

## MODELING METHODOLOGY

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## Using GCorr Macro for Multi-Period Stress Testing of Credit Portfolios

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### Abstract

This document presents a credit portfolio stress testing method that analytically determines multi-period expected losses under various macroeconomic scenarios. The methodology utilizes Moody's Analytics Global Correlation Model (GCorr®) Macro model within the credit portfolio modeling framework. GCorr Macro links the systematic credit factors from GCorr to observable macroeconomic variables. We describe the stress testing calculations and estimation of GCorr Macro parameters and present several validation exercises for portfolios from various regions of the world and of various asset classes.

This stress testing method can be useful for regulatory-style stress testing initiatives, such as the Federal Reserve Comprehensive Capital Analysis and Review (CCAR), which is based on expected loss projection under nine-quarter scenarios.

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## 1. Introduction

Moody's Analytics GCorr is a multi-factor model for credit correlations. The systematic factors in GCorr perform well in explaining systematic risk of a credit portfolio. However, these factors are latent or unobservable, which poses a challenge when using GCorr for stress testing exercises. GCorr Macro links GCorr systematic credit risk factors to macroeconomic variables, resolving this issue. It is worth emphasizing that the variables do not replace the GCorr factors in modeling a portfolio's systematic credit risk. Variables usually considered in stress testing scenarios, such as market indices or GDP, are broad, economy-wide indicators that, in contrast to GCorr factors, do not capture industry-specific effects.<sup>1</sup> The set of macroeconomic variables we consider in GCorr Macro includes the CCAR variables, as well as additional U.S. and international variables.

This document describes a method for multi-period stress testing credit portfolios. The method employs GCorr Macro within the Moody's Analytics credit portfolio modeling framework to analytically calculate stressed expected losses under multi-quarter macroeconomic scenarios. The stress testing method is useful for addressing regulatory-style stress testing initiatives, such as CCAR.<sup>2</sup> Calculating stressed expected losses on credit portfolios over a nine quarter period is the essential aspect of the CCAR exercise.

The GCorr Macro stress testing calculations follow the structure of Moody's Analytics credit portfolio modeling. In the first step, we determine the distribution of systematic credit risk factors, given a macroeconomic scenario. Subsequently, we use this distribution to produce stressed values of instrument-level credit risk parameters—probability of default (PD) and loss given default (LGD). In the final step, we obtain the stressed expected losses at the instrument- and portfolio-level. All of these calculations are analytical and do not require Monte Carlo simulation. In addition, this document describes estimating GCorr Macro parameters and model validation. Specifically, we conduct several exercises with Commercial & Industrial portfolios, designed to validate the stress testing method together with the GCorr Macro parameters. The exercises are based on historical scenarios. We also use GCorr Macro to calculate losses on the portfolios under CCAR and other hypothetical scenarios.

While Commercial & Industrial portfolios are the focus of this paper, GCorr Macro is compatible with a wide range of other asset classes, including Commercial Real Estate, Retail Credit, Sovereign, and others. Effectively, GCorr Macro can be applied to any asset class, as long as the systematic risk of the asset class can be described by GCorr factors. Section 7.3 presents GCorr Macro validation for portfolios with U.S. commercial real estate exposures.

Given the estimation methodology and validation exercises, we can conclude that GCorr Macro is suitable for scenarios similar to recent economic episodes, such as the financial crisis.

The remainder of this paper is organized as follows:

Section 2 introduces GCorr Macro and explains how it fits into the credit portfolio modeling framework.

Section 3 describes analytical calculations of stressed expected losses with GCorr Macro.

Section 4 explains how we estimate and validate GCorr Macro parameters.

Section 5 summarizes the GCorr Macro parameters, including correlations between GCorr Corporate factors and macroeconomic variables, variable selection, and illustrates how we use these parameters to conduct stress testing.

Section 6 describes the smoothing function applied to realign losses predicted by the macroeconomic shock.

Section 7 presents validation exercises of GCorr Macro with C&I, CRE portfolios under various historical scenarios.

Section 8 shows the losses projected by GCorr Macro for the C&I, CRE portfolios under Fed's CCAR scenarios.

Section 9 concludes.

Appendix A lists the macroeconomic variables included in GCorr Macro.

Appendix B describes instrument-level parameters that must be specified to use the analytical calculations presented in this paper.

Appendix C presents variable selection results.

<sup>1</sup> For example, the U.S. CCAR macroeconomic variables can explain around 60% of variation in the U.S. GCorr Corporate systematic credit risk factors.

<sup>2</sup> See "Comprehensive Capital Analysis and Review 2013: Assessment Framework and Results" by Board of Governors of the Federal Reserve System.

## 2. GCorr Macro

The GCorr Macro model links systematic credit risk factors of the Moody's Analytics GCorr model to macroeconomic variables.<sup>3</sup> GCorr Macro allows for various types of credit portfolio analyses, such as stress testing, reverse stress testing, and risk integration.<sup>4</sup>

This section introduces GCorr Macro and its basic properties. We briefly explain how GCorr Macro fits into the RiskFrontier™ credit portfolio modeling framework, and we discuss various ways to use GCorr Macro. We then focus on the main subject of this paper, using GCorr Macro for multi-period, analytical stress testing.

We begin by describing the GCorr model and the RiskFrontier framework. GCorr is a multi-factor model used to estimate correlations among credit quality changes (asset returns) of obligors in a credit portfolio. GCorr includes correlation estimates across a variety of asset classes: listed corporates (GCorr Corporate), private firms, small- and medium-sized enterprises (SMEs), U.S. commercial real estate (GCorr CRE), U.S. retail (GCorr Retail), and sovereigns.

In GCorr, a borrower's credit quality is affected by a systematic factor and an idiosyncratic factor. The systematic factor represents the state of the economy and summarizes all the relevant systematic risks that affect the borrower's credit quality. GCorr defines the systematic factor as a weighted combination of 245 correlated geographical and sector risk factors, where the weights can be unique to each borrower.<sup>5</sup> The idiosyncratic factor represents the borrower-specific risk that affects the borrower's credit quality. While borrowers with the same weights to the 245 factors are exposed to the same systematic shock, the borrower-specific factor is unique to each borrower. By construction, the systematic factor is independent of the idiosyncratic factor, and both are modeled with a standard normal distribution. Two borrowers correlate with one another when both are exposed to correlated systematic factors.

The RiskFrontier framework uses GCorr to estimate a distribution of credit portfolio losses on a horizon.<sup>6</sup> We next briefly summarize the framework components, depicted in the top shaded area in Figure 1.

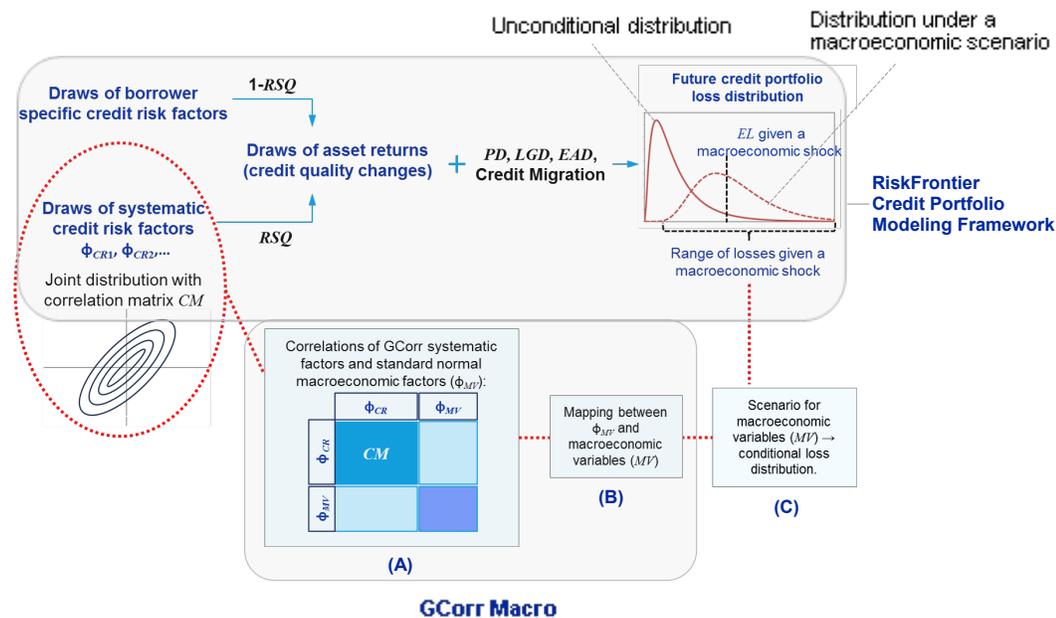
<sup>3</sup> "Modeling Credit Correlations: An Overview of the Moody's Analytics GCorr Model," Huang, et al. (2012) provides an overview of the GCorr model and the paper "Modeling Correlations Using Macroeconomic Variables," Pospisil, et al. (2012) introduces the concept of expanding GCorr by adding macroeconomic variables.

<sup>4</sup> For more details and examples of GCorr Macro uses, see "Applications of GCorr Macro: Risk Integration, Stress Testing, and Reverse Stress Testing," Pospisil, et al. (2013).

<sup>5</sup> The set of 245 factors consists of three asset class related subsets: 110 corporate factors (49 country factors and 61 industry factors), 78 U.S. commercial real estate factors (73 MSA factors and 5 property type factors), and 57 U.S. retail factors (51 state factors and 6 product type factors).

<sup>6</sup> For an introduction to the RiskFrontier credit portfolio modeling framework, see "An Overview of Modeling Credit Portfolios," Levy (2008).

Figure 1 RiskFrontier framework with GCorr Macro.



RiskFrontier employs a bottom-up approach to estimating portfolio value distribution at a future time horizon. Such an approach begins with modeling the credit quality of an individual borrower, which is affected by GCorr systematic and idiosyncratic factors. A parameter called R-squared ( $RSQ$ ) represents the proportion of the borrower's credit quality change attributable to the systematic factor. Returns on the systematic and idiosyncratic factors together establish the borrower's credit quality at horizon.

Because all borrowers are exposed to a set of correlated factors, the credit quality changes across borrowers are correlated. A Monte Carlo simulation engine generates random draws of these correlated credit quality changes. A valuation framework is applied in each simulation trial to determine the value of every instrument based on the credit quality of the corresponding borrower at horizon. The value depends on several input parameters, such as probability of default ( $PD$ ), loss given default ( $LGD$ ), credit migration matrix, and so forth. A portfolio value at horizon is given by the sum of the instrument values. Therefore, a distribution of the portfolio values can be estimated by running a large number of these simulations and calculations.

Figure 1 also depicts the role of the GCorr Macro model. GCorr Macro captures the relationship between GCorr systematic credit risk factors  $\phi_{CR}$  ( $CR$ —credit risk) and macroeconomic variables  $MV$  in two steps:

- >> The GCorr systematic factors  $\phi_{CR}$  and standard normal macroeconomic factors  $\phi_{MV}$  are linked by a Gaussian copula model with a correlation matrix, as displayed in box (A) in Figure 1.
- >> Mapping functions transform values of observable macroeconomic variables  $MV$  to the corresponding values of the standard normal macroeconomic factors  $\phi_{MV}$ . The mapping functions are represented by box (B) in Figure 1.

We emphasize that the GCorr Macro model does not change the loadings of borrowers' asset returns to systematic and idiosyncratic GCorr credit risk factors. In other words, borrower asset returns are linked to macroeconomic variables only through their loadings to the existing GCorr factors.

## 2.1 Ways to use GCorr Macro

GCorr Macro can be used in two principal ways:

- >> Simulation-based approach
- >> Multi-period, analytical stress testing

### SIMULATION-BASED APPROACH

We first comment on the simulation-based approach.<sup>7</sup> This analysis involves running the RiskFrontier Monte Carlo simulation engine and generating draws of standard normal macroeconomic factors together with GCorr factors. Thus, the simulation allows us to link the macroeconomic factors to portfolio losses on a trial-by-trial basis. Subsequently, we can analyze relationships between portfolio losses and macroeconomic variables and conduct stress testing and reverse stress testing exercises. Box (C) in Figure 1 illustrates how stress testing fits into the framework of RiskFrontier with GCorr Macro. The mapping functions allow us to translate a macroeconomic scenario into conditions on the standard normal macroeconomic factors  $\phi_{MV}$ . These conditions then imply a conditional distribution of losses under the scenario, depicted in the top right chart in Figure 1.

The simulation-based approach also facilitates the risk aggregation across risk types as well as risk allocation. The advantage of the simulation-based approach is that it generates the full loss distribution and employs the RiskFrontier valuation modules, which account for various cash flow profiles, and it can model optionalities. Furthermore, the portfolio losses produced by RiskFrontier software account for both defaults and credit migrations. The main drawback is computational time, especially for large portfolios or when the analysis is performed over multiple periods.

### MULTI-PERIOD ANALYTICAL STRESS TESTING

Multi-period, analytical stress testing produces stressed expected losses on a credit portfolio, under a specific scenario, over multiple quarters. Losses account only for defaults and, unlike in the simulation-based approach, do not include mark-to-market losses due to credit quality changes. The main advantage of multi-period analytical stress testing is the calculation time; calculations are run using analytical formulas and do not require Monte Carlo simulation.

The chart in the top right corner of Figure 1 shows one difference between the two approaches. While the simulation-based approach can describe the entire loss distribution given a macroeconomic shock (the dashed curve), the multi-period analytical stress testing approach can provide the expected loss given the shock (the dashed vertical line) with higher speed and over multiple quarters. The dispersion around the expected losses given the shock indicates that the macroeconomic variables in the scenario do not completely explain the systematic risk in the portfolio.

In this document, we focus on the multi-period analytical stress testing method in detail. We present the formulas used in calculations, illustrate how they work in practice, and show examples of stressed expected losses under various historical and hypothetical scenarios.

We use a simple example to illustrate how the analytical stress testing approach works. Assuming a fixed LGD, the stressed expected loss for a counterparty is given by the stressed PD. Stressed expected loss and stressed PD refer to conditional quantities under a macroeconomic scenario. To begin the stressed PD calculation, we provide the well-known Equation (1) for the conditional PD given a value of systematic credit risk factor  $\phi_{CR}$ .<sup>8</sup>

$$PD(\phi_{CR}) = N\left(\frac{N^{-1}(PD) - \sqrt{RSQ} \times \phi_{CR}}{\sqrt{1 - RSQ}}\right) \quad (1)$$

However, a macroeconomic scenario is not defined in terms of GCorr credit risk factors, but in terms of macroeconomic variables. With GCorr Macro, we can determine the conditional distribution of a credit risk factor given macroeconomic variables. A univariate example is provided in Equation (2). Function  $f$  maps the macroeconomic variable  $MV$  to a standard normal macroeconomic factor  $\phi_{MV}$ . Since the joint distribution of the factors  $\phi_{CR}$  and  $\phi_{MV}$  is normal, the conditional distribution is also normal.

$$\begin{aligned} \rho &= \text{corr}(\phi_{CR}, \phi_{MV}), \phi_{MV} = f(MV) \\ \phi_{CR} | MV &\sim N(\rho \times f(MV), 1 - \rho^2) \end{aligned} \quad (2)$$

Implied by GCorr Macro

<sup>7</sup> For more information, see "Applications of GCorr® Macro within the RiskFrontier™ Software: Stress Testing, Reverse Stress Testing, and Risk Integration," Lanfrancioni, et al. (2014).

<sup>8</sup> See "Loan portfolio value," Vasicek (2002).

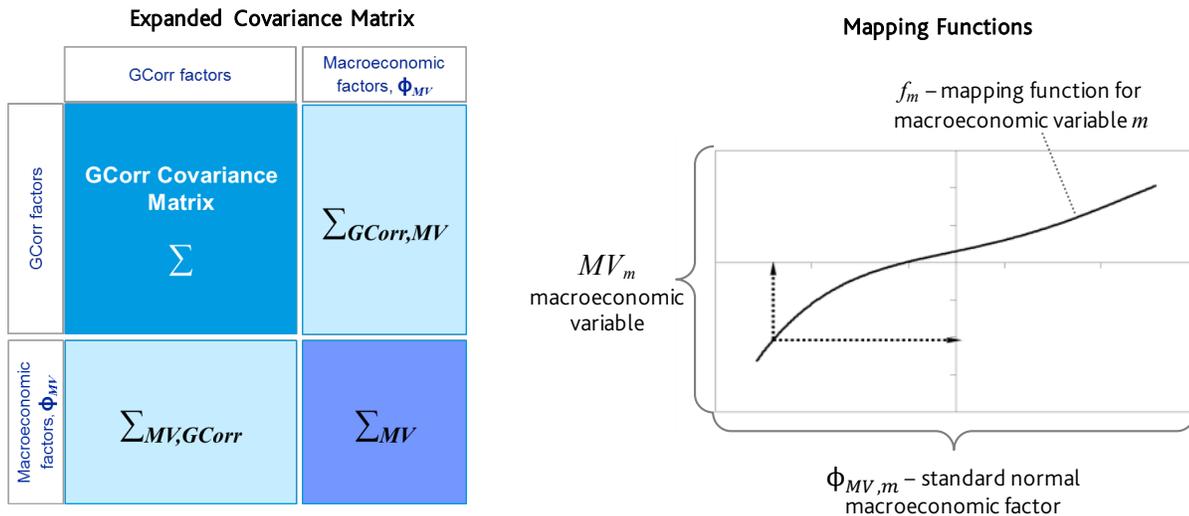
With the conditional distribution in place, we can derive the stressed PD according to Equation (3)

$$PD(MV) = \int_{-\infty}^{\infty} PD(\phi_{CR}) d(\phi_{CR} | MV) = N \left( \frac{N^{-1}(PD) - \sqrt{RSQ} \times \rho \times f(MV)}{\sqrt{1 - RSQ \times \rho^2}} \right) \quad (3)$$

Equation (3) allows us to calculate the stressed PD, given a scenario value of a macroeconomic variable. In Section 3, we generalize this example to include multiple macroeconomic variables, to determine stressed PD over multiple periods, and to calculate stressed LGD.

To conclude this section, we comment on the format in which GCorr Macro components are specified. Figure 2 shows an expanded covariance matrix linking GCorr factors (geographical and sector factors) to standard normal macroeconomic factors. The systematic factors affecting counterparty asset returns, denoted by  $\phi_{CR}$  and called custom indexes or composite factors, are linear combinations of the geographical and sector factors. Thus, the matrix implies the correlation between any custom index and a macroeconomic factor. The figure also shows a mapping function transforming a macroeconomic variable to a standard normal macroeconomic factor. In Section 4, we describe how we estimated the two GCorr Macro components displayed in Figure 2.

Figure 2 GCorr Macro components: the expanded covariance matrix and mapping functions.



### 3. Using GCorr Macro for Multi-Period Stress Testing

This section provides a detailed description of multi-period stress testing using GCorr Macro. The objective is to determine quarterly stressed expected losses on a portfolio, over a period of several quarters. We use the term “stressed expected losses” to mean conditional expected losses under a macroeconomic scenario.

The expected losses in our stress testing framework account for possible defaults in the future. From this perspective, the framework can accommodate many types of credit instruments, such as loans, bonds, or revolving lines of credit, as long as the user specifies the appropriate exposure at default through two quantities: commitment and usage given default (UGD).

An important feature of the framework is that the stressed expected loss calculations are carried out at the individual instrument level. We then determine the portfolio-stressed expected loss as the sum of the instrument-level stressed expected losses. A homogeneous pool of instruments can be represented as one instrument in our framework.

Figure 3 Flowchart of stress testing calculations based on GCorr Macro.

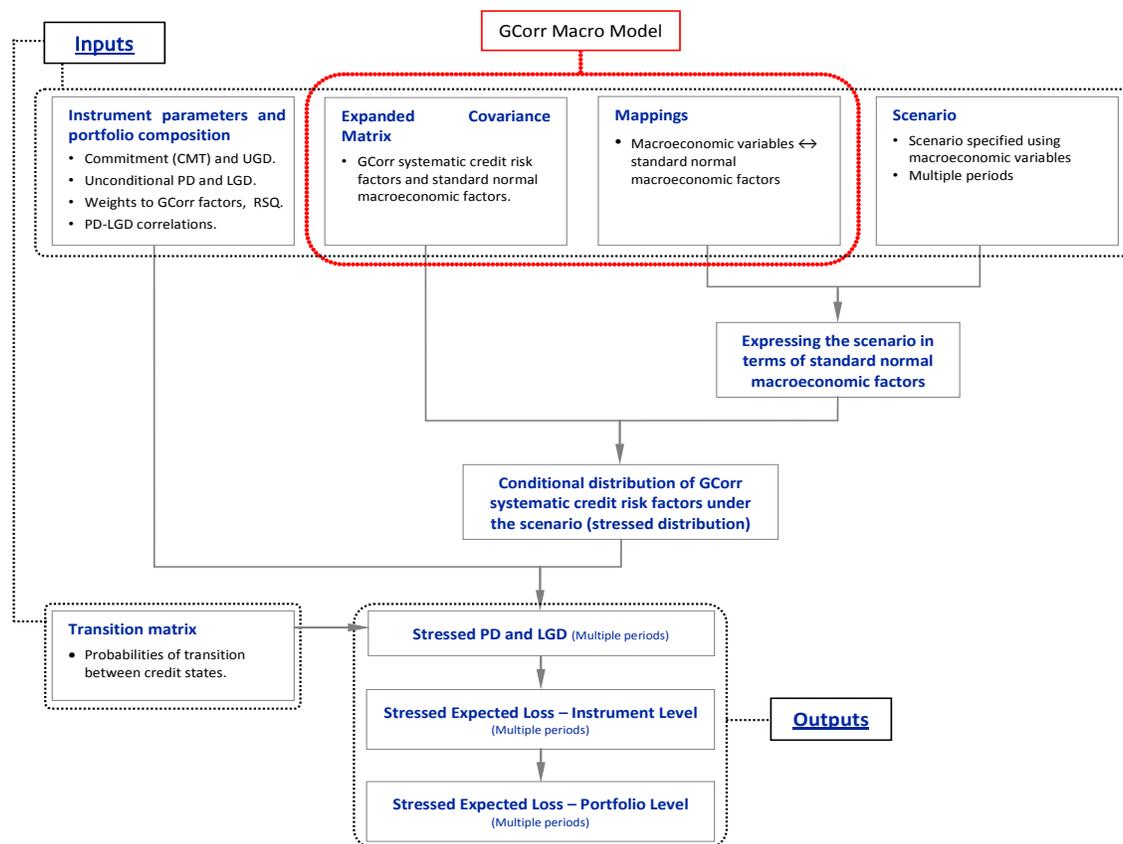


Figure 3 presents the structure of the stress testing calculations with GCorr Macro. On the input side, a user must specify the portfolio and the scenario. The GCorr Macro model is given by two components:

- >> An expanded covariance matrix linking GCorr credit risk factors with standard normal macroeconomic factors
- >> Mapping functions converting values of macroeconomic variables to values of standard normal macroeconomic factors

The calculations also require a matrix of quarterly transition probabilities between credit states, which allows us to fully account for the multi-period nature of the scenario.

