

September 2013

# Modelling and Stressing the Interest Rates Swap Curve

---

**Prepared by**

Juan M. Licari  
[Juan.Licari@moodys.com](mailto:Juan.Licari@moodys.com)  
Senior Director

Olga Loiseau-Aslanidi  
[Olga.Loiseau-Aslanidi@moodys.com](mailto:Olga.Loiseau-Aslanidi@moodys.com)  
Asst. Director

Jose Suarez-Lledo  
[Jose.Suarez-Lledo@moodys.com](mailto:Jose.Suarez-Lledo@moodys.com)  
Director

---

**Contact Us**

Email  
[help@economy.com](mailto:help@economy.com)

U.S./Canada  
+1.866.275.3266

EMEA (London)  
+44.20.7772.5454  
(Prague)  
+420.224.222.929

Asia/Pacific  
+852.3551.3077

All Others  
+1.610.235.5299

Web  
[www.economy.com](http://www.economy.com)

---

---

**Abstract**

We present a two-step modelling and stress-testing framework for the term structure of interest rates swaps that generates sensible forecasts and stressed scenarios out of sample. Our methodology is also able to replicate two important features of the data: the dynamics of the spread across maturities and the alignment of the key swap rates tenor points to their corresponding government yields. Modern models of the term structure of interest rates typically fail to reproduce these and are not designed for stress-testing purposes. We present results for the euro, the U.S. dollar, and British pound swap curves.

# Modelling and Stressing the Interest Rates Swap Curve

BY JUAN M. LICARI, OLGA LOISEAU-ASLANIDI, AND JOSE SUAREZ-LLEDO

We present a two-step modelling and stress-testing framework for the term structure of interest rates swaps that generates sensible forecasts and stressed scenarios out of sample. Our methodology is also able to replicate two important features of the data: the dynamics of the spread across maturities and the alignment of the key swap rates tenor points to their corresponding government yields. Modern models of the term structure of interest rates typically fail to reproduce these and are not designed for stress-testing purposes. We present results for the euro, the U.S. dollar, and British pound swap curves.

## Executive summary

In recent years, modelling and forecasting interest rates and yields has acquired a central role for central banks, policymakers, regulators and practitioners. Broadly speaking, there are two mainstream approaches to modelling the term structure. Both leverage the correlated structure of the cross section of maturities to decompose the interest rates curve into a reduced number of factors and their corresponding loadings. They differ, however, both on the structure they place on the factors and loading and on the way they model the interactions with the macroeconomy. The first approach is more common in the finance world, while the second is known as the macro-finance approach, more common in central banks and policy institutions.

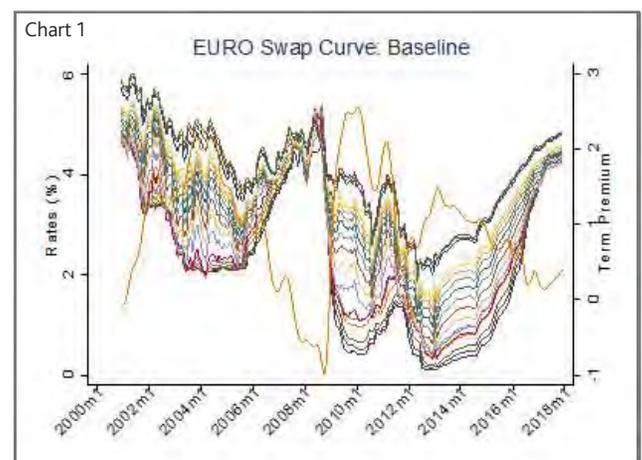
Our main contribution is on the realm of methodology for forecasting and stress-testing the interest rates curve. Modern models of the term structure of interest rates typically fail to reproduce important features of the data. Furthermore, these models are not designed for stress-testing purposes and can generate sensible forecasts for only a very short time horizon. However, practitioners would normally need to forecast and stress-test the term structure for longer horizons: two, three, even five years. We present here a two-step

approach to modelling and stressing the interest rates curve over long horizons that is also capable of generating sensible forecasts by targeting two features of the data: the dynamics of the spread across maturities (term premium) as economic conditions evolve and the alignment of key swap rates tenor points to their corresponding government yields.

We proceed first by designing a macroeconomic model that generates the different paths for the key macroeconomic variables under baseline and alternative scenarios. We then design a model for the term structure that takes those macroeconomic paths as drivers for the purpose of stress-testing. This approach is also supported by results in academic literature. In contrast with modern models of the term structure, we impose structure neither on the loadings nor on the factors, other than the latter being independent, which will actually help in avoiding biases in the scenarios. This is achieved by means of Principal Component Analysis.

The model for the dynamics of the interest rates

swap curve will be defined by an autoregressive structure of the factors (also interpreted as the level and slope of the curve), which are also a function of the economic drivers. In particular, the level of the curve is closely linked to the money market rate and the 10-year government yield, since the level factor normally encapsulates medium-term inflation expectations. The slope factor is, rather, related to shorter-term reactions of the economy and is thus driven by changes in GDP as well as changes in the term premium. We present here results for the euro and U.S. dollar swap curves. Chart 1 is an example of the euro swap curve under the baseline scenario.



### Introduction

In recent years, modelling and forecasting interest rates and yields has acquired a central role for central banks, policymakers, regulators and practitioners. It is of crucial importance for central banks and policymakers to understand the effects of their actions on the different segments of the interest rates curve, especially the short and long ends, that will ultimately anchor expectations and transmit monetary and fiscal policy. Needless to say, that interest rate risk and the movements of the full term structure are among the more important areas of risk management and stress-testing for banks and regulators.

The academic literature has developed a non-negligible number of models of the term structure that have been later adopted by practitioners. These models could be divided into two groups whose foundation is the reduction of the dimension of the cross section of maturities to a lower number of unobserved factors that summarizes the dynamic properties of the whole cross section. However, these two approaches differ on the assumptions about the underlying determinants of the term structure as well as on their technical treatment. The first group of models stemmed from the work of Vasicek (1977) and Cox, Ingersoll and Ross (1985) are built on risk neutrality and the no-arbitrage condition.

