

MODELING METHODOLOGY

Applications of GCorr[®] Macro within the RiskFrontier[™] Software: Stress Testing, Reverse Stress Testing, and Risk Integration

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Abstract

The GCorr Macro model expands the Moody's Analytics Global Correlation Model (GCorr) for credit risk by linking systematic credit risk factors from GCorr to macroeconomic variables. These macroeconomic variables can include standard indicators of economic activity (e.g., GDP, Unemployment Rate), financial market variables (e.g., Stock Market Index, Interest Rates), price indexes (e.g., House Price Index, Oil Price) and others. In addition, the variables can represent various geographies.

This paper describes how GCorr Macro, within the RiskFrontier credit portfolio modeling framework, lends itself to several functions that facilitate cohesive and holistic risk management, such as stress testing, reverse stress testing, and risk integration. Specifically, GCorr Macro within the RiskFrontier software provides insights into relationships between portfolio losses and macroeconomic variables, and enables single period stress testing analysis.

The outputs of stress testing using GCorr Macro within the RiskFrontier software include not only the conditional expected loss under a macroeconomic scenario, but the entire conditional loss distribution. It is worth noting that these losses reflect credit downgrades and are based on the valuation techniques in the RiskFrontier software. A reverse stress testing analysis characterizes the scenarios associated with a given level of losses, such as losses in the tail of a distribution. This analysis allows for better understanding of portfolio vulnerabilities in different parts of the loss distribution. The analyses based on GCorr Macro can be applied to various types of credit portfolios, from global corporates to U.S. residential mortgages.

Risk integration is another application of GCorr Macro discussed in this paper. GCorr Macro links systematic credit risk factors to certain macroeconomic variables that represent market or commodity risk factors, allowing for aggregation and allocation of credit, market, and other risks using a factor-based model.

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1. Introduction

Credit correlations are typically best described through factor models, with factors that characterize the credit environment.¹ An example is the Moody's Analytics GCorr factor model, which defines sets of systematic credit risk factors for several asset classes: corporates, U.S. commercial real estate (CRE) exposures, and U.S. retail exposures. Corporate factors are based on firms' asset returns, segmented by industry and country classifications. The model recognizes that the credit environment can be specific to those classifications. For example, the recent crisis was particularly impactful for Europe and the U.S., but did not seem to affect China as much. Alternatively, the recent crisis hit financial institutions particularly hard, while the technology downturn in the early 2000s severely struck the telecom and software industries. While useful in describing credit correlations, GCorr factors can be abstract and are not as intuitive as macroeconomic variables when communicating credit portfolio results throughout an organization. GCorr Macro, which links GCorr systematic credit risk factors to macroeconomic variables, fills this void.

In this paper, we describe applications of GCorr Macro within the RiskFrontier software, a Moody's Analytics framework for analyzing credit portfolios.² These applications include stress testing, reverse stress testing, and risk integration. The main purpose of the applications is to address several economic needs and some regulatory initiatives

Within the context of this paper, macroeconomic variables can include economic activity variables (e.g., GDP, Unemployment Rate), financial market variables (e.g., Stock Market Index, Interest Rates), price indexes (e.g., House Price Index, Oil Price) and others. Note that some of the macroeconomic variables can serve as risk factors themselves; for example, Stock Market Index and Interest Rates can be interpreted as market risk factors. The 2013 version of GCorr Macro contains 62 macroeconomic variables, including variables from the U.S., Eurozone, the UK, Canada, other developed economies, and several emerging countries. It is worth pointing out that GCorr Macro is a flexible model that can be customized to include additional macroeconomic variables meeting specific needs of a financial institution.

GCorr is a multi-factor model that describes the correlation structure across a wide range of credit entities, including large corporates, U.S. CRE, private firms including small to medium-sized enterprises (SMEs), U.S. retail, and sovereigns.³ It is important to emphasize that the reason for extending GCorr to include macroeconomic variables is not to increase explanatory power as far as describing credit correlations, given that the systematic credit factors are already designed to explain credit correlations. To consider one example of a typical macroeconomic variable, GDP is a broad brush measure and does not provide insight regarding the nature of a credit crisis and which sectors of the economy were affected most. This is particularly true when we recognize that the turn of the century recession in the U.S. was associated with the technology sector, and the more recent crisis was related to the financial and retail sectors. With that in mind, GDP is an intuitive measure that is useful as a communication vehicle.

Let us describe at a high level the idea of using GCorr Macro within the RiskFrontier software. In each trial, the RiskFrontier Monte Carlo simulation engine generates draws of the GCorr systematic credit risk factors and the counterparty-specific factors. The value of each instrument in a credit portfolio at a future horizon is determined by combining draws of the relevant systematic and specific factors. The total of instrument values gives the portfolio value on the horizon, which can be converted to the portfolio loss amount between analysis date and horizon. The portfolio losses across all trials constitute an estimate of the loss distribution. We can interpret this simulation as generating trials across "all possible economic scenarios" over the next year, without focusing on a specific scenario. We therefore refer to it as the unconditional simulation. Similarly, we refer to the distribution as the unconditional distribution and to the relevant statistics (such as expected loss) as unconditional statistics. Because GCorr Macro relates the systematic credit risk factors to macroeconomic variables, we can associate each trial with values of macroeconomic variables. In this way, the RiskFrontier software can provide us with values of both portfolio losses and macroeconomic variables across all trials. In this paper, we refer to this list as the Monte Carlo simulation output. It allows us to study relationships between losses and macroeconomic variables, which is useful for understanding various dynamics in the portfolio, and when selecting macroeconomic variables for a scenario. We can quantify these relationships, which enables us to determine to what degree a given set of macroeconomic variables spans the systematic risk of a credit portfolio. In addition, we can filter the simulation output in two ways:

- » Select trials with certain values of macroeconomic variables and then study the corresponding losses (stress testing)
- » Select trials with certain levels of losses and study the corresponding values of macroeconomic variables (reverse stress testing).

¹ See "Factor Models for Portfolio Credit Risk" (Schönbucher, P. J., 2000).

² For details, see "An Overview of Modeling Credit Portfolios" by Levy (2008).

³ For more information, see "Modeling Credit Correlations: An Overview of the Moody's Analytics GCorr Model" by Huang et al. (2012).

As we discuss in this paper, it is not convenient to conduct the stress testing by filtering trials because there may not be a sufficient number of trials to select for a given scenario. For this reason, the RiskFrontier software contains a conditional simulation feature, which utilizes GCorr Macro to determine the conditional distribution of the systematic credit risk factors given a macroeconomic scenario. It then generates draws from this conditional distribution and produces a distribution which we refer to as conditional distribution because it assumes a specific economic scenario. It is worth highlighting that both the unconditional and conditional distributions are determined using all RiskFrontier features: the losses reflect credit downgrades as well as defaults; the various RiskFrontier valuation techniques are employed; and the simulation can account for PD-LGD correlations.

Outputs of both the stress testing and reverse stress testing analyses provide insights into the risk of the credit portfolio. The stress testing analysis produces not just the stressed expected loss, but the entire loss distribution. This allows us to determine various statistics under the scenario, such as dispersion or the probability that the losses exceed a certain threshold under the scenario. Overlaying the conditional and unconditional distributions enables one to understand how much the scenario affects the given portfolio. We explain in this paper that the scenario impact is a combination of two effects: severity of the scenario and the degree to which the selected macroeconomic variables span the systematic risk of the portfolio. Reverse stress testing helps identify the possible scenarios that are associated with a certain level of losses, with losses in the tail of the unconditional distribution being typical of most interest. We emphasize that in most cases, a loss level is associated with an entire range of scenarios as opposed to just one scenario. Similar to stress testing, results of reverse stress testing can be attributed to a mix of effects, from portfolio characteristics to relationships between macroeconomic variables and the factors driving the systematic factors affecting the portfolio. All these analyses are applicable to all types of portfolios, from small concentrated portfolios to large diversified portfolios. If a portfolio includes various asset classes such as global corporates and residential mortgages, the diversification effects are accounted for in the resulting loss distributions through the RiskFrontier factor structure.

A recent interest in stress testing in the United States is related to CCAR⁴, a regulatory initiative by the Federal Reserve. The stress testing within the CCAR framework requires calculation of stressed expected losses (EL) over multiple quarters. Given that the RiskFrontier software is a single period tool, it is not feasible to use it directly for this task. However, we utilize the GCorr Macro framework within the RiskFrontier software to develop a system of analytical calculations which produce stressed probability of default, stressed loss given default, and stressed EL over multiple quarters. The analytical calculations are possible because the statistics of interest in CCAR are expected values only. We implemented the calculations in an application called GCorr Macro Stressed EL Calculator. A detailed description of the analytical methodology can be found in the paper by Pospisil, et al. (2014).

GCorr Macro can also help facilitate the risk integration processes. During the (unconditional) simulation, the RiskFrontier software with GCorr Macro can generate draws of systematic credit risk factors and macroeconomic variables, which can be interpreted as market risk factors, commodity risk factors, or risk factors of other type. After these risk factors are simulated, ALM risk systems or any risk system driven by these factors can be used to quantify relationship between losses on relevant portfolios (such as a stock portfolio or a portfolio of Treasury bonds) and the factors. When this relationship has been established, it can be employed to determine losses on the market or commodity risk driven portfolios for each trial. Because the RiskFrontier software provides losses on credit risk portfolios for each trial, this approach allows us to approximate the joint distribution of losses on portfolios of various risk types (credit, market, etc.), and thus aggregate capital across risk types and allocate the capital to the individual portfolios. This approach can be considered one of the top-down risk integration methods. The paper by Chen et al. (2010) explains how the approach fits into the range of various risk integration methods.

We summarize all the GCorr Macro applications in Table 1.

The remainder of the document is organized as follows:

- » Section 2 introduces the GCorr Macro model, describes unconditional and conditional Monte Carlo simulation with GCorr Macro, and illustrates what kind of information the simulation output provides.
- » Section 3 details how stress testing and reverse stress testing analyses are conducted in the RiskFrontier framework with GCorr Macro. The section presents several examples and discusses interpretation of the results.
- » Section 4 describes risk integration with GCorr Macro.
- » Section 5 concludes this paper.
- » Appendix A presents an alternative on modeling the relationship between portfolio losses and macroeconomic factors.
- » Appendix B contains the list of the pre-selected macroeconomic variables for each of the five portfolios studied.

⁴ See "Comprehensive Capital Analysis and Review: Methodology and Results for Stress Scenario Projections (CCAR)" (Board of Governors of the Federal Reserve System 2012).

- » Appendix C discusses the estimation of the GCorr Macro parameters, with emphasis on mappings between annual changes in macroeconomic variables and standard normal factors generated in simulation.
- » Appendix D explains how we validate the GCorr Macro model over a one-year horizon.

Table 1

Summary of GCorr Macro applications⁵

TYPE OF ANALYSIS	USE CASE	TOOL	REFERENCE
Relationships between portfolios losses and macroeconomic variables	Understanding the relationship between portfolio losses and various sets of macroeconomic variables. Providing guidance in selecting macroeconomic variables for stress testing a portfolio. Quantifying the extent to which macroeconomic variables span portfolio risk.	The RiskFrontier software Unconditional simulation	Section 3.1 and Section 3.2
Single period simulation based stress testing	Generating conditional portfolio loss distribution under a scenario over a single period. The losses incorporate migrations and valuation methodologies from the RiskFrontier software. Generating statistics for the conditional loss distribution – percentiles, stressed EL, probabilities that conditional loss exceeds a given thresholds (such as economic capital). Comparing single period conditional and unconditional portfolio loss distributions.	The RiskFrontier software Conditional simulation	Section 3.3
Reverse stress testing	Identifying economic scenarios that are associated with portfolio losses in a given range, for example in the tail of the distribution. Analyzing which GCorr systematic credit risk factors and macroeconomic variables are most closely associated with the tail event.	The RiskFrontier software Unconditional simulation	Section 3.4
Risk integration	Aggregating capital across portfolios of various risk types (credit, market, and others) and allocating the overall capital to risk types.	The RiskFrontier software Unconditional simulation	Section 4
Multi period analytical stress testing	Generating multi period instrument and portfolio level stressed PD, LGD, and EL under a scenario. Designed for CCAR-style stress testing.	GCorr Macro Stressed EL Calculator	Paper by Pospisil et al. (2014)

2. Using GCorr Macro within the RiskFrontier Framework

In this section, we first provide an overview of GCorr Macro and then describe how GCorr Macro is used in the RiskFrontier software. Specifically, we focus on the role of GCorr Macro in the Monte Carlo simulation of credit risk factors as it relates to the applications outlined in Section 1.

2.1 What is GCorr Macro?

The GCorr Macro model links the systematic credit risk factors of the Moody's Analytics GCorr model to macroeconomic variables. Before we discuss how GCorr Macro fits into the RiskFrontier framework, let us summarize the main components of that framework. The RiskFrontier software uses a bottom-up approach to estimate portfolio loss distribution at a future time horizon. Such an approach begins with modeling the credit quality of an individual borrower. A borrower's credit quality is affected by a systematic factor and an idiosyncratic factor. The systematic factor represents the state of the economy and summarizes all the relevant systematic risks that affect the borrower's credit quality. Sensitivity of a borrower's credit quality to the systematic factor is given by the R-squared value (RSQ).

In the RiskFrontier application, the systematic factors are typically modeled using GCorr, a multi-factor model for credit correlations. GCorr defines the systematic factor as a weighted combination of geographical and sector risk factors for a given asset class (corporates, U.S. CRE exposures, or U.S. retail exposures). The weights are unique to each borrower, depending on borrower's characteristics.⁶ The idiosyncratic factor represents the borrower-specific risk that affects the borrower's credit quality. Credit quality changes of two borrowers are correlated whenever both are exposed to correlated systematic factors. By

⁵ Another GCorr Macro application not discussed in the paper is limit setting. Based on stressed expected losses, a limit can be set on, for example, a particular industry or name.

⁶ GCorr contains in total 245 factors that can be broken down into three asset classes: 110 corporate factors (49 country factors and 61 industry factors) constituting the GCorr Corporate model, 78 U.S. commercial real estate factors (73 MSA factors and five property type factors) constituting the GCorr CRE model, and 57 U.S. retail factors (51 state factors and six product type factors) constituting the GCorr Retail model.

construction, the systematic factors are independent of idiosyncratic factors and both are modeled with a standard normal distribution.

The RiskFrontier Monte Carlo simulation engine generates random draws of the systematic and idiosyncratic factors from a standard normal distribution. Each draw of the factors implies certain credit qualities of all borrowers at a future horizon. Default of a borrower is defined as the event when its credit quality falls below a certain level. The RiskFrontier valuation framework is then applied to determine the value of every instrument based on the credit quality of the corresponding borrower at horizon. The value depends on several input parameters, such as probability of default (PD), loss given default (LGD), credit migration matrix, and so forth. A portfolio value at horizon is given by the sum of the instrument values. Therefore, a distribution of the portfolio values at horizon can be estimated by running a large number of these simulations and calculations. The value distribution can then be converted to the distribution of portfolio losses between the analysis date and horizon. We refer to this simulation as unconditional simulation, and to the loss distribution as unconditional distribution. This is because drawing the factors from standard normal distribution should be interpreted as simulating possible states of the economy at horizon, without considering a specific economic scenario. We have depicted the RiskFrontier framework in the top part of Figure 1.

The losses generated by the RiskFrontier software account not only for defaults, but also for changes in instrument values due to worsening credit qualities of borrowers. In addition, the losses of defaulted instruments can be linked to systematic factors through the Moody's Analytics PD-LGD correlation model.⁷

Apart from the standard portfolio statistics, the RiskFrontier software produces various instrument level characteristics: EL, unexpected loss (UL) defined as instrument value standard deviation, and two statistics describing the instrument's relation to the rest of the portfolio, namely the risk contribution (RC) and tail risk contribution (TRC).⁸

Figure 1 RiskFrontier framework with GCorr Macro

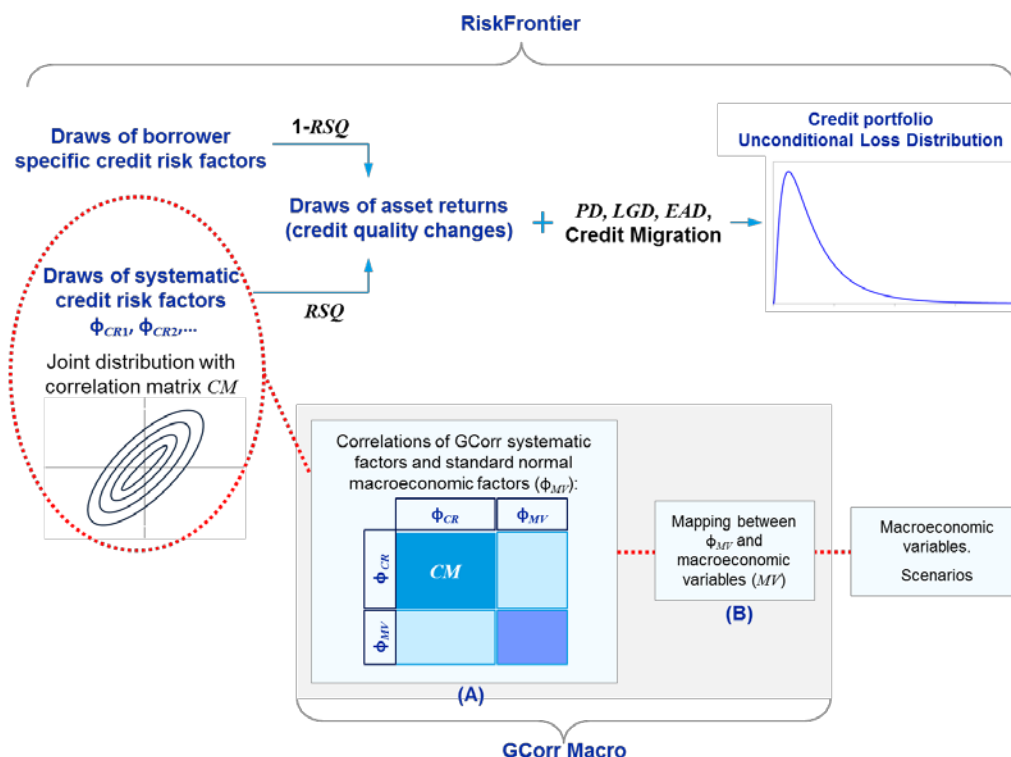


Figure 1 also illustrates the role of the GCorr Macro model within the RiskFrontier application. We comment first on how we transform macroeconomic variables when we include them into GCorr Macro. We need to correlate the macroeconomic variables with other factors and use historical data to estimate their distributions. This means that macroeconomic variables in this paper must be transformed into their stationary versions; we need to consider returns on Stock Market Index, as opposed to Stock

⁷ For details on the Moody's Analytics PD-LGD correlation model, see the paper by Levy and Hu (2007).