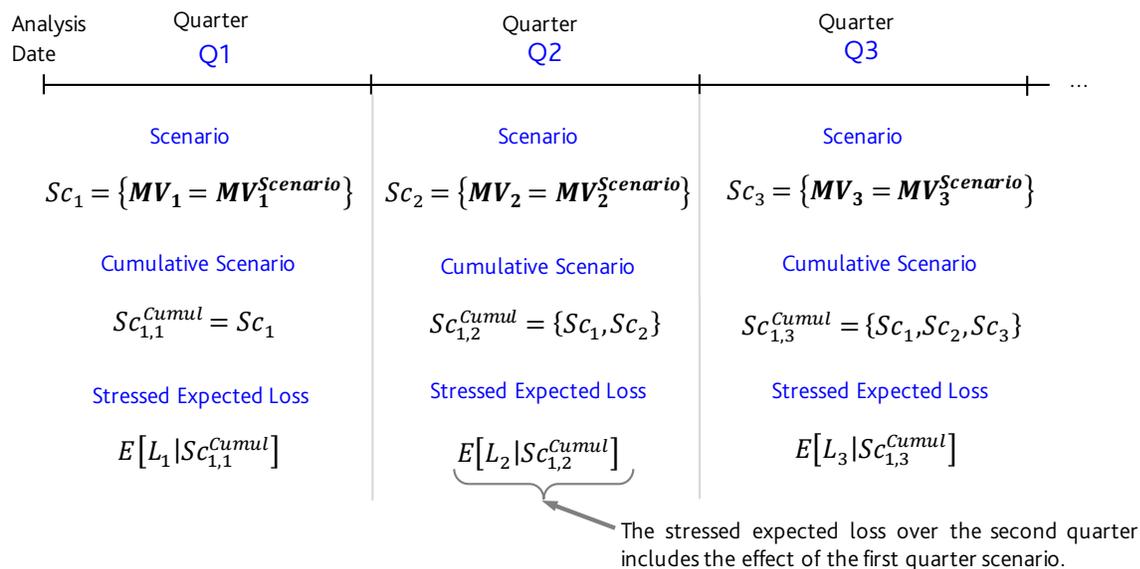
The GCorr Macro components and the transition matrix can be estimated using various data sources. In this paper, we work with the expanded covariance matrix and mapping functions that we estimated according to the approach described in Section 4. We use the DD transition matrix from RiskFrontier software which includes 29 non-default credit states and one default credit state.<sup>9</sup>

The user specifies a portfolio by providing the instrument-level parameters, shown in Figure 3 and listed in more detail in Appendix B. The weight of an instrument in the portfolio is given by its commitment (CMT) and usage given default (UGD), which, together, imply exposure at default for each quarter. By considering a term structure of these two quantities, we can account for changes in the exposure over time, such as when the instrument matures, is amortized, or as the financial institution adds new volume. Specifically, the role of UGD is to capture exposure dynamics of revolving lines of credit. The framework presented in this paper determines stressed PD and LGD parameters based on the scenario, while commitment and UGD remain unchanged. The user can account for this assumption by providing commitment and UGD values on the input that already incorporate the effects of the scenario. Note, it is possible to generalize the framework to include a stressed UGD calculation.

Further instrument-level parameters required for the stress testing calculations are term structures of unconditional PD and LGD, weights of the counterparty's systematic factor to GCorr factor, and the asset R-squared value, which represents sensitivity of the counterparty's asset return to the systematic factor. While the multi-period stress testing methodology requires a term structure of PD and LGD parameters, one can assume a flat term structure,<sup>10</sup> if the parameters are not available for a grid of tenors. The flat term structure is specified with a single value of the parameter.

LGD is stressed through the Moody's Analytics PD-LGD correlation model. When calculating stressed expected losses, it is possible to either stress LGD together with PD or to assume that LGD does not change under the scenario (for example, use a constant downturn LGD).

**Figure 4** Scenarios and stressed expected losses over multiple future quarters.



We assume the scenario is defined using conditions on quarterly stationary macroeconomic variables over a given number of quarters. We discuss stationarity transformations of macroeconomic variables in Section 4.1. An example scenario may be the stock market index drops by 20% during the second quarter from the analysis date. If the index is the third macroeconomic variable, we write this condition as  $MV_{2,3} = -20\%$ . We denote the vector of macroeconomic variables over the second quarter included in the scenario as  $MV_2$  and the set of values of these macroeconomic variables that the scenario prescribes as  $MV_2^{Scenario}$ .  $SC_2$  refers to the scenario over the second quarter and  $SC_{1,2}^{Cumulative}$  to the cumulative scenario through quarter 2 (i.e., the scenarios over quarters 1 and 2).

<sup>9</sup> For details, see "Modeling Credit Portfolios, RiskFrontier™ Methodology," Moody's Analytics (2013).

<sup>10</sup> For PD values, a flat term structure means that the unconditional instrument PD values are instrument-specific and do not vary over time. Note, unconditional PD values are only one input of stressed PD calculations, and a flat term structure for unconditional PD values does not imply a flat term structure for stressed PD values.

Figure 4 summarizes the definitions of scenarios over multiple quarters. Let us emphasize that a scenario can include an arbitrary number of macroeconomic variables from GCorr Macro. However, the variables included in the scenario should constitute a reasonable model describing the portfolio that will be stress tested. Choosing an appropriate set of macroeconomic variables is called a variable selection process. Later in this section, we present tools for variable selection, and in Section 5.3 we discuss how to conduct variable selection in practice.

As Figure 3 shows, the first step of the calculation involves mapping the values of stationary macroeconomic variables given by the scenario to conditions on standard normal macroeconomic factors. We perform this step with the mapping functions specified as a component of the GCorr Macro model. Let  $f_m$  denote the mapping function for a macroeconomic variable  $m$ . We can represent the mapping of macroeconomic variable  $m$  for quarter  $t$  as follows:

$$\phi_{MV,m,t}^{Scenario} = f_m(MV_m^{Scenario}) \quad (4)$$

The result of the mapping is a value of the standard normal macroeconomic factor  $m$  for quarter  $t$ . For example, 20% drop in a stock market index might be mapped to a value of  $-1.9$  in a standard normal space:  $\phi_{MV,2,3}^{Scenario} = -1.9$ .

In the next step, we determine the stressed distribution of GCorr factors. Unconditionally, the GCorr factors ( $r_{GCorr}$ ) have a joint normal distribution with covariance matrix  $\Sigma$ . Assuming that the standard normal macroeconomic factors over quarter  $t$  have values  $\phi_{MV,t}^{Scenario}$ , we can use the expanded covariance matrix from Figure 2 to derive the stressed distribution of the geographical and sector factors:<sup>11</sup>

$$r_{GCorr,t} | S c_t \sim N(\Sigma_{GCorr,MV} \times \Sigma_{MV}^{-1} \times \phi_{MV,t}^{Scenario}, \Sigma - \Sigma_{GCorr,MV} \times \Sigma_{MV}^{-1} \times \Sigma_{MV,GCorr}) \quad (5)$$

The counterparty's systematic factor (custom index) can be expressed as a linear combination of the GCorr factors ( $r_{GCorr}$ ), with a vector of weights  $w$ . Equation (6) implies the stressed distribution of the custom index for quarter  $t$ .<sup>12</sup>

$$\phi_{CR,t} | S c_t \sim N\left(s \times \underbrace{w^T \times \Sigma_{GCorr,MV} \times \Sigma_{MV}^{-1} \times \phi_{MV,t}^{Scenario}}_{E[\phi_{CR,t} | S c_t]}, 1 - \rho^2\right) \quad (6)$$

Note, the stressed expected value of the custom index is a linear function of the scenario values of the standard normal macroeconomic factors. Meanwhile, the stressed variance does not depend on the specific scenario values, and it is impacted only by the choice of which macroeconomic variables are included in the scenario. Parameter  $\rho$  can be interpreted as the multivariate correlation of the custom index with the standard normal macroeconomic factors in the scenario. If these macroeconomic factors explain a large portion of the custom index variability, parameter  $\rho$  is large and the stressed variance is low. The maximum possible value the parameters can attain is one, which corresponds to the case when the macroeconomic variables completely determine the custom index.

In effect, we can consider Equation (6) a linear regression style relationship

$$\beta^T = s \times w^T \times \Sigma_{GCorr,MV} \times \Sigma_{MV}^{-1} \quad (7)$$

Now we can express the stressed expected value of the custom index as:

$$E[\phi_{CR,t} | S c_t] = \beta^T \times \phi_{MV,t}^{Scenario} \quad (8)$$

<sup>11</sup> We use the standard formula for a conditional normal distribution: see "Multivariate Statistical Methods" by Morrison (2004).

<sup>12</sup> Scaling factor  $s$  ensures that the unconditional distribution of the custom index is standard normal:

$$s = \frac{1}{std(w^T \times r_{GCorr,t})}$$

This representation allows us to conduct variable selection in exactly the same way as in a multi-variety regression. In this context,  $\rho^2$  can be now interpreted as the R-squared in regular regression models, defined as the ratio of the explained variance to the total variance of the dependent variable. But the difference here is that unlike R-squared in a regression,  $\rho^2$  is based on our modeled correlation matrix, which is why we refer to  $\rho^2$  as pseudo R-squared. In the variable selection process, we use adjusted pseudo R-squared.<sup>13</sup>

As described in Section 5, in practice, parameter  $\rho^2$  does not reach value one if we focus on economy-wide macroeconomic variables, such as stock market index or GDP. This is because the GCorr factors can capture various industry effects, which cannot be described by broad economic indicators.

The statistical significance of macroeconomic variables in a model is essential later for the variable selection process. The t-statistics which describes the significance of  $i$ -th macroeconomic variable can be derived from Equation (6):

$$t - Stat_i = \sqrt{n} \frac{\beta_i}{\sqrt{1 - \rho^2} \sqrt{\chi_{ii}}} \tag{9}$$

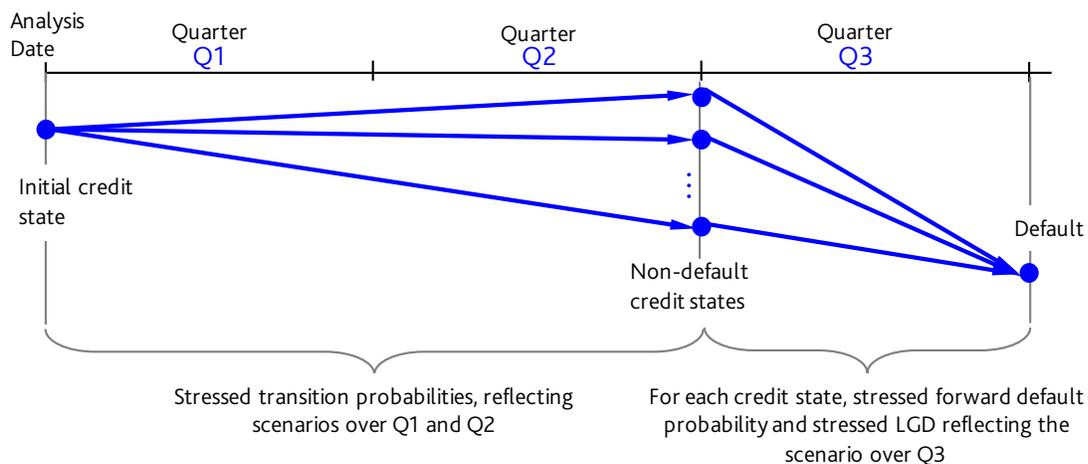
Where  $\chi_{ii}$  is the  $i$ -th diagonal term of the inverse of the correlation matrix of the macroeconomic factors  $\Sigma_{MV}$ ,  $n$  is the number of observations, and  $\beta_i$  is the  $i$ -th element in the vector  $s \times w^T \times \Sigma_{GCorr, MV} \times \Sigma_{MV}^{-1}$ .

Note that this is not the same t-statistics as in an empirical regression. The estimation is based on a correlation matrix, which is not a purely empirical correlation matrix, but is subject to certain economic adjustment described in Section 4.

Having determined the stressed distribution of a custom index, we calculate stressed expected losses. As noted earlier, the two parameters of the expected value that we stress are PD and LGD.

The instrument level inputs are specified as of the analysis date. An important question is how to account for losses over future quarters, beyond quarter one after the analysis date. We resolve this by considering the effect of credit migration, illustrated in Figure 5. For example, when calculating stressed expected loss for the third quarter after the analysis date, we determine the stressed PD and LGD for the third quarter for each non-default credit state in which the counterparty resides at the beginning of that quarter. As shown in Figure 5 the stressed PD and LGD depend on the stressed custom index distribution for the third quarter, which is given by the scenario over the third quarter. In addition, we compute stressed transition probabilities that the counterparty will migrate from an initial credit state, known on the analysis date, to a credit state at the beginning of the third quarter. These stressed transition probabilities account for the scenarios over the first and second quarters. Based on information available on the analysis date, we can calculate the stressed expected loss for the third quarter by combining the third quarter stressed credit risk parameters and stressed transition probabilities between the analysis date and the third quarter.

Figure 5 Example of incorporating credit migration: stressed credit parameters for the third quarter from the analysis date.



<sup>13</sup> The adjusted pseudo R-squared is defined as follows

$$Adjusted \rho^2 = 1 - (1 - \rho^2) \frac{n - 1}{n - K - 1}$$

where  $n$  is the number of observation and  $K$  is the number of macroeconomic variables used in the model.

The advantage of this approach is that it incorporates the full path of a scenario and does not rely solely on the scenario for a given quarter. For example, if a scenario assumes an adverse economic shock over the first two quarters, the counterparty's credit quality is likely to deteriorate. The stressed transition probabilities will reflect this fact and the counterparty will likely be in a bad credit state at the beginning of the third quarter. As a consequence, its stressed default probability over the third quarter will be higher than if a benign scenario is assumed over the first two quarters.

Next, we present equations for the stressed credit risk parameters. Equation (10) provides the stressed forward default probability (FPD) for quarter  $t$ , assuming that the counterparty is in a credit state  $cs$  at the beginning of  $t$ . The stressed FPD depends on the input parameters and the stressed custom index distribution for quarter  $t$ . Specifically,  $FPD_{t,cs}$  is the unconditional forward default probability for quarter  $t$  from the credit state  $cs$ , which can be calculated from the input PD term structure and the transition matrix.

$$FPD_{t,cs}(S_{C_t}) = N \left( \frac{N^{-1}(FPD_{t,cs}) - \sqrt{RSQ} \times E[\Phi_{CR,t}|S_{C_t}]}{\sqrt{1 - RSQ \times \rho^2}} \right) \quad (10)$$

Equation (11) shows how to determine the stressed LGD for quarter  $t$  and a credit state  $cs$  at the beginning of  $t$ . This equation is based on the Moody's Analytics PD-LGD correlation model.<sup>14</sup> The parameters  $a(cs)$ ,  $b$ , and the function  $p_{cs}(z, \Phi_{MV,t}^{Scenario})$  depend on the input parameters and the stressed GCorr factor distribution. Function  $p$  represents the density of the counterparty's recovery return, corresponding by variable  $z$ , given default and given the scenario over quarter  $t$ . Function  $L_{cs}(z, LGD_t)$  converts the recovery return  $z$  to a variable within the range  $0$  to  $1$ , which has, unconditionally, a Beta distribution.<sup>15</sup> Parameter  $k$ , specified as an input, characterizes the variance of the Beta distribution. The integral in Equation (11) must be evaluated using numerical techniques.

$$LGD_{t,cs}(S_{C_t}) = \int_{-\infty}^{\infty} L_{cs}(z, LGD_t) \times p_{cs}(z, \Phi_{MV,t}^{Scenario}) dz \quad (11)$$

$$L_{cs}(z, LGD_t) = Beta^{-1} \left( 1 - N_{a(cs),b}(z), (k-1)LGD_t, (k-1)(1-LGD_t) \right)$$

Stressed transition probabilities over a quarter  $t-1$  can be calculated according to Equation (12). Symbols  $cs_{t-1}$  and  $cs_t$  denote the credit states at the beginning of quarter  $t-1$  and quarter  $t$ , respectively.  $TP_{t-1 \rightarrow t, cs_{t-1} \rightarrow j}$  are unconditional transition probabilities coming from the transition matrix adjusted in order to be consistent with the input PD term structure.

$$TP_{t-1 \rightarrow t, cs_{t-1} \rightarrow cs_t}(S_{C_{t-1}}) = TP^*(cs_t) - TP^*(cs_{t-1})$$

$$TP^*(cs_t) = N \left( \frac{N^{-1} \left( \sum_{j=1}^{cs_t} TP_{t-1 \rightarrow t, cs_{t-1} \rightarrow j} \right) - \sqrt{RSQ} E[\Phi_{CR,t-1}|S_{C_{t-1}}]}{\sqrt{1 - RSQ \times \rho^2}} \right) \quad (12)$$

Equation (13) provides an iterative procedure for calculating cumulative stressed transition probabilities. We denote the initial credit state as  $cs_0$ .

$$TP_{1 \rightarrow t, cs_0 \rightarrow cs}^{Cumul}(S_{C_{1,t-1}}^{Cumul}) = \sum_{cs_{t-1}} TP_{1 \rightarrow t-1, cs_0 \rightarrow cs_{t-1}}^{Cumul}(S_{C_{1,t-2}}^{Cumul}) \times TP_{t-1 \rightarrow t, cs_{t-1} \rightarrow cs}(S_{C_{t-1}}) \quad (13)$$

<sup>14</sup> For information about the Moody's Analytics PD-LGD correlation model, see "Modeling Credit Portfolios, RiskFrontier™ Methodology," "Incorporating Systematic Risk in Recovery: Theory and Evidence," Levy and Hu (2007), and "Implications of PD-LGD Correlation in a Portfolio Setting," Meng, et al. (2010).

<sup>15</sup>  $Beta^{-1}$  denotes inverse of the cumulative distribution function of a Beta distribution.

After calculating the stressed credit risk parameters for an instrument, we can determine the stressed expected loss for quarter  $t$  according to Equation (14). The condition  $Sc_{1,t}^{Cumul}$  highlights the fact that the entire path of the scenario through quarter  $t$  impacts the loss.

$$\begin{aligned} E[L_{i,t} | Sc_{1,t}^{Cumul}] \\ = CMT_t \times UGD_t \times \sum_{cs} [TP_{1 \rightarrow t, cs_0 \rightarrow cs}^{Cumul}(Sc_{1,t-1}^{Cumul}) \times FPD_{t,cs}(Sc_t) \times LGD_{t,cs}(Sc_t)] \end{aligned} \quad (14)$$

The stressed expected loss can be compared to the unconditional expected loss, determined with the unconditional quarterly PD,  $PDq$  (implied by the input PD term structure), and the unconditional LGD.

$$E[L_{i,t}] = CMT_t \times UGD_t \times PDq_t \times LGD_t \quad (15)$$

The portfolio-stressed expected loss and unconditional expected loss are given by the sum of the corresponding instrument-level quantities.

$$\begin{aligned} E[L_{Portfolio,t} | Sc_{1,t}^{Cumul}] &= \sum_{i \in instruments} E[L_{i,t} | Sc_{1,t}^{Cumul}] \\ E[L_{Portfolio,t}] &= \sum_{i \in instruments} E[L_{i,t}] \end{aligned} \quad (16)$$

The expected losses in Equation (16) are expressed in cash terms. If we need to normalize the losses, we scale them by the total portfolio exposure:

$$\frac{E[L_{Portfolio,t} | Sc_{1,t}^{Cumul}]}{\sum_{i \in instruments} CMT_{i,t} \times UGD_{i,t}} \quad \frac{E[L_{Portfolio,t}]}{\sum_{i \in instruments} CMT_{i,t} \times UGD_{i,t}} \quad (17)$$

## 4. Estimating and Validating GCorr Macro Parameters

In this section, we explain our approach to estimating parameters of GCorr Macro, including the input data, the specific estimation techniques, and the challenges faced. In addition, we discuss the steps taken to validate the parameters. This process involves, for example, understanding parameter sensitivity to the estimation period and time series transformations. In Section 5, we discuss the resulting parameter values and their implications for stress testing.

Section 2 describes the structure of the GCorr Macro model. The parameters to be estimated are the expanded covariance matrix and the mappings of macroeconomic variables to standard normal macroeconomic factors. Specifically, for the expanded covariance matrix, we must determine covariances of the standard normal macroeconomic factors with GCorr factors and correlations among the standard normal macroeconomic factors.

First, we focus on the data used for estimation: the time series of macroeconomic variables (Section 4.1) and credit risk factors (Section 4.2). We then explain the process of estimating the expanded covariance matrix (Section 4.3) and the mapping functions that transform macroeconomic variables to standard normal factors (Section 4.4).

### 4.1 Macroeconomic Data

In Appendix A, we provide the list of macroeconomic variables, including data sources. We obtain quarterly time series for each variable from either 1970–2015 or for a shorter period if data availability is limited.

We choose the quarterly frequency, because many economic scenarios, such as the Fed's CCAR stress test, are based on quarterly values of macroeconomic variables. For the variables available at a higher than quarterly frequency, we select the last observation for a quarter. This choice makes the data consistent with the credit risk factor time series, which can be interpreted as returns between end-of-quarter time points.

For estimation purposes, we need to transform the macroeconomic time series into a stationary time series. In addition to stationarity, the transformations should produce time series with empirical distributions suitable for calibrating the mappings to standard normal distributions. For price index variables, such as a stock market index, we choose log-differencing as the most appropriate transformation.<sup>16</sup> We also apply log-differencing to rate variables, such as unemployment rate and interest rates. We choose log-differencing over plain differencing because these variables are bounded by zero from below, which introduces a bound on the possible range of differences. Such a bound would make the mapping to standard normal distribution more challenging; in log-differencing, we do not see this issue.

We also perform detrending, meaning that we calculate deviations of time series values from a trend. The trend is defined as the moving average of the time series values over a window of a given length.<sup>17</sup> For some time series, the detrending transformation helps us obtain a stationary time series that can be more naturally linked to corporate credit risk factors and corporate defaults. For example, real GDP growth time series reaches different levels during the economic growth periods of the late 1990s, the mid-2000s, and the aftermath of the financial crisis. However, from a corporate credit risk perspective, these periods are equivalent, because they experienced comparably low levels of defaults and C&I loan losses. By considering deviations of the real GDP growth from a trend, we make the time series more consistent with corporate credit risk dynamics.

We have explored the impact of various transformations on both the stationarity of the resulting time series as well as on correlations with GCorr factors. We summarize the transformations applied to individual macroeconomic variables in Appendix A. As a result of the transformations, we obtain macroeconomic time series, which we consider stationary, and we use them for the GCorr Macro estimation.

### 4.2 Credit Risk Data

The GCorr model provides the covariance matrix,  $\Sigma$ , of 245 credit risk factors that we expand with macroeconomic variables in Section 4.3.<sup>18</sup> GCorr Corporate includes 49 country factors and 61 industry factors. The dataset used to estimate these factors and their covariances contains firm-level historical time series of weekly asset returns, interpreted as credit quality changes, for the period 1999Q3–2015Q1.

<sup>16</sup> If  $X_t$  is a time series, differencing leads to the time series  $Y_t = X_t - X_{t-1}$ , log-differencing to  $Y_t = \log(X_t/X_{t-1})$ , calculating percentage changes to  $Y_t = (X_t - X_{t-1})/X_{t-1}$ .

<sup>17</sup> Detrending a time series  $Y_t$  with a time window of length  $K$  can be represented as

$$Y_t - \frac{1}{K} \sum_{k=1}^K Y_{t-k}$$

<sup>18</sup> For details on the GCorr model, see "Modeling Credit Correlations: An Overview of the Moody's Analytics GCorr Model," Huang, et al. (2012).

The first steps of the estimation process involve creating time series of country and industry factors from the firm level data. Next, we estimate common factors, orthogonal time series describing the co-movements in the country and industry factors. We use a similar approach for GCorr CRE and GCorr Retail factors: their co-movements are captured by orthogonal common factors. The final set of common factors describes relationships of factors not only within each asset class, but also across asset classes. Any GCorr factor can be represented by loadings  $\beta$  to the set of common factors  $f_n^{Common}$  and a residual  $\varepsilon_j$ , reflecting the portion of the GCorr factor unexplained by the common factors. The representation is described in Equation (18).

$$r_{GCorr,j} = \sum_{n=1}^N \beta_{j,n} f_n^{Common} + \varepsilon_j \quad (18)$$

$$var(r_{GCorr,j}) = \sigma_j^2, var(f_n^{Common}) = \sigma_{f,n}^2, var(\varepsilon_j) = \sigma_{\varepsilon,j}^2$$

$$cov(f_n^{Common}, f_k^{Common}) = 0, \sigma_j^2 = \sum_{n=1}^N \beta_{j,n}^2 \sigma_{f,n}^2 + \sigma_{\varepsilon,j}^2$$

We conduct several validation exercises in which we analyze relationships between U.S. macroeconomic variables and other measures of systematic credit risk in the U.S., in addition to the GCorr Corporate factors. One alternative is the time series of factors implied by corporate defaults and C&I loan delinquencies.<sup>19</sup> Our ultimate goal is to use GCorr Macro to project losses that would be consistent with the past behavior of delinquencies, so understanding how these time series co-move with macroeconomic variables helps us calibrate GCorr Macro. Separately, we also examine time series of systematic credit risk factors implied by corporate CDS data and how they relate to macroeconomic time series.<sup>20</sup>

It is worth highlighting that GCorr Corporate factors represent systematic credit risk at the level of 61 industries for each country. In contrast, the corporate default rates and CDS data can be properly used only at a coarser level, either for broad sectors or as an economy-wide index. The sample sizes for this data are too small to allow for more granular classifications. The C&I delinquency rate is only available at the national level.