To the second group belongs the so-called macro-finance stream of models that do not necessarily impose risk neutrality or the no-arbitrage condition but explicitly model the relationship of the macroeconomic variables with the term structure of yields and interest rates. These models stem from the dynamic version of the Nelson and Siegel (1987) work and are well-represented by Diebold and Li (2006) or Diebold, Rudebusch and Aruoba (2004). Even though both streams started early on and seemed to not intersect, they were eventually connected by Christensen, Diebold and Rudebusch (2009), who show how the Dynamic Nelson-Siegel models of the term structure can be extended to be made arbitrage-free and therefore equivalent to the term structure models used in the risk-neutral finance area.

Therefore, in this paper we will review only the methodology followed by the macro-finance approach.

Our main contribution is in the realm of methodology for forecasting and stress-testing the interest rates curve. Although great progress has been made in understanding interest rates, and refined models have been developed, their forecasting and stress-testing performance remains less encouraging. During the last decade, efforts have been made in several directions to incorporate macroeconomic factors to models of the term structure—Ang and Piazzesi (2003), Diebold and Li (2006), Diebold et al (2006), Ang et al (2007), and Rudebusch and Wu (2008). Such efforts were initially undertaken in order to relate movements in the curve to factors that were more easily interpretable and to increase the in-sample fit. However, no attempt at forecasting or stress-testing for a significant time horizon and in a dynamic environment was made at that stage.<sup>1</sup>

In fact, whether for business planning or for regulatory compliance, practitioners would normally need to forecast and stress-test the term structure for longer horizons: two, three or even five years. We present here a two-step approach to modelling and stressing the interest rates curve over long horizons. We try to develop a methodology that is capable of generating sensible forecasts by targeting two features of the data. On the one hand, current models appear to have difficulty in reproducing the dynamics of the spread across maturities as economic conditions evolve. In particular, it is observed in the data that under certain conditions the spread across maturities widens considerably, whereas in other environments the spread is significantly reduced. On the other hand, to the best of our knowledge, no methodology for interest rates swap curves looks at the fact that certain swap rates tenor points bear a close relationship to their corresponding government yield tenor. We believe that it would be a desirable property

that the outcome from the model reflected this relationship.

### Methodology

The nature of a stress-test exercise is unidirectional, as defined by regulation, modelling a risk metric as a function of the economic variables. This approach implies allowing for the economic drivers to impact the swap rates in this case, but not otherwise. More important, there is evidence from different setups that there is a significant effect from macroeconomic variables on the term structure but not so much in the reverse direction (Diebold et al [2006], Ang et al [2007], Dewachter and Lyrio [2002], and Rudebusch and Wu [2003]).

Furthermore, Joslin, Priebsch and Singleton (2012) argue that current macro-finance models may impose strong and counterfactual constraints on how the macroeconomy interacts with the term structure. They maintain that one should model macroeconomic risks that are distinct from yield curve risks, and they propose an asymmetric treatment of yields and macro variables in which the economic factors are not spanned by any portfolio of bond yields.

In line with these observations, our proposed framework to conduct stress-testing of swap rates is a two-stage process. The first stage involves forecasting the dynamic paths of key macroeconomic indicators such as GDP, money rates and government yields under different scenarios. These projections are generated by means of a macroeconometric model that will be discussed below. The dynamics of these macro models are driven by a set of simultaneous equations built upon economic theory and econometric methods. By including some key financial variables such as government yields, we account for the presence of feedback loops between the macroeconomy and the financial sector. In the second stage, we develop a factor model for the full curve of interest rates that explicitly integrates the macroeconomic drivers generated in the first stage. Because these drivers are forecast under alternative assumptions, we will be able to project the term structure of interest rates over those different scenarios.

<sup>1</sup> Some models such as Ang and Piazzesi feature static factors, while models with dynamic factors and macroeconomic variables perform out-of-sample exercises for only very short forecast horizons (Pooter et al [2007]).

As part of our exercise, we will compare the forecasting properties of our modelling approach with other dynamic models of the term structure such as Diebold and Li (2006). That model imposes functional forms on the way the different maturities load on the factors while leaving the factors free.<sup>2</sup> We do not impose any structure on either loadings or the factors.

### Macroeconomic scenarios

Part of the literature on interest rates generates forecasts for the macroeconomic factors along with those for the interest rates by estimating them jointly in a vector autoregressive system. This branch of the literature often focuses purely on short-term forecasting accuracy. However, our main interest in this paper lies in stress-testing, and for that purpose we will consider conditional forecasts. In short, the interest rates curve will be linked to a set of economic factors whose forecasts under alternative scenarios are derived separately.

In order to forecast macroeconomic variables, we employ a macroeconomic model represented by the system of simultaneous equations inspired by the Cowles Commission's approach.<sup>3</sup> Such models are still widely used among practitioners despite some criticisms (Simon, Pouliquen, Monso, Lalanne, Klein, Erkel-Rousse and Cabannes [2012]) thanks to their practical usefulness and a balance between consistency with economic theory and actual data fit. These are nonstructural models in that they are built from many equations that describe relationships derived from empirical data, yet they are structural models in that they also use economic theory to postulate the relationships.

In the broadest sense, the macro model we use describes aggregate economic activity determined by the intersection of aggregate demand and supply. In the short run, fluctuations in economic activity are primarily

ly determined by shifts in aggregate demand, while the level of resources and technology available for production is taken as a given. Prices and wages adjust slowly to equate aggregate demand and supply. In the long run, changes in aggregate supply determine the economy's growth potential. The rate of expansion of the resource and technology base of the economy is the principal determinant of the pace of economic growth.