⁸ Definitions of these statistics can be found in the Moody's Analytics document "Modeling Credit Portfolios, RiskFrontier Methodology".

Market Index levels, or changes in Unemployment Rate, as opposed to Unemployment Rate levels. Throughout this paper, we use the term “macroeconomic variables” to refer to these stationary transformations.

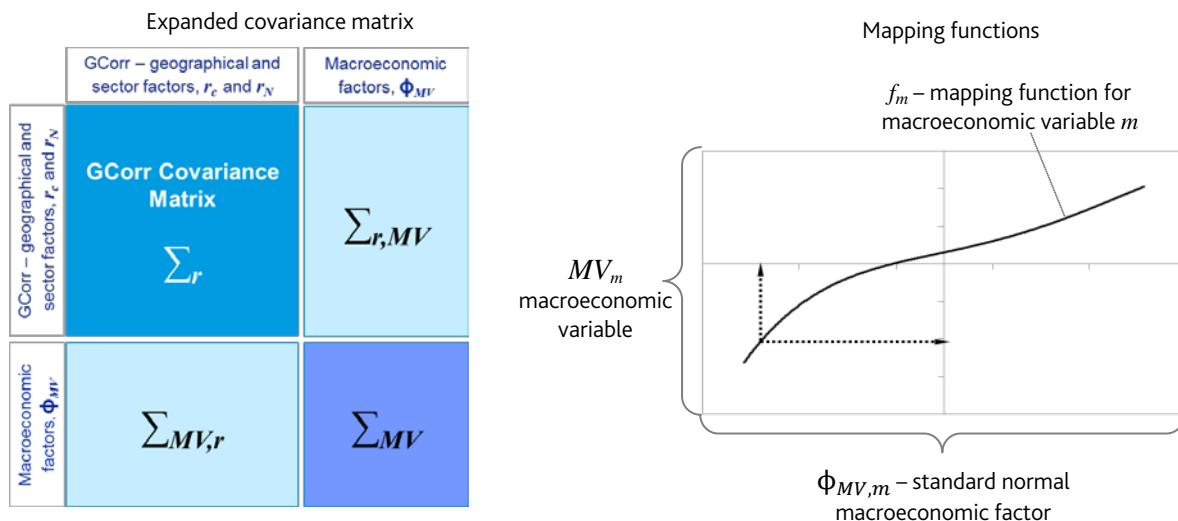
GCorr Macro captures the relationship between GCorr systematic credit risk factors ϕ_{CR} (CR–credit risk) and macroeconomic variables MV ⁹ in two steps:

- » First, the GCorr systematic factors ϕ_{CR} are correlated with standard normal macroeconomic factors ϕ_{MV} through a correlation matrix denoted by (A) in Figure 1.
- » Second, mapping functions transform values of the standard normal macroeconomic factors ϕ_{MV} to the corresponding values of observable macroeconomic variables MV . The mapping functions are represented by box (B) in Figure 1. The purpose of the mapping functions is to accommodate the fact that macroeconomic variables (GDP growth or returns on a Stock Market Index) do not have empirically normal distribution. However, although we transform marginal distributions of macroeconomic variables, their joint distribution and relationship to GCorr factors is still described through a Gaussian copula function.

Horizon is an important setting for the credit portfolio analysis with GCorr Macro. After the analysis horizon is selected, the mapping functions estimated for that horizon must be used. For example, for a one-year horizon, we need to utilize mappings reflecting one year changes in macroeconomic variables. We have estimated mapping functions for two horizons: one year (relevant for standard RiskFrontier analysis) and one quarter (relevant for CCAR style stress testing in the GCorr Macro Stressed EL Calculator). We discuss this topic in detail in Appendix C, where we explain the role of horizon in estimation and uses of the mapping functions and the correlation matrix.

We emphasize that the GCorr Macro model does not change the loadings of borrowers' asset returns to the GCorr systematic and idiosyncratic credit risk factors. In other words, borrower asset returns are linked to macroeconomic variables only through their loadings to the existing GCorr factors.

Figure 2 Components of GCorr Macro: the expanded covariance matrix and mapping functions



To summarize, GCorr Macro is defined by two sets of parameters: the correlation matrix linking GCorr systematic credit risk factors ϕ_{CR} with standard normal macroeconomic factors ϕ_{MV} and the mapping functions relating values of standard normal macroeconomic factors ϕ_{MV} to values of observable macroeconomic variables MV . In Figure 2, we show how these parameters are specified in practice. The relationship between ϕ_{CR} and ϕ_{MV} is given by the expanded covariance matrix that links the

⁹ The notation MV refers to macroeconomic variables after a stationarity transformation.

geographical and sector factors (r_C and r_N) to ϕ_{MV} .¹⁰ While the top left block of the matrix, Σ_r , comes from the GCorr model, the remaining blocks are estimated as a part of GCorr Macro. Since the systematic credit risk factors ϕ_{CR} are a linear combination of r_C and r_N , the expanded covariance matrix from Figure 2 implies the expanded correlation matrix (A) from Figure 1. In Figure 2, we also display a mapping function f_m for the m -th macroeconomic variable MV_m . We note that each macroeconomic variable has its own mapping function. In practice, the mapping functions are specified using lookup tables where the value of the standard normal macroeconomic factor can be looked up based on the value of the macroeconomic variable.¹¹

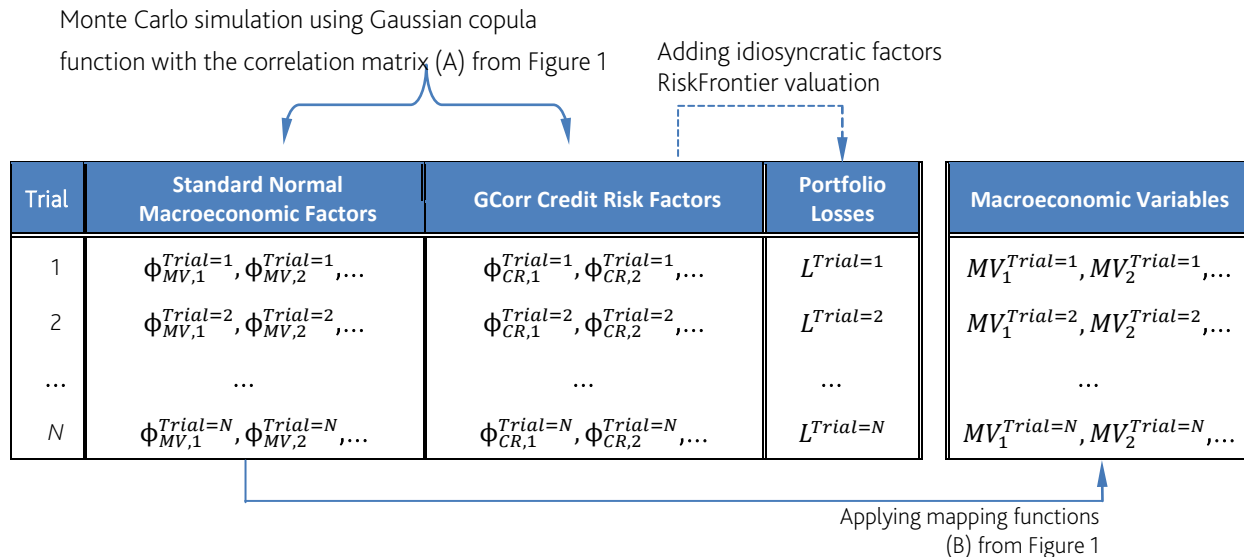
2.2 Monte Carlo Simulation Output

Many of the analyses presented in the paper (Section 3 and Section 4) are based on a RiskFrontier Monte Carlo simulation output. We have displayed the format of the output in Figure 3. It contains trial-by-trial values of the GCorr systematic credit risk factors and macroeconomic factors. The marginal distributions of simulated values of both sets of factors are standard normal and their joint distribution is given by the Gaussian copula model with correlation matrix (A) from Figure 1. In each trial, the RiskFrontier software combines the simulated values of the systematic factors and borrower level idiosyncratic factors to calculate instrument values, portfolio value, and portfolio loss for that trial. The portfolio loss is also part of the Monte Carlo simulation output. The interpretation of losses depends on the models and methods employed to conduct the calculations. For example, if the PD-LGD correlation model is used during the calculations, losses in the simulation output reflect systematic risk in recovery. If credit migration is used, the losses include not only defaults, but also decreases in instrument values due to borrowers' credit quality deterioration.

Values of standard normal macroeconomic factors from each trial can be translated into observable macroeconomic variables using the mapping functions. It is important to ensure that the horizon of the mapping functions is consistent with the horizon used to calculate the losses. In a typical RiskFrontier use case, the horizon considered would be one year.

The Monte Carlo simulation output in is the result of unconditional simulation because the factors were generated from a standard normal distribution. In the RiskFrontier framework, this represents a range of all possible scenarios, with their corresponding probabilities, between analysis date and horizon. No conditioning on a specific scenario is involved.

Figure 3 RiskFrontier Monte Carlo simulation output



¹⁰ A factor table is an alternative way to specify the relationship between r_C , r_N , and ϕ_{MV} . By a factor table we mean a factor model which defines common orthogonal factors that explain co-movements in r_C , r_N , and ϕ_{MV} :

$$r_{C,j} = \sum_{n=1}^N \beta_{j,n} f_n^{Common} + \varepsilon_j, \quad r_{N,j} = \sum_{n=1}^N \beta_{j,n} f_n^{Common} + \varepsilon_j, \quad \phi_{MV,j} = \sum_{n=1}^N \beta_{j,n} f_n^{Common} + \varepsilon_j$$

$$cov(f_n^{Common}, f_k^{Common}) = 0, \quad cov(f_n^{Common}, \varepsilon_j) = 0, \quad cov(\varepsilon_j, \varepsilon_k) = 0, \quad var(f_n^{Common}) = \sigma_{f,n}^2, \quad var(\varepsilon_j) = \sigma_{\varepsilon,j}^2$$

Note that this factor model implies the expanded covariance matrix in Figure 2.

¹¹ If a value of the macroeconomic variable that we want to map falls between two tabulated values, we use interpolation.

A practical question is how many trials are needed for efficient analyses of the Monte Carlo simulation output. When calculating Economic Capital (EC) at a 10bps target probability, the necessary number of trials is hundreds of thousands and ideally millions.¹² In analyses that involve using the simulation output to fit and understand relationships between portfolio losses and credit risk or macroeconomic factors, it is not necessary to generate too many extreme events and therefore the number of trials can be lower – such as tens of thousands or 100,000.¹³ On the other hand, if we want to carry out an analysis by selecting a certain subset of trials, for example only the trials with extreme losses, the simulation must be conducted with enough trials such that there are sufficient trials even after the selection. Similarly to the EC case, if we want to select losses exceeding the 10bps quantile, we may need close to a million of trials, or more, from the simulation.

As Figure 3 shows, the simulation output provides portfolio level losses. Due to technology constraints it might not be possible to analyze trial-by-trial losses on individual instruments. First, technological requirements to process instrument level data depend on the size of the data, which is given as number of instruments times number of trials.¹⁴ Thus, outputting instrument level information for millions of trials if a portfolio contains hundreds of thousands of instruments might not be feasible unless the organization is prepared to meet the big technology demands. However, it is possible to conduct instrument level analysis in specific cases. One case is addressing how individual instruments react to a scenario, which can be determined based on the output of conditional simulation as we discuss in Section 2.3. Another case is individual instrument behavior in the portfolio loss distribution tail,¹⁵ such as the question of which instruments have defaulted or how they are related to macroeconomic variables. In this case, we can generate instrument-level information only for trials in which portfolio loss is in the tail. This is feasible as long as the number of these trials is limited to, for example, 1000. If the objective is to have instrument level losses across all trials from the unconditional simulation, then either the portfolio size must be very limited or it is possible to utilize the RiskFrontier Deal Analyzer® (RFDA), which can produce instrument level trial-by-trial losses for one instrument at a time.

If the analysis is to be conducted for a large, diversified portfolio (such as a portfolio across asset classes or across geographies), it might be convenient to run the Monte Carlo simulation not only for the aggregate portfolio, but also for (non-overlapping) sub-portfolios. The Monte Carlo simulation output can then be used to perform analysis for the aggregate portfolio and the (non-overlapping) sub-portfolios. Having sub-portfolio losses allows us to understand how strongly the sub-portfolios are associated with macroeconomic variables, as well as how interactions and potential diversification of the sub-portfolios effect impact the dynamics of the aggregate portfolio.

2.3 Conditional Simulation

The objective of conditional simulation is to estimate distribution of portfolio losses given a macroeconomic scenario. In theory, the distribution could have been estimated from the Monte Carlo simulation output in Figure 3, by selecting only the trials representing the given scenario. While this approach is feasible if the scenario is defined using one macroeconomic variable and is not too extreme relative to the number of trials used for the simulation, it is not practical for typical stress testing analyses. In such analyses, multiple macroeconomic variables are usually considered, and for adverse scenarios, the variables are assumed to have extreme values. In this case, there might be very few trials representing this scenario even if the number of trials used in the simulation is large.¹⁶ For this reason, we developed the conditional simulation to allow for an efficient estimation of the conditional loss distribution.

An important part of a stress testing exercise is deciding which macroeconomic variables to use when defining a scenario. In effect, any stress testing model, including GCorr Macro, links portfolio losses to multiple macroeconomic variables. Therefore, the role of a macroeconomic variable is determined by its marginal impact, which is determined not only by the relationship of that variable to the portfolio, but also by interactions among the macroeconomic variables. We must account for this consideration when selecting macroeconomic variables to define a scenario. A reasonable scenario should be defined using a set of macroeconomic

¹² When calculating EC in practice, it is possible to use the Importance Sampling (IS) technique in the RiskFrontier software to reduce the necessary number of trials. IS modifies the joint distribution of GCorr systematic credit risk factors.

¹³ The values provided in this discussion should be considered examples that apply to stylized portfolios. The choice of the number of trials is a portfolio specific decision and certain portfolios might require more trials for a reasonable analysis, while others less.

¹⁴ We note that the portfolio level simulation output does not face this challenge; it contains data of dimension “one” (portfolio loss) times the number of trials. Even if the number of trials is several million, it is possible to process such a file in software packages like SAS or MATLAB.

¹⁵ Note that this analysis is very closely linked to the concept of Tail Risk Contribution (TRC) – the expected instrument loss given a certain range of portfolio level losses. The RiskFrontier software currently provides TRC measures without Monte Carlo simulation output.

¹⁶ The expected number of trials representing the scenario can be expressed as $N^{Sim} \cdot p^{Scenario}$, where N^{Sim} is the number of trials used in the simulation and $p^{Scenario}$ is the probability of a scenario. To understand the dynamics of $p^{Scenario}$, let us consider scenario which prescribes that n macroeconomic factors have values between -2.1 and -1.9 standard deviations. We assume that the macroeconomic factors have pair-wise correlations of -2 . For two factors ($n = 2$), the probability of the scenario is $p^{Scenario} = 0.051\%$, however for four factors ($n = 4$), it drops to $p^{Scenario} \approx 0.00037\%$ because we added further restrictions in the scenario definition. In this case, about thirty million trials would be needed to obtain about one hundred trials representing the scenario – and one hundred trials is not sufficiently large to describe the conditional loss distribution well.

variables that has an overall strong association with the portfolio. In addition, each variable from that set must have an economically meaningful marginal impact on the portfolio. As these requirements suggest, the variable selection procedure is highly portfolio-specific. We discuss variable selection in detail in Section 3.2. In this section, we assume that the macroeconomic variables that define a scenario for a portfolio were selected.