### 4.3 Expanded Covariance Matrix

The expanded covariance matrix links standard normal macroeconomic factors to the GCorr factors. This section describes in detail how we estimate the matrix, with a focus on the GCorr Corporate country and industry factors.

We use the quarterly macroeconomic time series from Section 4.1 and quarterly credit risk factor data discussed in Section 4.2. First, we analyze time series relationships between the macroeconomic variables and credit risk factors. For example, Figure 6 shows dynamics of U.S. unemployment rate changes, a GCorr composite factor representing systematic credit risk in a U.S. industry and the U.S. C&I loan delinquency rate. In line with economic intuition, the credit risk measures move together with the unemployment rate, especially during times of economic stress. We quantify these relationships and use the results to determine a general level of correlations between credit risk factors and each macroeconomic variable. We refer to these correlation levels as target correlations.

Although we rely primarily on the GCorr Corporate factor time series to analyze the relationships, the delinquency rate-implied factors and CDS-implied factors introduced in Section 4.2 help us validate and, in some cases, adjust the target correlations. We provide two examples: unemployment rate and stock market variables (value index and VIX). The asset return time series underpinning GCorr Corporate are based on the Vasicek-Kealhofer methodology for EDF estimation.<sup>21</sup> As a result, the asset returns for a firm depend on a combination of equity returns, interest rate changes, and the firm's balance sheet characteristics, such as leverage.

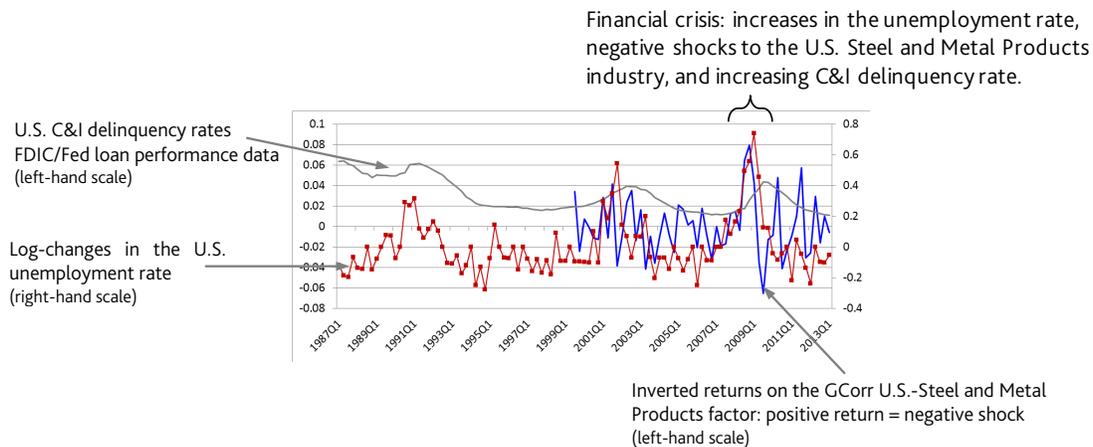
<sup>19</sup> The specific method that allows us to imply credit risk factor time series from time series of default or delinquency rates is similar to Equation (8) in "Modeling Credit Correlations: An Overview of the Moody's Analytics GCorr Model," Huang, et al. (2012). We use two time series for this exercise: default rate of U.S. large listed non-financial corporates, based on Moody's Analytics data, and an FDIC/Fed delinquency rate on C&I loans originated by U.S. commercial banks.

<sup>20</sup> We use U.S. CDS corporate data from Markit. Time series of CDS spreads are converted into asset returns proxies by applying a methodology described in "CDS-implied EDF™ Credit Measures and Fair-value Spreads," Dwyer, et al. (2010). The time series of these asset return proxies are used to estimate CDS-implied systematic credit risk factors.

<sup>21</sup> See "Modeling Credit Portfolios, RiskFrontier™ Methodology" and "Understanding 2006 Correlations," by Moody's Analytics.

Given that GCorr factors are based on asset returns (which incorporate stock market information), we observe relatively high correlations of the GCorr factors with the stock market variables, and relatively low—although significantly positive—correlation with the unemployment rate changes, compared to other variables. One reason for the low correlation with the unemployment rate may be a timing issue because the unemployment rate is more closely linked to past stock market returns. However, time series of corporate delinquencies exhibits a stronger association with unemployment rate than with the stock market. We use this information as an input when adjusting the correlations of GCorr factors and these macroeconomic variables.

**Figure 6** Example of time series dynamics of a macroeconomic variable and a credit risk factor.



GCorr 2015 Corporate provides factor time series over the period 1999Q3–2015Q1. The delinquency rates and the macroeconomic variables are available over longer periods of time, which allows us to analyze how correlations of credit risk factors with macroeconomic variables and correlation among macroeconomic variables vary over time. While some relationships, such as between U.S. real GDP growth and unemployment changes, are relatively stable over time, others strongly depend on the economic environment. For example, relationships among interest rates, stock market, consumer price index, and credit risk factors are contingent on whether the economy is in a high or low inflation environment. As a result, correlations estimated from the period of the financial crisis, when consumer price inflation was not an issue, would differ from correlations based on the 1970s data, when the U.S. economy experienced high inflation.

Our objective is to estimate an expanded covariance matrix that reflects relationships among variables over the recent period, including the effects of the financial crisis. The reason is that typical stress testing exercises, such as CCAR, are based on scenarios that mimic the financial crisis episode to some degree. Therefore, we focus on the period 1999–2015 for the estimation. We can consider this choice a trade-off between the need for a sufficient number of quarterly observations and the objective to include data describing mainly the recent period.

We determine the expanded covariance matrix from the loadings of the GCorr factors, and macroeconomic variables to the orthogonal common factors  $f^{Common}$  from Section 4.2 and additional principal components which represent commonalities in macroeconomic variables unexplained by  $f^{Common}$ .<sup>22</sup> To obtain the loadings of the macroeconomic variables, we regress the quarterly macroeconomic time series on quarterly versions of the common factors, as well as on the additional principal components from 1999Q3–2015Q1.<sup>23</sup> Subsequently, we adjust certain loadings so that the general level of correlations between systematic credit risk factors and macroeconomic variables matches the target correlations introduced earlier in this section.<sup>24</sup>

Using a common factor representation to determine the expanded covariance matrix is consistent with the philosophy of a general GCorr approach. The assumption behind this approach is that the common factors can explain dependencies among the

<sup>22</sup> In addition to the loadings, we need information regarding standard deviations of the common factors and standard deviations of the residuals unexplained by the common factors.

<sup>23</sup> In some cases, we regress a macroeconomic variable on common factors as well as a country-specific factor. We choose this approach when we want to make sure that the correlation of the macroeconomic variable with the country's composite factors is higher than with composite factors of the other countries. Examples where we used this approach are: UK equity market and GDP, South Africa equity market, and GDP.

<sup>24</sup> Another way to link the GCorr factors to macroeconomic variables would be to regress the factors on macroeconomic variables. As we point out earlier, the macroeconomic variables do not completely explain variation in the factors. In addition, the residuals of the regressions would still be correlated. In other words, the macroeconomic variables cannot capture the correlation structure of the GCorr factors either.

composite credit risk factors and macroeconomic variables. To validate this assumption, we compare the general level of empirical time series correlations and the factor table implied correlations of the variables. The levels of these two sets of correlations are close.

The approach based on common factors has several advantages. It imposes a structure on dependencies between macroeconomic variables and GCorr factors, which allows us to ensure that the cross-sectional variation in correlations meets certain economic conditions. For example, the common factor representation implies that the Eurozone equity market or GDP macroeconomic variables are more strongly correlated with composite credit risk factors representing Eurozone industries, as opposed to industries from other countries. In addition, the common factor approach mitigates the issue of outliers in empirical time series correlations.

Note, we estimate the expanded covariance matrix of GCorr factors and standard normal macroeconomic factors based on stationary macroeconomic time series, without transforming them to have a normal distribution. We have conducted several exercises that show, for some variables, such distributional transformation does not substantially impact the resulting correlation patterns. However, these transformations can lead to lower correlations in some other cases, because they typically mute the impact of extreme observations in the macroeconomic time series from the period of the recent financial crisis. For example, the U.S. unemployment rate increased substantially during the crisis, as Figure 6 shows, and the credit risk factors experienced a negative shock at the same time. These time series exhibited lower volatility and less co-movement during times of economic growth. Therefore, the extreme crisis observations lead to a higher correlation in this case, compared to a correlation based on a benign period only.

Viewed from a distributional perspective, the financial crisis was not an extreme event, with respect to period 1999Q3–2015Q1. Its two most adverse quarters, 2008Q4 and 2009Q1, are the two worst observations out of only 63 observations. Replacing the time series with their standard normal equivalents obtained by using such a distributional transformation would mute the impact of the crisis observations and lead to lower correlations. That may not be desirable, because we want to keep the effect of the financial crisis unmitigated, so that the matrix can be used for scenarios representing severe economic conditions. For this reasons, we do not apply further distributional transformations to estimate the expanded covariance matrix.

In Section 7, we demonstrate that the expanded covariance matrix together with other parameters provides adequate levels and patterns in projected losses under various historical scenarios.

#### 4.4 Mapping Macroeconomic Variables to Standard Normal Factors

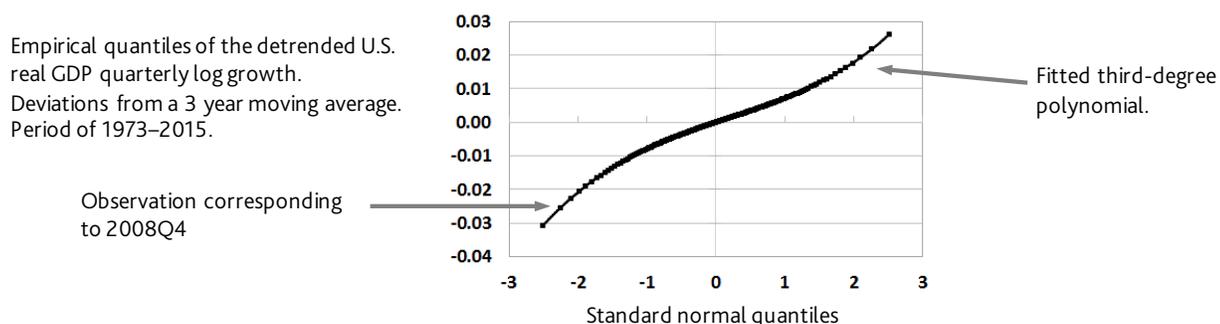
This section describes estimating the mapping functions, which convert scenarios specified using stationary macroeconomic variables to scenarios based on standard normal factors. For example, if a scenario prescribes a real GDP decline by 2.6% from a trend, the mapping function may imply that this value corresponds -2.3 shock in the standard normal space.

The quarterly stationary macroeconomic time series from Section 4.1 serves as the input dataset for estimation of the mappings. We estimate a mapping for each macroeconomic variable separately. First, we assign standard normal quantiles to values of a time series using the empirical quantile method. Specifically, we determine the empirical probability that the macroeconomic variable will be lower than a given value in the time series. The empirical probability is implied by the rank on the value in the time series. Subsequently, we convert the empirical probability into a standard normal quantile. Figure 7 shows an example of empirical quantile mapping for U.S. real GDP growth.

We need to map any scenario value to a standard normal factor, not just the historical values. Therefore, we fit a function to the empirical quantile mappings. Our analyses indicate that third-degree polynomials provide the best fit for most variables.

The fitted third-degree polynomials are the mapping functions we use to map quarterly macroeconomic variables to standard normal factors, and vice-versa.

**Figure 7** Example of a mapping calibration: U.S. Real GDP growth versus the corresponding standard normal quantiles.



For the mapping estimation, we use the period of the early 1970s through 2015 or the longest possible period for variables with limited data. We conducted exercises to examine the impact of this choice on the estimated mappings and losses projected by GCorr Macro. We find that the period we ultimately select is the most suitable, because it provides us more observations in the tail to fit a polynomial than a shorter period allows. Moreover, the selected period leads to the satisfactory validation results discussed in Section 7.

## 5. Understanding GCorr Macro Parameters

This section provides an overview of the GCorr Macro parameters estimated in Section 4 and illustrates their role in calculating stressed credit risk parameters. More specifically, Section 5.1 summarizes correlations between macroeconomic variables and GCorr composite factors, implied by the estimated expanded covariance matrix.<sup>25</sup> We are interested in the general correlation levels, as well as in cross-sectional patterns of the correlations across industries and countries. In Section 5.2, we show how the expanded covariance matrix and the mappings of macroeconomic variables to standard normal factors determine the stressed distributions of credit risk factors. We present examples showing the magnitude of stress associated with some historical macroeconomic observations. In section 5.3, we discuss how to select macroeconomic variables for a scenario and a given portfolio. Section 5.4 describes how the stressed credit risk parameters depend on the interactions of GCorr Macro parameters and the unconditional instrument-level inputs, such as PD or asset R-squared value.

### 5.1 Correlations of Macroeconomic Variables with GCorr Factors

Table 1 presents summary statistics of correlations between several U.S. macroeconomic variables and 61 GCorr composite factors representing U.S. industries. These correlations are implied by the GCorr Macro expanded covariance matrix we estimated in Section 4.3.

Table 1

#### Summary Statistics of Correlations between Select U.S. Macroeconomic Variables and 61 GCorr Composite Factors Representing U.S. Industries

CATEGORY	MACROECONOMIC VARIABLE	CORRELATION WITH THE 61 U.S. GCORR			
		CUSTOM INDEXES			
		AVERAGE	STD. DEV.	RANGE: 5TH – 95TH PERCENTILES	
Economic activity	Real GDP	42%	3%	36%	45%
	Nominal GDP	41%	3%	36%	44%
	Unemployment rate	-43%	3%	-46%	-37%
	Industrial production	35%	5%	27%	42%
Financial markets	BBB Spread	-48%	3%	-51%	-41%
	Dow Jones Total Stock Market Index	57%	4%	50%	61%
	VIX – Stock Market Volatility	-41%	3%	-43%	-35%
Real estate markets	House price index	27%	2%	23%	29%
	CRE price index	28%	2%	24%	29%
Interest rates	3-Month Treasury rate	16%	2%	12%	19%
	10-Year Treasury rate	18%	6%	11%	27%

<sup>25</sup> Going forward, we use the terms "correlations of macroeconomic variables with credit risk factors" and "correlations of standard normal macroeconomic factors with credit risk factors" interchangeably. Within the GCorr Macro stress testing framework, we assume the expanded covariance matrix links the standard normal macroeconomic factors to GCorr Corporate factors. However, the expanded covariance matrix was estimated based on the stationary macroeconomic time series, and therefore both terms refer to this matrix.

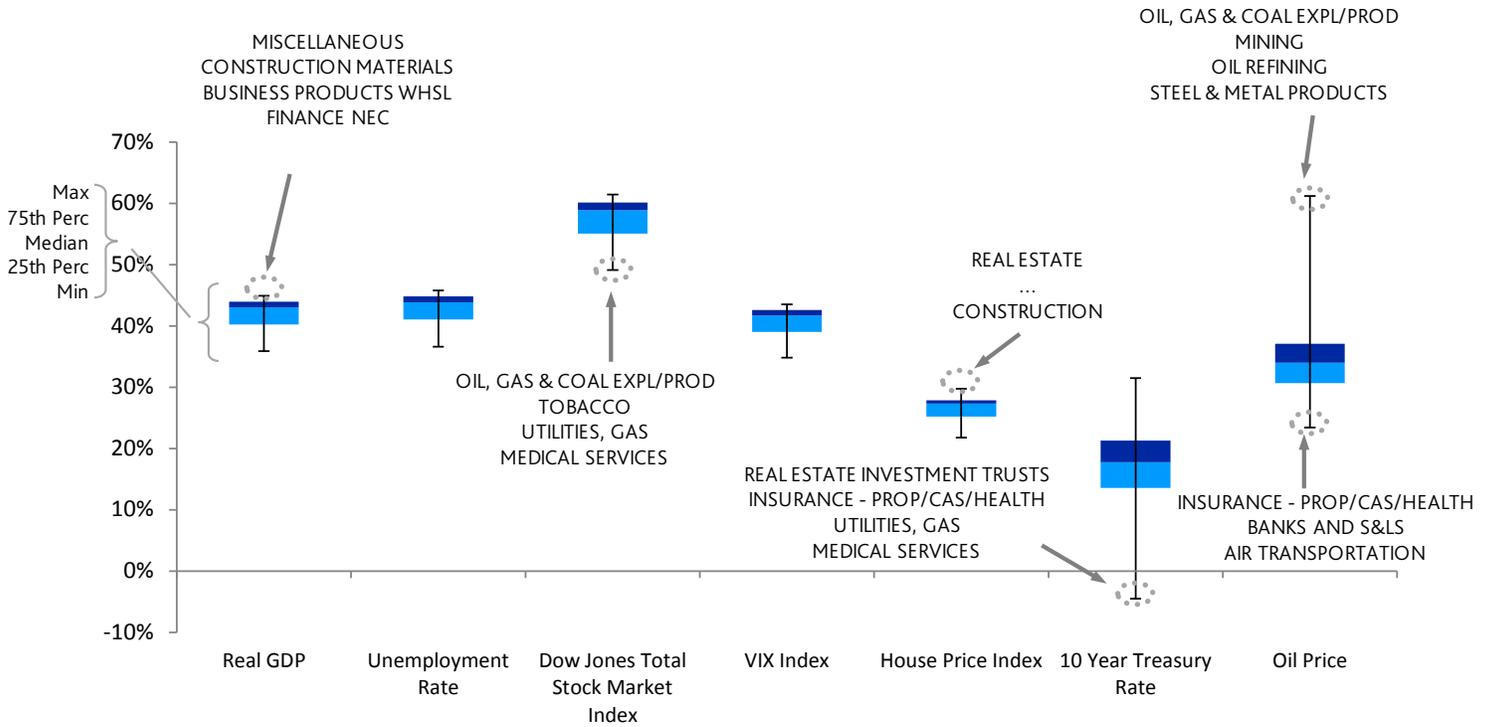
	Mortgage rate	19%	7%	7%	30%
Commodity	Oil price	35%	7%	25%	47%
Consumer prices	Consumer price index	18%	1%	16%	19%
Disposable income	Real personal disposable income	-7%	4%	-13%	-1%
	Nominal personal disposable income	-4%	5%	-10%	6%

The variables in Table 1 are divided into several categories according to their definition and interpretation. As the table indicates, some variable types are more strongly related to the credit risk factors than others. Namely, some economic activity variables (GDP, unemployment rate, etc.) and some financial market variables (stock market index, VIX, corporate spread) exhibit the strongest association with the factors. Both the magnitude and signs of the correlations are consistent with economic intuition.

Other variables have very low correlations with the credit risk factors, such as real and nominal personal disposable income. One reason stems from the fact that disposable income incorporates effects of government policies, such as tax rebates included in past government stimulus packages, which were approved in response to worsening economic conditions in 2001, 2008, and 2009. As a result, disposable income might increase during quarters when the economy deteriorates and credit risk factors experience negative shocks, which leads to the low and negative correlations.

We now discuss the cross-sectional variation in correlations across industries. The range of correlations is given in Table 1. Figure 8 visually displays the variation in correlations for several macroeconomic variables. We observe a high variation for 10 Year Treasury rate and Oil Price. The pattern in relationships between the 10-Year Treasury rate and GCorr custom indexes follows from the definition of asset returns. Interest rate changes are incorporated into asset returns, and their impact is given by the firm's leverage. As a result, the dispersion in leverage across industries leads to the dispersion in the correlations of interest rate changes and asset return-based factors.

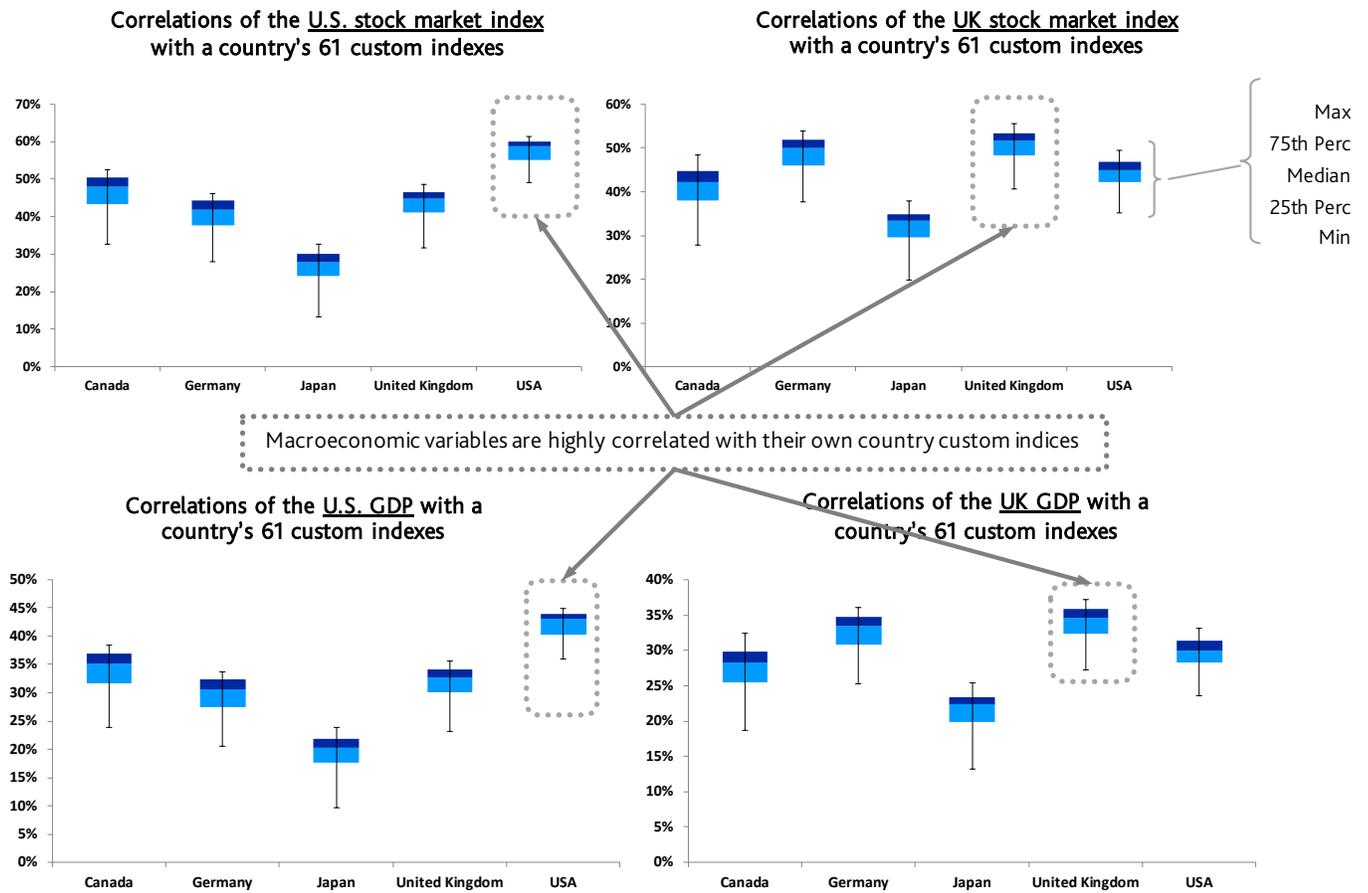
**Figure 8** Cross-sectional variation in correlations of select U.S. macroeconomic variables and GCorr composite factors representing 61 U.S. industries (correlations of Unemployment rate and VIX are scaled by "-1").



The variation in correlations with Oil Price also has an economic interpretation. The Oil, Gas & Coal Expl/Prod and Mining industries, with revenues linked to oil and commodity prices, show the highest correlations with Oil Price. At the other end of the spectrum, we see the Airline industry with low positive, but insignificant, correlation.