Our model is composed of a set of equations for "core" and "auxiliary" endogenous variables. The core variables are the most important and decisive variables such as GDP and its components, trade, labor market, prices, and monetary policy. The system also includes exogenous variables such as population growth, global GDP, and global energy prices, which are forecast outside the macro model.<sup>4</sup> These exogenous variables relate to foreign demand, international competitiveness and foreign prices affecting a small, open, domestic economy and are the starting point of our forecast process. Also important, they are key sources of where exogenous shocks could originate from. In turn, the auxiliary variables may be driven by the core and exogenous variables but are not allowed to determine the core variables. Examples of such second-tier endogenous variables are price deflators and industrial production.

Formally, the reduced form for the system of simultaneous equations can be written as:

$$Y_t = \sum_{k=1}^K \beta_k Y_{t-k} + \sum_{p=0}^P \beta_p X_{t-p} + u_t$$

where  $Y_t$  is the vector of endogenous variables;  $X_t$  is the vector of exogenous variables, and  $\beta_k, \beta_p$  are coefficient matrices. The specification of each individual equation is selected based on statistical properties, back-cast performance, evaluation of short-term and long-term forecasts, the system's stability, and parsimony. The whole macro model is also shocked with stress scenarios of exogenous or endogenous variables to

ensure that the responses of the system to impulses are within a reasonable range.<sup>5</sup>

Forecasts are obtained from simulations on these models where regressions are used to estimate coefficients based on historical relationships and theoretical a priori. Our scenario generation begins with our baseline forecast, from which we develop the basic outlines of alternative scenarios by running multiple simulations to develop a probability distribution of economic outcomes. We then produce alternative scenarios that align with this probability distribution. On page 4 are some examples for both the euro zone and U.S. economies.

### Modelling swap rates

When modelling the term structure, the correlated dynamics of the cross section of maturities plays an important role, as it allows data to be compressed into a lower-dimensional vector of unobserved factors. A very popular specification frames the interest rates in a state-space form

$$R_t = A + LF_t + \varepsilon_t \quad (1)$$

$$F_t = \alpha + \sum_{k=1}^K \beta_k F_{t-k} + v_t, \quad v_t \sim N(0,1) \quad (2)$$

The first equation models the different interest rates as a function of N factors, F, and the second equation models the dynamics of the swap rates curve through a number, K, of lags of the factors.  $R_t = [r_t(1), \dots, r_t(M)]$  denotes a  $(M \times 1)$  vector of swap rates observed at time t for M different maturities;  $F_t$  denotes a  $(N \times 1)$  vector of factors obtained from the interest rates data with  $N < M$ . A is a constant matrix that may generally be zero; and L is the matrix that defines how the interest rates depend on the factors.  $\varepsilon_t$  are approximation errors that will be described below, and  $v_t$  are standard regression errors.  $\varepsilon_t$  and  $v_t$  are mutually orthogonal.

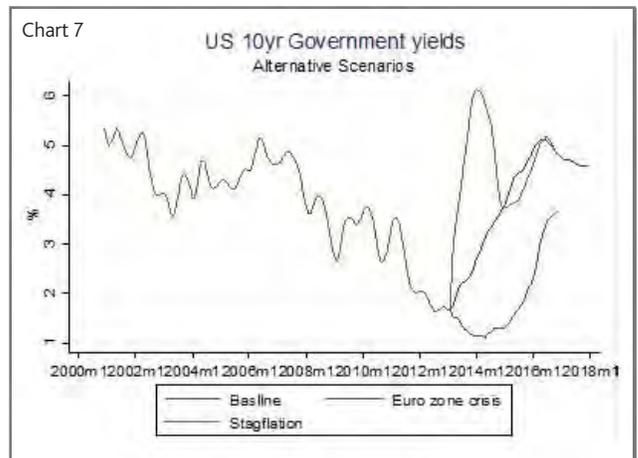
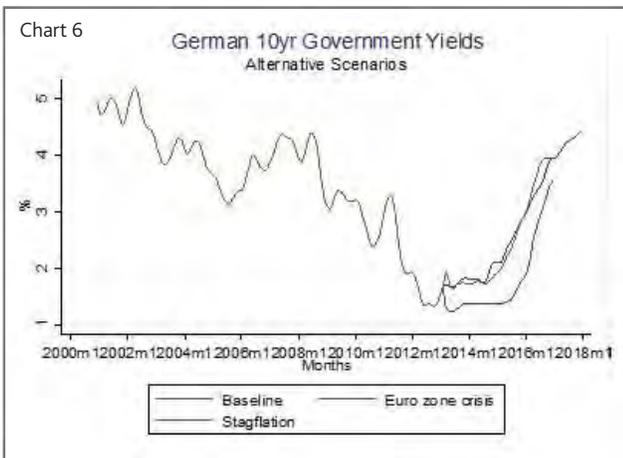
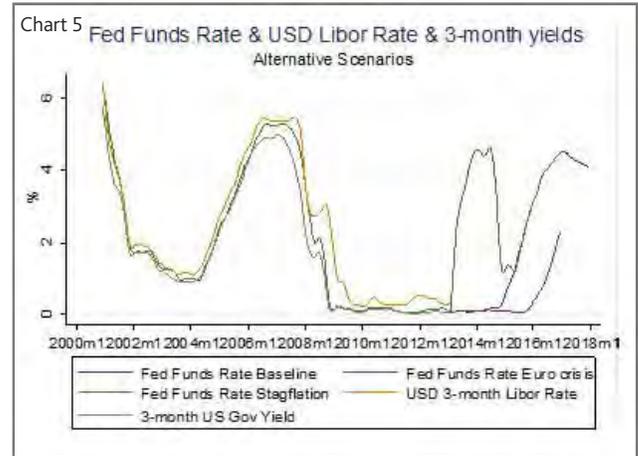
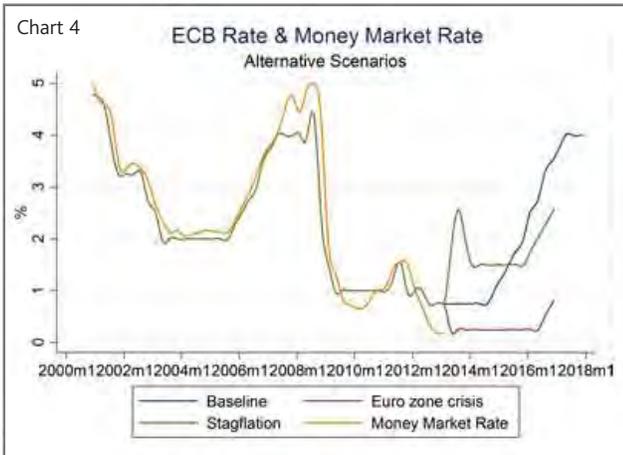
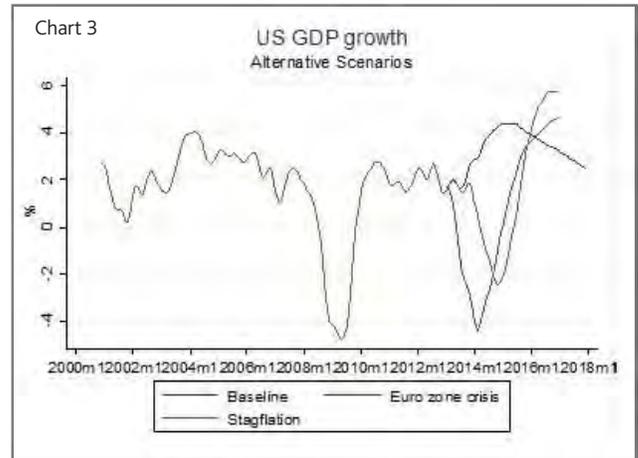
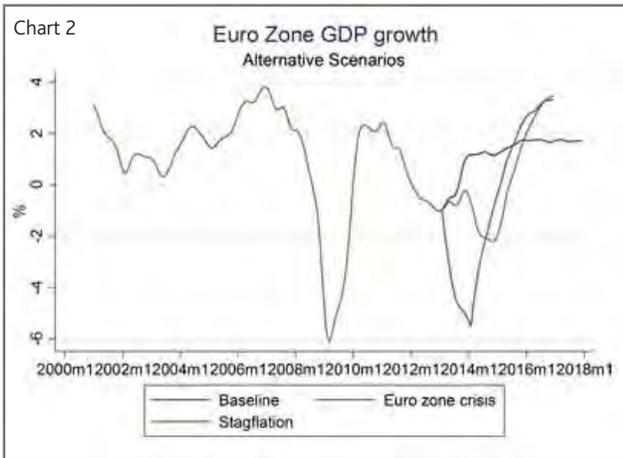
The state space representation in (1) and (2) nests most of the existing models for modelling and forecasting the term structure commonly used in the literature as well as by practitioners. In our model, however, we include a set of