The conditional simulation works in the following way. First, we assume that there are N_{MV} macroeconomic variables included in an economic scenario. Each of those selected variables is assigned a specific value $MV_m^{Scenario}$ ($m = 1, \dots, N_{MV}$), which represents the hypothetical change in the value of the macroeconomic variable between the analysis date and the horizon of interest (typically, but not exclusively, one year). For example, if the third ($m = 3$) macroeconomic variable of a scenario is the U.S. Stock Market Index and the scenario hypothesizes that the index will drop by 0.5 over the next year (i.e., the log return will be -0.5), then we have $MV_3^{Scenario} = -0.5$. The scenario value of each macroeconomic variable, $MV_m^{Scenario}$, must be converted to the corresponding value of the standard normal macroeconomic factor, $\phi_{MV,m}^{Scenario}$, using the specific mapping function for this variable, f_m (an example of the mapping function is in the right hand chart of Figure 2):

$$\phi_{MV,m}^{Scenario} = f_m(MV_m^{Scenario}), \quad m = 1, \dots, N_{MV} \quad (1)$$

We denote the column vector of scenario values of the standard normal macroeconomic factors as

$$\Phi_{MV}^{Scenario} = [\phi_{MV,1}^{Scenario}, \dots, \phi_{MV,N_{MV}}^{Scenario}]^T.$$

The next step uses the expanded covariance matrix from Figure 2 in order to determine the conditional distribution of all GCorr geographical and sector factors (vectors \mathbf{r}_C and \mathbf{r}_N) given the scenario. Only the scenario macroeconomic factors are kept in the matrix; the rest of the macroeconomic factors are removed. The conditional distribution is normal and its parameters can be written in terms of blocks of the expanded covariance matrix:¹⁷

$$\mathbf{r}_C, \mathbf{r}_N | \Phi_{MV}^{Scenario} \sim N(\Sigma_{r,MV} \times \Sigma_{MV}^{-1} \times \Phi_{MV}^{Scenario}, \Sigma_r - \Sigma_{r,MV} \times \Sigma_{MV}^{-1} \times \Sigma_{MV,r}) \quad (2)$$

Any GCorr systematic credit risk factor ϕ_{CR} is a linear combination of the geographical and sector factors: $\phi_{CR} = \mathbf{w}^T \times \begin{bmatrix} \mathbf{r}_C \\ \mathbf{r}_N \end{bmatrix}$. The vector \mathbf{w} contains the weights of the systematic factor to the geographical and sector factors. These weights are specific for each borrower; if a borrower is a corporate operating in the U.S. Auto industry, then the weights for this borrower are defined so that its systematic factor is a sum of the U.S. country factor and the Auto industry factor. As a result, we can derive the conditional distribution of ϕ_{CR} given the macroeconomic scenario:¹⁸

$$\phi_{CR} | \Phi_{MV}^{Scenario} \sim N\left(\underbrace{s \times \mathbf{w}^T \times \Sigma_{GCorr,MV} \times \Sigma_{MV}^{-1} \times \Phi_{MV,t}^{Scenario}}_{E[\phi_{CR,t} | \Phi_{MV}^{Scenario}]}, 1 - \rho_{CR,MV}^2\right) \quad (3)$$

Let us make two comments about the conditional distribution in formula (3):

- » The conditional expected value, $E[\phi_{CR,t} | \Phi_{MV}^{Scenario}]$, is a linear function of the scenario values of the standard normal macroeconomic factors.
- » The conditional variance is expressed using a parameter $\rho_{CR,MV}$, which represents the multivariate correlation of the systematic factor and the macroeconomic factor.¹⁹ If the correlation equals one, $\rho_{CR,MV} = 1$, then the macroeconomic variables selected for the scenario perfectly span the systematic risk of the borrowers with the systematic factor ϕ_{CR} . After conditioning on these variables, there is no residual variation in the systematic factor and therefore its conditional variance is

¹⁷ We use the standard formula for a conditional normal distribution. See "Multivariate Statistical Methods" by Morrison (2004).

¹⁸ Scaling factor s ensures that the unconditional distribution of the custom index is standard normal:

$$s = \frac{1}{std(\mathbf{w}^T \times \begin{bmatrix} \mathbf{r}_C \\ \mathbf{r}_N \end{bmatrix})}$$

¹⁹ In a hypothetical regression context, ρ^2 would be the R-squared value after regressing systematic credit risk factor on standard normal macroeconomic factors selected for the scenario.

zero ($1 - \rho_{CR,MV}^2 = 0$). The lower the multivariate correlation, the more variance is left in the systematic factor after conditioning on the macroeconomic variables.

After the conditional distributions of systematic factors for all counterparties are determined, the Monte Carlo simulation engine can generate random draws of the factors from these distributions. These draws are then combined with the draws of idiosyncratic factors²⁰ to obtain asset returns of individual counterparties. These asset returns then reflect the scenario through their systematic risk component. Note that the sensitivity of asset returns to the scenario is a combination of sensitivity of the asset return to ϕ_{CR} (given by RSQ) and sensitivity of ϕ_{CR} to the scenario (given by $\rho_{CR,MV}$). When the asset returns are generated in this way, the RiskFrontier software can employ its valuation techniques to calculate values of all instruments, and in turn portfolio loss, for each trial. The portfolio losses across all trials then constitute an estimate of the conditional loss distribution. We also refer to this simulation, which uses the conditional distribution of systematic factors, as conditional simulation.

We emphasize that in addition to changing the distribution of the systematic factors, all other RiskFrontier calculations are the same in conditional and unconditional simulation, as described here and in Section 2.2. Thus, the losses from the conditional simulation include all RiskFrontier features that can be used in unconditional simulation: mark-to-market losses due to credit deterioration, PD-LGD correlation, and so forth. This fact is important because it allows us to directly compare the conditional and unconditional loss distributions.

In addition to the conditional loss distribution, we are interested in various portfolio and instrument level statistics that describe behavior of the portfolio under the scenario. At the portfolio level, the informative summary statistics include the expected value of the conditional distribution (referred to as stressed expected loss or stressed EL), various percentiles of the distribution, its standard deviation (stressed unexpected loss or stressed UL), and probabilities that the loss will exceed a certain threshold under the scenario, such as economic capital. Because the conditional distribution is built from the bottom-up using instrument values, the RiskFrontier software can also provide instrument level characteristics.²¹ These include stressed EL, stressed PD, stressed LGD, stressed UL, and stressed risk contribution (stressed RC). Some statistics mentioned here can be calculated analytically without the use of simulation, such as the stressed PD and stressed LGD (see the paper by Pospisil et al. (2014)), or the stressed EL for loan-type instruments.

While the conditional simulation works for a scenario with multiple macroeconomic variables and a portfolio exposed to multiple systematic factors, we use a single variable and a single factor example to illustrate the steps of the calculations. We consider a portfolio of corporates in the U.S. oil industry that are exposed to one systematic credit risk factor – $\phi_{US,oil}$. We examine a scenario which is based on oil price as the single macroeconomic variable, and assumes that oil price drops by two standard deviation: $\phi_{\Delta OilPrice} = -2$. We note that this value can be obtained by mapping a hypothetical drop in oil price (such as a drop by 60% over a one year period) to the standard normal space.

We assume that the correlation between the systematic factor and the oil price factor is $\rho_{CR,MV} = 41\%$; in other words, an increase (decrease) in oil price tends to be associated with an increase (decrease) in credit qualities of firms in the industry, as economic intuition suggests. The effect of this scenario is shown in Figure 4. First, the negative oil price shock impacts the systematic factor distribution. In the absence of a scenario, this distribution is standard normal. Under the scenario, it stays normal, but its mean and variance change and their new values depend on the scenario ($\phi_{\Delta OilPrice}$) and the correlation of the systematic and macroeconomic factors ($\rho_{CR,MV}$). In the single variable setting, the calculation of the conditional mean and variance is straightforward. After the conditional factor distribution is determined, it is possible to draw values from this distribution and estimate the conditional loss distribution.

Figure 4. shows that as a result of the drop in oil price, the conditional distribution of the systematic factor has a negative expected value of -0.82, which represents an adverse systematic shock to the oil industry firms. Consequently, the conditional loss distribution puts more probability on the larger losses compared to the unconditional distribution. This is because an adverse shock to the industry means more defaults and more credit downgrades, which reduce values of exposures to the borrowers in the industry. All these effects lead to larger losses. The conditional standard deviation of the factor is 0.91, down from the unconditional standard deviation of one. The variation given by the conditional standard deviation represents systematic shocks to the oil industry unexplained by oil price movements. Although the correlation of the oil price factor and the systematic credit risk factor is positive, it is less than one because effects other than oil price movements impact the oil price industry, such as state of the economy. On average, an oil price drop leads to an adverse shock to the industry, which is in line with historical experience; for example, the recent financial crisis has seen a drop in oil price as a result of declining economic activity. However, this is not the case in every situation. . For example, if the price drop occurs as result of a new oil field discovery during a booming economy, the drop in price can be compensated by higher demand that can keep or even increase profits for the industry. This range of

²⁰ The assumption of GCorr Macro is that the macroeconomic variables are related to systematic factors driving counterparties' credit qualities, not to the idiosyncratic factors. Thus, the idiosyncratic factors are still drawn from standard normal distributions independently of the scenario.

²¹ The term "stressed" in this context and throughout this paper should be understood as "conditional on the macroeconomic scenario."

possibilities associated with an oil price drop is captured by the conditional factor distribution. For well-diversified portfolios, dispersion in the conditional loss distribution is driven by this variation in the conditional distribution of systematic credit risk factors. In the extreme case when the macroeconomic factor had a perfect correlation with the systematic factor, the conditional distribution of the systematic factor and of the portfolio loss would be concentrated in a single point and would therefore have no dispersion.

Figure 4 An example of conditional loss distribution with a single factor and a single macroeconomic variable. Assuming a portfolio of corporates operating in the U.S. Oil industry driven by the GCorr systematic factor $\phi_{US,Oil}$. A scenario is defined using a single standard normal macroeconomic factor representing oil price change: $\phi_{\Delta OilPrice}$.

Assumption: $\rho_{CR,MV} = \text{corr}(\phi_{US,Oil}, \phi_{\Delta OilPrice}) = 41\%$

Conditional distribution of the systematic factor: $\phi_{US,Oil} | \phi_{\Delta OilPrice} \sim N(\rho_{CR,MV} \cdot \phi_{\Delta OilPrice}, 1 - \rho_{CR,MV}^2)$

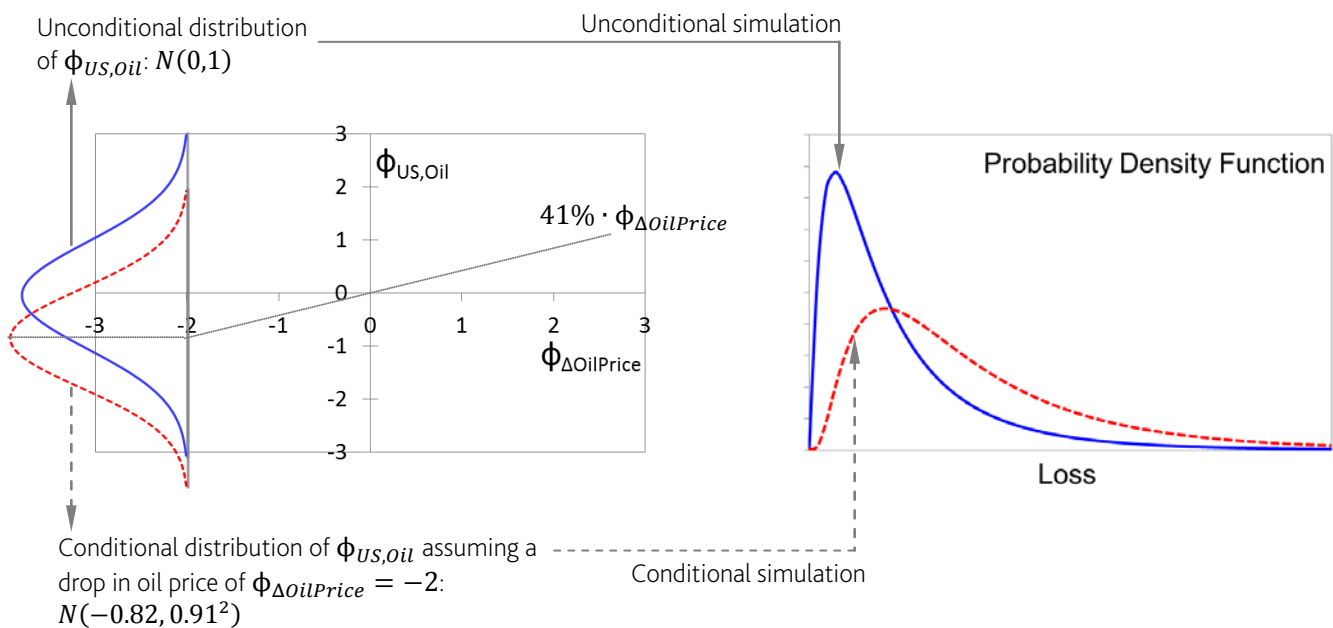
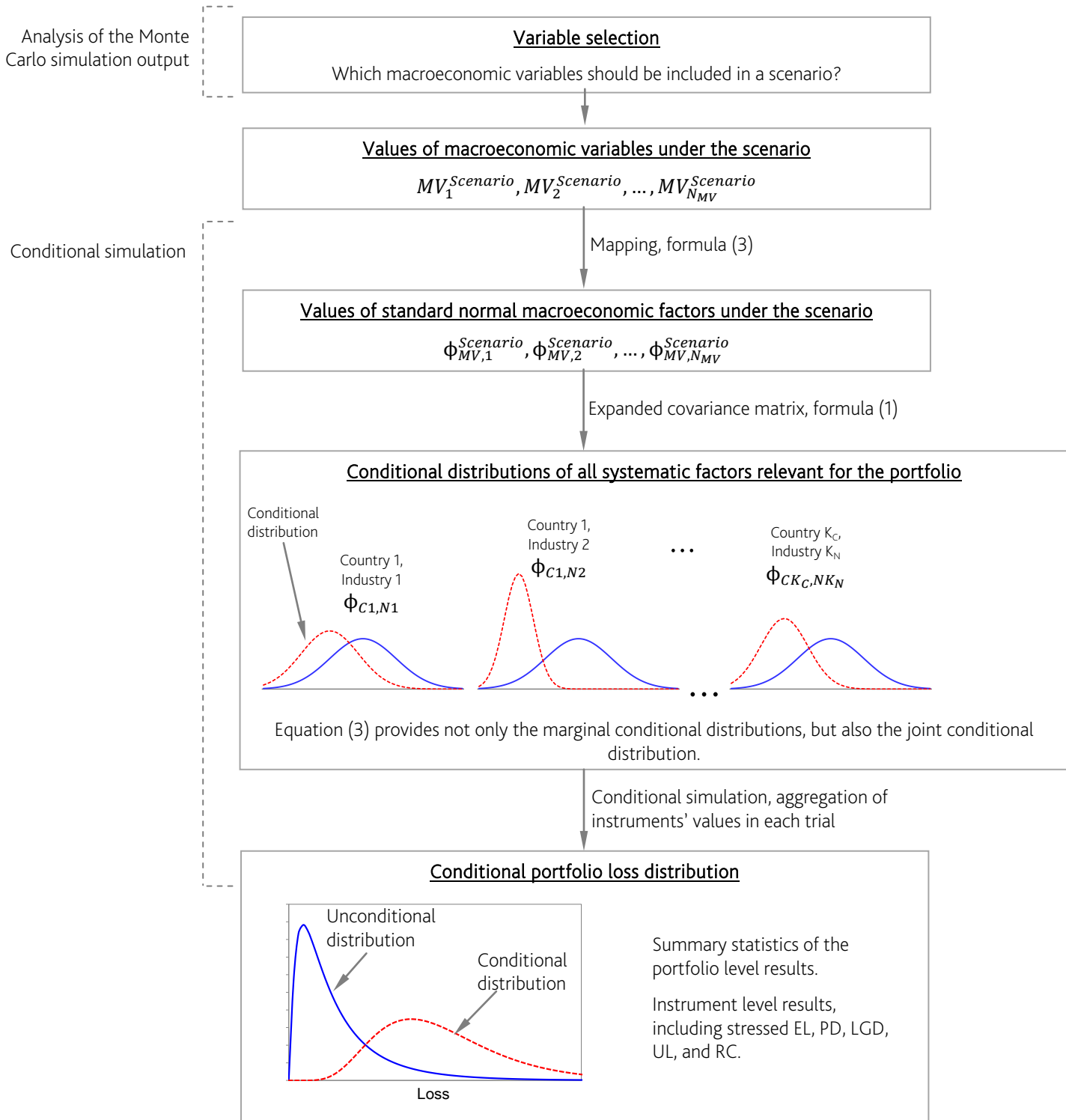


Figure 5 summarizes the steps of the conditional simulation, from variable selection through estimate of the conditional distribution.

Figure 5 Summary of the conditional simulation calculations



3. Stress Testing and Reverse Stress Testing with GCorr Macro

In this section, we define test portfolios and use them to illustrate how to conduct analyses with the Monte Carlo simulation output and to run a conditional simulation. Specifically, in Section 3.1 we focus on relationships between portfolio losses and individual macroeconomic variables, which provide insight into portfolio properties by indicating which variables are most closely linked to a portfolio, and which have weak or insignificant correlations with it. However, in stress testing we want to know how a set of macroeconomic variables impacts portfolio losses. To answer this question, we must select appropriate macroeconomic variables for a given portfolio. In Section 3.2 we present a variable selection procedure which relies on standard regression model techniques. After we select a set of appropriate macroeconomic variables, we can run conditional simulation, the results of which we show in Section 3.3.

We can also use the Monte Carlo simulation output for reverse stress testing analysis, as we demonstrate in Section 3.4. When presenting the examples, we put emphasis on the interpretation of the results, which is an important part of the analysis. Understanding questions such as why two highly correlated variables should not be used in conditional simulation, how the mapping functions impact the conditional distribution, or where the dispersion in the conditional distribution comes from are crucial to correctly interpret the results.

Table 2 introduces four test portfolios:²² the U.S. corporate portfolio, the Global corporate portfolio, the U.S. CRE portfolio, and the U.S. retail portfolio. In addition, we analyze the aggregate portfolio, which combines these four portfolios.

Table 2

Characteristics of test portfolios

PORTFOLIO	U.S. CORPORATE PORTFOLIO	GLOBAL CORPORATE PORTFOLIO	U.S. CRE PORTFOLIO	U.S. RETAIL PORTFOLIO
Composition	2,266 exposures to 1,133 U.S. corporate counterparties Diversified across industries (Auto, Oil, Telephone, Paper, Banks, etc.)	3,734 exposures to 1,867 non-U.S. corporate counterparties Location: Japan, Europe, Australia. Industry diversification.	130 U.S. commercial real estate exposures. Diversified across U.S. Metropolitan Statistical Areas (MSA)	250 homogeneous pools of U.S. residential mortgage exposures. Diversified across U.S. states.
Correlation model	GCorr 2013 Corporate	GCorr 2013 Corporate	GCorr 2013 CRE	GCorr 2013 Retail
Weighted* Avg. PD ²³	1.38%	1.56%	1.30%	79 bps
Weighted* Avg. LGD	40%	40%	25%	40%
Weighted* Avg. Asset R-squared value	36%	39%	34%	7.4%
Weighted* Avg. Recovery R-squared value	34%	34%	20%	20%
% of total exposure accounted for by the top 10 exposures	16%	8%	66%	39%
EL	70 bps	84 bps	41 bps	35 bps
Capital w.r.t. EL (10 bps target probability)	12.10%	10.81%	14.98%	8.06%
Weight in the aggregate portfolio	39%	29%	16%	16%

* Mark-to-market exposures are used as weights

²² IACPM (The International Association of Credit Portfolio Managers) has created a publicly available realistic corporate portfolio for testing purposes. The U.S. exposures from the IACPM portfolio constitute our U.S. corporate portfolio and the non-U.S. IACPM exposures constitute our Global corporate portfolio.