Shifting our focus to patterns across countries, in Figure 9, we summarize correlations of two U.S. and UK macroeconomic variables with custom indexes of several countries. The U.S. macroeconomic variables tend to be more closely correlated with U.S. credit risk factors than with other countries' factors, in-line with economic intuition. Moreover, the U.S. macroeconomic variables have a larger impact on, for example, Canadian factors than Japanese or German factors. The UK macroeconomic variables also have high correlations with the UK factors relative to the other countries' factors. Although the figure shows macroeconomic variables for two countries only, we can also draw similar conclusions for other countries.

Figure 9 Cross-sectional variation in correlations of two U.S. and UK macroeconomic variables with GCorr composite indexes representing 61 industries in several countries.

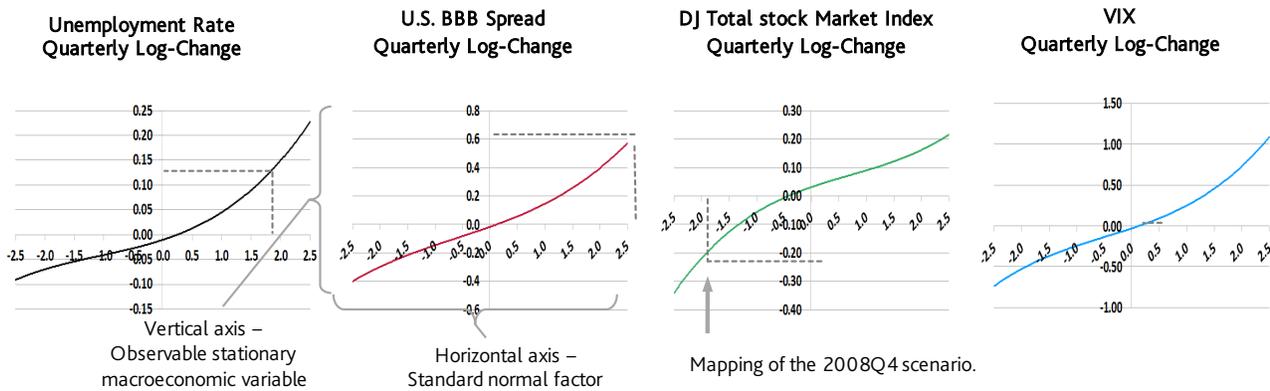


### 5.2 Stressed Distribution of Credit Risk Factors

With the estimated GCorr Macro parameters in place, we can specify the stressed distribution of systematic credit factors, in other words, the conditional distribution given a macroeconomic scenario.

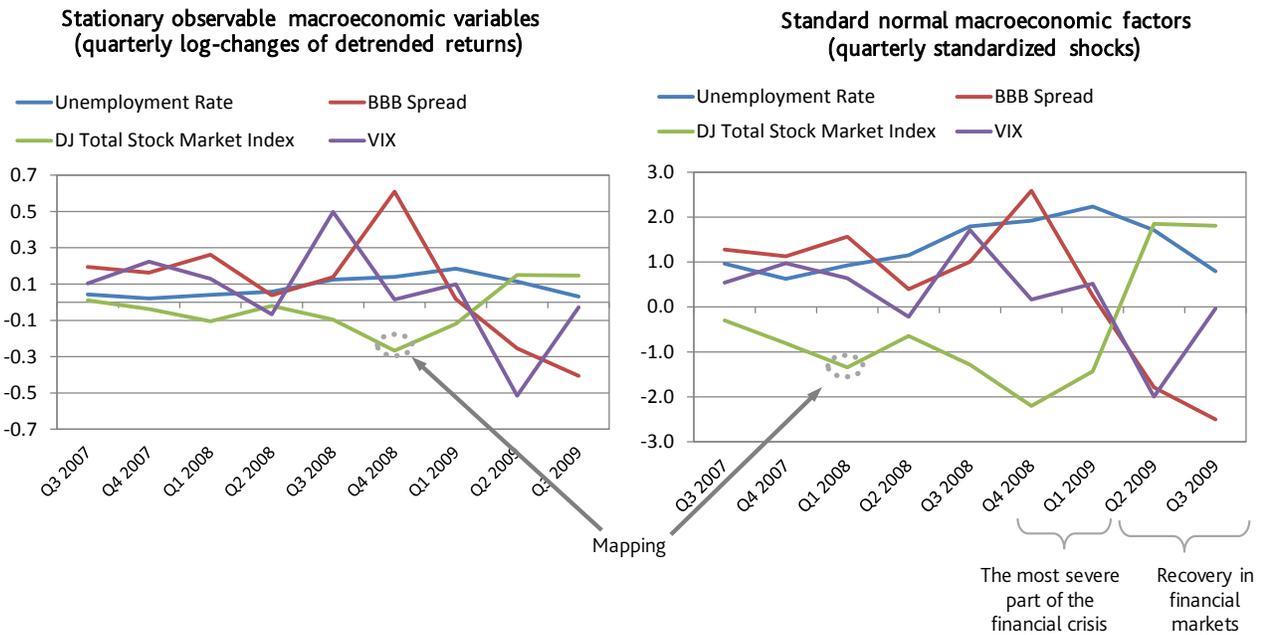
First, we must use the mapping functions to convert the scenario values of stationary macroeconomic variables to the corresponding values of standard normal macroeconomic factors. Figure 10 shows examples of mapping functions for four U.S. macroeconomic variables: Unemployment Rate, U.S. BBB Spread, Dow Jones Total Stock Market Index, and VIX Index. Note, if a scenario is specified, for example, using the unemployment rate level, it must be transformed into the stationary version, in this case, quarterly log-change in unemployment rate. Appendix A lists the transformations.

Figure 10 Mapping function examples.



In the left-hand chart in Figure 11, we plot a historical scenario over period 2007Q3–2009Q3 defined with the four U.S. macroeconomic variables, after the stationarity transformations (quarterly log-changes). The right-hand chart shows the corresponding values of the standard normal factors. Therefore, the vertical axis scale should be interpreted as standard normal distribution values. For example, the worst quarter of the financial crisis, 2008Q4, is mapped to standard normal values of about +2 or -2 for two of the variables.

Figure 11 Mapping an historical scenario representing quarters 2007Q3–2009Q3.



The scenario values of standard normal macroeconomic variables together with the expanded covariance matrix imply the stressed distribution of credit risk factors. Figure 12 shows the credit risk factor representing the U.S. Steel & Metal Products industry and the historical scenario based on the four macroeconomic variables. The stressed expected value of the credit risk factor can be represented as a linear combination of the standard normal macroeconomic factor under the scenario. The coefficients to the macroeconomic variables are derived from the expanded covariance matrix.<sup>26</sup> The left-hand chart in Figure 12 shows coefficients linking the stressed expected value of a credit risk factor to standard normal macroeconomic factors. Based on this chart, we can conclude that the signs of the coefficients are in-line with economic intuition. For example, a rise in the

<sup>26</sup> For more information, see Section 3.

unemployment rate, while keeping the other variables unchanged, will negatively impact the factor's stressed expected value. Comparing the magnitudes of the coefficients, the U.S. Equity variable has the largest impact within this scenario. It is important to realize that the magnitudes also depend on the industry.

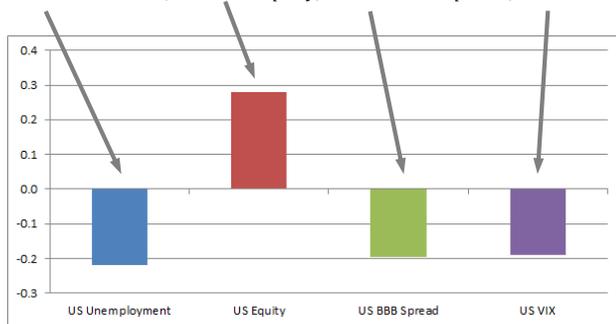
In addition to the coefficients, we are interested in the parameter  $\rho$  which provides information about the explanatory power of the four macroeconomic variables for the U.S. Steel & Metal Products credit risk factor. Statistically, the parameter represents multivariate correlation of the credit risk factor with the standard normal macroeconomic factors. As discussed in Section 3, value  $\rho^2$  has an equivalent interpretation as the R-squared coefficient of a regression of the systematic credit risk factor on the four standard normal macroeconomic factors.

The right-hand chart in Figure 12 shows the path of the credit risk factor's stressed expected value over the period 2007Q3–2009Q3. The standard deviation around those values can be determined as  $\sqrt{1-\rho^2}$  recall that, unconditionally, the factor has a normal distribution with the mean of zero and the standard deviation of one.

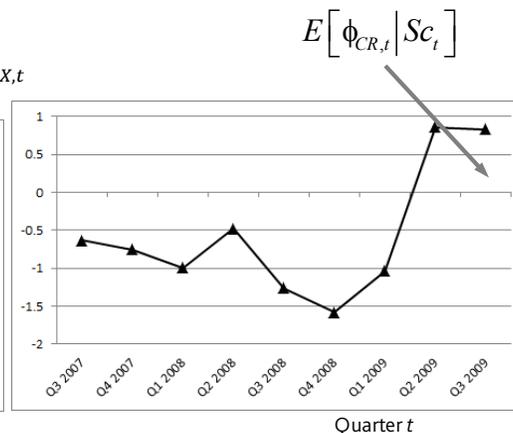
**Figure 12** Stressed expected value of the credit risk factor representing U.S. Steel & Metal Products industry, based on the historical scenario over 2007Q3–2009Q3.

**Coefficients of the stressed expected value of the U.S. Steel & Metal Products credit risk factor to the standard normal macroeconomic factors. The coefficients are implied by the expanded covariance matrix.**

$$E[\phi_{CR,t} | Sc_t] = \beta_1 \phi_{UnemRate,t} + \beta_2 \phi_{Equity,t} + \beta_3 \phi_{BBBSpread,t} + \beta_4 \phi_{VIX,t}$$



**Stressed expected value of the U.S.–Steel & Metal Products credit risk factor for quarter t:**



Parameter  $\rho = 69\%$ : multivariate correlation of  $\phi_{CR}$  with the standard normal macroeconomic factors.

Average  $\rho$  across all U.S. industries = 65%

Standard deviation of the factor under the scenario for each quarter:  $\sqrt{1 - \rho^2} = 73\%$

### 5.3 Variable Selection

To run stress testing analysis for a given portfolio, we first choose the macroeconomic variables to include in the analysis. Note, the set of selected variables should reflect the composition of the credit portfolio. For example, the selected variables for a portfolio of U.S. corporate exposures might be different from those selected for a U.S. retail portfolio or those for a Eurozone corporate portfolio.

The calculation outputs are stressed expected losses that are additive quantities, so exposures can be grouped into portfolios based on the most relevant sets of macroeconomic variables. At the end of the stress testing analysis, the results can be aggregated across portfolios. For example, a loan book containing U.S. SME lending and U.S. consumer loans can be split into U.S. SME portfolio and U.S. consumer loan portfolios because these two portfolios are likely to be driven by different sets of macroeconomic variables. At the end, the stressed expected losses across these portfolios can be aggregated.

In this section, we describe a variable selection procedure which relies on standard regression model techniques in the GCorr Macro context, presented in Section 3. The set of selected macroeconomic variables should meet several criteria:

- >> The set should statistically explain a sufficient portion of variation in the systematic credit risk factors.
- >> The model must be parsimonious, in the sense that it must achieve high explanatory power with as few variables as possible to avoid multicollinearity and reduce noise in parameter estimates.
- >> There should be an economic narrative explaining why the selected variables are relevant for the given portfolio. This includes ensuring that the direction and strength of the relationship between each variable in the model and portfolio losses is in-line with economic intuition.

Our variable selection procedure has three steps:

1. Select a subset of the 91 macroeconomic variables included in GCorr Macro 2015. The idea is to narrow the set of potential candidates from which the final macroeconomic variables will be selected. This subset is chosen based on economic intuition. For example, we should expect U.S. Unemployment or U.S. CRE Index to be potential candidates for the U.S. CRE portfolio while UK Unemployment or Eurozone GDP to be candidates for a UK and Eurozone portfolio, respectively.
2. Identify among the pre-selected set of macroeconomic variables the variables to which the analyzed portfolio is most sensitive. This is done using a univariate style analysis, in which we quantify how the systematic credit risk factor for each instrument in the portfolio is related to each individual macroeconomic variable. In particular, we run the EL calculator with a stress scenario including only that single macroeconomic variable and estimate the variable's portfolio determine the coefficient  $\beta$  (see Equation (7)) for each systematic credit risk factor with respect to each individual macroeconomic variable and then average them across the instruments in the portfolio.<sup>27</sup> This gives us an indication of the strength of the relationship between systematic factors driving the portfolio and each macroeconomic variable. Note, the coefficient is derived from the expanded covariance matrix. From the expanded covariance matrix, we can also calculate a t-statistic for each coefficient (Equation (9)) and average the t-statistics across instruments to assess whether the relationship between the portfolio and a macroeconomic variable is statistically significant. Using the t-statistic, we discard all the macroeconomic variables that are not significant<sup>28</sup> together with those that have a coefficient with an economically unintuitive sign. During the variable selection procedure, we must ensure that relationships between the macroeconomic variables and systematic credit risk factors are economically meaningful. For example, Unemployment Rate should have negative sign because if the Unemployment Rate increases, the systematic factor return should be negative. Similarly, GDP should have positive sign because the systematic factor return should be positive if GDP grows. For some variables, one may not have a prior assumption on the direction of relationship (for example, for Oil Price).
3. Calculate different models combining the macroeconomic variables that passed the second step. In particular, we consider all possible combinations of three to five macroeconomic variables. For each combination, we determine coefficients of the instruments' systematic credit risk factors to the macroeconomic factors included in the model, the corresponding t-statistics and the adjusted pseudo R-squared value. Then we average<sup>29</sup> the coefficients, t-statistics, and R-squared values across instruments to obtain portfolio level quantities. Of all the candidate models, we exclude those that fail at least one of the following two tests:
  1. At least one estimated coefficient in the model is insignificant according to its t-statistic.<sup>30</sup> This restricts the number of variables in the model to only the ones that contribute to explaining variation in the credit risk factors, and, thus, keeps the model parsimonious.
  2. At least one coefficient has an unintuitive sign. This eliminates models with economically unintuitive relationships.

We rank the models that pass the two criteria according to their explanatory power measured by their adjusted pseudo R-squared. The adjusted pseudo R-squared captures the trade-off between how well the model fits and the number of parameters estimated.

However, the statistical measures (selecting the model with the highest adjusted pseudo R-squared) does not have to be the only criterion to select the best model. It is also important to include economic considerations. For example, if several models pass the

<sup>27</sup> Specifically, we calculate weighted average of the instrument level coefficients, where the weights are instruments' exposures at default.

<sup>28</sup> Statistical significance is determined by performing t-test.

<sup>29</sup> These are weighted averages, where the weights are given by instruments exposures at default.

<sup>30</sup> The t-test is carried out in the same way as in the univariate analysis.

tests and have all high-adjusted pseudo R-squared value, it might make sense to select the one that offers the most compelling economic narrative even if it does not have the highest explanatory power among the models.

The previous step focused only on models with up to five variables. If the best model from Step 2 contains exactly five variables, we must test whether including an additional variable leads to a model which passes our two tests: significant coefficients and intuitive signs of all coefficients. If not, the best model from the second step is considered the final model. If yes, we should repeat the test adding another variable.

Table 2 presents the final sets of selected macroeconomic variables for each portfolio analyzed in this paper. We perform the selection based on the variable selection procedure described in this section. For each portfolio, we considered three criteria when deciding on the final models: adjusted pseudo R-squared, economic narrative, and backtesting performance. For more information about these criteria, see Section 7.

Table 2

**Selected Macroeconomic Variables**

U.S. LARGE CORPORATES AND SME	U.S. CRE PORTFOLIO	EUROZONE LARGE CORPORATES PORTFOLIO	JAPAN LARGE CORPORATES PORTFOLIO
U.S. Unemployment Rate	U.S. Real GDP	Euro Area Equity	Japan Real GDP
U.S. Dow Jones Total Stock Market Index	U.S. Dow Jones Total Stock Market Index	Eurozone Spread	Japan Equity Index
U.S. Market Volatility Index(VIX)	U.S. CRE Price Index	Eurozone GDP	
U.S. BBB Spread			

Table 3 provides detailed examples of the top macroeconomic models for the U.S. large corporates and U.S. SME portfolios, ranked by adjusted pseudo R-squared that passed the variable selection procedure.<sup>31</sup> We include the coefficients and the t-statistics. The results for other portfolios are presented in Table 11 in Appendix C.

Table 3

**Examples of Top U.S. Macroeconomic Models for U.S. Large Corporates and SME Portfolios After Variable Selection**

MODEL #	MACRO VARIABLE 1	MACRO VARIABLE 2	MACRO VARIABLE 3	MACRO VARIABLE 4	COEF 1 (T-STAT)	COEF 2 (T-STAT)	COEF 3 (T-STAT)	COEF 4 (T-STAT)	ADJUSTED R-SQUARED
1	U.S. Unemployment	U.S. Equity	U.S. VIX	U.S. BBB Spread	-0.220 (-2.074)	0.281 (2.172)	-0.191 (-1.764)	-0.196 (-1.724)	38.0%
2	U.S. Unemployment	U.S. BAA Yield	U.S. Equity	U.S. VIX	-0.2166 (-2.040)	-0.166 (-1.710)	0.369 (3.131)	-0.196 (-1.812)	38.0%
3	U.S. Unemployment	U.S. Equity	U.S. VIX	U.S. Corporate Profits	-0.224 (-2.113)	0.358 (3.013)	-0.190 (-1.748)	0.147 (1.502)	37.6%
4	U.S. Unemployment	U.S. BAA Yield	U.S. Equity	U.S. Industrial Production	-0.170 (-1.573)	-0.147 (-1.499)	0.442 (4.162)	0.158 (1.567)	37.6%
5	U.S. Equity	U.S. VIX	U.S. Industrial Production	U.S. BBB Spread	0.337 (2.703)	-0.183 (-1.683)	0.185 (1.836)	-0.179 (-1.535)	37.5%

<sup>31</sup> In this study, we focus on BBB Spread instead of Yield due to its superior forecasting capabilities in describing variation in default probabilities and expected losses. Moreover, Spread is the component of the Yield most closely related to default risk, unlike the risk-free interest rate.

Finally, in general the value of the adjusted pseudo R-squared does not typically exceed 40%, which means that a variation in the custom indexes is not completely explained by the macroeconomic variables we consider. Comparing the nature of GCorr factors and the macroeconomic variables, this result is economically intuitive. For example, the GCorr Corporate factors are latent factors constructed to represent systematic credit risk of countries and industries. However, the macroeconomic variables in the examples are economy-wide indicators which cannot explain industry effects.

#### 5.4 Stressed Credit Parameters

As the equations in Section 2 and Section 3 show, the values of the stressed credit risk parameters depend on both the stressed credit risk factor distribution and the input unconditional parameters. We illustrate this point with two examples, shown in Figure 13 and Figure 14.

In Figure 13, we plotted the stressed PD as a function of the unconditional PD for different asset R-squared values. We assume an adverse economic shock which translates into the stressed custom index expected value of  $-2$ . The explanatory power of the macroeconomic variables is given by  $\rho = 75\%$ . The figure shows that both the PD and the R-squared value strongly impact the value of the stressed PD. In terms of direction, the stressed PD is an increasing function of the unconditional PD and of the R-squared value.

Figure 13 highlights the point that unconditional PD is not the only parameter that determines stressed PD. In addition, we need to know the counterparty's asset R-squared value and its custom index, which is given by its geographical location and sector. This information provides additional granularity that allows the model to further differentiate borrowers with the same unconditional PD level.

**Figure 13** Impact of unconditional PD and asset R-squared value on the stressed PD over a single period.

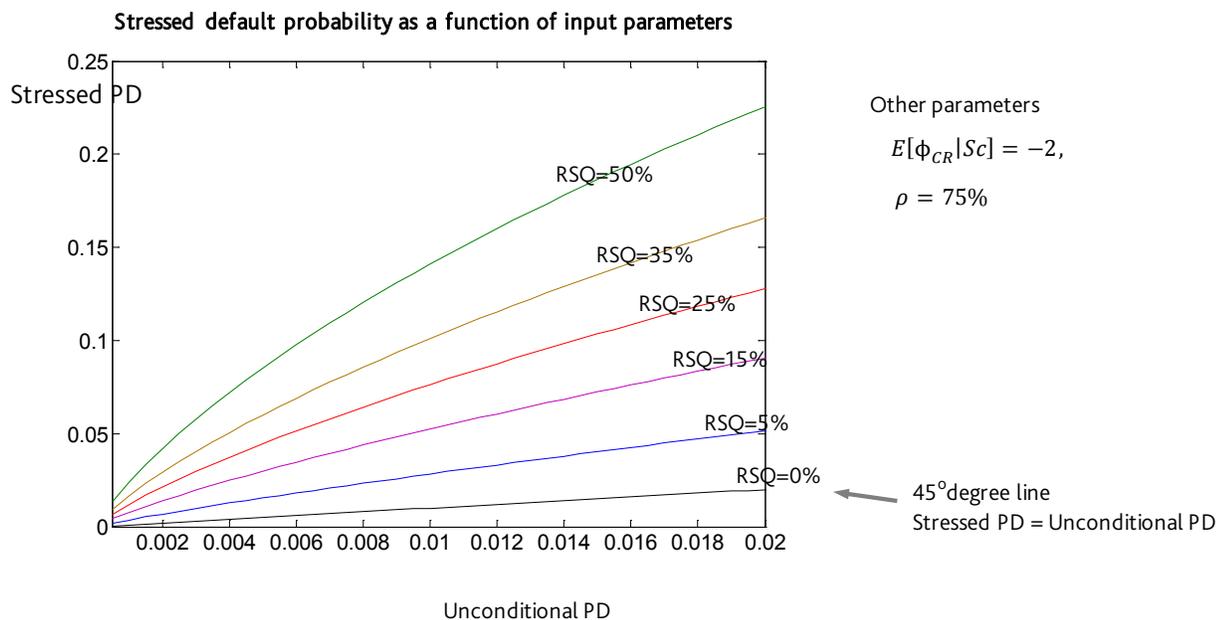
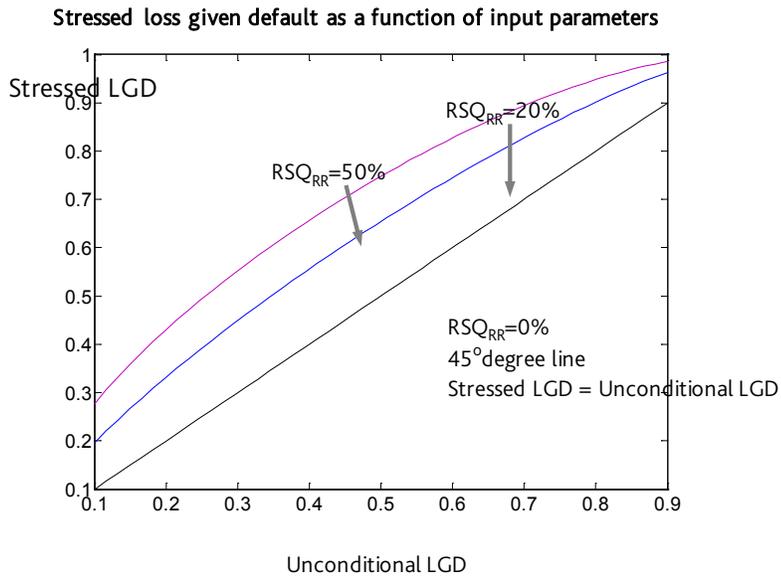


Figure 14 shows the dependence of stressed LGD on the unconditional LGD and the recovery R-squared value. Stressed LGD also depends on other parameters, namely, the unconditional PD, asset value R-squared, correlation of asset return and recovery return, and variance parameter of the unconditional LGD distribution. We assume the same stressed credit risk factor distribution as in the previous example. For the parameters considered here, the stressed LGD is an increasing function of the recover R-squared.<sup>32</sup>

<sup>32</sup> For certain input parameter combinations, stressed LGD may become a decreasing function of recovery R-squared.

Figure 14 Impact of unconditional LGD and recovery R-squared value on the stressed LGD value over a single period.