<sup>2</sup> The models commonly used in finance place further structure by restricting both loadings and factors.

<sup>3</sup> Cowles Commission approach can be thought of as specifying and estimating approximations of the decision equations (Simon, Pouliquen, Monso, Lalanne, Klein, Erkel-Rousse and Cabannes [2012]).

<sup>4</sup> The forecasts of exogenous variables such as population projections are sourced from international agencies including the International Monetary Fund and the World Bank.

<sup>5</sup> Cointegration and error correction methods are used when appropriate in separate equations.



economic drivers,  $E_t$ , obtained from our macro models, that enter the second equation as exogenous determinants of the factors dynamics:

$$F_t = \alpha + \sum_{k=1}^K \beta_k F_{t-k} + \sum_{j=0}^J \beta_j E_{t-j} + v_t, \quad v_t \sim N(0,1) \quad (3)$$

The system (3) is then estimated as a VAR of typically order 1 ( $K=1$ ). While in most of the literature the macroeconomic drivers and their relationships with the factors are estimated as endogenous variables in the VAR  $F_t = (F_{1t}, \dots, F_{Nt}, E_{1t}, \dots, E_{Ht})$  for a number  $H$  of economic drivers, we focus here on the stronger directional causality from macroeconomic variables to the interest rates curve, as discussed before and reported in the literature.

Even though most modern models of the term structure consider three factors, that are interpreted as the level, slope and curvature of the interest rates curve, we will follow here more recent studies that consider only the first two of those factors, as the curvature factor tends to show little variability and almost no relation to economic variables. Although such is the most widely used approach, modern models differ in the way they extract the factors and the loadings of the different maturities on those factors. The macro-finance approach streaming from Diebold and Li (2006) and Diebold et al (2006) places structure on those loadings, leaving the factors to be determined in the following system of equations:

$$r_t(m) = L_t + \frac{1-e^{-m\lambda}}{m\lambda} S_t + \left( \frac{1-e^{-m\lambda}}{m\lambda} - e^{-m\lambda} \right) C_t \quad (4)$$

where  $r_t$  is the interest rate at time  $t$  for maturity  $m$ ;  $L_t$ ,  $S_t$ , and  $C_t$  are the level, slope and curvature factors; and  $\lambda$  is a parameter controlling the decay of the dependence on the factors.

In contrast, the Principal Component Analysis does not place any structure on the loadings or on the factors, other than the latter being orthogonal. This technique extracts the factors through the diagonalization of the correlation matrix of the data—that is, they are the eigenvectors of the data covariance matrix and therefore are purely data-driven. Thus, interest rates are a linear combination of these eigenvectors (factors):

$$R_t = \Lambda' V_t^\lambda \simeq L F_t, \quad (5)$$

where  $V_t^\lambda$  is the matrix of eigenvectors and  $\Lambda$  is the matrix of the loadings of the eigenvectors on the interest rates. PCA produces orthogonal factors by construction, therefore  $\Lambda' = \Lambda^{-1}$ . The set of  $M$  eigenvectors explains all the variance in the set of  $M$  interest rates. However, since our aim is to reduce the dimension of the model, we want to consider only the set of first  $N$  eigenvectors (factors,  $F_t$ ) that would still explain most of the variance of the dataset. The choice of PCA is based on the fact that the orthogonality of the factors allows the reduction of the dimension without generating a bias from omitting some of the factors, or from modelling rates as a function of factors that are not independent. Also, using independent factors extracted from the correlation matrix will better capture the underlying structural relationships in the data, and each factor will explain a different part of the data.