²³ Note that while the U.S. corporate portfolio contains a cross section of investment grade and high yield exposures, the U.S. retail portfolio includes mortgages that are predominantly related to prime borrowers, and therefore the low PD observed.

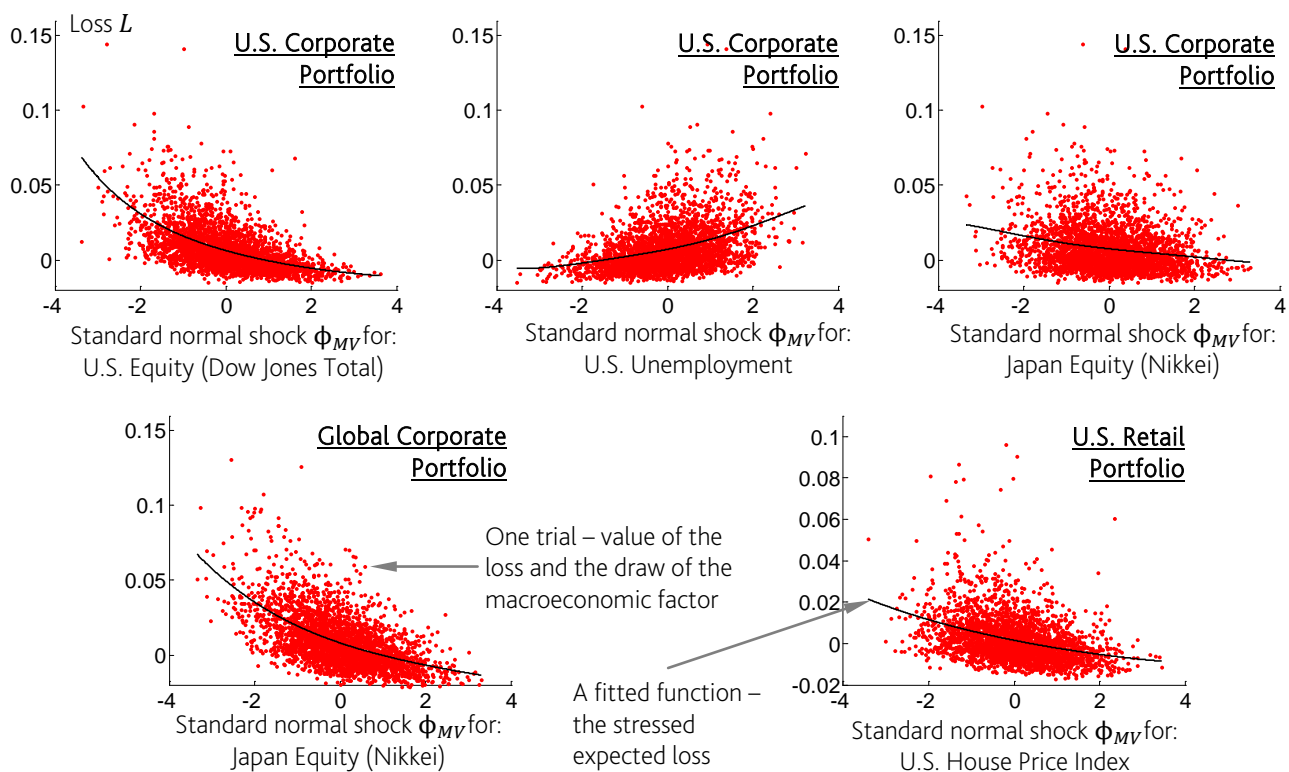
We run the RiskFrontier application for the four portfolios using unconditional simulation to obtain the Monte Carlo simulation output file in the form of Figure 3. We use the GCorr 2013 model, and choose one-year horizon for the RiskFrontier simulation. We consider losses due to counterparty credit deterioration as well as from defaults. The loss distribution also accounts for systematic risk recovery through the Moody's Analytics PD-LGD correlation model.

3.1 Univariate Analysis: Portfolio Losses versus Single Macroeconomic Variables

The first question we examine is how strongly portfolio losses are correlated with individual macroeconomic variables. Such an analysis will allow us to identify the macroeconomic variables that are relevant for the portfolios and that should be used in a stress testing exercise.

In Figure 6, we plot losses L on selected portfolios against draws of selected standard normal macroeconomic factors ϕ_{MV} across the first 3,000 trials in the Monte Carlo simulation output file. We are interested in the direction and strength of the relationship. As expected, losses on the U.S. corporate portfolio tend to be higher for negative shocks to the U.S. equity factor (decreases in the stock market index) and for positive shocks to the U.S. unemployment factor (increases in the unemployment rate). There are some variables where the direction of the relationship may go either way. For example, if we consider a corporate portfolio and the oil price, the relationship depends on which industries play a bigger role in the corporate portfolio: the oil industry (higher losses with increasing oil price) or the airline industry (lower losses with increasing oil price). To indicate the direction of the relationship, we fitted a function in each plot (black line). These functions can be interpreted as stressed expected loss on the portfolios given a certain shock to the selected macroeconomic factors.

Figure 6 Univariate analysis – portfolio losses versus individual standard normal macroeconomic factors using the Monte Carlo simulation output



The dispersion of losses around the lines representing stressed expected losses can be attributed to several effects. First, the macroeconomic variables do not span the systematic risk in the portfolios. Therefore, even if we condition on a shock in a macroeconomic variable, there is still a residual variation in the GCorr systematic factors (for example, due to some industry effects unrelated to the overall state of the economy) and this variation drives the dispersion in losses around the stressed expected loss. Second, there might be some counterparty idiosyncratic effects which are not diversified away. Because these effects are unrelated to the state of the economy, they contribute to variation in losses under a macroeconomic scenario. The idiosyncratic effects can be present in either small portfolios or in portfolios with substantial name concentration.

We can also compare the charts according to the strength of the relationship. The U.S. corporate portfolio shows a stronger relationship with the U.S. equity factor than with the Japan equity factor, which is ensured by the fact that GCorr U.S. corporate factors drive the systematic risk of the portfolio, and are more strongly correlated with the U.S. equity than the Japan equity factor.

The next part of the univariate analysis involves quantifying the relationship between losses and individual macroeconomic variables. The approach we employ here calculates the correlation between transformed losses and macroeconomic factors. Losses are transformed to linearize the relationship between losses and macroeconomic factors. In particular, we use the normal inverse transformation, i.e., $N^{-1}(L)$ ²⁴. We are not assuming that the transformed losses are normally distributed, but are looking for a transformation leading to a distribution as normal as possible. In Appendix A, we demonstrate that the relationship between the transformed losses and macroeconomic factors is close to linear. We also show that the results of this analysis are not sensitive to the choice of the normal inverse transformation. Moreover, it is important to note that the method proposed is portfolio specific. There might be portfolios for which applying the normal inverse transformation on losses is not convenient and additional options should be considered.

Table 3 shows the correlation between the transformed losses for each portfolio and some selected macroeconomic factors, i.e., $\text{corr}(N^{-1}(L), \phi_{MV})$. We find losses on the U.S. corporate portfolio to be highly correlated with U.S. financial markets variables such as U.S. Equity and U.S. VIX. Moreover, U.S. Unemployment together with U.S. CRE and House Price Indices exhibit high correlation with losses in the U.S. CRE and U.S. retail portfolio. Regarding Global portfolio losses, Japan Equity shows the highest correlation.

Table 3

Correlations of transformed losses with macroeconomic factors

	U.S. CORPORATE PORTFOLIO	GLOBAL PORTFOLIO	U.S. CRE PORTFOLIO	U.S. RETAIL PORTFOLIO	AGGREGATE PORTFOLIO
U.S. GDP	-42%	-43%	-43%	-29%	-47%
U.S. Unemployment	41%	51%	54%	35%	53%
U.S. Equity (Dow Jones Total)	-60%	-39%	-33%	-32%	-51%
U.S. House Price Index	-36%	-52%	-47%	-40%	-51%
U.S. CRE Index	-44%	-45%	-53%	-38%	-52%
U.S. VIX	47%	29%	15%	25%	37%
Japan Equity (Nikkei)	-23%	-59%	-15%	-20%	-38%

In this section, we presented the univariate analysis between losses and macroeconomic factors. This analysis helps identify the macroeconomic variables that are relevant for the portfolios and should be used in a stress testing exercise. The next section describes the methodology used to select a set of multiple macroeconomic variables for stress testing, and presents the results for the five portfolios analyzed.

3.2 Selecting Macroeconomic Variables for Stress Testing

One GCorr Macro application is stress testing using conditional simulation, which provides a distribution of portfolio losses for a given macroeconomic scenario. To perform this type of calculation, we must first select the appropriate set of macroeconomic variables for the analyzed portfolio (we sometimes refer to this set of variables as “model”). The set of selected variables is portfolio specific. For example, the selected variables for a portfolio of U.S. retail exposures might be different from those selected for a globally diversified portfolio. This section presents a variable selection procedure which relies on standard regression model techniques. We apply this procedure to the five portfolios studied in this paper and present the final sets of macroeconomic variables selected for each portfolio.

The selected set of macroeconomic variables should meet several criteria:

- » The set should statistically explain a sufficient portion of variation in the portfolio losses.
- » The model must be parsimonious in the sense that it must achieve high explanatory power with as few variables as possible to avoid multicollinearity and reduce noise in parameter estimates.

²⁴ To account for positive negative losses the transformation is actually calculated as $N^{-1}(\text{Loss}_{\text{trial}} - \min(\text{Loss}_{\text{trial}}) + 0.1\text{bps})$.

- » There should be an economic narrative as to why the selected variables are relevant for the given portfolio. This includes making sure that the direction and strength of the relationship between each variable in the model and portfolio losses is in line with economic intuition.

The statistical tool used in variable selection is the following regression model, estimated across Monte Carlo simulation trials from Figure 3:

$$N^{-1}(Loss_{Trial}) = \alpha + \sum_{k=1}^K \beta_k \phi_{MV,k,Trial} + \varepsilon_{Trial} \quad (4)$$

In this model, $Loss_{Trial}$, represents the loss on a portfolio in a given trial, and $\phi_{MV,1,Trial}, \dots, \phi_{MV,K,Trial}$ are the draws of the K macroeconomic factors that were included in the model.

Regarding the significance of the estimated coefficients $\hat{\beta}_k$, it is important to note that it is not appropriate to use the directly calculated t-statistic. Equation (4) is estimated using the Monte Carlo simulation output described in Section 2.2, which contains a large number of simulated values based on a correlation model estimated using a much smaller number of observations. In particular, the model was estimated using 55 observations (number of quarters between 1999Q3 and 2013Q1), while the Monte Carlo simulation output includes tens of thousands of trials. As a result, the t-statistic should be adjusted using the following formula:²⁵

$$tstat_{adj} = tstat_{sim} \sqrt{\frac{N^{est}}{N^{sim}}} \quad (5)$$

where $tstat_{sim}$ is the t-statistic calculated in the regression model, N^{sim} is the number of simulated values from the Monte Carlo simulation output, and N^{est} is the number of observations used to estimate the correlation model. It is important to mention that this adjustment method is only an approximation to indicate the statistical significance of the coefficients, and it does not determine the precise t-statistic. We can conduct significance tests by comparing the t-statistic value $tstat_{adj}$ to the critical value of the Student t-distribution with $N^{est} - K - 1$ degrees of freedom: $t_{N^{est}-K-1, 1-\alpha}$ in the case of one-sided test and $t_{N^{est}-K-1, 1-\frac{\alpha}{2}}$ for a two-sided test, where α is the significance level. The use of one-sided or two-sided depends on whether we have an assumption about the sign of the coefficient.

During the variable selection procedure, we must ensure that relationships between the macroeconomic factors and losses are economically meaningful, in other words, all the coefficients estimated in (4) should have intuitive signs. For example, unemployment rate variables should have positive sign because the higher the unemployment rate, the higher the losses. Similarly, GDP variables should have negative signs because losses should be smaller if GDP grows:

$$\beta_{Unemployment} > 0, \quad \beta_{GDP} < 0, \quad (6)$$

Thus, it is necessary to have the conditions like (6) in place before variable selection. For some variables, one may not have a prior assumption on the direction of relationship (for example, for oil price).

The following steps comprise one possible approach to variable selection, which can be altered to fit the needs of a specific analysis:

- » The first step is to select a subset of the 62 macroeconomic variables included in GCorr Macro 2013. The idea is to narrow the set of potential candidates (pre-selected variables) from which the final macroeconomic variables will be selected.²⁶ This subset can be chosen based on economic intuition as well as the univariate analysis. For example, we should expect U.S. Unemployment or U.S. CRE Index to be potential candidates for the U.S. CRE portfolio while Japan GDP or Germany Unemployment to be candidates for the Global portfolio. We can also leverage the univariate analysis described in the

²⁵ We note that this significance test is based on several simplifying assumptions. The chief among them is that the only estimation error in the model comes from correlations between systematic credit risk factors and the standard normal macroeconomic factors. In other words, we assume that the error in estimates of other parameters, including PDs, LGDs, counterparty's R-squared, can be omitted in this context.

²⁶ In theory, it is possible to run through all possible models based on subsets of 62 macroeconomic variables. However, this approach is not practical because it requires too much computational time.

previous section. For example, we can run a univariate regression in (4) for each variable and test the significance of the coefficient using the t-statistic. We can then create a set of pre-selected variables by including only the variables that passed the significance test.²⁷ Note that this procedure is equivalent to choosing only the macroeconomic variables for which the correlation $\text{corr}(N^{-1}(L), \phi_{MV})$ from Section 3.1 is significantly different from zero. The question is to what degree one should rely on economic considerations or on the statistical procedure when pre-selecting macroeconomic variables. It depends on specifics of the analysis, including whether there are prior economic assumptions, which variables are relevant, and for which variables projections under a scenario are available.

- » In the second step, we estimate models (4) based on all possible subsets of up to K^* pre-selected macroeconomic variables. The value K^* , for example $K^* = 6$, limits the number of models estimated and thus the computational time. We examine larger models in the next step. Of all the models, we exclude those that fail at least one of the following two tests:
 - At least one estimated coefficient in the model is insignificant according to its t-statistic. This restricts the number of variables in the model to only the ones that contribute to explaining variation in portfolio losses, and thus keeps the model parsimonious.
 - At least one coefficient does not meet restriction from (6). This eliminates models with economically unintuitive relationships.

We rank the models that pass the two criteria according to their explanatory power measured by the regression adjusted R-squared value²⁸. The selection of the best model among them should include both statistical (selecting the model with the highest adjusted R-squared value) and economic considerations. This means focusing on several top models based on the adjusted R-squared value, comparing the variables included in those models, comparing the relative magnitudes of the estimated coefficients, and choosing the model which lends itself to the most reasonable economic narrative for relationship between its variables and the portfolio losses. In this approach, we use the adjusted R-squared value and t-statistics to find a balance between explanatory power and model simplicity. One alternative approach is to use the Akaike's Information Criterion (AIC)²⁹. As the adjusted R-squared, the AIC captures the trade-off between the goodness of fit of the model and the number of parameters estimated. The preferred model is the one with the lowest AIC. It is important to mention that both approaches will imply quite similar results, in some cases including additional macroeconomic variables if we use only AIC.

- » The previous step focused only on models with up to K^* variables. If the best model with that step contains exactly K^* variables, we need to test whether including more variables to that best model leads to a model which passes our two tests: significant coefficients and intuitive signs of all coefficients. In that case, we can consider this expanded model as the final model. If not, the best model from the second step will be considered the final model.

The variable selection procedure provides us with the final model, and thus the final set of macroeconomic variables that we will use for stress testing in the next section.

Let us turn to our test portfolios introduced in Table 2. Appendix B contains the list of the pre-selected macroeconomic variables for each of the five portfolios. For example, the pre-selected set for the U.S. corporate portfolio includes U.S. variables reflecting economic activity such as GDP and Unemployment, and also financial variables such as BBB Yield³⁰, Equity and VIX. We excluded variables such as CRE Index or non-U.S. variables since from an economic perspective they seem to be less relevant for U.S. corporate exposures. Similarly, for the U.S. CRE and retail portfolio we pre-selected variables such as U.S. GDP and U.S. Unemployment, and some financial variables such as U.S. Equity, but we exclude U.S. BBB Yield since we consider this variable more related to corporate exposures rather than CRE or retail exposures. In addition to this, for the U.S. Retail portfolio we also

²⁷ It is important to realize that even if a macroeconomic variable is excluded in the pre-selection step because of its insignificant univariate relationship with portfolio losses, it can still have a significant impact on losses in a multivariate regression model.

²⁸ As for the t-stat, when adjusting the R-squared we use the number of observations used to estimate the correlation model and not the number of simulations, in other words:

$$AdjRSQ = RSQ - (1 - RSQ) \frac{K}{(N_{est} - K - 1)}$$

where RSQ is the regression R-squared, K is the number of macroeconomic variables included in the regression and N_{est} is the number of observations used to estimate the correlation model.

²⁹ Because the model was estimated using 55 observations, we actually use the corrected AIC (AICc) which is used in for small samples:

$$AIC = \log(\hat{\sigma}_e^2) + 2 \frac{(K + 1)}{(N_{est} - K - 2)}$$

where $\hat{\sigma}_e^2$ is the variance of the regression residuals, K is the number of macroeconomic variables included in the regression and N_{est} is the number of observations used to estimate the correlation model.