Other parameters

$$E[\phi_{CR}|Sc] = -2, \rho = 75\%$$

$$\rho_{A,RR} = \sqrt{RSQ \times RSQ_{RR}}$$

$$PD = 1\%, RSQ = 10\%, k = 4$$

## 6. Realigning Stressed Expected Losses

GCorr Macro calculates quarterly stressed expected losses given a specific macroeconomic scenario in that quarter. In reality, the defaults caused by macroeconomic shocks are usually realized over several quarters instead of within one quarter. For this reason, GCorr Macro applies a smoothing function to realign the losses predicted by the macroeconomic shock in one quarter over several quarters. Specifically, for four different asset classes, we estimate a smoothing function so that the resulting losses show the same time series patterns as historically observed losses. The four asset classes covered are U.S. Large Corporates, U.S. Small-and Medium-Sized Enterprises, U.S. Commercial Real Estate (CRE), and U.S. Residential Mortgages.

### 6.1 Calibration

We calibrate the smoothing function so that the stressed expected losses produced by GCorr Macro under historical scenarios match the time series dynamics of the historical default rate. During the calibration process, we assume that LGD is 100%, and, therefore, time series movements in expected losses are driven by PD. The calibration process begins by using GCorr Macro to project losses for the next nine quarters, beginning 2006Q1. We repeat this process for each quarter through 2010Q1. We then run a panel regression to determine the proper weights (coefficients) so that the smoothed expected loss most closely matches the historical default rate.

We define the smoothed quarterly stressed PD as a weighted average of the quarterly stressed PD values in the same quarter and the previous  $N^{Smooth} - 1$  quarters:

$$PDq_{i,t}^{Smooth}(Sc_{1,t}^{Cumul}) = c_{PD}^{Smooth} \times \left( \sum_{k=0}^{N^{Smooth}-1} w_k^{Smooth} \times PDq_{i,t-k}(Sc_{1,t-k}^{Cumul}) + w_* \right), t = 1, \dots, T$$

Here,  $w_k^{Smooth}$  is the weight to the  $k^{\text{th}}$  lagged quarterly stressed PD and  $w_*$  is a constant term added to all the quarters. The calibration of the weights is described in the next section.  $N^{Smooth}$  represents the number of quarters to use in the weighted average and is set to 4 for the exercises in this paper. Finally,  $c_{PD}^{Smooth}$  is a scaling factor that makes the smoothed stressed cumulative PD over  $T$  quarters equal to the original stressed cumulative PD:

$$c_{PD}^{Smooth} = \frac{\sum_{t=1}^T PDq_{i,t}(Sc_{1,t}^{Cumul})}{\sum_{t=1}^T \left( \sum_{k=0}^{N^{Smooth}-1} w_k^{Smooth} \times PDq_{i,t-k}(Sc_{1,t-k}^{Cumul}) + w_* \right)}$$

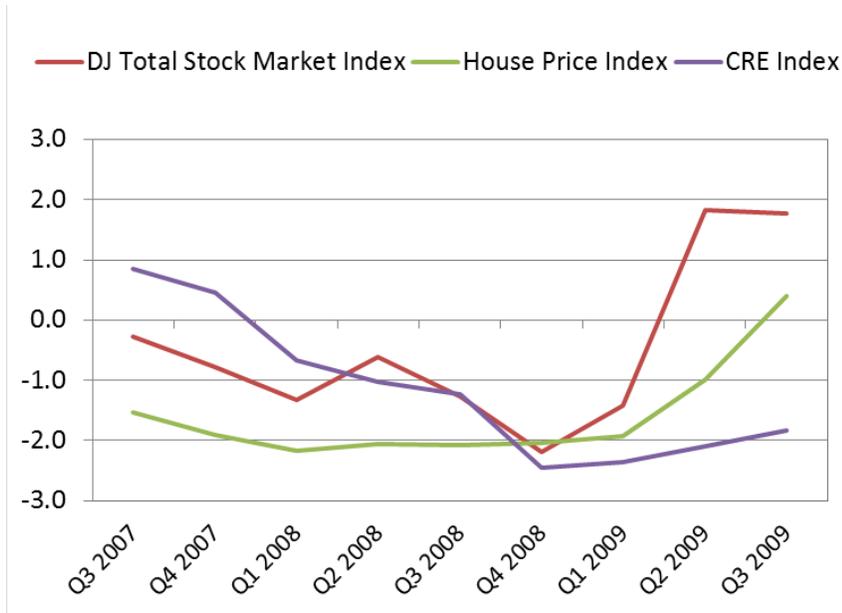
The unconditional PD over the first quarter is used as  $PDq_{i,t}(Sc_{1,t}^{Cumul})$  for  $t < 0$ .

We run a panel regression across all the windows (from 2006Q1–2010Q1) to fit the smoothed stressed PD to the observed default rate and obtain the weights  $w_k^{Smooth}$  and  $w_*$ .

We also smooth the stressed LGD using the same weights as we do for PD. The smoothed LGD values are rescaled so that the nine-quarter smoothed cumulative stressed expected loss equals the cumulative loss before smoothing.

There is a separate calibration for each asset class, because the losses in each asset class can be driven by different macroeconomic variables. For example, the CRE index has a stronger relationship with the losses in a CRE portfolio, while the Dow Jones has a stronger relationship with a corporate portfolio. Figure 15 shows the market shocks of several variables through the financial crisis in standard normal space.

Figure 15 Market shocks of selected macroeconomic variables.

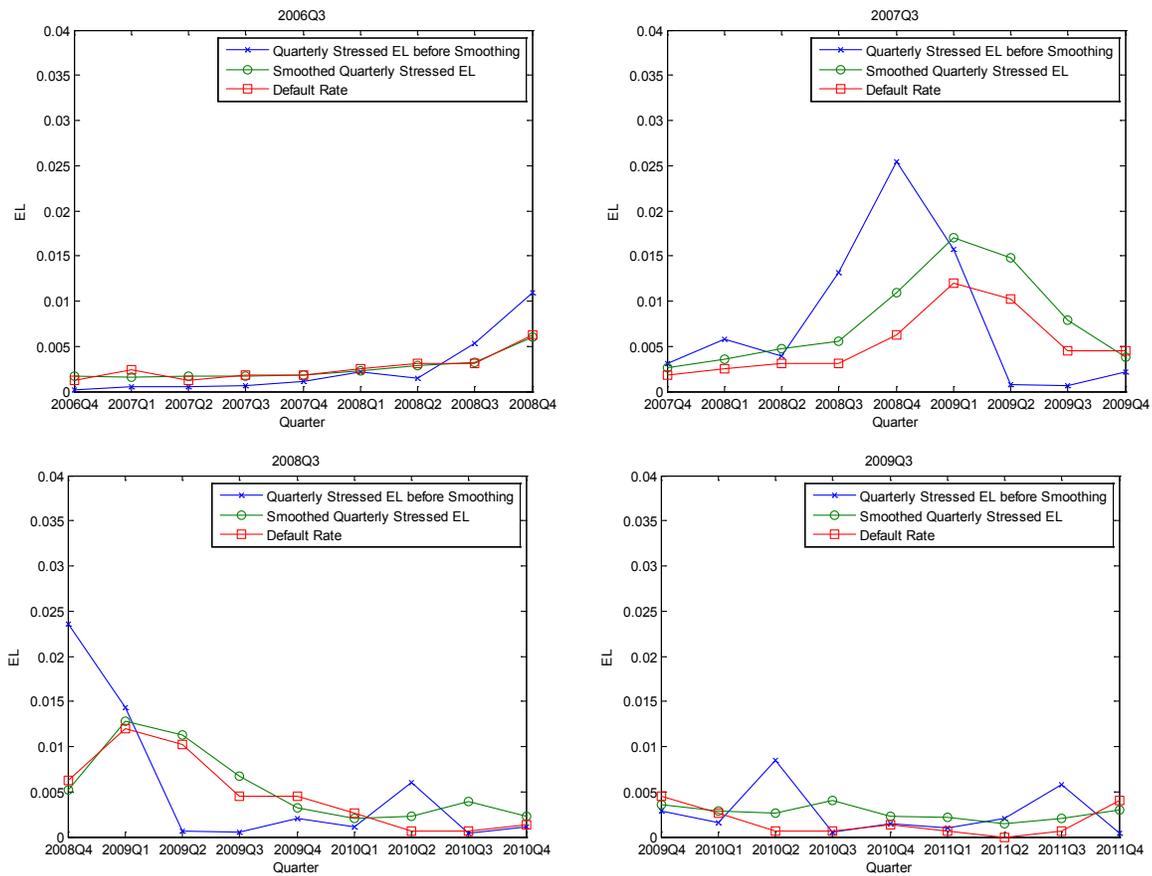


The CRE index shows a series of increasingly severe shocks followed by a gradual recovery, whereas, the house price index declined in the quarters before the crisis, followed by a quicker recovery. Both variables behave differently than the Dow Jones, which is a major driver in corporate losses. The realized default rates also have different peaks and patterns, so it is important to estimate a different set of coefficients for each asset class.

## 6.2 Validation

To validate the smoothing coefficients, we compare the smoothed stressed expected losses with the default rate for various windows. For all four asset classes, we observe the comparison in periods before the crisis, during the crisis, and after the crisis. Figure 16 shows this comparison for the large corporates portfolio. The blue line indicates the quarterly stressed expected loss resulting from GCorr Macro with no smoothing applied. The green line represents the smoothed losses, and the red line is the benchmark default rate. The top-left plot refers to the pre-crisis period, the top-right and bottom-left plots refer to the financial crisis, and the bottom-right plot refers to the post-crisis period.

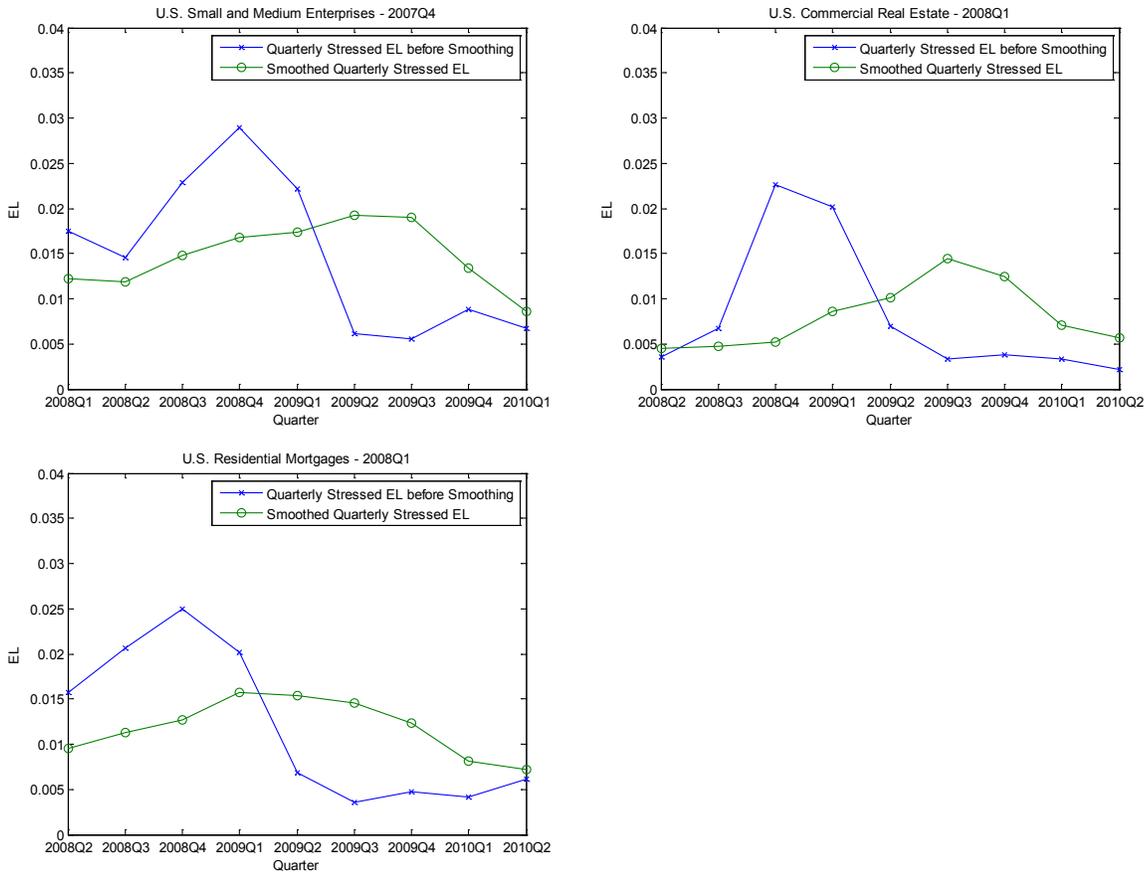
Figure 16 Validation of loss realignment for Large Corporates (analysis dates are specified at the top of each chart).



In each window, the stressed loss is a leading indicator of the default rate. After smoothing, the losses have similar time series dynamics as the default rate. The smoothed losses match the default rate well in all economic environments.

For the other asset classes, the default rates are proprietary, so we may show only the stressed expected losses before and after smoothing. Figure 17 compares the quarterly stressed losses before and after smoothing for the U.S. Small- and Medium-Sized Enterprises portfolio, U.S. Commercial Real Estate (CRE) portfolio, and U.S. Residential Mortgages portfolio. Analysis dates are specified at the top of each chart.

Figure 17 Smoothed Quarterly Stressed EL for U.S. Small- and Medium-Sized Enterprises, U.S. Commercial Real Estate, and U.S. Residential Mortgages.



Similar to the large corporates portfolio, we find that stressed losses are a leading indicator of the default rate and, once smoothed, they have similar time series dynamics as the default rate.

## 7. Validation of GCorr Macro with Historical Scenarios

This section presents several analyses illustrating levels and patterns in credit portfolio losses produced by GCorr Macro over recent economic episodes. Our objective is to assess how the losses and stressed probability of default compare to various benchmarks (in other words, we conduct backtesting) and to understand how different aspects of the modeling framework impact the losses. This type of analysis contributes to the process of GCorr Macro validation.

For this analysis, we must first select the appropriate set of macroeconomic variables for a given sample portfolio. We refer to this set of variables as "model." The selected model and its performance for historical scenarios are portfolio-specific, so the analysis is carried out for nine sample portfolios across various regions and asset classes (U.S. Large Corporates, U.S. SME, U.S. Commercial Real Estate, Eurozone Large Corporates and Japan Large Corporates). For analysis on U.S. Retail portfolios, see "Understanding GCorr 2015 Retail," Huang, et al.

We organize this section as follows:

Section 7.1 presents the results for the U.S. Large Corporates and SME portfolios.

Section 7.2 presents the results for several international portfolios (Eurozone Large Corporates and Japan Large Corporates).

Section 7.3 presents the results for U.S. Commercial Real Estate.

### 7.1 U.S. Large Corporate and SME Portfolios

We use two stylized credit portfolios to validate GCorr Macro for U.S. Corporates: a portfolio of exposures to U.S. large listed corporates (U.S. Large Corporates portfolio) and a portfolio of exposures to U.S. Small- and Medium-sized Enterprises (U.S. SMEs portfolio). Table 4 summarizes portfolio characteristics.

Table 4

#### Stylized Portfolios Used for Validation

PORTFOLIO	U.S. SME PORTFOLIO	U.S. LARGE CORPORATES PORTFOLIO
Types of Counterparties	U.S. Small- and Medium-sized Enterprises (non-financial)	U.S. large listed corporates (firms constituting 99% of total liabilities issued by listed firms)
Exposure Pooling	130 pools of loans Loans are pooled by 13 U.S. sectors and 10 risk levels <sup>33</sup>	61 pools of loans Loans are pooled by 61 GCorr industries <sup>34</sup>
R-squared	Weighted average R-squared = 6.1% <sup>35</sup>	Weighted average R-squared = 31.6% <sup>36</sup>
Probability of Default	two cases - Time varying PD <sup>37</sup> - Fixed PD: <sup>38</sup> Weighted average PD = 2.03% (annualized)	two cases - Time varying PD <sup>39</sup>
Loss Given Default for Projections	- LGD = 50% <sup>40</sup>	two cases - LGD = 100% (used for PD benchmarking) - LGD = 40% (used for projections)

<sup>33</sup> Pool weights proportional to the firm counts by the U.S. sector/risk level categories in the CRD database.

<sup>34</sup> Pool weights proportional to the large firm counts by GCorr industries in GCorr 2015 Corporate.

<sup>35</sup> Source: U.S. SME correlation model, R-squared values by sectors.

<sup>36</sup> Source: GCorr 2015 Corporate, large firm average R-squared values by industries.

<sup>37</sup> Average pool level RiskCalc U.S. CCA EDF. The time varying PD is used for back-testing.

<sup>38</sup> This level of PD is used for loss projections and one back-testing exercise. The pool-level PD and in turn the weighted average PD match the corresponding values used for a portfolio in the paper "Stress Testing Probability of Default for Private Firm C&I Portfolios: RiskCalc Plus Stress Testing PD and LGD Model (granular approach) – United States v4.0 Corporate Model" by Chen, et al., 2014. The paper introduces a stress testing methodology, Moody's Analytics Stressed PD Model, which we use as one of the benchmarks for GCorr Macro as we explain in Section 7.2. The PD values used in that paper are RiskCalc CCA EDF levels from 2011 Q3, the date of the CCAR 2012 exercise. It is worth noting that the CCA EDF levels in 2011 were comparable to the pre-crisis levels in 2006–2007.

<sup>39</sup> Average pool level CreditEdge U.S. EDF. The time-varying PD is used for backtesting.

<sup>40</sup> The LGD of 50% matches the LGD from the paper "Stress Testing Probability of Default for Private Firm C&I Portfolios: RiskCalc Plus Stress Testing PD and LGD Model (granular approach) – United States v4.0 Corporate Model" by Chen, et al., 2014, which we use as one of the benchmarks for GCorr Macro.

All calculations are based on the GCorr Macro model, estimated by expanding GCorr 2015, explained in Section 3. In the analyses where we stress LGD, the LGD variance is parameterized with  $k=4$ . Furthermore, we set the PD-LGD correlation parameters to  $RSQ_{RR}=34\%$  and  $\rho_{A,RR}=33\%$ . In case of unstressed LGD analyses, we fix the LGD level either at 40% in case of U.S. Large Corporates, or 50% for the U.S. SME portfolio.

We start this section with a discussion of the top macroeconomic models for U.S. large corporate and SME portfolios ranked by adjusted pseudo R-squared.<sup>41</sup> Next, we analyze the historical fluctuations of U.S. EDF rate. This is followed by performing two types of validation exercises. First, we determine GCorr Macro stressed expected losses based on historical economic episodes over 2001–2015. We want to find out how the losses vary across the economic episodes within this period. The second part of the validation analysis focuses on the levels of stressed expected losses projected by GCorr Macro and compares them to losses provided by various benchmarks. When summarizing results, we report nine quarter cumulative losses which correspond to the time horizon considered by the CCAR document.

#### VARIABLE SELECTION RESULTS

Before we begin the validation exercises, we must select the models with the highest explanatory power. To do so, we conduct a univariate statistical analysis of the economically relevant macro variables. We discard all macro variables with factor coefficients that are either insignificant or have an unintuitive sign. The remaining macro variables will be used in a multivariate analysis. Again, we remove models with unintuitive signs or insignificant coefficients (using 10% significance level) and rank the models that pass these tests according to their adjusted pseudo R-squared.

In Section 5.3, we provide a more detailed description of the selection procedure and an overview of the top macroeconomic models for the U.S. Large Corporates and U.S. SME portfolios, ranked by adjusted pseudo R-squared that passed the variable selection procedure.<sup>42</sup>

#### TIME SERIES PATTERNS IN STRESSED EXPECTED LOSSES

In the first set of validation exercises, we compare stressed expected losses produced by GCorr Macro across various economic episodes. In particular, we focus on the levels of losses during the recent financial crisis.

We consider the period 2001–2015, which includes four distinct episodes:

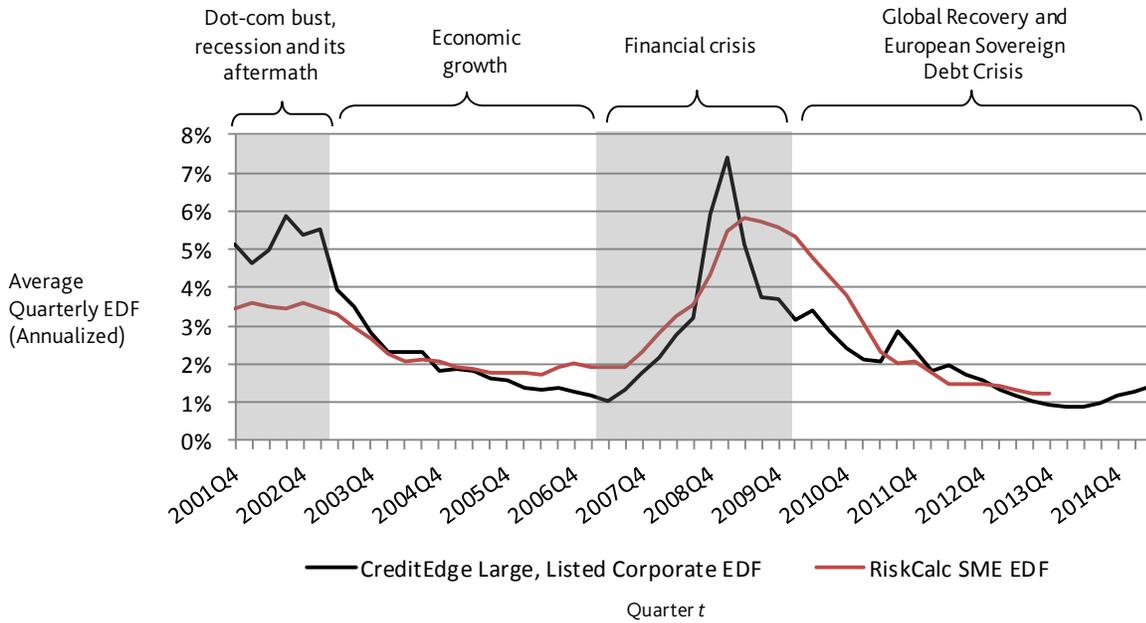
- >> Dot-com bust, the recession of 2001, and its aftermath
- >> Period of economic growth, approximately mid-2003–mid-2007
- >> 2008–2009 financial crisis
- >> Global recovery and Eurozone sovereign debt crisis of 2010–2015

Figure 18 displays these economic episodes using the average CreditEdge™ benchmark EDF for U.S. large public firms and the average RiskCalc™ EDF value for the U.S. SME portfolios. EDF values are high during periods of economic distress, while they drop to low levels during periods of economic growth. Comparing both measures, we see that the overall variability and the spikes during economic downturns are far more pronounced for CreditEdge EDF measures of U.S. large corporates.

<sup>41</sup> The concept of adjusted, pseudo R-squared for variable selection is introduced in Section 3.

<sup>42</sup> In this study, we focus on Baa Spread instead of Baa Yield due to its superior forecasting capabilities in describing variation in default probabilities and expected losses.

**Figure 18** Average quarterly CreditEdge benchmark EDF value for U.S. large public firms and average RiskCalc EDF value for U.S. SME firms.



Our first goal is to understand the sensitivity of the expected losses of U.S. SME and U.S. Large Corporate portfolios to changing macroeconomic conditions. In Figure 19 and Figure 20, we plot the nine-quarter cumulative stressed expected losses estimated with GCorr Macro starting from 2001, while using an unconditional PD term structure. Controlling for fluctuations in unconditional PD values allows us to isolate the effect on expected losses that is purely coming from the change in the historical macroeconomic environment. The scenarios are defined using historical values of the macroeconomic variables from U.S. Unemployment, U.S. Equity, U.S. VIX, and U.S. BBB Spread. The portfolio characteristics remain the same across the period.

**Figure 19** Cumulative nine-quarter expected losses, unconditional and stressed, for the U.S. SME portfolio—flat PD term structure and stressed LGD in stress scenario.