In line with most of the recent literature, we find that the first two factors (level and slope) account for about 98% of the variance in the data, and therefore we will focus on the modelling of these two. This implies that  $F_t = (Level_t, Slope_t)$  with  $N=2$ . Thus, estimating the curve of interest rates,  $R_t$ , as a function of these two factors—equation (1)—will always carry an approximation error,  $\varepsilon_t$ , as there will always remain a small fraction (about 2%) of the data unaccounted for. Finally, depending on the default transformations to the matrix of loadings,  $\Lambda$ , applied by the different software, it might be convenient to re-estimate the linear function in equation (1),  $L$ , that relates the interest rates to the two factors.<sup>6</sup> We will come back to this in the forecasting section below.

It is important to note that the factors extracted in the macro-finance literature are not guaranteed to be independent. Also, factors estimated through the Kalman filter may impose normality. PCA instead is a neutral technique in that sense, respecting the prop-

erties of the data whatever they may be. As a final note, this approach based on PCA is silent about the no-arbitrage condition. We follow here the advice in Duffee (2012) and Diebold and Rudebusch (2013)<sup>7</sup> that if the no-arbitrage condition is embedded in the data, imposing it does not improve the forecasts, whereas if it is not present in the data, imposing it will create a bias. It is precisely in stressed times that no-arbitrage may be less likely to hold, and we are interested here in developing a methodology for stress-testing.

## Estimating the dynamics of the curve

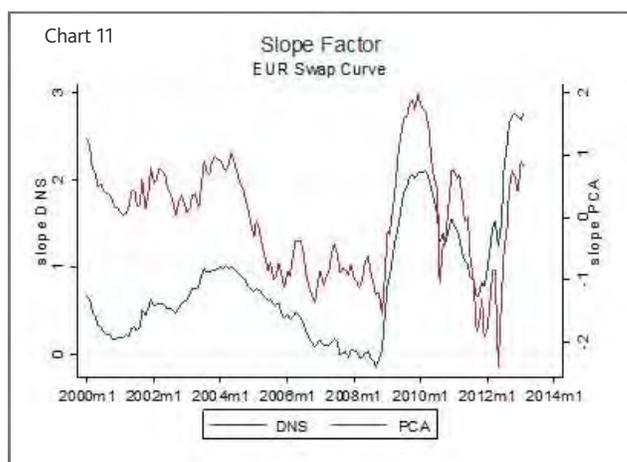
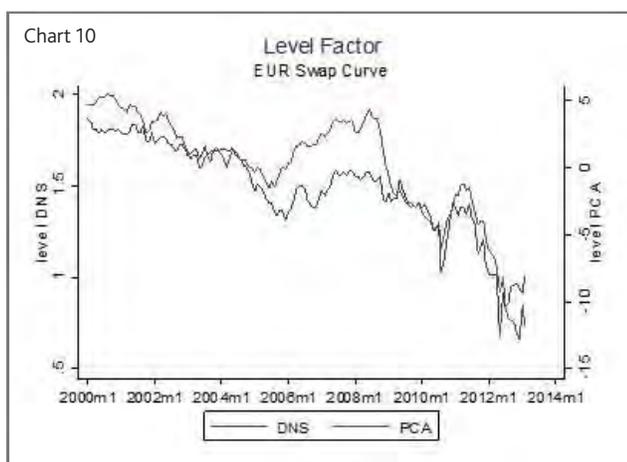
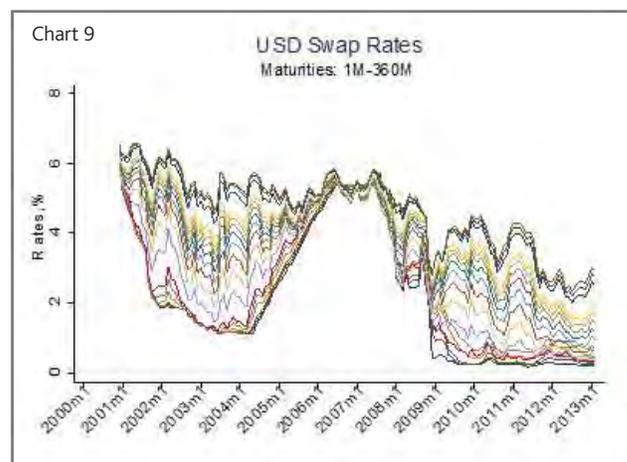
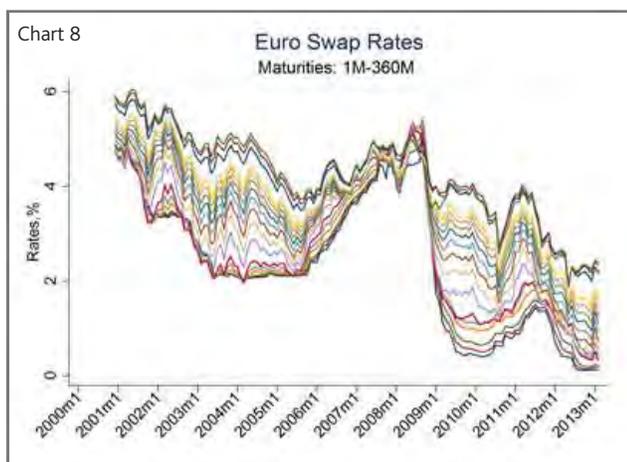
We consider monthly data for interest rates swaps for the euro. The sample period is 2000:1 to 2013:2. The cross section of maturities includes the spot swap contract rates for tenor points one, two, three, six and nine months, and forward swap contract for one-, two-, three-, four-, five-, six-, seven-, eight-, nine-, 10-, 15-, 20- and 30-year tenor points. Data have been retrieved from Bloomberg. We model the rates in logs in order to ensure strictly positive forecasted interest rates. Chart 8 illustrates the evolution of euro and GBP interest rates swaps over the sample period (see Charts 8-9 on page 6).