³⁰ We consider BBB yield in this example because it is the corporate yield variable included in the CCAR scenarios. However, the client can also consider scenarios including Baa yield because GCorr Macro also provides a mapping function for the Baa yield.

exclude the CRE Index. Finally, for the Global portfolio we only consider non-U.S. variables since it includes only non-U.S. exposures. In particular, we include Equity, GDP and Unemployment variables for some European countries, Japan, and Australia. The pre-selected set for the aggregate portfolio includes all the pre-selected variables for the four portfolios.

Table 4 presents the final sets of selected macroeconomic variables after running the variable selection procedure described earlier in this section. For each portfolio, we selected the set providing the maximum adjusted regression R-squared value, after eliminating models failing our criteria on variable significance and economic intuition. Estimated coefficients, adjusted t-statistics and regression R-squared and adjusted R-squared are included.

Table 4

Selected macroeconomic variables

	U.S. CORPORATE PORTFOLIO	GLOBAL PORTFOLIO	U.S. CRE PORTFOLIO	U.S. RETAIL PORTFOLIO	AGGREGATE PORTFOLIO
Intercept	-1.996*** (-88.223)	-1.819*** (-83.753)	-1.826*** (-73.010)	-2.065*** (-86.644)	-1.947*** (-114.832)
U.S. Unemployment	0.067*** (2.767)				
U.S. BBB Yield	0.059*** (2.532)				0.075*** (4.273)
U.S. Equity	-0.074** (-2.282)		-0.046** (-1.760)	-0.053** (-2.203)	
U.S. House Price Index			-0.074*** (-2.750)	-0.073*** (-3.020)	-0.037** (-1.919)
U.S. CRE Index			-0.087*** (-3.130)		-0.083*** (-4.372)
U.S. VIX	0.059** (1.903)				0.043** (2.096)
Eurozone Equity					-0.055*** (-2.600)
Japan Equity		-0.133*** (-6.089)			
UK Unemployment		0.065*** (2.986)			
RSQ	50.3%	43.7%	40.0%	22.9%	64.4%
Adjusted RSQ	46.3%	41.5%	36.4%	19.9%	63.7%

Note: adjusted t-statistic in brackets.

It is worth mentioning that if we relax the restriction of all coefficients to be significant and select the model based on maximizing the adjusted R-squared or minimizing the AIC, additional macroeconomic variables are included for some of the portfolios. For example, for the U.S. retail portfolio, U.S. Unemployment is also selected, and for the Global portfolio Japan GDP is included. The variables selected for the U.S. corporate and U.S. CRE portfolio do not change.

This section describes a procedure to select the macroeconomic variables to be included in the final model used for stress testing. This procedure can be partly automated, but still requires judgment when assessing, for example, which models have a meaningful economic interpretation based on the type of portfolio analyzed.

3.3 Stressed Distribution of Portfolio Losses

After we select the macroeconomic variables for a given portfolio to use for stress testing, the loss distribution can be computed conditional on any macroeconomic scenario based on these variables. Section 2.2 describes the methodology to estimate the conditional loss distribution. Because the entire conditional distribution is estimated, we can determine both the stressed expected loss and other statistics, such as various percentiles of the distribution, its standard deviation representing the stressed unexpected loss, and probabilities that the loss will exceed a certain threshold (for example, economic capital) under the scenario. This section presents stress testing results and their interpretation for three of the portfolios analyzed in this paper: U.S. corporate portfolio, U.S. CRE portfolio and U.S. retail portfolio.

The macroeconomic scenario we study is the CCAR 2014 Severely Adverse Scenario and returns on macroeconomic variables are computed over a one-year horizon (from 2013 Q3 to 2014 Q3). In the first step, the scenario shocks need to be translated into values of a standard normal distribution. We estimate mapping functions for each of the 62 macroeconomic variables included in GCorr Macro and use them to convert the return on each variable under the scenario into a standard normal shock. Please refer to

Appendix C for a detailed description of the mapping functions. Table 5 presents the one-year horizon shocks for the selected macroeconomic variables under the CCAR 2014 Severely Adverse scenario.

Table 5

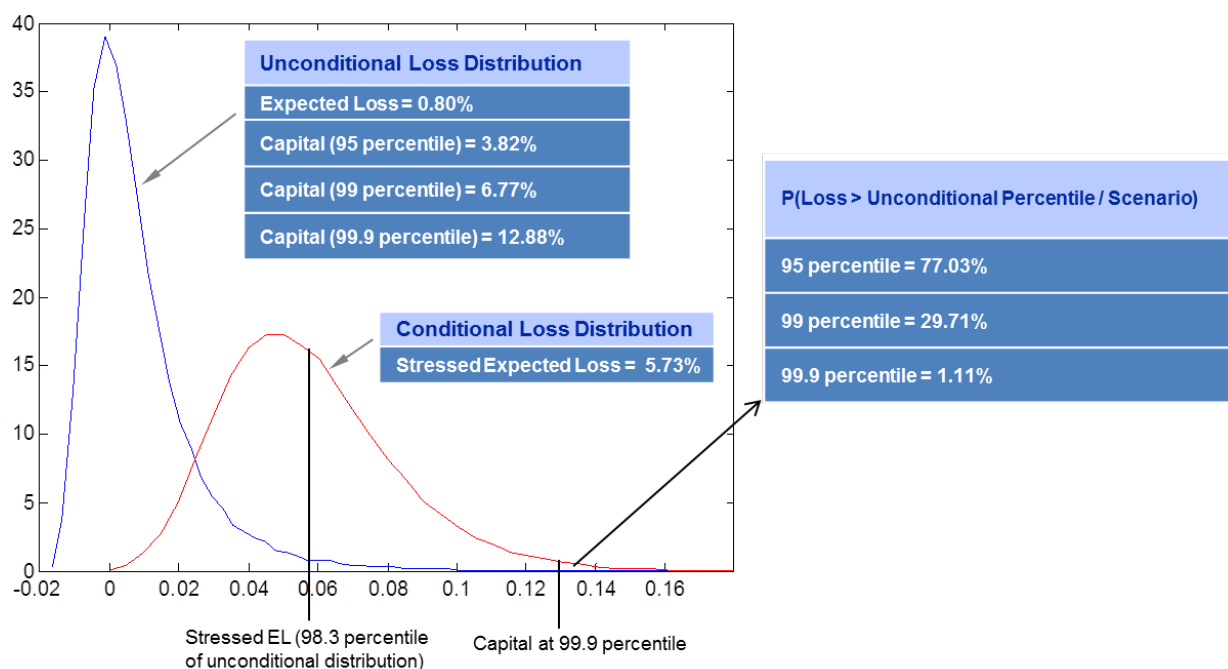
CCAR 2014 Severely Adverse Scenario: one-year horizon shocks

MACROECONOMIC VARIABLE	2013Q3	2014Q3	SCENARIO SHOCK (LOG-RETURNS)	STANDARD NORMAL SHOCK
U.S. Unemployment	0.073	0.107	0.3824	1.70
U.S. BBB Yield	0.049	0.062	0.2353	1.54
U.S. Equity	17718.3	8943.3	-0.6837	-2.31
U.S. House Price Index	158.8	139.1	-0.1325	-1.79
U.S. CRE Index	217	175.5	-0.2123	-1.69
U.S. VIX	17	57.9	1.2255	2.52

It is important to remark that the losses from the conditional simulation include all RiskFrontier features that can be used in unconditional simulation (mark-to-market losses due to credit deterioration, various valuation methods, PD-LGD correlation and so forth), which allows us to directly compare the conditional and unconditional loss distributions.

Figure 7 illustrates the conditional and unconditional loss distributions for the U.S. corporate portfolio. Based on the results from Section 3.2, we use the following variables for stress testing of this portfolio: U.S. Unemployment, U.S. BBB Yield, U.S. Equity and U.S. VIX. The regression R-squared is 50%, implying that the macroeconomic variables selected do not completely span the systematic risk of this portfolio.³¹

Figure 7 Conditional loss distribution: U.S. Corporate Portfolio



As expected, due to the severe shocks assumed in the scenario for all variables, the conditional loss distribution shifts to the right, putting more probability on the larger losses compared to the unconditional distribution. The stressed expected loss is seven times higher than the unconditional expected loss. In addition to the severity of the shocks, the high counterparty R-squared values in this portfolio³² also contribute to this result—the higher the R-squared values, the more downgrades and defaults given a severe

³¹ The U.S. corporate portfolio is a large diversified portfolio without substantial name concentration, so the residual variation in losses does not stem from idiosyncratic effects of few counterparties.

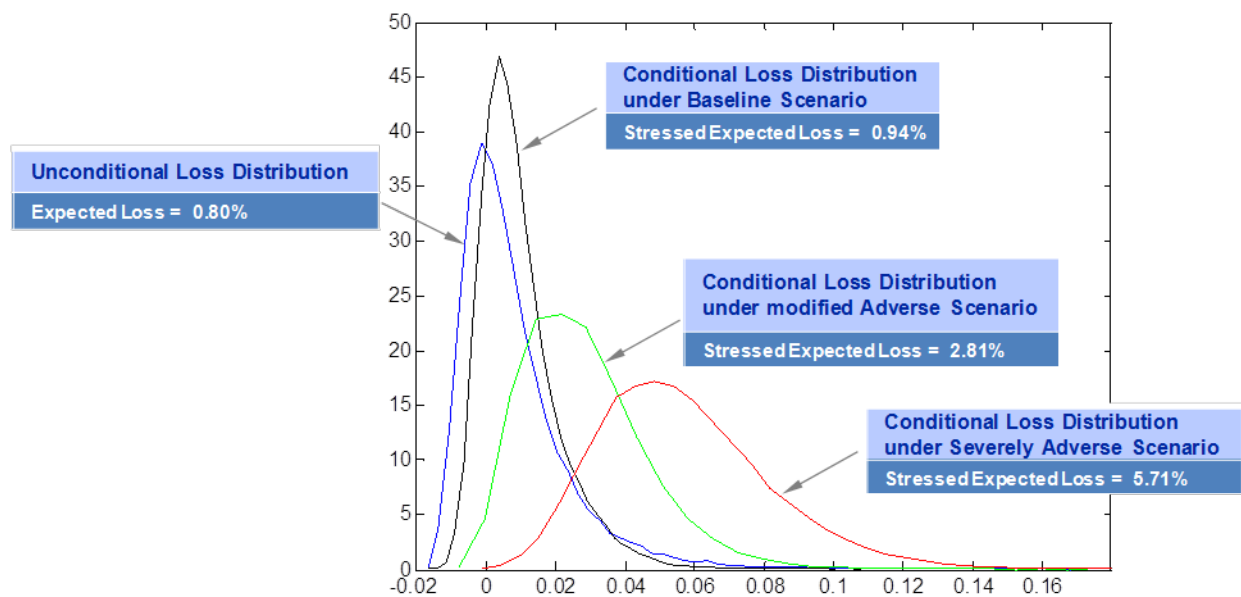
³² See Table 2.

shock. The stressed unexpected loss (measured by the standard deviation of the conditional distribution) is equal to 2.5%, 1.5 times higher than the unconditional unexpected loss.³³ Regarding economic capital, if we define it as the 99.9 percentile of the unconditional loss distribution, the probability of exceeding it under the stressed scenario is 1.11%. Recall that the annual shocks assumed in this scenario are quite severe, where for example the log-return on the stock market is -0.70 and the log-return on unemployment rate is almost 0.40 (equivalent to -50% and 47% returns, respectively).

Figure 8 presents the conditional loss distributions under the three CCAR 2014 scenarios: Baseline, Adverse and Severely Adverse. Note that we have adjusted the BBB yield shock assumed in the Adverse scenario. The Adverse scenario assumes the BBB yield to increase from 4.9% in 2013 Q3 to 9.2% in 2014 Q3. This implies an annual log return equal to 0.63 compared to 0.24 for the Severely Adverse scenario. The increase in the BBB yield is driven mainly by the increase assumed in the reference rate and not the credit spread, which is the more relevant component of yield in the context of default risk.³⁴

The CCAR 2014 Adverse scenario assumes an economic environment of increasing interest rates. For example, the five-year yield is assumed to increase from 1.5% in 2013 Q3 to 4.5% in 2014 Q3, and the 10-year yield from 2.7% to 5.7%. If we use the 10-year yield as the reference rate, the spread is assumed to increase only from 2.2% to 3.5%. To account for this, we adjusted the BBB yield shock assumed in the Adverse scenario. In particular, we use as the reference rate the 10-year yield values assumed in the Severely Adverse scenario and add the spread values assumed in the Adverse scenario. As a result, the BBB yield declines from 4.9% in 2013 Q3 to 4.6% in 2014 Q3, implying an annual log return equal to -0.06.

Figure 8 Conditional loss distribution under CCAR 2014 scenarios: U.S. Corporate portfolio



As expected, the conditional loss distribution under the Baseline scenario is concentrated around lower losses than the other two scenarios. On the other hand, the distribution under the Severely Adverse scenario is centered around the highest losses. The stressed expected losses under the Baseline, Adverse, and Severely Adverse scenarios are 0.9%, 2.8%, and 5.7%, respectively.³⁵

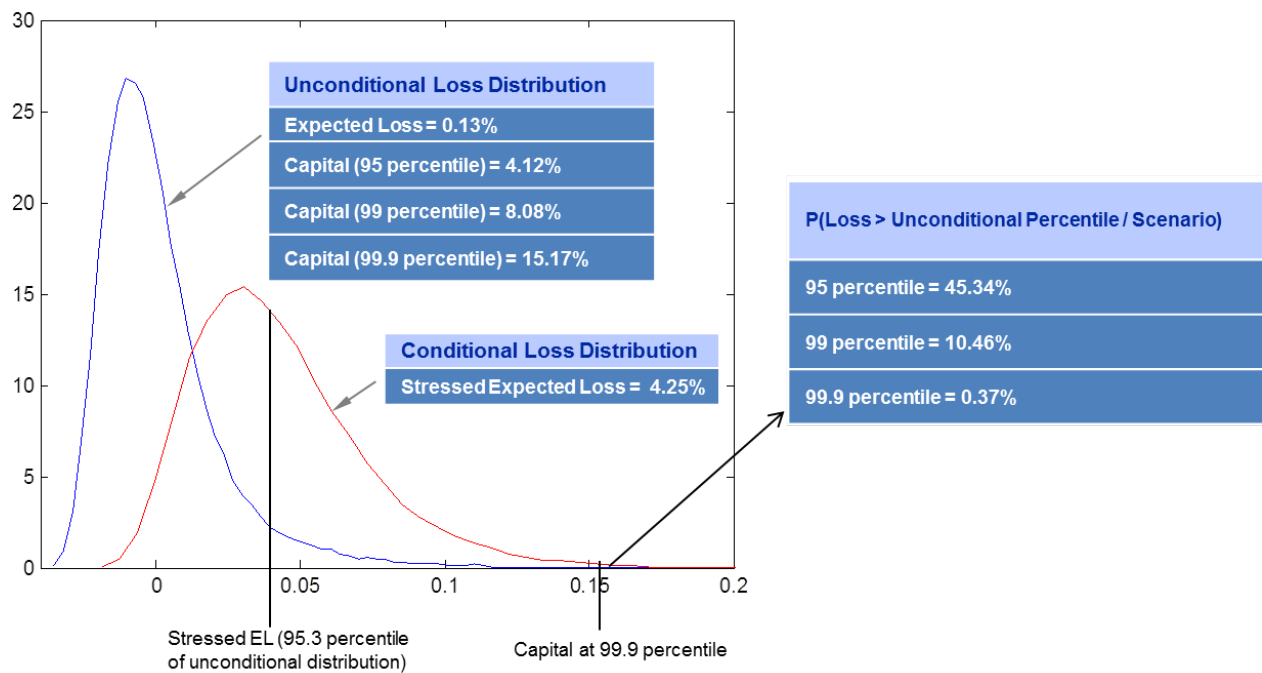
Figure 9 illustrates the unconditional and conditional loss distributions for the U.S. CRE portfolio. The scenario studied is the CCAR 2014 Severely Adverse scenario. The macroeconomic variables selected for this portfolio are U.S. Equity, U.S. House Price Index and U.S. CRE Index, and the regression R-squared is 40%.

³³ We note that while the systematic credit risk factors ϕ_{CR} have a lower standard deviation under a scenario than unconditionally, as formula (3) implies, this does not have to be true for losses. The reason is that losses are a non-linear function of the factors.

³⁴ We make this statement based on empirical correlations between credit risk factors on the one side with log changes in credit spread or in reference interest rate on the other side. Over the recent period (1999–2013), the correlation with credit spread has been substantially higher than with interest rate.

³⁵ Note that without modifying the Adverse scenario, the corresponding loss distribution would lie closer to the Severely Adverse distribution because the BBB yield shock would be higher due to increasing interest rates.