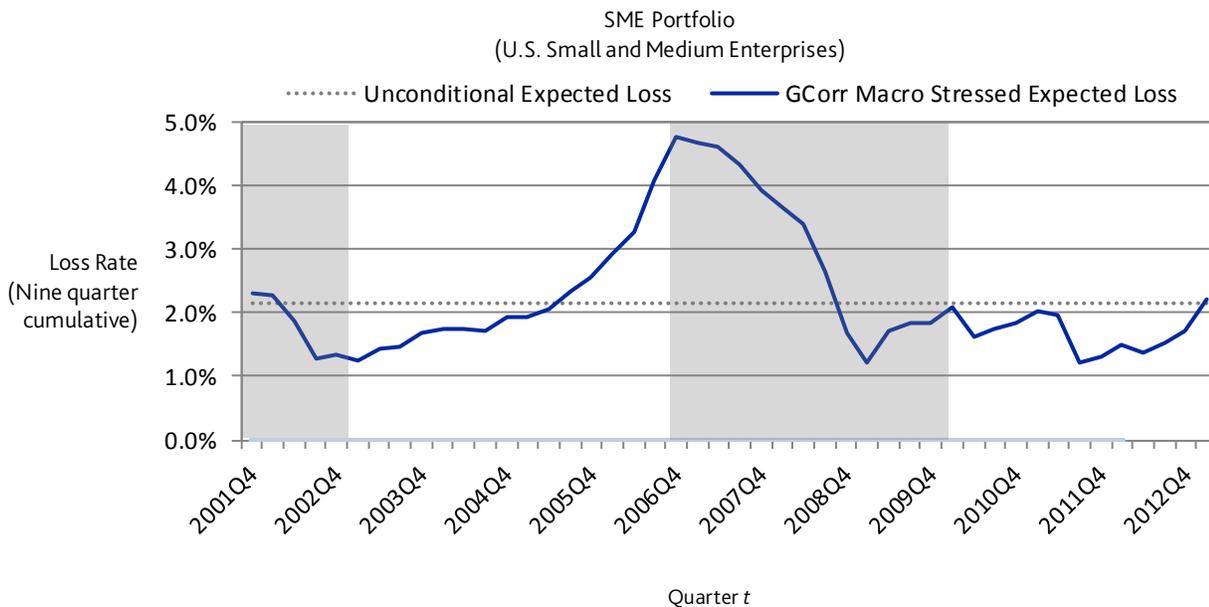


Figure 20 Cumulative nine-quarter expected losses, unconditional and stressed, for the large U.S. Large Corporates portfolio — flat PD term structure and stressed LGD in stress scenario.

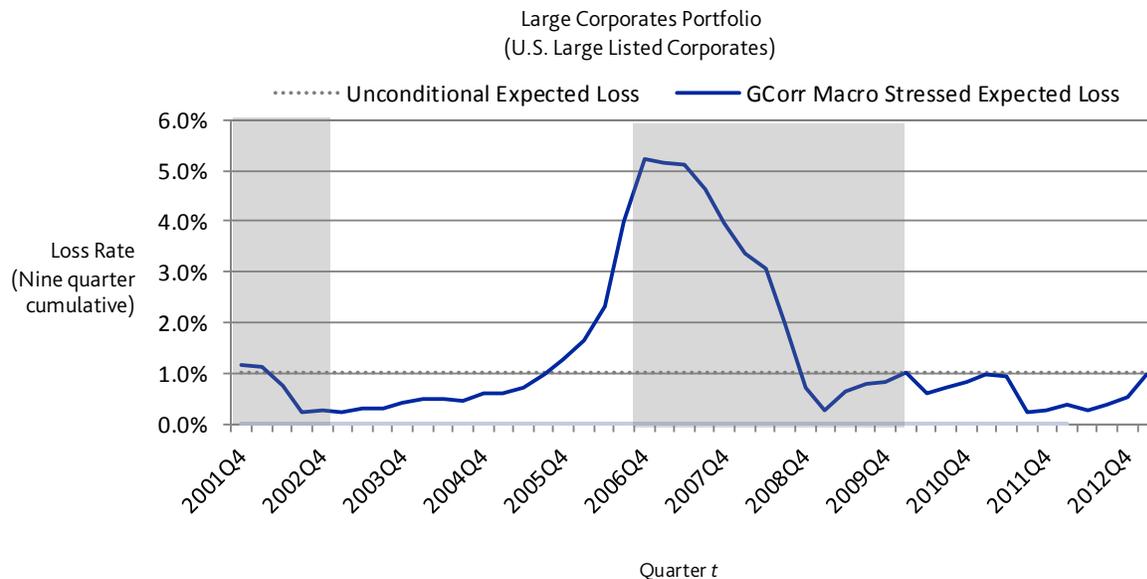


Figure 19 and Figure 20 show similar patterns, in terms of time series dynamics:

- >> Losses are higher during periods of economic distress and lower during the period of economic growth.
- >> Note, GCorr Macro produces high stressed credit parameters and expected losses for the exactly those nine-quarter periods when the scenario assumes most negative shocks to the macroeconomic variables. As a result, the series in Figure 19 and Figure 20 peak for the nine quarter period 2007Q1–2009Q1.
- >> The stressed expected losses are higher during the recent financial crisis than during the early 2000s recession. We attribute this result to macroeconomic variable dynamics during these two episodes. As an example, we show time series of log changes in the Unemployment Rate and the Dow Jones Total Stock Market Index in Figure 21. Both variables, but especially Unemployment Rate, experienced larger quarterly shocks during the recent financial crisis. This also applies to the other two variables in the scenario: U.S. BBB Spread and U.S. VIX.

Figure 21 Quarterly log changes in Unemployment Rate, Dow Jones Total Stock Market Index, and BBB Spread.

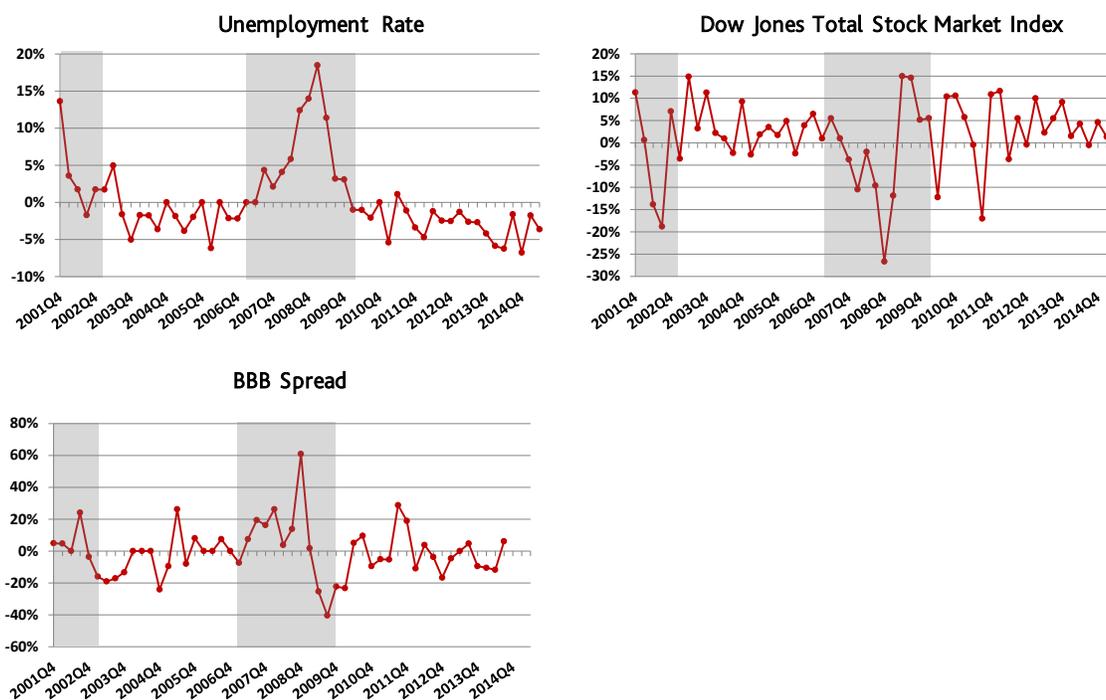


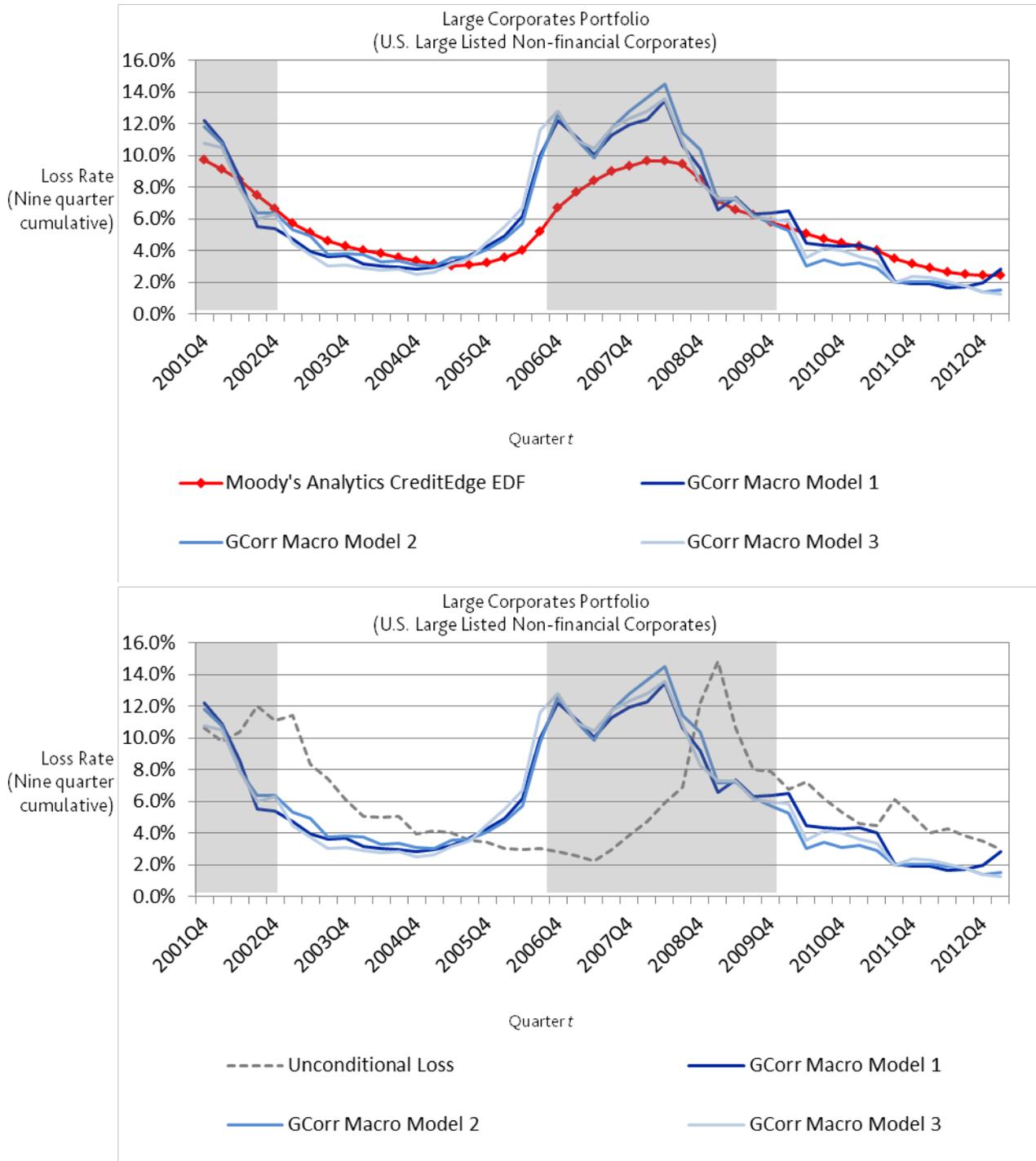
Figure 19 and Figure 20 also indicate that the time series of losses for the Large Corporate portfolio fluctuate more than for the SME portfolio. Specifically, the stressed expected losses on the large portfolio became about five times larger than the unconditional expected losses during the financial crisis. For the SME portfolio, they were about two times larger. We can attribute this difference to the difference in R-squared values of the portfolios. The U.S. large corporates have substantially higher average R-squared values than the SMEs, 31.6% and 6.1%, respectively, which implies that economic distress of a given magnitude will have a larger impact on U.S. Large Corporates.

Next, we evaluate historical performance of GCorr Macro and the selected models for U.S. Large Corporates and SMEs, conducting a backtesting exercise. In this exercise, we study how well the predicted model results match up with the historical behavior of certain benchmarks. The aim is to validate whether our model can explain observed historical movements in stressed expected losses and probability of default proxies in a plausible way.

#### BENCHMARKING FOR U.S. LARGE CORPORATES PORTFOLIO

Beginning with the U.S. Large Corporates portfolio, Figure 22 shows the backtesting results of the stressed probability of default. Here, we focus on three of the models in Table 3 and compare them to the historical movement of the nine-quarter average CreditEdge benchmark EDF values for large U.S. public firms. We obtain the GCorr Macro stressed PD by computing the stressed expected loss with a constant LGD of 100% and a time-varying unconditional (input) PD for each of the portfolio's loan pools. We determine the unconditional PD for each nine-quarter period using EDF values as of the beginning of that period. The Moody's Analytics CreditEdge EDF value is used as the benchmark for these stressed PD values. It is constructed by computing for each quarter the average EDF value for the sample's U.S. firms. Afterward, for each point in time, we cumulate losses over the next nine quarters.

Figure 22 Backtesting GCorr Macro Stressed Expected Loss from top U.S. Models 1, 2, and 3 of U.S. large corporate portfolio — time varying input PD and LGD=100%<sup>43</sup>



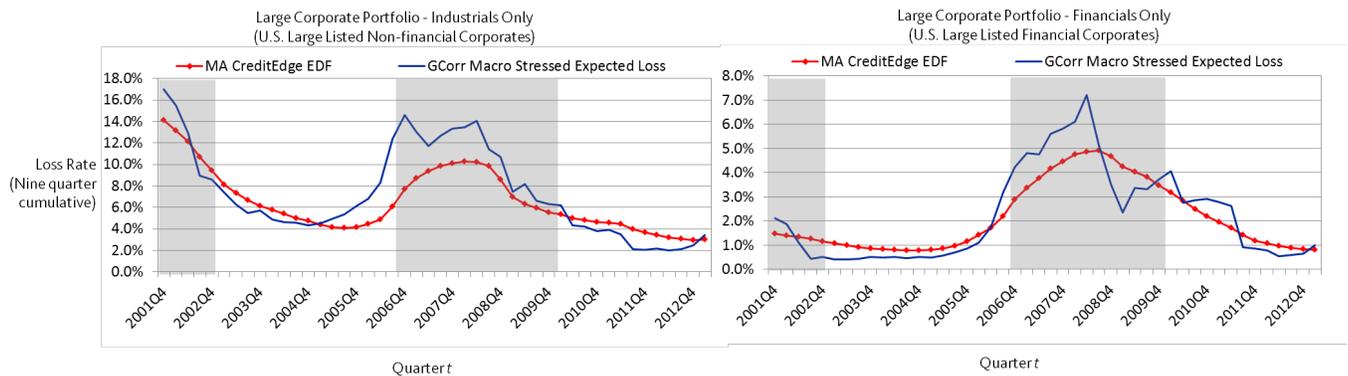
Assessing Figure 22, we see that the overall time series patterns of the benchmark are matched by the top-three GCorr macro models. Two periods of high stressed PD, in 2001 and 2007–2008, are observable for the GCorr Macro models, with the first one

<sup>43</sup> The models selected include the top U.S. model by adjusted pseudo RSQ and the next best two models by adjusted pseudo RSQ that include new macro variables not contained in the top model. We ignore, hereby, models that contain both Unemployment Rate and U.S. GDP, due to capturing similar underlying economic relationships.

arising from the dot-com bust and the latter occurring during the financial crisis. In both cases, the GCorr Macro stressed PD provides a conservative fit to the observed EDF values, with the stressed PD spikes being slightly higher. The unconditional loss shows a time lag in its spike compared to the stressed variables, as, by design, it is not taking into account the future macroeconomic environment of the following quarters.

Comparing the top-three U.S. models, we observe that all three models show similar trends, due to all of them sharing the variables U.S. Equity, U.S. VIX, and U.S. Unemployment Rate.

**Figure 23** Backtesting of GCorr Macro Stressed Expected Loss — top U.S. model of large U.S. Industrial and Financial Portfolio with time-varying PD term structure and LGD = 100%.



In Figure 23, we split the U.S. Large Corporates portfolio into Financials and Industrials sub-portfolios. We observe several differences in the stressed PD behavior over time. First, we see that the dot-com bust represents the most significant stressed PD increase for the industrials sub-portfolio, overtaking the increase seen during the financial crisis. For the Financials sub-portfolio, the dot-com bust leads to a moderate increase of roughly 1.5% stressed PD, and it is overshadowed by the huge and prolonged increase during the financial crisis. While the overall stressed PD level for the Industrials sub-portfolio is much higher than the Financials level (e.g. 4.5% vs. 1%, respectively, during the economic growth period, following the dot-com bust), variability is far higher for the Financials portfolio. Compared to the Industrials sub-portfolio, the Financials sub-portfolio shows a significantly larger sensitivity to the stress that occurred during the aftermath of the financial crisis.

We can explain higher stressed PD levels observed outside of economic downturns via the weighted-average EDF value for the Industrials sub-portfolio, which is approximately 1.5 times higher than the Financials sub-portfolio. In the case of the higher Financials variability, one of the primary drivers is the higher weighted average R-squared (35% for Financials vs. 29% for Industrials), which leads to higher sensitivity to macroeconomic shocks. Moreover, the Financials sub-portfolio shows a higher sensitivity to the U.S. Unemployment variable, which posts a particularly large increase during the financial crisis.

#### BENCHMARKING FOR U.S. SME PORTFOLIO

In the next set of validation exercises, we study the U.S. SME portfolio, and we compare the expected losses produced by GCorr Macro to various benchmarks.

Moody's Analytics has developed a methodology for stressing PD values based on the RiskCalc modeling framework for private firm PD values.<sup>44</sup> We refer to this model as the "Stressed PD Model" and use it as a benchmark for GCorr Macro. The portfolio setup of both models remains the same. In Figure 24, we plot time series of nine-quarter cumulative losses for the U.S. SME portfolio projected by the Stressed PD Model and GCorr Macro. Note, losses are based on stressed PD projections only; we assume LGD to be constant. Unconditional PD values for the instruments use a flat term structure. Time series patterns of losses from the two models are similar. The one difference is that GCorr Macro losses peak somewhat precede peaks of Stressed PD Model losses. The reason: GCorr Macro projects the highest losses over the nine-quarter period associated with the most adverse economic shocks, while the Stressed PD Model has built-in features that cause a delay between a shock and losses (for example, the model links default probabilities to lagged returns of certain variables, as opposed to contemporaneous returns). The Stressed PD Model

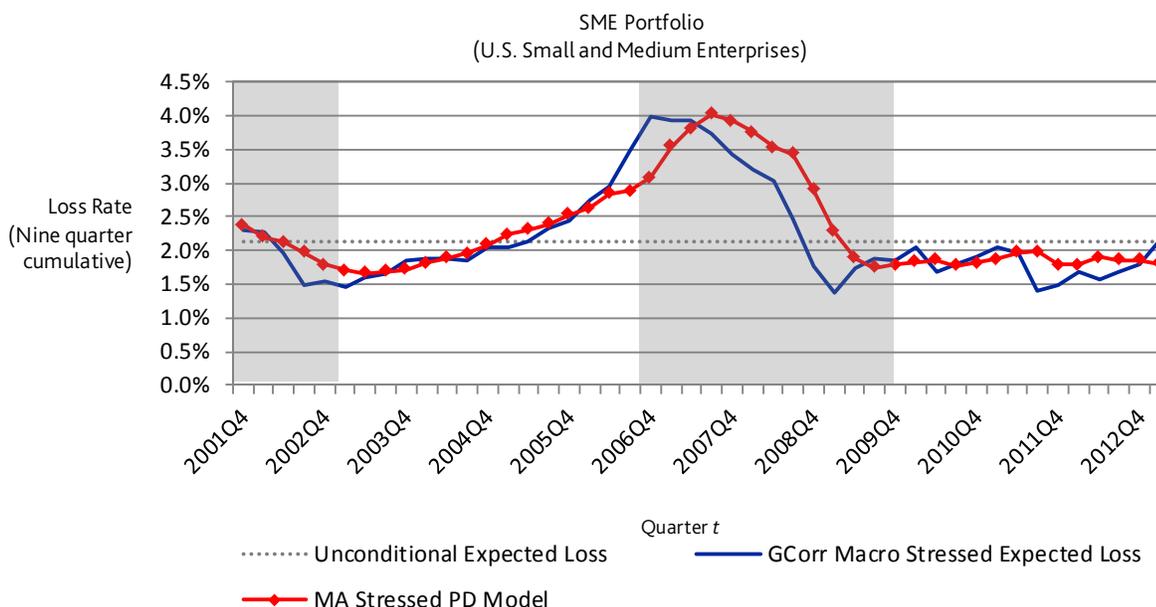
<sup>44</sup> See the paper "Stress Testing Probability of Default for Private Firm C&I Portfolios: RiskCalc Plus Stress Testing PD and LGD Model (granular approach) – United States v4.0 Corporate Model" by Chen, et al., 2014.

and GCorr Macro respond similarly to different economic episodes, such as projecting higher losses for the period of financial crisis than for the recession during the early 2000s.<sup>45</sup>

An important observation is that the two models provide comparable levels of nine-quarter projected losses, as Figure 24 shows.

Comparing the GCorr Macro projected losses from Figure 19 and Figure 24 allows us to assess the impact of stressing LGD. The impact is especially pronounced during the financial crisis as the losses with stressed LGD reached a level of around 5%, while it was approximately 4% without stressed LGD.

**Figure 24** Cumulative, nine-quarter expected losses for the SME Portfolio: GCorr Macro, Moody's Analytics Stressed PD Model, and Unconditional Expected Loss — Flat PD term structure and fixed LGD of 50%.



Let us summarize the validation exercises presented in Section 7.1. First, we show how losses produced by GCorr Macro vary across economic episodes during the period 2001–2015. Second, we compare loss levels from GCorr Macro to various benchmarks, with an emphasis on the financial crisis period. Validation exercises demonstrate that GCorr Macro can differentiate economic episodes according to their severity, and that the cumulative losses it projects over nine quarters for pre-crisis portfolios under the financial crisis scenario are broadly in-line with the benchmarks. These conclusions are relevant for CCAR style analyses, as financial institutions stress test their portfolios with the current risk parameters (in other words, parameters from 2014–2015), assuming a severely adverse economic scenario created by the Federal Reserve, which is similar to the financial crisis episode.

Another GCorr Macro feature underscored by the validation exercises is the model's ability to handle both large and small firm portfolios. The assumption we need to make is that both types of firms load to the same set of factors: GCorr Corporate systematic factors. However, the R-squared parameter that plays an important role in the GCorr Macro calculations allows us to account for the different sensitivities of various firms to the factors and, in turn, macroeconomic variables.

## 7.2 International Corporate Portfolios

After we conduct the validation exercises for the U.S. SME and U.S. Large Corporate portfolios, we turn our attention to how well the GCorr Macro multi-period stress testing methodology performs for Eurozone and Japan Large Corporates. Table 5 summarizes the stylized credit portfolios of the aforementioned regions.

<sup>45</sup> The types of macroeconomic variables used in the Stressed PD Model are the same as in our exercises: Unemployment Rate, Baa Corporate Yield, a stock market index, and the VIX Index.