Sharp upswings in the euro short-term rates between 2006 and 2008 reflected the European Central Bank's controlling of thriving euro zone's economies with tight-money policies. In this expansionary period the spread between short- and long-term rates is very narrow. Following the peak in 2008, short-term rates fell sharply with economies in recession and policy rate cuts, while the longer-term rates formed a relatively smoother downtrend. This created a wider spread between short- and long-term rates, increasing sharply the slope of the swap rates curves, that is the difference between the long- and short-term rates. It is this behaviour of the spread across maturities that other models fail to capture and what we will use as a criterion of the forecasting ability of our approach.

In this section we want to compare the estimation and forecasting results of the

<sup>7</sup> Christensen, Diebold and Rudebusch (2009) adjust the Nelson-Siegel model to make it consistent with arbitrage-free models. Although they show that it forecasts well out-of-sample, Carriero, Kapetanios and Marcellino (2009), using a longer forecasting sample, report that the performance of the arbitrage-free DNS model is not that different from the two-step Nelson-Siegel model.

<sup>6</sup> Dauwe and Moura (2011) mention that "any set of vectors that can span the subspace generated by the loadings is then equivalent to the loadings without loss of accuracy".



macro-finance family of models, based on the Dynamic Nelson-Siegel approach, with the results from our model. We will also analyze our results in terms of our ability to capture both the dynamics of the spread across maturities and the alignment of the key swap rates to the corresponding yields.

Chart 10 shows that there may be significant differences between the two main factors, level and slope, extracted from the DNS model and those extracted via PCA. The time series of the DNS factors are extracted as described in equation (4) using the cross section of yields for each month, while fixing lambda<sup>8</sup> (see Charts 12-13 on page 7).

The following figures display connections between the latent factors and macroeconomic variables, providing some intuitive support for our models for the level and slope. Charts 14 through 21 show that the level factor appears to be closely linked to money market rate and 10-year sovereign yields. They also show the relation of economic growth and the term premium (defined here as the difference between the 10-year yield and the three-month money market rate) with the PCA slope factor (see Charts 14-21 on page 7-8).

We now model the dynamics of the factors in (6) following different approaches: (a) separate autoregressive integrated moving average models for each factor, (b) separate ARIMA models with autoregressive conditional heteroskedasticity innovations, and (c) VAR models for the factors with the economic variables as exogenous drivers and

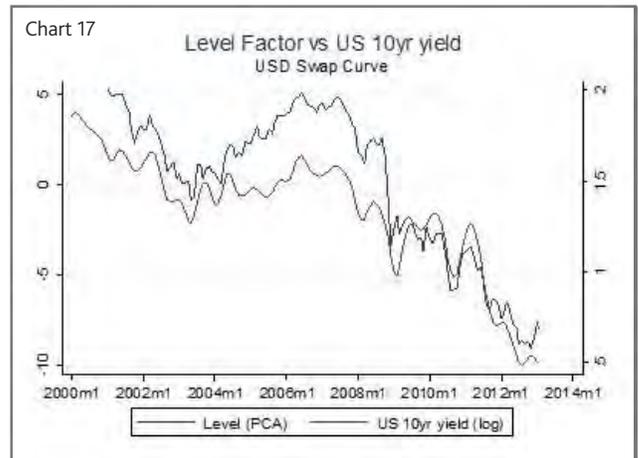
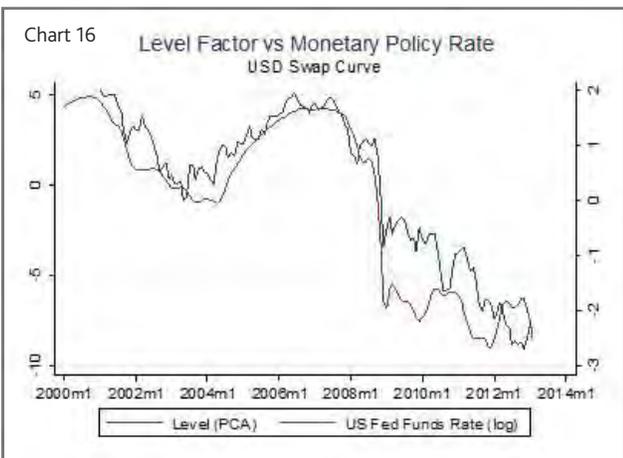
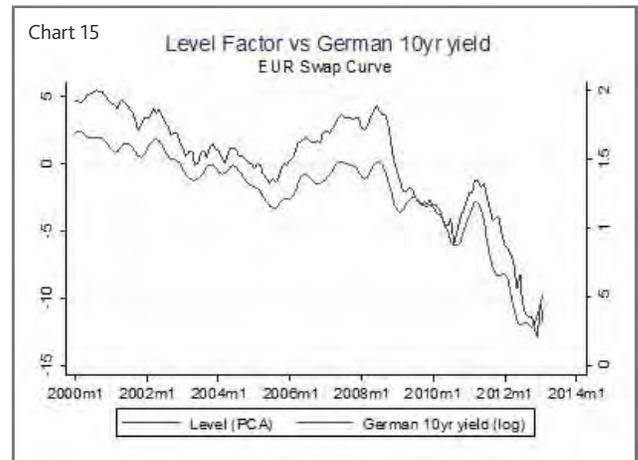
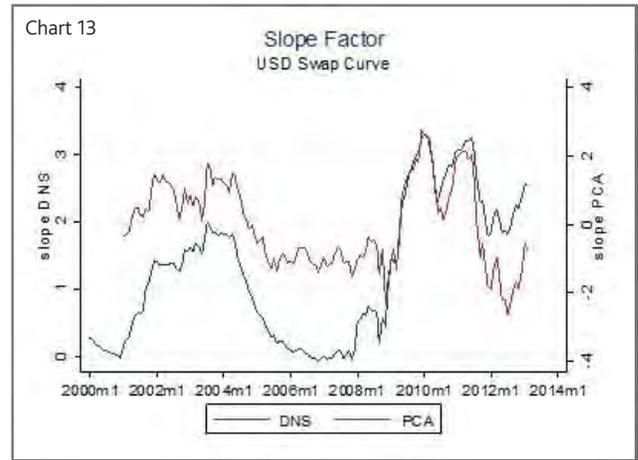
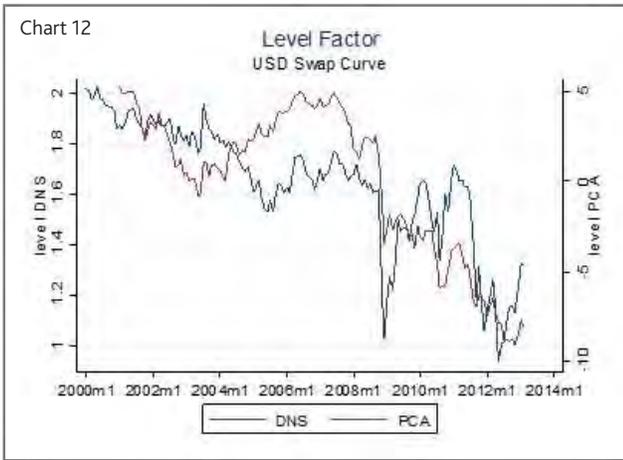
the first lag of the factors. The following system is representative of the models tested:

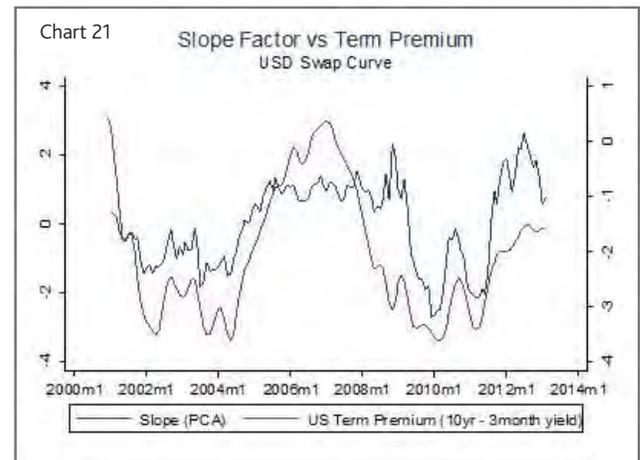
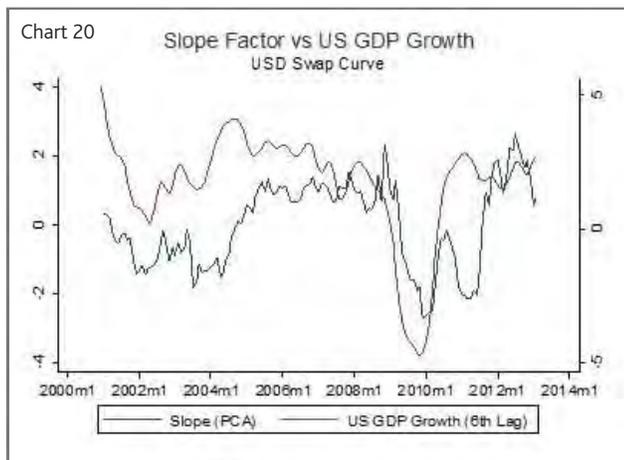
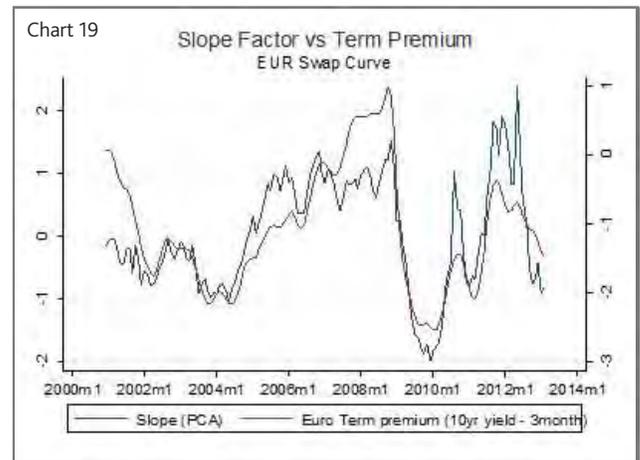
$$L_t = \beta_0 L_{t-1} + \beta_1 \text{Money rate}_t + \beta_2 \text{10yr rate}_t + \varepsilon_{L,t} \quad (6)$$

$$S_t = \varphi_0 S_{t-1} + \varphi_1 \text{GDP Growth}_t + \varphi_2 (\text{Term Premium}_t) + \varepsilon_{S,t} \quad (7)$$

Tables in the Appendix report parameter estimates of these models for the euro PCA factors. The parameter estimates signs and magnitude are mostly as expected by economic theory. Both the level and slope factors are highly persistent. The long-term and short-term interest rates are significant determinants of the level factor, which is typically interpreted as reflecting the evolution over time of the perceived medium-term inflation target. By doing this we also achieve the calibration of the short end of the swap curve to the short-term bond yields, as the money market rate moves very closely with the three-month yield rate. Moreover, 10-

<sup>8</sup> The main role played by lambda is to determine the maturity at which the loading on the curvature factor is at its maximum. In Diebold and Li (2006), the value of lambda that maximizes the curvature loading at 30 months is 0.0609.





year sovereign yields are also incorporated as part of this equation, as they reflect the longer-term inflationary expectation, which also allows aligning the long end of the curve.