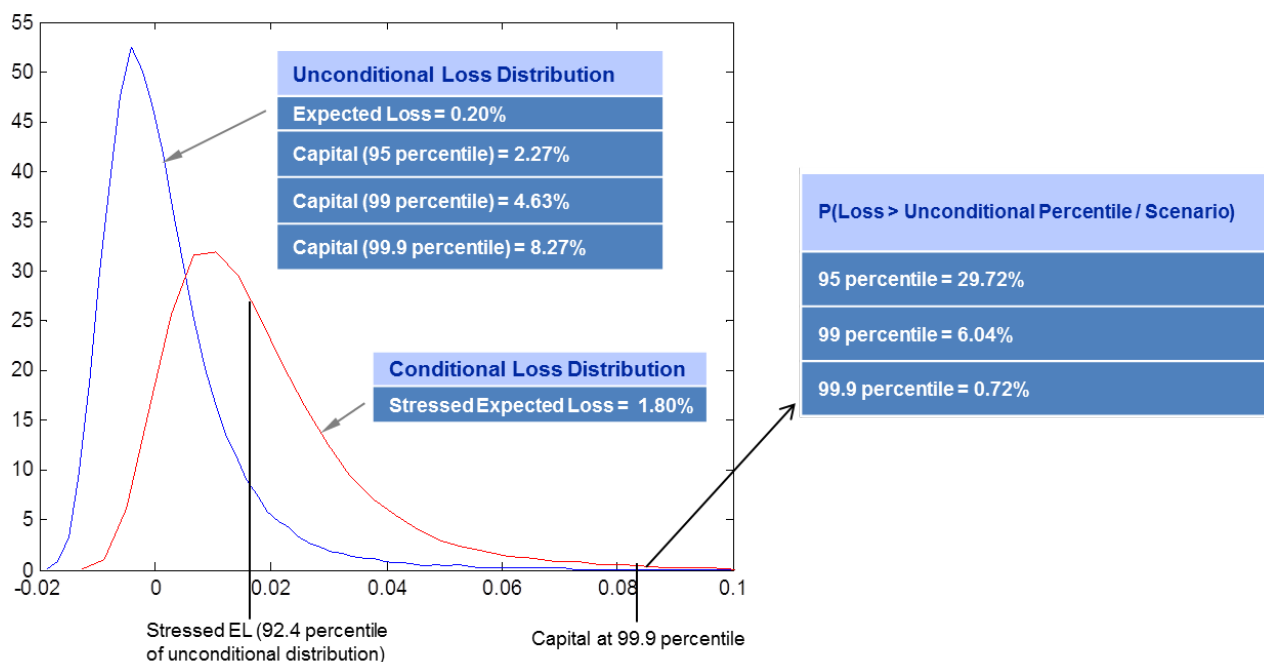
Figure 9 Conditional loss distribution: U.S. CRE Portfolio



The expected and unexpected stressed losses are 4.28% and 2.98%, compared to 0.13% and 2.18% for the unconditional loss distribution. In terms of economic capital, the probability of exceeding the 99.9th percentile of the unconditional loss distribution under the stressed distribution is around 0.37%.

Finally, Figure 10 presents the unconditional and conditional loss distribution for the U.S. retail portfolio. In this case, the selected macroeconomic variables are U.S. Equity and U.S. House Price Index and the regression R-squared is equal to 23%.

Figure 10 Conditional loss distribution: U.S. Retail Portfolio



For the U.S. retail portfolio, the expected loss increases from 0.20% for the unconditional distribution, to 1.80% for the stressed distribution. Moreover, unexpected loss increases from 1.16% to 1.66%. If we define economic capital as the 99.9th percentile of the unconditional loss distribution, the probability of exceeding it under the stressed loss distribution is equal to 0.72%.

In this section, we presented the results of the conditional simulation, where loss distributions are estimated conditional on a given macroeconomic scenario. In particular, we compare the unconditional and stressed loss distribution for the U.S. corporate, U.S. CRE and U.S. retail portfolios, analyzing not only the expected values and standard deviations of the distributions, but also some statistics related to the tail of the distributions. The next section describes the GCorr Macro model reverse stress testing application.

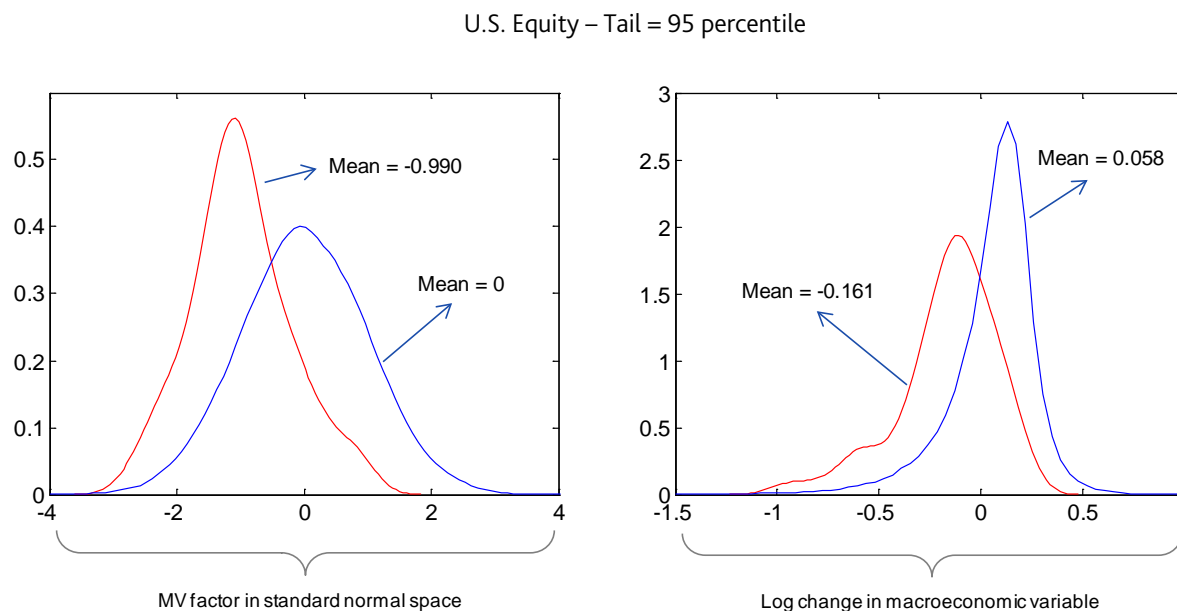
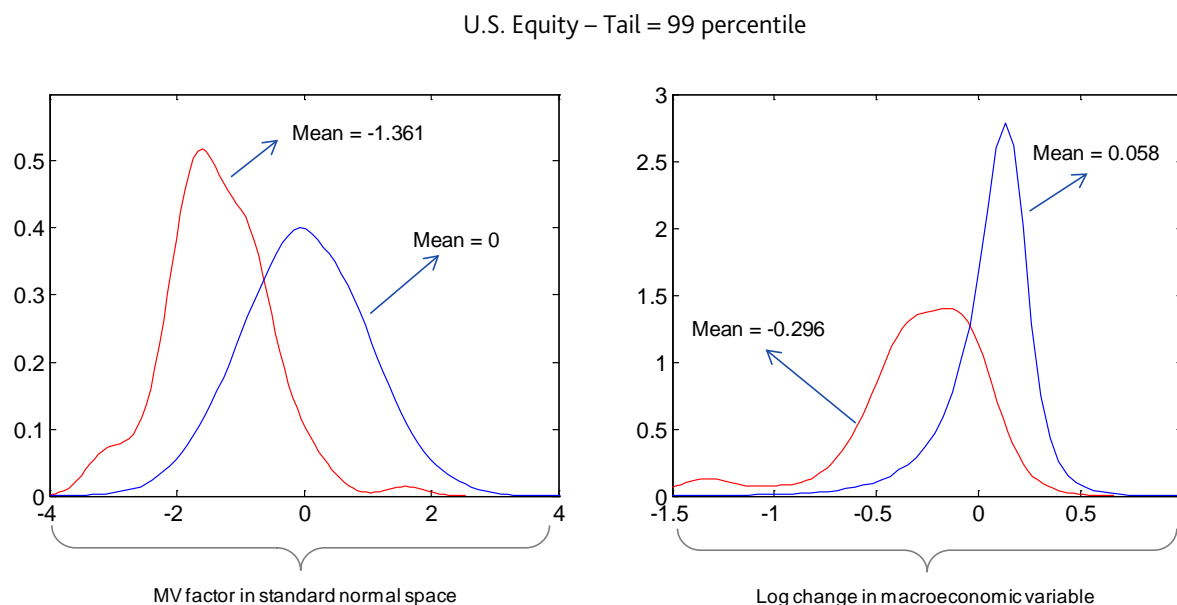
3.4 Reverse Stress Testing

In addition to the stress testing exercises described in Section 3.3, we can also use the Monte Carlo simulation output to characterize the macroeconomic scenarios associated with certain levels of portfolio losses, typically extreme losses. This type of analysis is called reverse stress testing.

We define losses as extreme if they are equal to a certain value, which in general represents a certain percentile of the loss distribution, such as the 95th or 99th percentile. A reverse stress testing exercise can be conducted by selecting only those trials from the Monte Carlo simulation output in which the losses are equal (or sufficiently close) to the chosen value. Then the distribution of the macroeconomic variables across the select trials can be calculated and linked with the types of scenarios associated with the extreme losses.

In particular, assume we determine the 99th percentile as the value defining extreme losses. We then select the trials for which the losses are between the 98.95 and 99.05 percentiles. If the total number of trials is 100,000, then the number of selected trials is 100 (i.e., 0.1% x 100,000). Finally, we compute the distribution of standard normal macroeconomic factors and the corresponding values of macroeconomic variables across the selected trials and compare them to the unconditional distributions across all trials.

Figure 11 shows an example of a reverse stress testing analysis. It describes the distribution of the standard normal U.S. equity factor and U.S. Equity variable corresponding to extreme losses of the U.S. corporate portfolio (red lines), compared to the unconditional distributions (blue lines). All distributions are estimated from the Monte Carlo simulation output. The top charts refer to extreme losses associated with the 99th percentile and the bottom charts to extreme losses associated with the 95th percentile.

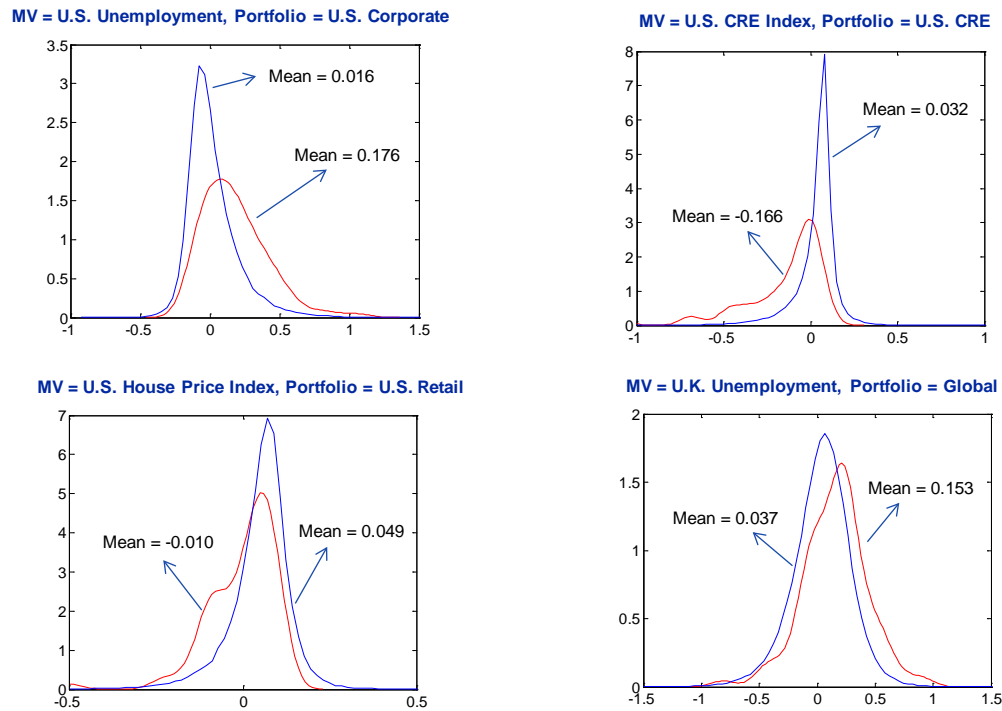
Figure 11 Reverse stress testing analysis: U.S. Corporate Portfolio³⁶

As expected, the extreme losses tend to be associated with large negative U.S. Equity returns, which is consistent with economic intuition.

Figure 12 presents the distribution for selected macroeconomic variables corresponding to extreme losses for the U.S. corporate, U.S. CRE, U.S. retail portfolios and Global portfolio. The extreme losses are defined as the 99th percentile. In all cases, the macroeconomic variables distributions conditional on extreme losses imply higher probability for more severe shocks. For example, the distribution of U.S. Unemployment shocks conditional on extreme losses for the U.S. corporate portfolio has a mean equal to 0.176 (log-return), more than ten times larger than the mean of the unconditional distribution.

³⁶ It is worth mentioning that the conditional distributions are in fact histograms based on 100 trials (i.e., 0.1% x 100,000). The more trials used for simulation, the more precisely the conditional distribution is estimated. We use kernel density estimation to get smooth density functions.

Figure 12 Reverse stress testing analysis: Selected macroeconomic variables distribution in the tail event



Regarding the reverse stress testing results, it is important to note that there is not a single macroeconomic scenario associated with extreme losses, but rather a range of scenarios. As Figure 11 shows, the U.S. Equity returns associated with 99th percentile of U.S. Corporate losses range from roughly -0.7 to 0.1. This range excludes strong positive returns, which means that those returns do not coincide with losses around the 99th percentile, as expected. However, it is possible to observe scenarios with zero or mildly negative equity returns and still large losses. This is because there is a sufficient residual variation in losses beyond the variation related to the macroeconomic variable. Therefore, a mild negative equity return in combination with large residual negative shocks (for example, industry specific adverse events) can take the loss to the 99th percentile. The dispersion of the conditional distributions in Figure 11 and Figure 12 is also related to the univariate analysis—the stronger the correlation between portfolio losses and a macroeconomic variable, the less dispersed the conditional distribution of the variable given a loss level.

Finally, in addition to the macroeconomic factor distribution associated with extreme losses, we can also compute the distribution of the GCorr systematic credit risk factors conditional on extreme losses. For example, let us consider the Global portfolio and define losses as the 99th percentile of the unconditional loss distribution. We can then select the trials where the losses are between the 98.95 and 99.05 percentiles, and compute the distribution of the GCorr systematic credit risk factors (or custom indexes) across these trials. Figure 13 presents the average shock of the GCorr factors representing country credit risk³⁷ across the trials corresponding to extreme losses.

³⁷ We determine the factor as the equally weighted average of all custom indexes for a given country. For example, in the U.S. case, we calculate the average of 61 U.S. custom indexes, each corresponding to one industry (U.S. – Aerospace & Defense, U.S. – Agriculture, etc.). We call this average of custom indexes as a country factor in this section, however we emphasize it is different from the country factors in GCorr (such as r_{US}) which are combined with industry factors to obtain custom indexes.

Figure 13 GCorr factors representing country credit risk: average shock when conditioning on extreme losses on the Global portfolio



The Global portfolio is mainly concentrated in Japan, Europe, and Australia. As a result, we observe the Japan and Australia country factors, together with several European country factors to exhibit the largest average shocks. This type of analysis indicates the countries which exhibit the largest credit risk shock for a given level of losses.³⁸ We note that the result does not depend only on distribution of commitments across countries, but also on risk profiles of exposures in individual countries as well as on correlations of country factors. For example, the U.S. factor shows a shock even though the Global portfolio is not exposed directly to the U.S. factor. This is the result of relatively high credit correlations of the U.S. and European countries: shocks to the U.S. country factors are likely to be associated with shocks to Europe, and vice versa.

4. Risk Integration with GCorr Macro

In this section, we discuss a top-down risk integration framework based on the GCorr Macro model in a multi-factor setting and its application to economic capital integration and allocation across risk types.³⁹

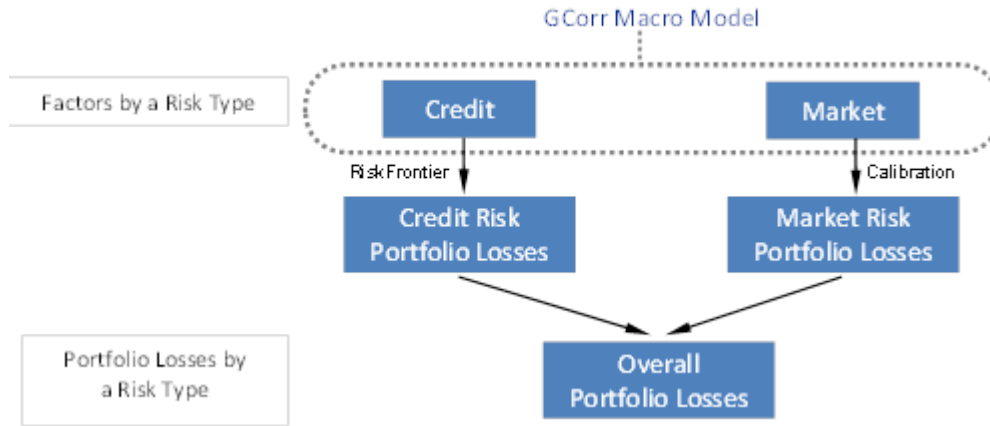
4.1 Integrating Market and Credit Risk Using a Top-Down Framework

Figure 14 provides an overview of the framework. For exposition, we consider two types of portfolios containing financial instruments: credit risk sensitive portfolios and market risk sensitive portfolios. Our objective is to estimate the joint distribution of losses on all of these portfolios through their exposure to credit risk factors and market risk factors. The GCorr Macro model links the factors across risk types. The estimated joint distribution allows us to determine the distribution of total losses across all portfolios. We can use this distribution to calculate the aggregate economic capital for the portfolios, and then allocate the capital to the individual portfolios while properly accounting for their contribution to the capital.

³⁸ We note a similarity of the interpretation with the Tail Risk Contribution (TRC) context in RiskFrontier. Aggregating TRC by countries could provide similar insights to those in Figure 12.

³⁹ Various top-down and bottom-up risk integration approaches are presented in "Risk Integration: New Top-down Approaches and Correlation Calibration" (Chen, N., A. Kaplin, A. Levy, and Y. Wang, 2010).

Figure 14 Overview of the risk integration framework with the GCorr Macro model



As explained in the Section 2, GCorr Macro provides a correlation matrix that links the GCorr credit risk factors ϕ_{CR} and macroeconomic variables ϕ_{MV} . Suppose a subset of the macroeconomic variables ϕ_{MV} are the relevant market risk factors which are divided into factor sets f_m . A set f_m contains F_m factors: $f_m = \{f_m^j, j = 1, \dots, F_m\}$. While each market portfolio is exposed to one factor set, there can be an overlap between two factor sets if two market risk portfolios are exposed to the same market factor.

The distribution of losses on the credit portfolios can be estimated using the RiskFrontier software. Meanwhile, market risk is analyzed through a market risk system with factors that overlap with GCorr Macro. We denote losses on credit portfolio m as L_m^{Credit} and denote losses on market portfolio m as L_m^{Market} .

