to de-lever equity volatility in the early 1990s.

A large corporate transaction—such as a merger or acquisition—may result in a permanent change in the volatility of the company. Post-event asset returns are likely to be more informative than pre-event asset returns for estimating empirical volatility. Consequently, empirical volatility is computed differently in the model following a corporate event. If there has been a large change in firm size or capital structure, then empirical volatility is computed by placing a larger weight on the post-event asset returns.

4.2.2. Modeled Volatility

Western Refining went public in January 2006. At the time of this publication, it is currently in the process of buying Giant Industries. Prior to the purchase, Western Refining was not rated by Moody's as it had no debt outstanding. It is likely to take on substantial debt to finance the purchase of Giant Industries. Practitioners need to be able to assess the risk of both Western Refining and Giant Industries to manage their portfolio.

When the equity price history is limited, we rely more on modeled volatility. Using empirical volatility for public firms as the dependent variable, we estimate volatility on the basis of the size, industry and geography of the firms, as well as some accounting ratios. For a new firm, the volatility used in the model is heavily based on modeled volatility. For firms with a long history of equity prices, the volatility used in the model is a combination of modeled volatility and empirical volatility.

In industry and geography aggregates, one observes empirical volatility changing over time in response to changing business conditions. Each month, modeled volatility is recalibrated so that on average modeled volatility is equal to empirical volatility. In this way, modeled volatility neither increases nor decreases changes in aggregate volatility that may occur as the result of changing business conditions.

4.3 Non-Gaussian Relationship Between Distance-to-Default and the EDF Value

As the VK model is a barrier model, the model relates the asset value, the default point and volatility to the default probability via a first passage through time formula. Vasicek has noted that the probability of default for a first passage through time model is approximately equal to:

$$2 \times \Phi(-DD) \tag{13}$$

where DD is the so-called Distance-to-Default and Φ is the cumulative normal distribution. Distance-to-Default can be defined as:

$$DD(V, X_T, \sigma_A, T, \mu, a) = \frac{\log(V / (X_T + aT)) + \left(\mu - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \tag{14}$$

Where V is the value of a firm's assets, X_T is the default point to the horizon, μ is the drift term, σ_A is the volatility of assets, T is the horizon and a represents cash leakages per unit time due to interest payments, coupons and dividends. The value of the firm's assets and volatility is computed as described above. The default point is computed as current liabilities plus a portion of long-term debt. For longer horizons, a larger portion of long-term debt is included in the default point to reflect that long-term debt becomes more important at longer horizons. Note that the DD varies considerably with the horizon under consideration. At longer horizons, the weight on volatility increases relative to the default point.

Empirically, there is a strong relationship between DD and the observed default rates—firms with a larger DD are less likely to default. Nevertheless, the actual default rate found in the data differs from the literal predictions of the model. Taken literally, the Brownian motion assumption on asset value implies a Gaussian relationship between DD and the EDF credit measure. Specifically, for a DD greater than 4, a Gaussian relationship predicts that defaults will occur 6 in 100,000 times. This would lead to one half of actual firms being essentially risk-free. This implication is not found in the data. Consequently, when implementing the model, we depart from the Gaussian assumption by implementing an empirical mapping.

5 CONCLUSION

The EDF 8.0 model release represents a major recalibration of the Moody's KMV pioneering structural model. One critical component of the model—the DD-to-EDF mapping—has been significantly enhanced. The new mapping increases the accuracy of the level of the EDF credit measure. Further, it extends the dynamic range of the model and improves granularity. By using DD dynamics to build the EDF term structure, our EDF credit measures for longer horizons are more useful for pricing purposes and more accurate for measuring the credit migration risk.

The EDF 8.0 model is also being released with improvements to the production process. We incorporated semi-annual financial statements for firms outside of North America. This will significantly improve the timeliness in feeding the most recent financial information into the model. Further, the interest rate used in the model is now updated on a monthly basis. The risk-free interest rate is an input to the VK model, which impacts the relationship between the market value of assets and the market value of equity. Until the EDF 8.0 model release, the risk-free rate was held constant as our research showed that updating the interest rate did not impact model performance. Nevertheless, for conceptual consistency we are updating the interest rates with the EDF 8.0 model release. As interest rates have a nonlinear impact on the model, updating the interest rates in the model impacts every observation somewhat differently. Consequently, users will observe that there is a tight band of the new EDF credit measures when plotted against the old EDF credit measures rather than the one-to-one mapping that is implied by Figure 2.

We document how the new calibration was done and the corresponding implementation decisions. The EDF credit measures were calibrated to a default database that is especially thorough, resulting in a highly accurate EDF credit measure. The model is regularly validated on other populations. Credit migration techniques are used to compute the expected default frequency at longer horizons. The use of credit migration techniques adjusts the EDF levels for censorship in the data. By adjusting for firms that disappear from the sample, the forward EDF credit measures are now larger, particularly for the lower-quality credits.

In this report, we described the impact of the model across many different applications of the model. We show how the EDF credit measures themselves have changed and how the model provides better granularity during both recessionary periods when default risk is elevated as well as in expansionary periods when default risk is low. We show that the new forward EDF credit measures are consistent with realized EDF credit measures in subsequent periods and the impact on the pricing curves: for high-quality credits the pricing curves are steeper while for low-quality credit the pricing curves are less downward sloping. We show the impact of the new EDF credit measures on required regulatory and economic capital for two different portfolios at the end 2005. As the impact on capital will differ from portfolio to portfolio and from time period to time period, users are encouraged to run such experiments on their own portfolios. Finally, we show that the increased granularity will improve the use of EDF Credit Categories (or EDF-implied Ratings) as a relative measure of default risk.

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