In Table 5, for all portfolios:

- » Types of Counterparties: Large listed corporates in a given region (firms constituting 99% of total liabilities of firms in the given region)
- » Exposure pooling: by industries within a given region
- » Pool weights: proportional to the large firm counts by GCorr industries in GCorr 2015 Corporate
- » Source of the R-squared values: GCorr 2015 Corporate, large firm average R-squared values by industries (applies to all portfolios)
- » Definition of PD: unconditional (input) PDs are time varying, average pool-level EDF values from CreditEdge

Table 5

### Stylized Portfolios Used for Validation

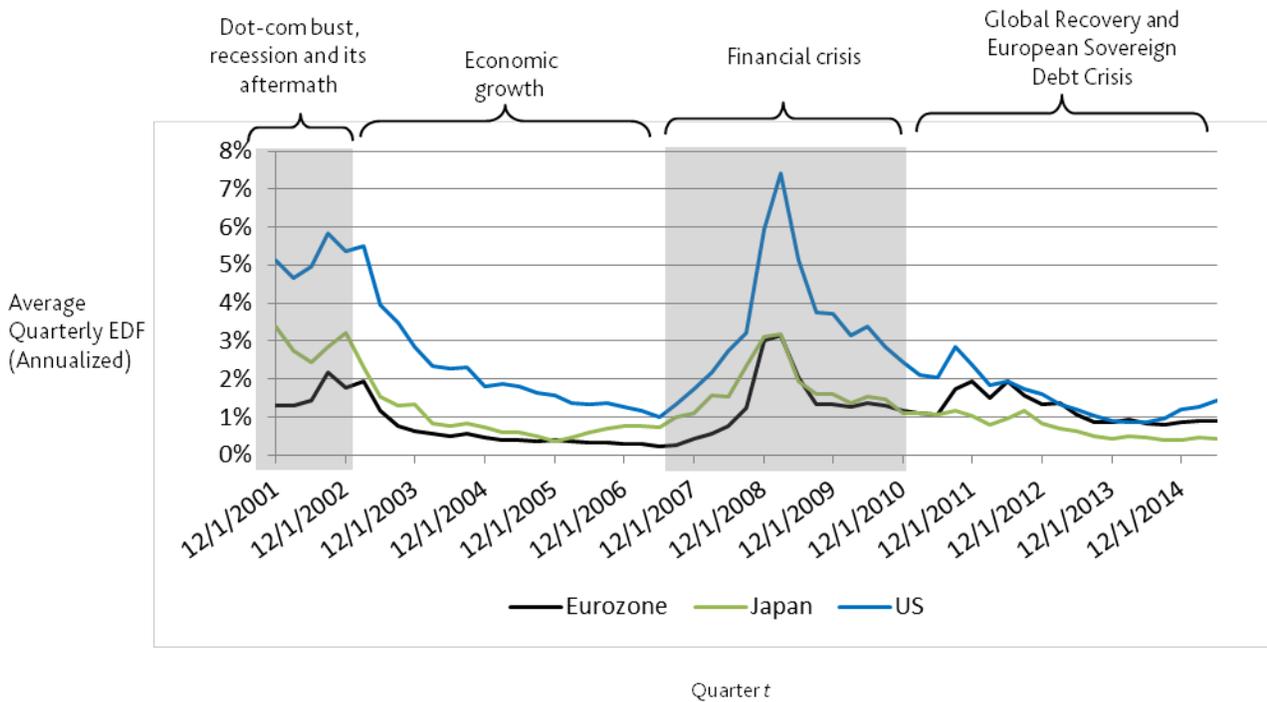
PORTFOLIO	EUROZONE PORTFOLIO	JAPAN PORTFOLIO
Exposure Pooling	60 pools of loans to Eurozone corporates Loans are pooled by 60 GCorr industries	60 pools of loans to Japanese corporates Loans are pooled by 60 GCorr industries
R-squared	Weighted average R-squared = 32.1%	Weighted average R-squared = 34.6%
Loss Given Default for Projections	LGD = 100%	LGD = 100%

Comparing these portfolios with each other and the large corporate portfolio for the U.S., we see that the Japanese portfolio bears the greatest systematic risk in its portfolio, followed by those for the Eurozone and the U.S. In terms of the default risk measured by the portfolio weighted average probability of default at the start of 2007, we find that the lowest risk by far is seen for the Eurozone portfolio. On the other hand, Japan and the U.S. possess high PD values compared to the other regional portfolios.

Figure 25 shows the average CreditEdge benchmark EDF value for large public firms for the Eurozone and Japan. Like the U.S., we again see large increases during the dot-com bust and the financial crisis. From 2010 on, no other economic downturns lead to notable EDF value increases in the U.S. and in Japan. For example, the impact of the 2011 Japanese earthquake lead to only smaller increases in average EDF levels. However, we do see an increase in the EDF time series during the Eurozone crisis for large public firms in the Eurozone.

Comparing the different country/regions during the dot-com bust and the financial crisis, we see that during the former, Japan has higher EDF levels. Overall, the EDF time series behavior from the U.S. is affected during both crisis periods. The EDF value spikes for the other international regions are lower throughout. Japan and Eurozone EDF levels show the lowest increase during the financial crisis, with comparable levels during the height of the dot-com bust.

Figure 25 Average CreditEdge benchmark EDF for large Eurozone, Japanese, and US public firms.



### 7.2.1 EUROZONE

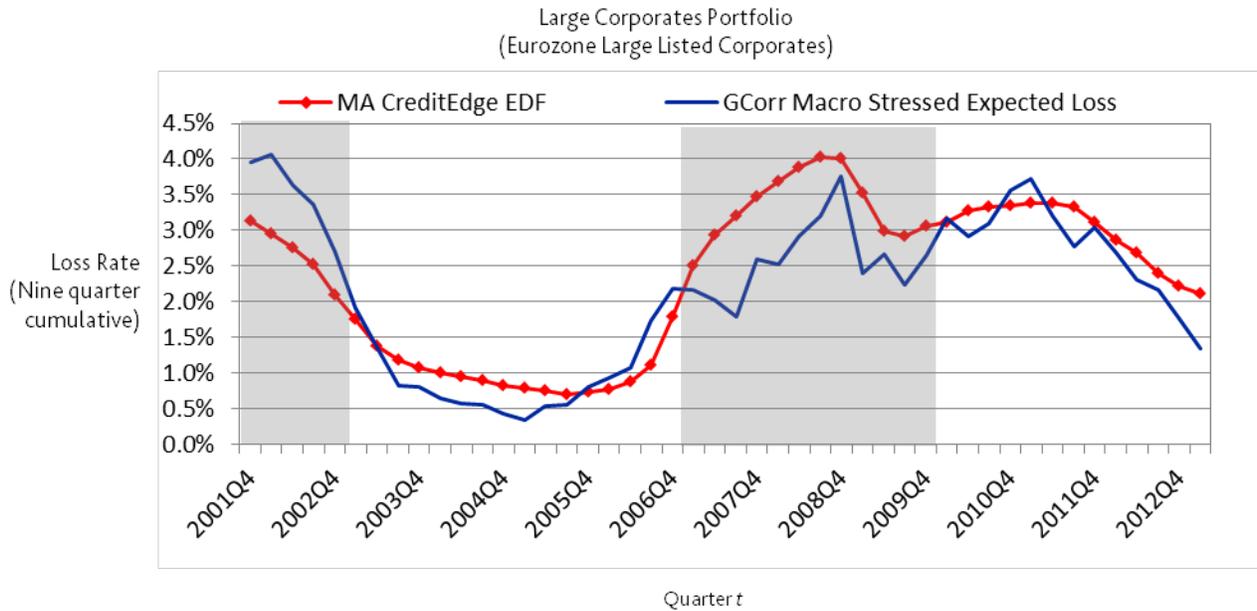
In the following analysis for Eurozone, we focus on the selected regional top model performance. The selection procedure follows similar rules as outlined in Section 5.3, with the added constraint of considering only region-specific macroeconomic variables.

The selected Eurozone model is comprised of the Eurozone Equity, Eurozone Spread,<sup>46</sup> and Eurozone GDP variables. Compared to the U.S., the adjusted pseudo R-squared of the model is lower (38.0% vs. 31.7%), which may be due to the greater regional diversity.

Similar to Figure 22, in the following chart we compare the backtesting results of the stressed probability of default from the Eurozone GCorr Macro top model to benchmark CreditEdge EDF measure.

<sup>46</sup> We emphasize that the Eurozone spread is a Eurozone corporate spread (similarly to the U.S. BBB Corporate Spread) as opposed to a measure of sovereign spread.

**Figure 26** Backtesting GCorr Macro Stressed Expected Loss from top GCorr Macro model of Eurozone Large Corporates portfolio with time-varying PD term structure and LGD=100%.

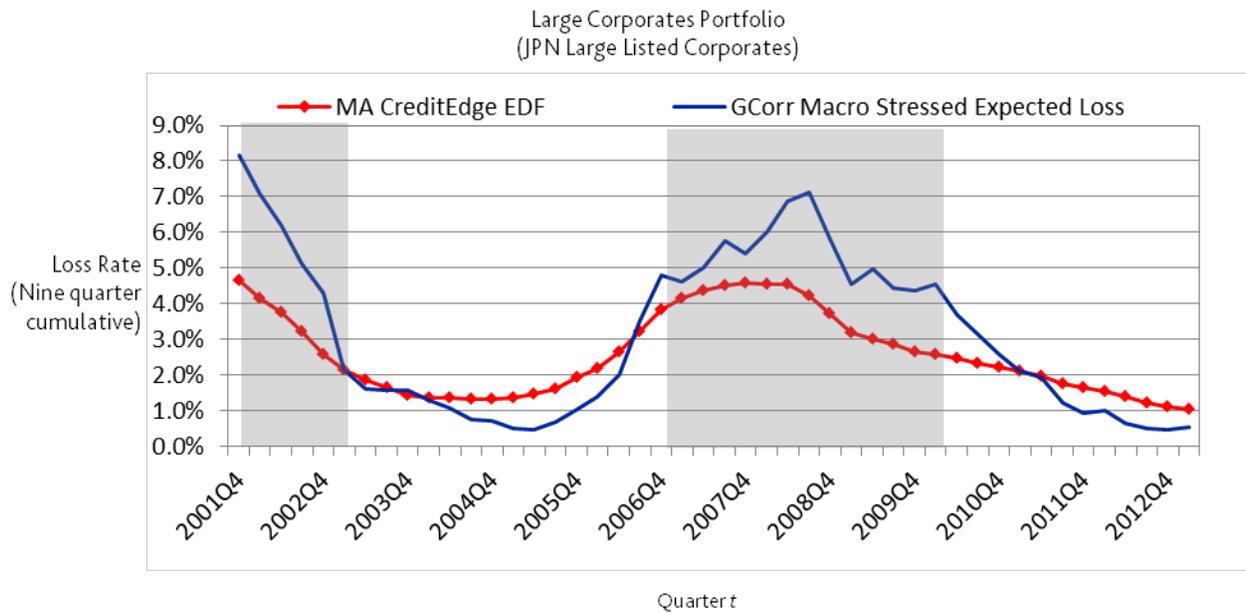


Looking at the pattern of the GCorr Macro stressed PD values and comparing it to the EDF benchmark, we see that they are aligned prior to the financial crisis. However, it remains below the benchmark from 2007–2008. One contributing factor is the prolonged period of low levels of quarterly unconditional input PD until 2008. Both the stressed PD and EDF remain at an elevated level during the Eurozone crisis.

**7.2.3 JAPAN**

The top model for Japan consists of two macroeconomic variables: Japan GDP and Japan Equity. Despite its small number of variables, its adjusted pseudo R-squared is the highest among all non-US regions.

**Figure 27** Backtesting GCorr Macro Stressed Expected Loss from Top GCorr Macro model of Japanese Large Corporates portfolio with time-varying PD term structure and LGD=100%.



As Figure 27 shows, the overall pattern for the stressed PD of Japan's top GCorr model follows the general behavior of the benchmark EDF rate. However, the spike during the financial crisis is more pronounced than is the case for the benchmark EDF rate. The Japanese portfolio is mainly influenced by Japanese Equity which showed significant drops in 2007 and 2008. The credit risk factors in the Japanese portfolio prove in turn highly sensitive towards the Japanese top model that includes the Equity variable. This leads to a slightly higher stressed PD from 2007–2008.

The Japanese earthquake in 2011 led only to a minor increase in EDF levels after the initial recovery from the financial crisis.

### 7.3 U.S. Commercial Real Estate Portfolios

In the following two sections, we demonstrate how to use GCorr Macro for non-corporate asset classes, namely for portfolios of U.S. Commercial Real Estate (CRE) exposures.

We first focus on CRE exposures. Figure 28 summarizes correlations of U.S. GCorr CRE factors with U.S. macroeconomic variables: Real GDP, U.S. Equity Index, and CRE Price Index.<sup>47</sup> For each property type (Hotels, Industrial, Multi-Family, Office, and Retail), the correlations vary across 73 U.S. Metropolitan Statistical Areas (MSAs). All three variables have significantly positive correlations with the factors, which is consistent with economic intuition. For example, economic conditions measured by Real GDP growth and direction of the commercial real estate market measured by CRE Price Index return affect a CRE portfolio's performance.

<sup>47</sup> GCorr CRE contains 73 MSA factors and five property type factors (Hotels, Industrial, Multifamily Housing, Office, and Retail) to measure systematic risk for commercial real estate properties.

Figure 28 Distribution of correlations of U.S. GCorr CRE factors and select U.S. macroeconomic variables.

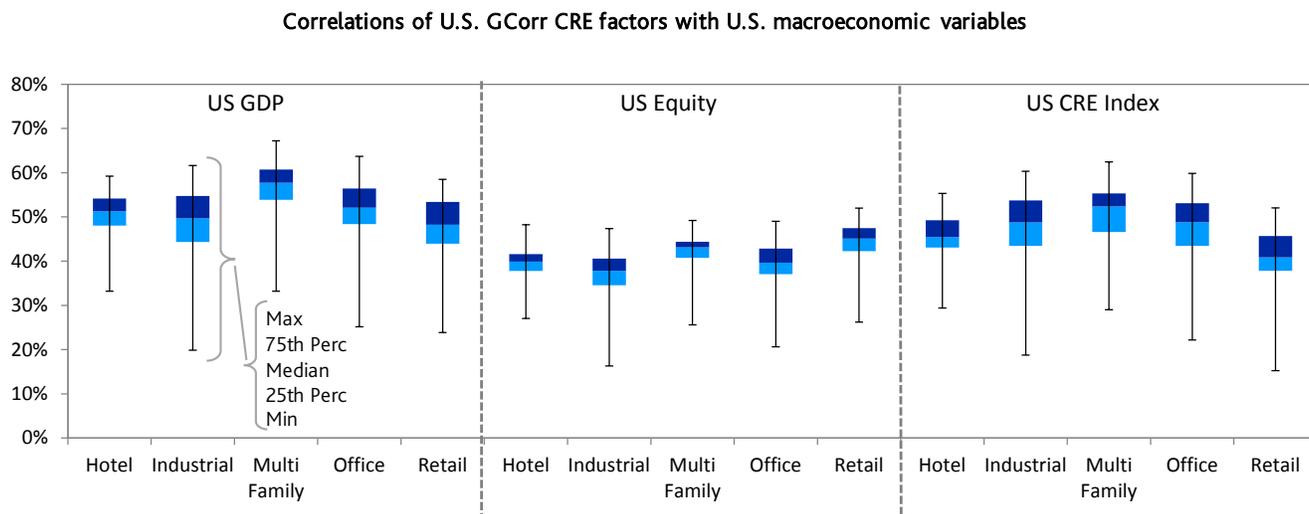


Table 6 summarizes the stylized credit portfolios we use to validate GCorr Macro for U.S. CRE. Unlike corporate portfolios, where point-in-time CreditEdge EDF measures for large corporates are well developed, setting the level of probability of default in a CRE portfolio over time can be quite challenging, without having a model that an institution uses to determine PDs.<sup>48</sup> To perform the exercise, we assume a constant PD through time to backtest model performance with a CRE portfolio. Pool-level probability of default is set to the pre-crisis default rate in 2008Q2, so we can understand how the model behaves during a financial crisis.

Table 6

**Stylized Portfolios Used for Validation**

PORTFOLIO	CRE PORTFOLIO
Types of Counterparties	Commercial Real Estate Debtors
Exposure Pooling	295 pools of loans Loans are pooled by 54-63 GCorr MSAs for each products
Pool Weights	Pool weights proportional to the CMBS initial balance by GCorr MSA in GCorr 2015 CRE
R-squared	Source: GCorr 2015 CRE Weighted average R-squared = 35.56%
Probability of Default	Fixed PD: <sup>49</sup> Weighted average PD = 0.81% (annualized)
Loss Given Default for Projections	LGD = 100%

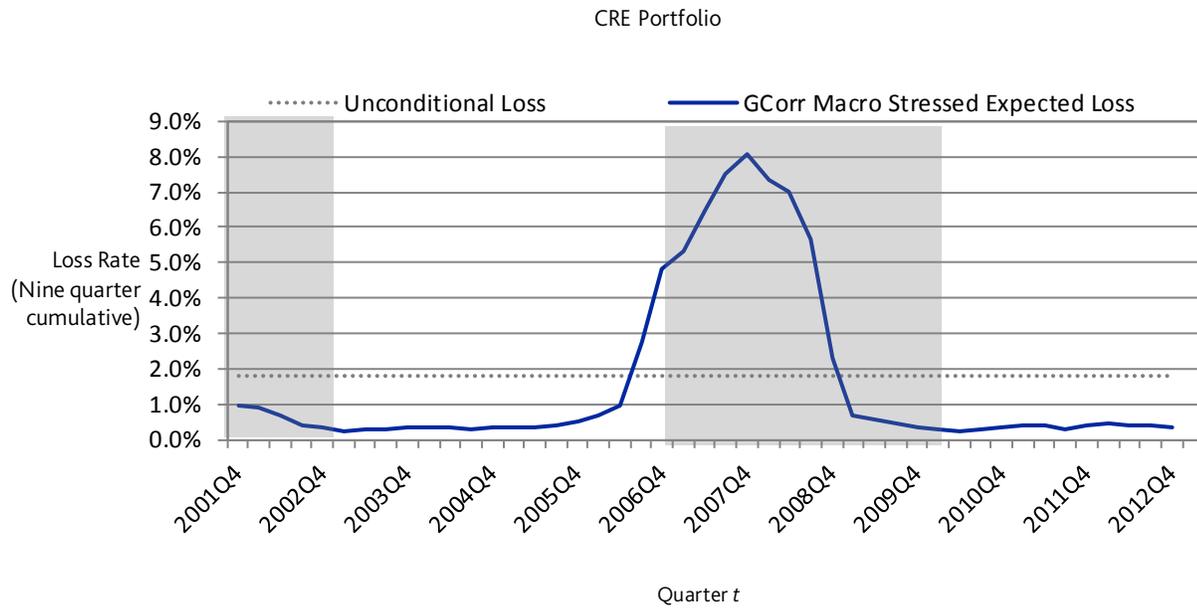
Figure 29 shows that the level of the spike in GCorr Macro stressed losses peaks at 8%, which roughly matches the peak of the historical default rate during the financial crisis (we do not display the historical default rate<sup>50</sup>). But the peak of GCorr Macro losses occurred in 2007Q4, preceding the historical 9Q accumulated default rate, which peaked in early 2009. One explanation for the timing difference is that the GCorr Macro model will exhibit the highest losses in the nine-quarter period of the most adverse economic shocks, while the realized loss by its nature is less fluctuating, and often lags behind economic indicators.

<sup>48</sup> One possible option would be to use Commercial Mortgage Matrix (CMM), where PD and LGD depend on various loan and property characteristic, like Loan-to-Value ratio or Debt Service Coverage Ratio.

<sup>49</sup> Empirical RMBS default rate.

<sup>50</sup> The default rate is defined as 90 days to 120 days delinquency rate in the projection.

Figure 29 Backtesting of GCorr Macro Stressed Expected Loss from Top GCorr Macro model of CRE Portfolio with constant PD term structure and LGD=100%.



## 8. Projected CCAR Losses Based on GCorr Macro

This section presents the results of several stress testing exercises. We use the portfolios introduced in the previous section to calculate the stressed expected losses with GCorr Macro under the CCAR scenarios. The questions of interest are how the projected losses compare across scenarios, across portfolios, as well as how they compare to the losses based on historical scenarios, described in Section 7.

### 8.1 U.S. Large Corporate and SME Portfolios

The Federal Reserve publishes three CCAR scenarios each year: Baseline, Adverse, and Severely Adverse. We choose four macroeconomic variables for projecting losses with GCorr Macro under the CCAR scenarios: U.S. Unemployment Rate, U.S. BBB Spread, Dow Jones Total Stock Market Index, and the U.S. VIX Index. Figure 30 and Figure 31 show the quarterly smoothed stressed expected losses for the U.S. SME portfolio and the U.S. Large Corporates portfolio, respectively. For the following exercises, we use the same portfolios as in Section 7.

**Figure 30** Quarterly smoothed losses projected by GCorr Macro for the U.S. SME portfolio under CCAR 2014, CCAR 2015, and CCAR 2016 scenarios.

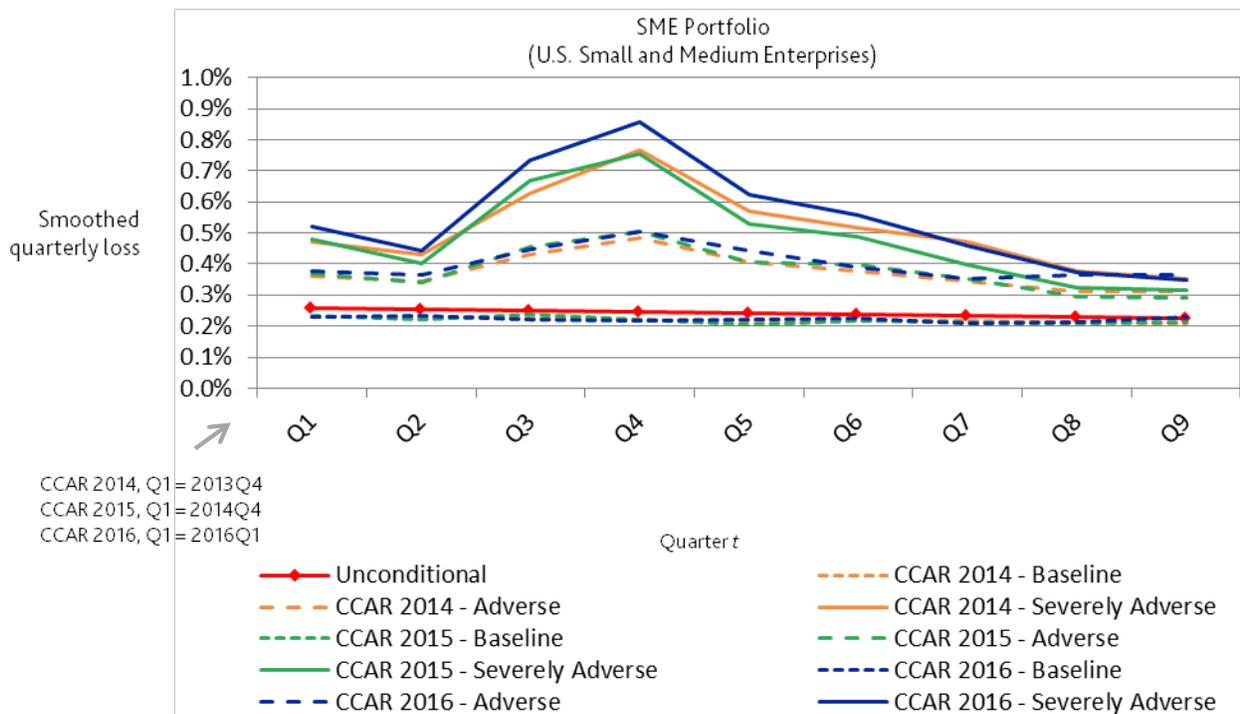


Figure 31 Quarterly smoothed losses projected by GCorr Macro for the Large Corporates portfolio under CCAR 2014, CCAR 2015, and CCAR 2016 scenarios.