The two risk systems can be linked through the following steps. First, a market risk platform is used to simultaneously generate draws of losses on a market risk portfolio and returns on the corresponding factor set. Second, one can use regression techniques to estimate parameters for a polynomial representation in equation (7):

$$L_{m,i}^{Market} = \beta_m^0 + \sum_{j=1}^{F_m} \sum_{p=1}^{N_p} \beta_m^{j,p} (f_{m,i}^j)^p + \sum_{j=1}^{F_m} \sum_{k=1}^{F_m} \beta_{m,Cross}^{j,k} (f_{m,i}^j f_{m,i}^k) + \sqrt{1 - R_m^2} \sigma_m \varepsilon_{m,i} \quad (7)$$

where N_p is the degree of the polynomial function.

In equation (7), $f_{m,i}^j$ is the realization of factor j from factor set f_m used in construction of the loss distribution for market portfolio m during trial i . Parameter R_m^2 represents the R-squared of the polynomial regression and σ_m the standard deviation of the loss distribution. Variable $\varepsilon_{m,i}$ is the idiosyncratic portion of the loss for trial i , with expected value equal to 0 and standard deviation equal to 1. Once the parameters from equation (7) are estimated, we can generate the calibrated loss on a market portfolio for any value of the market factors f_m as follows:

$$\widehat{L}_m^{Market}(f_m) = \widehat{\beta}_m^0 + \sum_{j=1}^{F_m} \sum_{p=1}^{N_p} \widehat{\beta}_m^{j,p} (f_m^j)^p + \sum_{j=1}^{F_m} \sum_{k=1}^{F_m} \widehat{\beta}_{m,Cross}^{j,k} (f_m^j f_m^k) + \sqrt{1 - \widehat{R}_m^2} \widehat{\sigma}_m \varepsilon_m^* \quad (8)$$

Symbols $\widehat{\beta}_m$, \widehat{R}_m^2 , $\widehat{\sigma}_m$ denote the estimated parameters, while ε_m^* stands for a random draw from a standard normal distribution, uncorrelated with the factors.⁴⁰

A model of type (7) can be considered suitable for our risk integration framework if R_m^2 is high, which means that the factors f_m explain most of the variation in the portfolio losses. If R_m^2 is low, the model can still be utilized, but only in the case when ε_m is uncorrelated with idiosyncratic factors of other portfolios. For example, gains and losses on a portfolio of U.S. treasury securities

⁴⁰ Alternatively ε_m^* can be sampled by bootstrapping the regression standard errors.

can be explained by several factors representing movements of several points on the treasury yield curve. In this case, we can expect the R_m^2 value to be high.

Now that we have introduced all components of the risk integration framework, we can combine them to estimate the joint distribution of losses on all portfolios as well as the total loss distribution. The credit and market risk factors are jointly simulated from a Gaussian copula within the RiskFrontier software. For a given draw of credit risk and market risk factors, we can determine the credit portfolio losses using the RiskFrontier software and the market portfolio losses using equation (8). For each trial, the total loss is given as the sum of losses across all portfolios:

$$L^{Total} = \sum_c L_c^{Credit} + \sum_m \widehat{L}_m^{Market}(f_m) \quad (9)$$

We summarize the estimation process and its output in Table 6.

Table 6

Estimating the joint distribution of portfolio losses and the total loss distribution

The diagram illustrates the process of estimating the joint distribution of portfolio losses and the total loss distribution. It shows a table with five columns: Trial, Simulated GCorr Credit Risk Factors, Simulated Market Risk Factor Sets, Losses on Credit Risk Portfolios, Losses on Market Risk Portfolios, and Aggregate Losses. The first two columns are linked by a bracket labeled 'GCorr Macro Model Gaussian Copula'. The last two columns are linked by a bracket labeled 'Sum'. Arrows indicate that the 'Simulated GCorr Credit Risk Factors' and 'Simulated Market Risk Factor Sets' are inputs to 'RiskFrontier Software', which then outputs 'Losses on Credit Risk Portfolios' and 'Losses on Market Risk Portfolios'. The 'Losses on Market Risk Portfolios' are also calculated using the 'Calibrated Model, Equation (2)'. The 'Losses on Credit Risk Portfolios' and 'Losses on Market Risk Portfolios' are then summed to produce the 'Aggregate Losses'.

Trial	Simulated GCorr Credit Risk Factors	Simulated Market Risk Factor Sets	Losses on Credit Risk Portfolios	Losses on Market Risk Portfolios	Aggregate Losses
1	$\phi_{k,1}, k = 1, \dots, N_k$	$f_{m,1}, m = 1, \dots, F_m$	$L_{c,1}^{Credit}, c = 1, \dots, N_c$	$\widehat{L}_m^{Market}(f_{m,1}), m = 1, \dots, F_m$	L_1^{Total}
2	$\phi_{k,2}, k = 1, \dots, N_k$	$f_{m,2}, m = 1, \dots, F_m$	$L_{c,2}^{Credit}, c = 1, \dots, N_c$	$\widehat{L}_m^{Market}(f_{m,2}), m = 1, \dots, F_m$	L_2^{Total}
...
N	$\phi_{k,N}, k = 1, \dots, N_k$	$f_{m,N}, m = 1, \dots, F_m$	$L_{c,N}^{Credit}, c = 1, \dots, N_c$	$\widehat{L}_m^{Market}(f_{m,n}), m = 1, \dots, F_m$	L_N^{Total}

In Section 4.2 we present portfolio analyses which utilize the output from Table 6 and, more generally, the multi-factor risk integration framework from Figure 14.

We conclude this section by discussing the features of the framework that distinguish it from other risk integration approaches. The framework requires estimation of two inputs: the GCorr Macro model linking factors across risk types, and calibrated models relating market portfolio losses to market risk factors. Compared to the traditional copula risk integration approaches, our framework requires estimation of more input parameters. However, the parameters have intuitive interpretations, whether it is the correlation between a shock to the credit quality of corporates within the U.S. air transportation industry and the U.S. stock market return, or the sensitivities of a U.S. Treasury securities portfolio to points on the treasury yield curve. Moreover, the rich and flexible factor structure of our framework allows for a more accurate description of correlations and concentrations across credit and market portfolios compared to simpler approaches relying on few parameters.

There are other methodologies that allow the combination of credit risk and market risk scenarios produced by different systems. For example, the paper by Morrison (2013) uses GCorr Macro to simulate credit and market risk scenarios using reordering techniques.⁴¹ The main idea behind this method is the existence of one (or a few) underlying risk factor(s) common between different risk systems that will allow one to describe the effects of interaction between the full sets of underlying risk factors in each system, while maintaining the stand-alone loss distributions unchanged. While this reordering approach might be relatively simple to implement, it also induces certain dependencies across underlying risk factors for various systems rather than specifying them directly. Therefore, it requires careful consideration and validation of a particular reordering procedure.

⁴¹ See "Aggregation of market and credit risk capital requirements via integrated scenarios" (Morrison, 2013).

4.2 Economic Capital Aggregation and Allocation

In this section, we discuss how we can use the simulation output described earlier with calibrated market portfolio losses to do the following:

- » Determine aggregate capital
- » Allocate the aggregate capital to individual portfolios based on their Risk Contribution (RC)/Tail Risk Contribution (TRC)

The analysis accounts for correlations and concentrations across risk types, geographies, sectors, etc., through factor correlations implied by GCorr Macro.

Financial organizations are subject to various risk sources and are typically required to assess overall risk. At the same time, the management of various risk sources is often siloed, and the respective organization units often have sophisticated risk systems in place to assess those risk sources separately. For example, a unit responsible for the trading book is likely to have a good idea of its market risk exposure, but their system will not typically account for the credit risk of the banking book. The top-down approach discussed earlier offers a solution where risk sources are analyzed separately and combined in order to arrive at the overall risk picture.

To illustrate, in this section we assume that the organization has four units managing four separate portfolios that capture the following four risk sources: U.S. Credit Risk, UK Credit Risk, U.S. Market Risk, and UK Market Risk. Furthermore, market portfolio losses can be approximated using a quadratic representation.

$$Loss_{US}^{Market} = \beta_0^{US} + \beta_1^{US} * S\&P500 + \beta_2^{US} * USRate + \beta_3^{US} * (S\&P500)^2 + \beta_4^{US} * (USRate)^2 + \beta_5^{US} * S\&P500 * USRate + \varepsilon_{US}^{Market} \quad (10)$$

and

$$Loss_{UK}^{Market} = \beta_0^{UK} + \beta_1^{UK} * FTSE100 + \beta_2^{UK} * UKRate + \beta_3^{UK} * (FTSE100)^2 + \beta_4^{UK} * (UKRate)^2 + \beta_5^{UK} * FTSE100 * UKRate + \varepsilon_{UK}^{Market} \quad (11)$$

To parameterize the coefficients in these calibrations, we make the following assumptions which are based on intuitive relationships.

With $\beta_0 = 0$ we assume that losses are in excess of expected loss and the factors (including the second order polynomial terms) are normalized to have zero means. We also assume that the market portfolio has positive exposure to the equity markets and thus β_1 is negative as we are modeling losses. Third, β_2 is the measure related to the modified duration of the market portfolio and we would expect it to be positive (value goes down as the rates go up leading to increase in losses or decrease in gains). β_3 is assumed to represent instruments which tend to have concave sensitivity to changes in market returns and is expected to be negative. β_4 is related to the convexity measure and we would expect it to be positive. β_5 captures the cross-moment effects, which we expect to be negative to compensate for the correlation effects between these two factors. We present the coefficients used in the subsequent examples in Table 7.

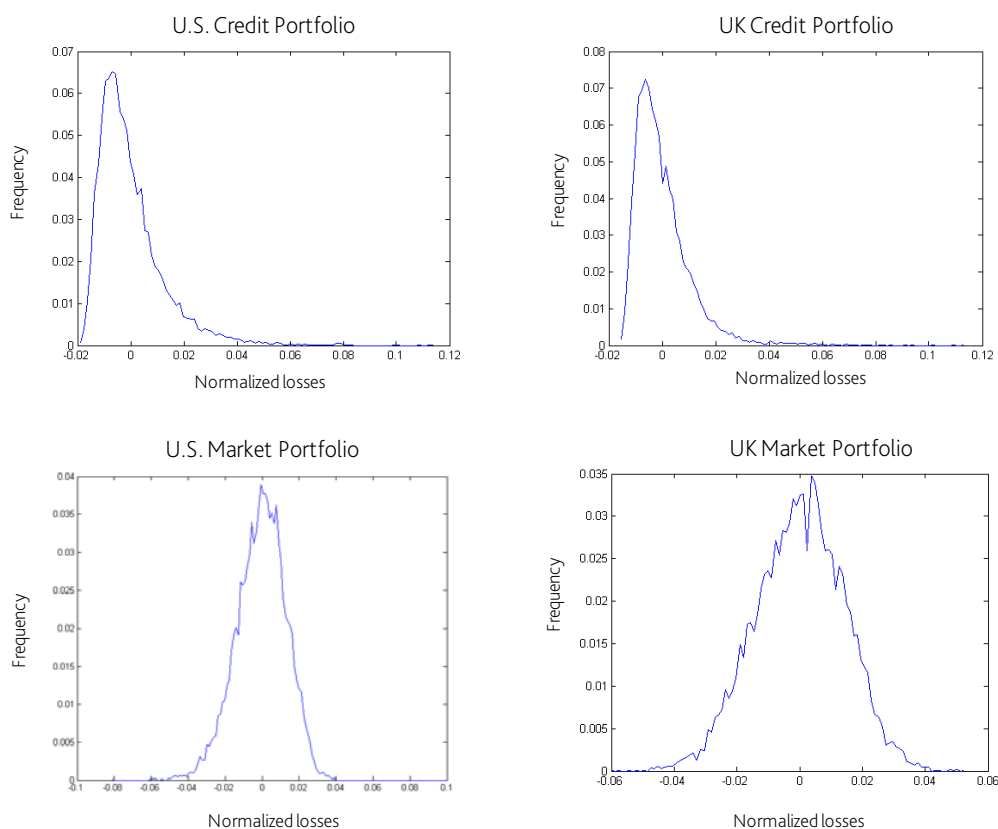
Table 7

Parameterization of the calibrated loss functions used in the example

COEFFICIENT	VALUE	COEFFICIENT	VALUE
β_1^{US}	-0.3	β_1^{UK}	-0.4
β_2^{US}	0.2	β_2^{UK}	0.15
β_3^{US}	-0.7	β_3^{UK}	-0.8
β_4^{US}	0.4	β_4^{UK}	0.3
β_5^{US}	-0.2	β_5^{UK}	-0.3

We can now use these parameterizations to calculate market portfolio loss distributions that are consistent with the credit portfolio loss distributions simulated by the RiskFrontier software with GCorr Macro⁴². Figure 15 demonstrates the stand-alone simulated and calibrated loss distributions in our exercise. Note that the loss distribution tail is much more pronounced for the credit portfolios compared to the market portfolios, as one would expect.

Figure 15 Stand-alone loss distributions simulated by the RiskFrontier software (U.S. Credit and UK Credit) and calculated using the calibrated loss functions based on simulated underlying factors (U.S. Market and UK Market)



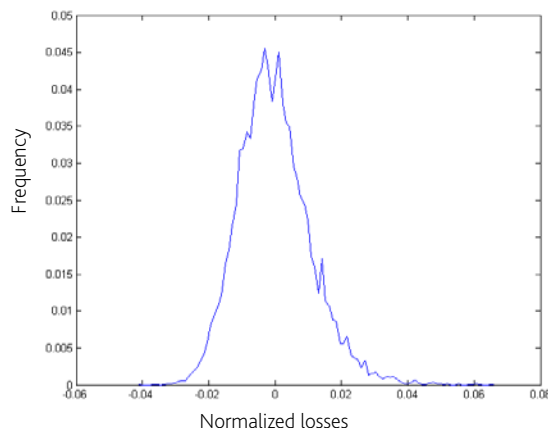
⁴² The credit portfolios used in the analysis are subsets of the IACPM portfolio, which is a diversified portfolio consisting of 3000 borrowers across seven developed countries and 60 industries. Specifically, the U.S. credit portfolio consists of 1,133 borrowers and the UK credit portfolio consists of 359 borrowers.

These stand-alone loss distributions are thus constructed consistently. Therefore, the aggregated loss distribution can be calculated by adding the loss realizations for the four loss distributions for each trial, guaranteeing that the underlying correlation structure is described by GCorr Macro.

$$Loss_i^{Aggregated} = Loss_{US,i}^{Credit} + Loss_{UK,i}^{Credit} + Loss_{US,i}^{Market} + Loss_{UK,i}^{Market} \quad (12)$$

Figure 16 displays the aggregated loss distribution.

Figure 16 Aggregate loss distribution combining market and credit risk



From the aggregate loss distribution, we can calculate the aggregate capital as a percentile of the distribution. Then the question arises: how much of that aggregate capital should be attributed to each sub-portfolio? To calculate these allocations one can use either Risk Contributions that measure the contribution of a particular portfolio to the aggregated portfolio's Unexpected Loss (or Standard Deviation of losses) or Tail Risk Contributions that measure the effect of various portfolios on the tail of the aggregated loss distribution.⁴³

$$RC^{RiskSource} = \frac{Cov(Loss^{RiskSource}, Loss^{Aggregated})}{UL^{Aggregated}} \quad (13)$$

$$TRC^{RiskSource} = E[Loss^{RiskSource} | LowerBound \leq Loss^{Aggregated} \leq UpperBound] \quad (14)$$

Because financial institutions often focus on tails (or extreme losses) of their portfolios, for this illustration we use Tail Risk Contributions that are calculated over the worst 1% of losses of the aggregate portfolio. Table 8 presents the standalone and allocated aggregate capital for each of the sub-portfolios. It demonstrates the magnitude of reduction in sub-portfolio capital coming from diversification effects within the aggregate portfolio.

Table 8

Standalone and reallocated capital

PORTFOLIO	PORTFOLIO MTM	STANDALONE CAPITAL	REALLOCATED CAPITAL	REDUCTION
U.S. Credit	36.284 B	7.93%	7.62%	3.91%
UK Credit	8.357 B	7.83%	4.46%	43.04%
U.S. Market	30.000 B	3.58%	2.86%	20.11%
UK Market	10.000 B	4.10%	3.65%	10.98%
Total	84.641 B	5.92%	5.15%	13.01%

Notice that the diversification effects are very different across sub-portfolios. For example, the U.S. Credit portfolio is the largest, and the dynamics of the aggregated portfolio are driven to a large extent by the dynamics of that portfolio. It is not surprising that this portfolio demonstrates the smallest diversification benefit. Meanwhile, the UK Credit portfolio exhibits the largest reduction.

⁴³ Tail Risk Contribution is usually calculated as contribution to capital, which represents discounted loss. In this case, the discount rate can be assumed to be zero.

5. Conclusion

This paper describes the different GCorr Macro model applications which facilitate a cohesive and holistic risk management. Specifically, we describe how GCorr Macro, within the RiskFrontier credit portfolio modeling framework, can be used for stress testing analysis, reverse stress testing, and risk integration.

This paper focuses on simulation-based methods for conducting one-period stress testing analysis. Within the RiskFrontier software, GCorr Macro provides insights into relationships between portfolio losses and macroeconomic variables. It is worth noting that the portfolio losses reflect credit downgrades and are based on valuation techniques in the RiskFrontier application. We introduce the conditional Monte Carlo simulation with GCorr Macro, and illustrate how the unconditional and conditional Monte Carlo simulation output can be used for one-period stress testing and reverse stress testing analysis. We present the results for various types of credit portfolios, namely U.S. and global corporates and U.S. residential and commercial mortgages.

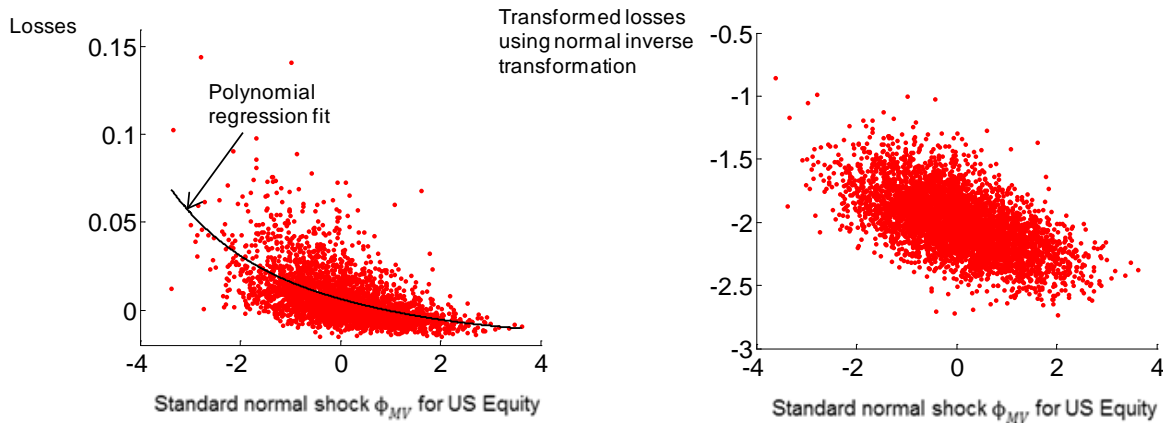
Risk integration is another application of GCorr Macro discussed in this paper. We describe how GCorr Macro can be used for risk integration including aggregation and allocation of capital across market and credit portfolios.

In addition to the applications described above, GCorr Macro provides further insights. While current GCorr factors are sufficient to explain systematic credit risk in a portfolio, these factors are latent. By expanding the GCorr model to include macroeconomic variables and other market risk factors, we can now use more intuitive factors to describe credit portfolio dynamics.

Appendix A Modeling Relationships Between Losses and Macroeconomic Factors

As discussed in Section 3.1, to quantify the relationship between portfolio losses and macroeconomic variables we first transform the losses using the normal inverse transformations to linearize the relationship between losses and macroeconomic factors. We then calculate the correlation between transformed losses and macroeconomic factors. We emphasize that we do not assume the transformed losses are normally distributed. Instead, we look for a transformation leading to a distribution as normal as possible.⁴⁴ Figure 17 illustrates the relationship between portfolio losses and macroeconomic variables. In particular, we plot the U.S. corporate portfolio losses and transformed losses versus the U.S. Equity factor.

Figure 17 U.S. Corporate losses vs. U.S. Equity factor



From Figure 17 we can observe that the relationship between losses and macroeconomic factors is not linear (left chart), while the relationship between transformed losses and macroeconomic factors is close to linear (right chart).

To assess whether the results from Table 3 are sensitive to the choice of the loss transformation, we try to apply an alternative approach to estimate the relationship between losses and macroeconomic factors. The alternative approach estimates a polynomial regression model. In this case, no transformations are applied to the losses and the following equation is estimated:

$$Loss_{Trial} = \alpha + \sum_{i=1}^I \beta_i \phi_{MV,Trial}^i + \varepsilon_{Trial} \quad (15)$$

where I is the order of the polynomial estimated.

This regression is estimated for each macroeconomic factor separately. Then, the regression R-squared is calculated and its square root is compared to the absolute values of correlation figures in presented in Table 3. Table 9 reports the square root of the regression R-squared for selected macroeconomic factors. We use a 5th order polynomial to obtain the results.

⁴⁴ The main objective of the transformation is to reduce skewness of the loss distribution.

Table 9

Square root of regression R-squared for selected macroeconomic factors

	U.S. CORPORATE PORTFOLIO	GLOBAL PORTFOLIO	U.S. CRE PORTFOLIO	U.S. RETAIL PORTFOLIO	AGGREGATE PORTFOLIO
U.S. GDP	39%	41%	40%	26%	45%
U.S. Unemployment	38%	50%	51%	32%	51%
U.S. Equity (Dow Jones Total)	56%	38%	30%	29%	50%
U.S. House Price Index	33%	51%	44%	36%	48%
U.S. CRE Index	41%	43%	49%	34%	50%
U.S. VIX	43%	28%	14%	23%	36%
Japan Equity (Nikkei)	21%	57%	13%	18%	36%

Comparing Table 3 and Table 9 we observe that the levels obtained from both approaches are comparable. Moreover, the rank ordering for these selected macroeconomic variables and portfolios is identical. For example, under both approaches the U.S. retail portfolio losses exhibit the highest correlation with the U.S. House Price Index factor, followed by U.S. CRE Index factor and U.S. Unemployment.

Appendix B Pre-Selected Macroeconomic Variables

In Section 3.2, we discuss variable selection and explain how we chose the best model from a set of pre-selected macroeconomic variables. The pre-selected variables include those variables that have—in a univariate context—statistically significant and economically intuitive relationship with portfolio losses. Table 10 presents the pre-selected macroeconomic variables for the five portfolios studied in this paper.

Table 10

Pre-selected macroeconomic variables

PORTFOLIO	PRE-SELECTED MVS
U.S. Corporate Portfolio	U.S. GDP, U.S. Nominal GDP, U.S. Unemployment, U.S. Short Rate, U.S. Long Rate, U.S. BBB Yield, U.S. Equity, U.S. House Price Index, U.S. VIX
Global Portfolio	Eurozone GDP, Japan GDP, UK GDP, Australia GDP, Eurozone Equity, Japan Equity, UK Equity, France Unemployment, Germany Unemployment, UK Unemployment, Australia Unemployment
U.S. CRE Portfolio	U.S. GDP, U.S. Nominal GDP, U.S. Unemployment, U.S. Short Rate, U.S. Long Rate, U.S. Equity, U.S. House Price Index, U.S. CRE Index, U.S. VIX
U.S. Retail Portfolio	U.S. GDP, U.S. Nominal GDP, U.S. Unemployment, U.S. Short Rate, U.S. Long Rate, U.S. Equity, U.S. House Price Index, U.S. VIX
Aggregate Portfolio	U.S. GDP, U.S. Nominal GDP, U.S. Unemployment, U.S. Short Rate, U.S. Long Rate, U.S. BBB Yield, U.S. Equity, U.S. House Price Index, U.S. CRE Index, U.S. VIX, Eurozone GDP, Japan GDP, Australia GDP, Eurozone Equity, Japan Equity, UK Equity, France Unemployment, Germany Unemployment, UK Unemployment, Australia Unemployment

Appendix C Estimating GCorr Macro Parameters

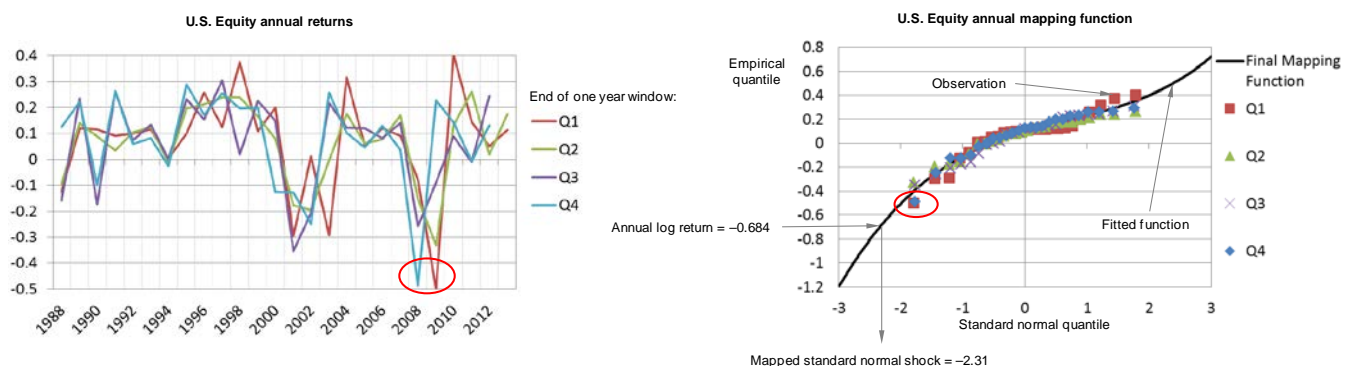
GCorr Macro is defined by two sets of parameters: (a) the correlation matrix linking GCorr systematic credit risk factors Φ_{CR} with standard normal macroeconomic factors Φ_{MV} , and (b) the mapping functions relating values of standard normal macroeconomic factors Φ_{MV} to values of observable macroeconomic variables MV .

As mentioned in Section 2, the mapping functions depend on the analysis horizon. For example, for a one-year horizon, we need to utilize mappings reflecting one year changes in macroeconomic variables. If the horizon is one quarter, we need mappings reflecting quarterly changes. We have estimated mapping functions for two horizons: one year (i.e., annual mapping) and one quarter (i.e., quarterly mapping). The annual mapping is relevant for standard RiskFrontier analysis, while the quarterly mapping is appropriate for CCAR style stress testing in the GCorr Macro Stressed EL Calculator. In this appendix, we describe the methodology used to estimate the annual mapping. The paper by Pospisil et al. (2014) describes estimation of quarterly mapping.

For the annual mapping estimation, we use macroeconomic time series from the early 1970s through 2013, or the longest possible period for variables with limited data. We estimate a mapping for each macroeconomic variable separately. The methodology used is identical to the one used for the quarterly mapping. In particular, we transform the macroeconomic variables to obtain stationary time series, and then map the quantiles of the empirical distribution of the transformed time series to the quantiles of a standard normal distribution at the same probability level. Because we need to map any scenario value to a standard normal factor, not just the historical values, we then fit a function to the empirical quantile mappings. Our analyses indicate that third degree polynomials provide the best fit for most variables.

The main challenge when estimating the annual mapping is that the sample period available includes only about 40 years of data. To utilize as much data as possible, we decide to compute annual returns at a quarterly frequency, that is, returns over a one year window rolled over a quarterly frequency. We try two different approaches. One approach computes a unique quarterly time series of annual returns for each macroeconomic variable and estimates the empirical quantile mapping using this series. The main problem with this approach is the high autocorrelation of the series, since the annual returns are computed from overlapping periods. As an alternative, for each macroeconomic variable we construct four separate time series of annual returns, so that each of the four series consists of returns over non-overlapping windows. We then estimate a mapping function for each of the four series and take the average of the four mapping functions as the final annual mapping function. Figure 18 illustrates this approach using the U.S. Equity as an example.

Figure 18 Annual mapping estimation for the U.S. Equity



The left hand side of Figure 18 shows the four time series of annual U.S. Equity returns.⁴⁵ From the four series, we can conclude that an annual drop in the stock market of more than 50% is an extreme shock based on historical information. The right hand side shows the final mapping function estimated. An annual drop in the stock market of 68% (shock assumed in the CCAR 2014 Severely Adverse Scenario from 2013 Q3 to 2014 Q3) is mapped to a standard normal shock of -2.31, an extremely rare event based on the historical data. Typically, we observe the worst observation over the past 25-40 years to be mapped to a standard normal quantile of around -2. A scenario much more extreme than the worst historical observation gets mapped to a shock of a bigger magnitude than -2.

To validate the annual mapping functions, we conducted different exercises to examine how the losses projected by GCorr Macro compared to the historical losses. It is important to mention that for all validation exercises, we used the final mapping functions (estimated according to the method from together with the quarterly correlation matrix described in the paper by Pospisil et al. (2014)). To ensure this approach is reasonable, we ran validation exercises for different portfolios and compared the GCorr Macro

⁴⁵ The time series start in 1988 since the data available for the Dow Jones U.S. Total Stock Market Index (i.e., U.S. Equity) starts in 1987 Q1.

losses to different benchmarks. We found the results reasonable both in terms of matching the levels during the last financial crisis and time series dynamics. Appendix D presents the validation exercise run for a U.S. corporate portfolio as an example.

Appendix D Validation of GCorr Macro over a One-Year Horizon

This appendix presents one of the exercises conducted to validate the GCorr Macro model over a one-year horizon. The goal is to illustrate the levels and patterns in credit portfolio losses produced by GCorr Macro over recent economic episodes and compare them to different benchmarks. In particular, we consider a portfolio of exposures to U.S. large listed non-financial corporates⁴⁶ and estimate the losses projected by GCorr Macro under various historical scenarios. Table 11 summarizes the portfolio characteristics.

Table 11

Stylized portfolio used for validation

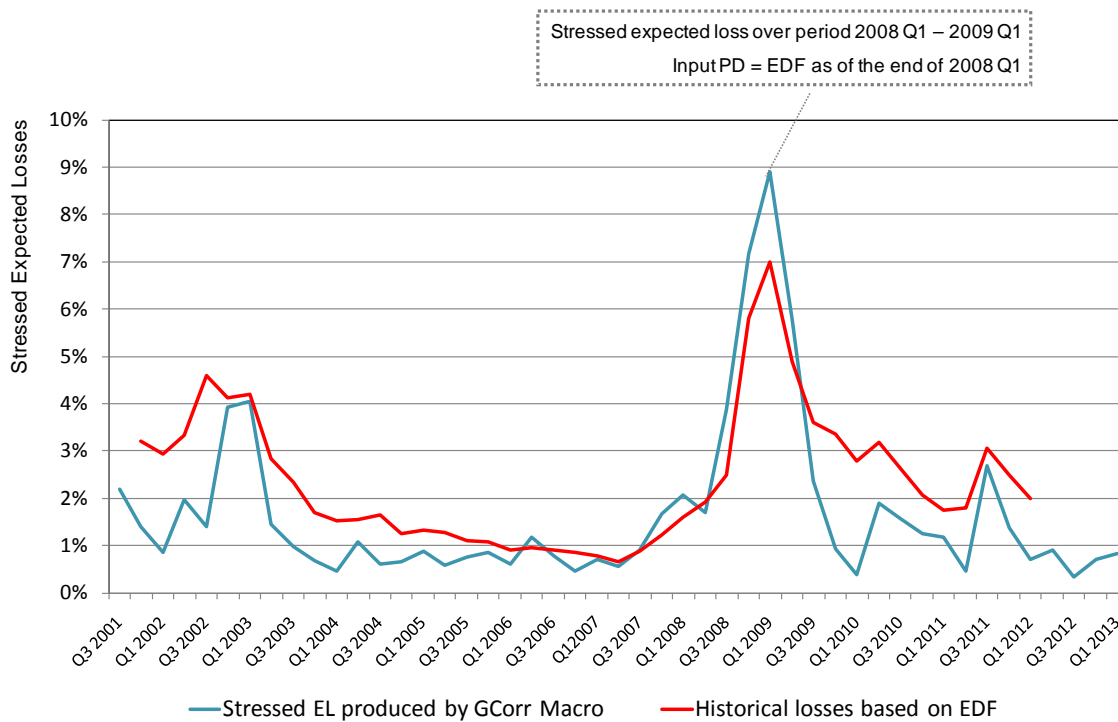
U.S. LARGE CORPORATES PORTFOLIO	
Types of Counterparties	Large U.S. listed non-financial corporates (size > 300 mil. USD)
Exposure Pooling	55 pools of loans Loans are pooled by 55 U.S. GCorr non-financial industries
Pool Weights	Pool weights proportional to the large firm counts by U.S. industries in GCorr 2012 Corporate
R-squared	Source: GCorr 2012 Corporate, large firm median R-squared values by industries Weighted average R-squared = 24.3%

As discussed in Section 3, to estimate the portfolio losses under any scenario we must first select the macroeconomic variables to be included in the scenario. In this case, we choose U.S. Unemployment, U.S. BBB Yield, U.S. Equity, and U.S. VIX. The historical scenarios are defined over a one-year horizon, at a quarterly frequency. For example, the scenario shock for U.S. Unemployment in 2009 Q1 refers to the U.S. Unemployment log return over the period 2008 Q1 to 2009 Q1 (i.e., the analysis date is the end of 2007 Q4). The mapping functions used for each selected variable are the annual mapping functions described in Appendix C. Finally, we use the quarterly correlation matrix described in the paper by Pospisil et al. (2014).

Figure 19 presents the stressed expected losses for the U.S. large corporate portfolio over the period 2001 Q3 to 2013 Q1. The input PD is defined as the EDF as of the analysis date, and we assume LGD to be equal to 40%.

⁴⁶ Note that this portfolio is different from the U.S. Corporate portfolio introduced in Section 3.

Figure 19 Stressed Expected Losses over historical scenarios: comparison of losses over one year windows



The period analyzed includes three distinct economic episodes:

- » Dot-com bust, the recession of 2001 and its aftermath.
- » Period of economic growth, approximately mid 2003– mid 2007.
- » Financial crisis and its aftermath.

The stressed expected losses projected by GCorr Macro show intuitive time series dynamics. Losses are higher during periods of economic distress and lower during the period of economic growth. Comparing the different crisis episodes, the stressed expected losses are higher during the recent financial crisis than during the early 2000s recession. We can attribute this to macroeconomic variable dynamics during these two episodes, where all macroeconomic variables have seen larger annual shocks during the recent financial crisis.

To validate GCorr Macro under a historical scenario, we can use historical EDF as a benchmark. In effect, we assume that historical losses were equal to the historical EDF level times a constant LGD of 40%. As Figure 19 shows, GCorr Macro projects somewhat higher losses compared to losses implied by historical EDF during the last financial crisis. Regarding time dynamics, we find that GCorr Macro losses and EDF time series exhibit similar patterns, experiencing the crisis peak at the same time, though GCorr Macro projects a faster recovery due to rapid improvements in some macroeconomic variables, such as U.S. Equity. We also make the comparison of the stressed expected losses from GCorr Macro with the default rate time series and found that GCorr Macro losses precede the default rate during the last financial crisis. This is because GCorr Macro produces high stressed credit parameters and expected losses for the same period when the scenario assumes a negative shock to the macroeconomic variables, while defaults are usually realized over several periods instead of within one period. Note that we spread expected losses over several quarters in the multi-period Stressed EL Calculator in order to match the default rate patterns. However, a one-period analysis is not intended to account for this type of dynamics.

Finally, it is important to remark that if we used another mapping function, the level of losses would be different. For example, using the mapping function derived from the unique quarterly time series of annual returns, we obtain higher level of losses during the last financial crisis.

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