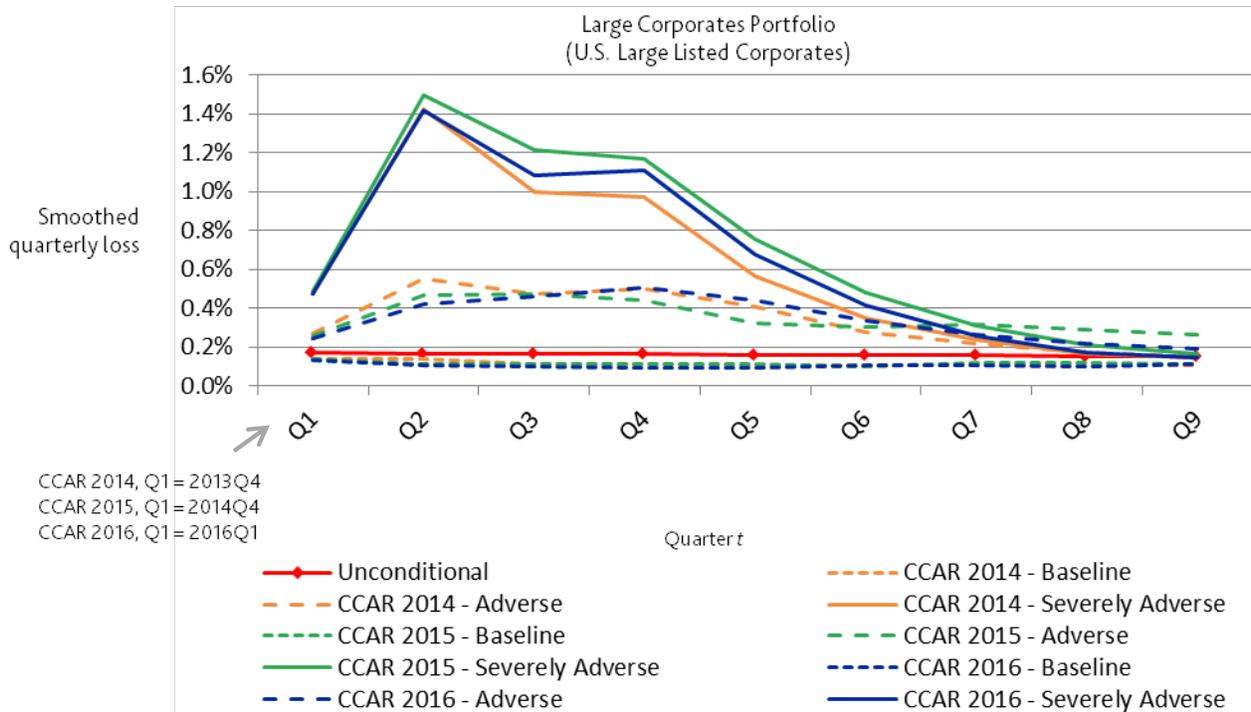


Figure 30 and Figure 31 exhibit similar patterns. The projected quarterly losses under the Adverse scenarios are higher during the earlier quarters. We can attribute this finding to the assumed paths of the macroeconomic variables; the adverse shocks occur early, which leads to large quarterly losses at the beginning, and the variables recover in later quarters, implying lower losses at the end.

Figure 32 presents the log changes for the four macroeconomic variables, under the CCAR 2014, CCAR 2015, and CCAR 2016 Severely Adverse scenarios.

Figure 32 Quarterly log changes of selected macroeconomic variables shocks under the CCAR 2014, CCAR 2015, and CCAR 2016 Severely Adverse scenarios.

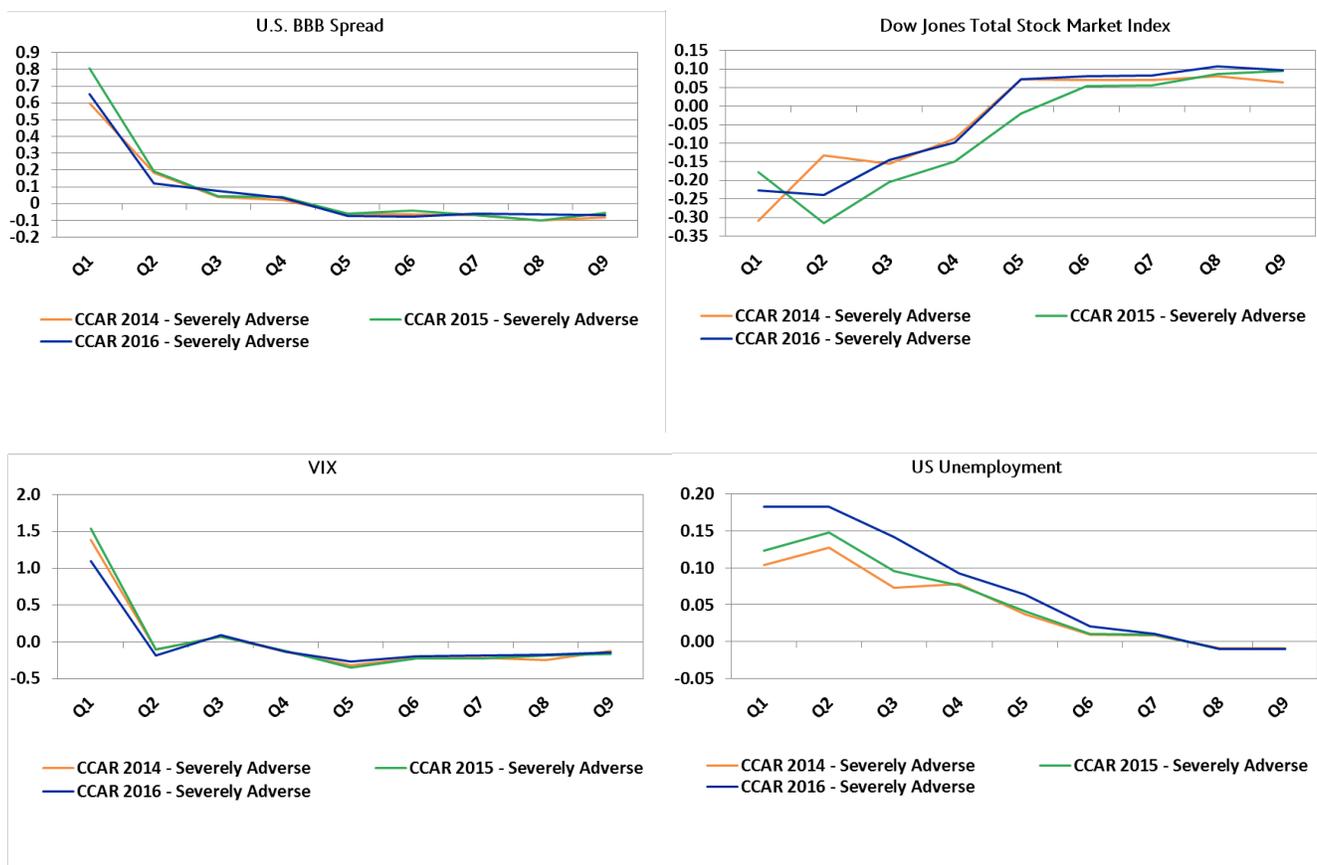


Table 7 describes the nine-quarter cumulative losses for both portfolios under the different CCAR scenarios. In-line with economic intuition, the Severely Adverse scenarios are associated with the largest projected losses among the scenarios. Comparing CCAR 2016 with CCAR 2014 and CCAR 2015, all portfolios exhibit similar cumulative losses for the Baseline and Adverse scenarios. For the Severely Adverse scenario, the CCAR 2016 scenario is slightly less severe than the CCAR 2015 scenario.

Comparing the paths of macroeconomic variables in Figure 32, the Severely Adverse scenario in CCAR 2015 is the most severe, due to the large drop in the equity scenario and the increase in the BBB spread scenario. The Severely Adverse scenarios for CCAR 2014 and 2016 are quite similar, except the 2016 scenario shows a higher increase in the unemployment rate. As a result, the 2016 scenario produces higher stressed losses than the 2014 scenario, but both are lower than the 2015 scenario.

Table 7

**Nine-Quarter Cumulative Expected Losses for CCAR 2014, CCAR 2015, and CCAR 2016 Scenarios. U.S. SME and Large Corporates Portfolio**

SCENARIO	SME	LARGE CORPORATES
Unconditional	1.71%	1.44%
CCAR 2014 Baseline	1.89%	1.04%
CCAR 2014 Adverse	3.26%	3.01%
CCAR 2014 Severely Adverse	3.44%	5.31%
CCAR 2015 Baseline	1.94%	1.03%

CCAR 2015 Adverse	3.53%	3.12%
CCAR 2015 Severely Adverse	4.16%	6.27%
CCAR 2016 Baseline	1.83%	0.93%
CCAR 2016 Adverse	2.99%	3.07%
CCAR 2016 Severely Adverse	3.73%	5.74%

Note, the CCAR 2014, 2015, and 2016 Baseline scenario losses are not substantially lower than the unconditional losses in Figure 30 and Figure 31. To explain this pattern, we examine the macroeconomic variable paths under the scenarios. While the unemployment rate decreases the stock market provides positive returns, and the BBB Spread decreases slightly; the scenario assumes mild increases in VIX.

Table 7 shows an interesting feature. The losses across scenarios are more dispersed for the U.S. Large Corporates portfolio than for the SME portfolio. As we point out in the context of the historical scenarios in Section 7, we can attribute this effect to the substantially different asset R-squared values of the portfolios: 31.6% for the Large Corporates portfolio and 6.1% for the SME portfolio.

### 8.2 U.S. Commercial Real Estate Portfolios

We choose the same set of variables (Real GDP, U.S. Equity, CRE index) and portfolios as in Section 7.3, except that we set LGD equal to 40%. Figure 33 shows the quarterly smoothed stressed expected losses for the U.S. CRE portfolio.

The CCAR 2015 Severely Adverse scenario leads to losses that are at somewhat higher the losses obtained under the Financial Crisis scenario. This follows because the projection for the CRE index in the Severely Adverse scenario is more severe than during the financial crisis.

Figure 33 Quarterly smoothed losses projected by GCorr Macro for the U.S. CRE portfolio under the CCAR 2014, CCAR 2015, and CCAR 2016 scenarios.

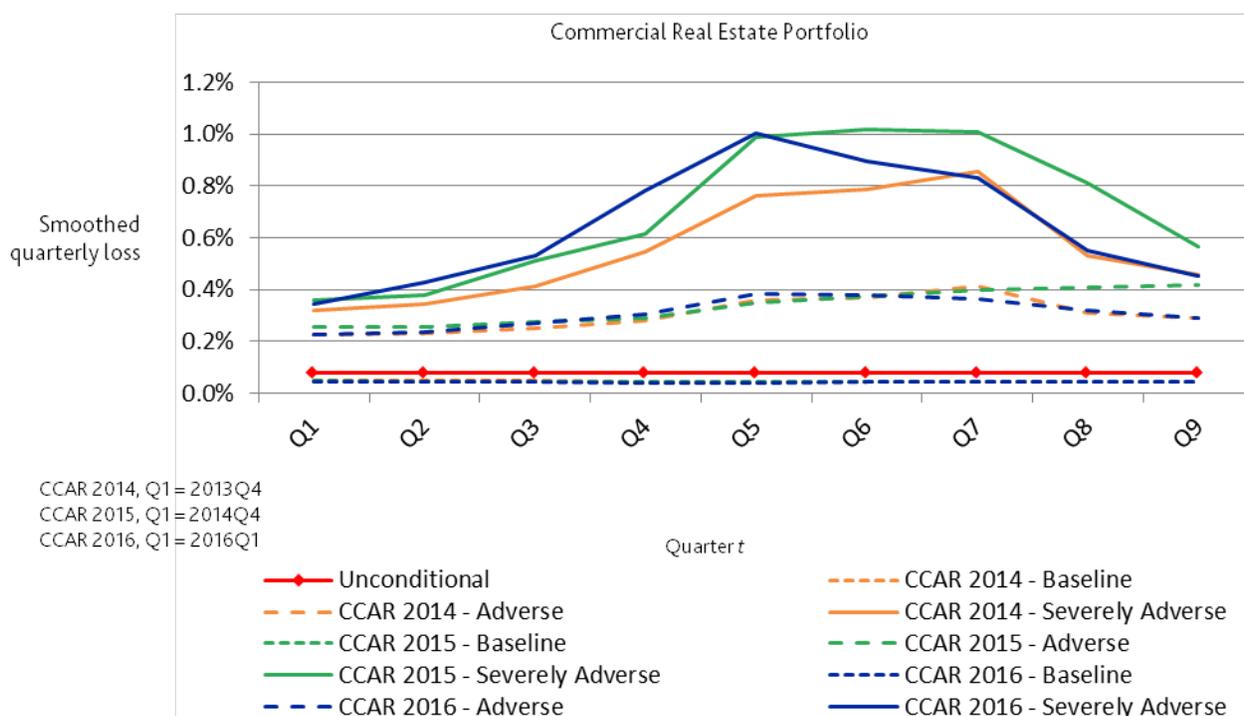


Table 8

**Nine-Quarter Cumulative Expected Losses for CCAR 2014, CCAR 2015, and CCAR 2016 Scenarios. U.S. CRE Portfolio**

SCENARIO	CCAR 2014	CCAR 2015	CCAR 2016
Unconditional	0.71%	0.71%	0.71%
Baseline	0.42%	0.41%	0.39%
Adverse	2.72%	3.03%	2.78%
Severely Adverse	5.02%	6.26%	5.83%

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## 9. Conclusion

This document presents a multi-period analytical stress testing methodology that can be applied to a credit portfolio to compute instrument- and portfolio-expected losses, period-by-period, under a given scenario. The approach allows for one modeling framework to be applied consistently across the entire portfolio. To capture losses in future periods, we use stressed transition probabilities to account for past macroeconomic shocks. Compared to simulation-based stress testing using GCorr Macro, the primary advantage of a multi-period analytical stress testing methodology is the calculation time; calculations are run using analytical formulas, and they do not require Monte Carlo simulation. Furthermore, the approach is consistent with the economic framework that underpins the Moody's Analytics RiskFrontier application.

The method also provides information regarding the extent to which the macroeconomic variables span the risks of the portfolio. Typical economy-wide variables included in scenarios do not explain all portfolio risk. As a result, users must realize that there is still dispersion in losses under the scenario around the stressed expected loss. The simulation-based method of stress testing discussed briefly in this paper can be used to help quantify the dispersion given a scenario.

We conduct several validation exercises in which we use the stress testing method to produce losses on Commercial & Industrial portfolios under various historical scenarios. As our results indicate, time series dynamics of these projected losses are in-line with economic intuition, and the loss levels are appropriate when compared to various benchmarks. Each stress testing model is suitable for some type of economic episode. Given how we estimate GCorr Macro and given the validation results, we can conclude that GCorr Macro performs well when considering scenarios that resemble recent economic episodes, especially the recent financial crisis. Using GCorr Macro for different types of economic environments, such as the stagflation experienced in the 1970s, may require a different parameterization of the model.

## Appendix A Macroeconomic Variables

Table 9 lists the 91 macroeconomic variables included in GCorr Macro 2015.

Table 9

### Macroeconomic Variables Included in GCorr Macro

REGION	MACROECONOMIC VARIABLE	TRANSFORMATIONS	SOURCE
U.S.	Real GDP	Log Change + Detrending (13-Quarter Window)	Bureau of Economic Analysis
U.S.	Nominal GDP	Log Change + Detrending (13-Quarter Window)	Bureau of Economic Analysis
U.S.	Real disposable income	Log Change	Bureau of Economic Analysis
U.S.	Nominal disposable income	Log Change	Bureau of Economic Analysis
U.S.	Unemployment rate	Log Change	Bureau of Labor Statistics
U.S.	CPI (Consumer Price Index)	Log Change + Detrending (Three-Quarter Window)	Bureau of Labor Statistics
U.S.	3-month Treasury yield / Federal Funds Rate	Log Change	CCAR
U.S.	10-year Treasury yield	Log Change	CCAR
U.S.	Baa corporate yield / BBB corporate yield (CCAR)	Log Change	Moody's Investors Service
U.S.	Mortgages rate	Log Change	Freddie Mac Commitment Rates
U.S.	Dow Jones Total Stock Market Index	Log Change	Dow Jones
U.S.	Market Volatility Index (VIX)	Log Change	Chicago Board Options Exchange
U.S.	Case-Shiller House Price Index / National House Price Index (CCAR)	Log Change	Case-Shiller
U.S.	Commercial Real Estate Price Index	Log Change	CCAR
Europe	Euro Area real GDP	Log Change + Detrending (13-Quarter Window)	Copyright European Communities
Europe	Euro Area Inflation	Log Change in the Index + Detrending (Three-Quarter Window)	CCAR
Europe	Euro Area Bilateral Dollar Exchange Rate (\$/Euro)	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Asia	Developing Asia Real GDP Growth	None	CCAR
Asia	Developing Asia inflation	None	CCAR
Asia	Developing Asia Bilateral Dollar Exchange Rate (F/U.S.D, index, Base=2000 Q1)	Log Change	CCAR

Japan	Japan Real GDP	Log Change + Detrending (13-Quarter Window)	Economic and Social Research Institute
Japan	Japan Inflation	Detrending (Three-Quarter Window)	CCAR
Japan	Japan Bilateral Dollar Exchange Rate (Yen/U.S.D)	Log Change	Moody's Analytics - Economic & Consumer Credit Analytics (www.economy.com)
UK	UK Real GDP	Log Change + Detrending (13-Quarter Window)	UK Office for National Statistics
UK	UK Inflation	Log Change in the Index + Detrending (Three-Quarter Window)	UK Office for National Statistics
UK	UK Bilateral Dollar Exchange Rate (U.S.D/Pound)	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
U.S.	Light Vehicle Sales	Log Change	Bureau of Economic Analysis
U.S.	Residential Housing Starts	Log Change	U.S. Census Bureau
U.S.	Corporate Profits with IVA & CCA	Log Change	Bureau of Economic Analysis
U.S.	Retail Sales	Log Change	U.S. Census Bureau
U.S.	FHFA All Transactions Home Price Index	Log Change	Federal Housing Finance Agency
UK	UK Home Price Index	Log Change	Nationwide Building Society
UK	UK CRE Index	Log Change	FTSE
UK	UK FTSE All Shares Equity Index	Log Change	FTSE
U.S.	U.S. Industrial Production	Log Change + Detrending (Three-Quarter Window)	Federal Reserve
Global	Oil Price	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Japan	Japan Equity Index	Log Change	Nikkei
Europe	Euro Area Equity Index	Log Change	STOXX
Canada	Canada GDP	Log Change + Detrending (13-Quarter Window)	STCA - Statistics Canada
Canada	Canada Equity Index	Log Change	Standard & Poor's
South Africa	South Africa GDP	Log Change + Detrending (13-Quarter Window)	Statistics South Africa
South Africa	South Africa Equity	Log Change	FTSE
Australia	Australia GDP	Log Change + Detrending (13-Quarter Window)	A.U.S.T

Brazil	Brazil GDP	Log Change + Detrending (13-Quarter Window)	IBGE
Mexico	Mexico GDP	Log Change + Detrending (13-Quarter Window)	INEGI
France	France Unemployment	Log Change	INSEE
Germany	Germany Unemployment	Log Change	German Federal Statistical Office
UK	UK Unemployment	Log Change	ONS
Hong Kong	Hong Kong Unemployment	Log Change	Census & Statistics Department
Brazil	Brazil Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Australia	Australia Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Canada	Canada Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Mexico	Mexico Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Hong Kong	Hong Kong Equity Index	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
China	China Equity Index	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Middle East	Middle East Equity Index	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Mexico	Mexico Equity Index	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Canada	Canada BBB yield	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Canada	Canada Bilateral Dollar Exchange Rate (U.S.D/CAD)	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Canada	Canada House Price Index	Log Change	Moody's Analytics - Economic & Consumer Credit Analytics (www.economy.com)
Canada	Canada Mortgage Rate	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Europe	Euro Area LIBOR	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
U.S.	U.S. BBB Spread	Log Change	CCAR
Canada	Canada BBB Spread	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Europe	Eurozone BBB Spread	Log Change	Moody's Analytics - Economic & Consumer Credit Analytics (www.economy.com)
Europe	Eurozone Unemployment	Log Change	Eurostat
South Africa	South Africa Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics (www.economy.com)
Thailand	Thai Private Consumption Expenditure	Log Change	NESDB
Thailand	Thai Export	Log Change	BOT
Thailand	Thai Investment	Log Change	BOT

Thailand	Thai FX (U.S.D/THB)	Log Change	BOT
Thailand	Thai House Price Index	Log Change	BOT
Thailand	Thai Household Debt to GDP	Log Change	BOT
Thailand	Thai Minimum Lending Rate	Log Change	BOT
Thailand	Thai Equity	Log Change	SET
U.S.	U.S. 5 Year Rate	Log Change	CCAR
U.S.	U.S. Prime Rate	Log Change	CCAR
Germany	Germany GDP	Log Change + Detrending (13-Quarter Window)	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Germany	Germany Equity	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
France	France GDP	Log Change + Detrending (13-Quarter Window)	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
France	France Equity	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Netherlands	Netherlands Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Netherlands	Netherlands GDP	Log Change + Detrending (13-Quarter Window)	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Netherlands	Netherlands Equity	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Spain	Spain Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Spain	Spain GDP	Log Change + Detrending (13-Quarter Window)	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Spain	Spain Equity	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Sweden	Sweden Unemployment	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Sweden	Sweden GDP	Log Change + Detrending (13-Quarter Window)	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Sweden	Sweden Equity	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )
Sweden	Sweden House Price Index	Log Change	Moody's Analytics – Economic & Consumer Credit Analytics ( <a href="http://www.economy.com">www.economy.com</a> )

## Appendix B Instrument-Level Inputs for Stress Testing

0 lists all instrument-level parameters that must be specified as inputs for the multi-period analytical stress testing methodology outlined in this paper.

Table 10

### Instrument-level input parameters

Instrument ID	Commitment, Term Structure	Usage Given Default, Term Structure
	$CMT_1, \dots, CMT_T$	$UGD_1, \dots, UGD_T$
1	100 (Currency), ..., 100 (Currency)	100%, ..., 100%
...	...	...

Instrument ID	Probability of Default, Term Structure	Loss Given Default, Term Structure	Parameter driving LGD variance
	$PD_1, \dots, PD_T$	$LGD_1, \dots, LGD_T$	$k$
1	0.01, ..., 0.01	40%, ..., 40%	4
...	...	...	...

Instrument ID	Asset R-squared	Custom index weights to GCorr factors	Parameterizing the PD-LGD correlation model	Recovery index weights to GCorr factors
	$RSQ$	$w_1, \dots, w_{NF}$	Specifying one of the sets of inputs <sup>51</sup>	
			$RSQ_{RR}$ and $\rho_{A,RR}$	$LGD^{Downturn}$ and $\alpha$
1	0.15	1, 0, ..., 0, 1, 0, ..., 0	0.3 and 0.4 or 70% and 0.05	
...	...	...	... or ...	

We note that, while the framework allows input to Commitment, UGD, PD, and LGD as a term structure, one can assume the term structure is flat if the parameters are not available for a grid of tenors. Under the flat term structure assumption, the framework requires only one value of each parameter: for example, PD over one year horizon, LGD for one-year horizon, and so forth.

LGD is stressed through the Moody's Analytics PD-LGD correlation model. As 0 shows, there are two ways to specify the model parameters: one relies directly on the recovery R-squared and asset-recovery correlation parameter; the other is based on a downturn LGD as the input.<sup>52</sup> When calculating stressed expected losses, it is possible to either stress LGD together with PD or to assume that LGD does not change under the scenario.

<sup>51</sup> Notation used for the PD-LGD correlation model parameters:  $RSQ_{RR}$  – recovery R-squared values;  $\rho_{A,RR}$  – correlation of the asset and recovery R-squared values;  $LGD^{Downturn}$  and  $\alpha$  – downturn LGD corresponding to a shock with magnitude given by probability level  $\alpha$ . Details on the PD-LGD correlation parameters can be found in the paper "Incorporating Systematic Risk in Recovery: Theory and Evidence," Levy and Hu (2007).

<sup>52</sup> Another set of inputs for the PD-LGD correlation model are weights of the counterparty's recovery index to the GCorr factors. Although the framework allows for these weights to differ from the custom index weights, we assume they are identical.

## Appendix C Variable Selection Results

Table 11

### Selected Macroeconomic Variables

	U.S. CRE PORTFOLIO	U.S. LARGE CORPORATES PORTFOLIO	EUROZONE LARGE CORPORATES PORTFOLIO	JAPAN LARGE CORPORATES PORTFOLIO
U.S. Real GDP	0.215* (1.504)			
U.S. Unemployment Rate		-0.220** (-2.074)		
U.S. BBB Spread		-0.196** (-1.724)		
U.S. Dow Jones Total Stock Market Index	0.269** (2.096)	0.281** (2.172)		
U.S. CRE Price Index	0.353*** (2.937)			
U.S. Market Volatility Index(VIX)		-0.191** (-1.765)		
Euro Area Equity			0.319*** (2.669)	
Eurozone Spread			-0.257** (-2.137)	
Eurozone GDP			0.188** (1.765)	
Japan Equity Index				0.573*** (5.553)
Adjusted R-squared	36.7%	38.0%	31.7%	31.7%

Note: t-statistic in brackets.

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