The slope factor responds with some lag to the output deviation from its trend as well as to the term premium. The latter is included in the slope equation to complete the calibration of the whole curve: the difference between 10-year and the three-month yield rates. In other words, the level is a medium-to-long-term variable, whereas the slope reflects adjustments to short-term fluctuations.

**Baseline forecasting and stress-testing**

Models of type (b) do not seem to bring much extra value that could not be captured through seasonal-type effects, so we focus on the results for models (a) and (c). As we discussed in an earlier section, the loadings in equation (1) are re-estimated with a simple

ordinary least squares regression of the swap rates at each maturity on the level and slope factors.<sup>9</sup> In contrast, the DNS swap rates are calculated using the fixed functional form associated with the factors defined in equation (4).

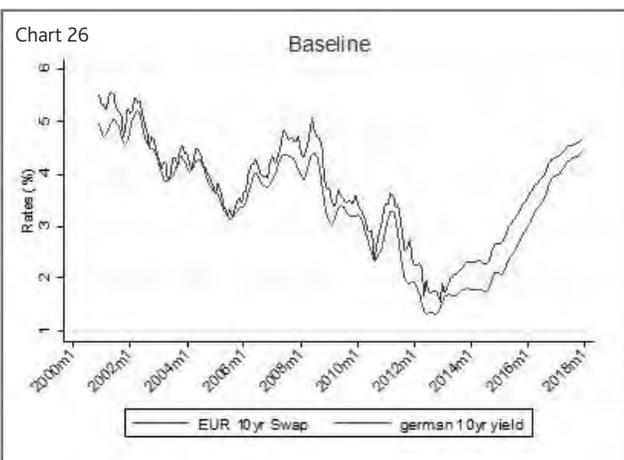
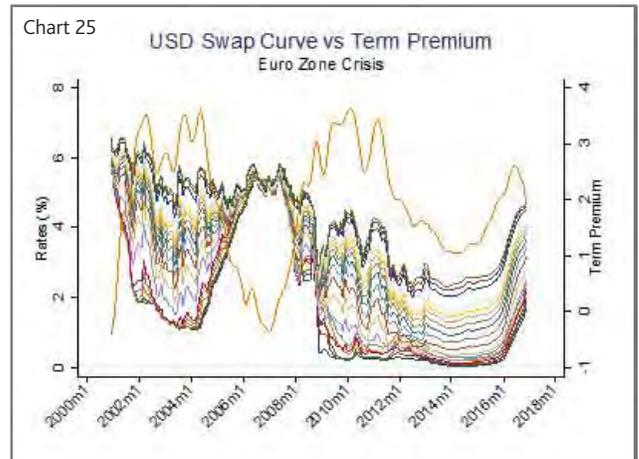
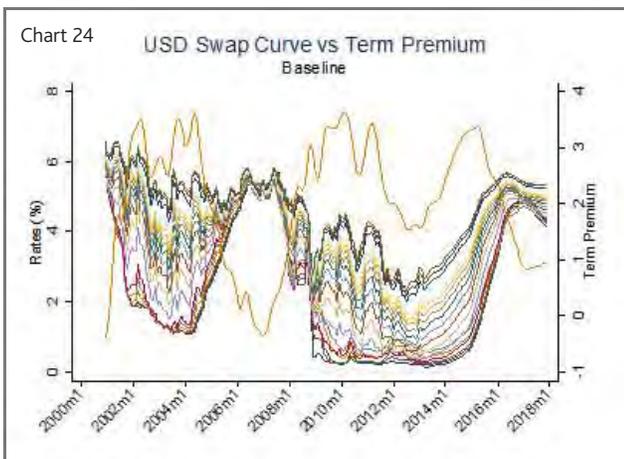
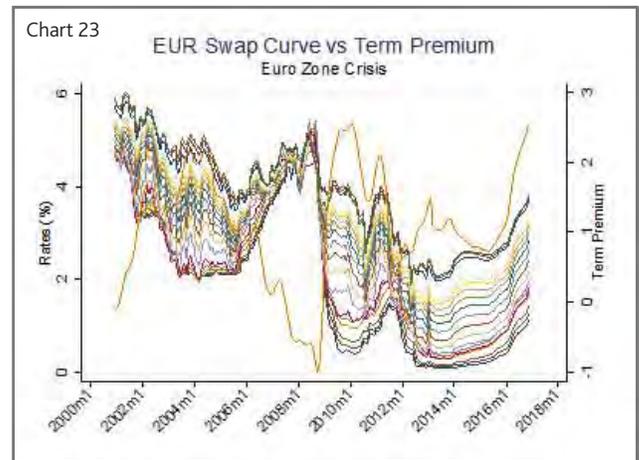
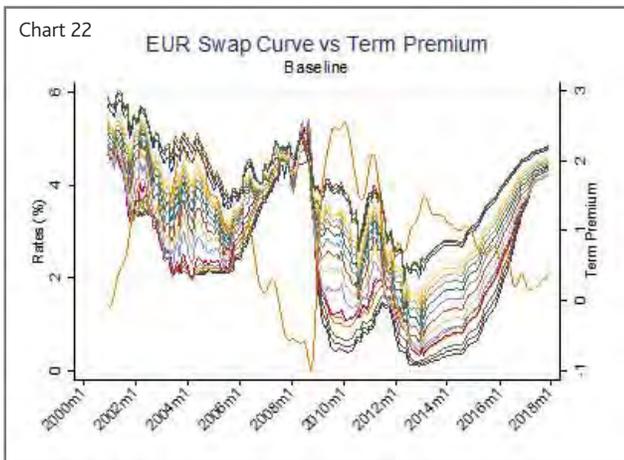
Given a set of parameter estimates from models (a) and (c) we compute *conditional* dynamic forecasts of endogenous variables (level and slope) for the period 2013:3 through 2018:3. Forecasts for the swap rates conditional on the macro variables projections under the baseline and the euro zone crisis scenarios are shown in Chart 22. The PCA approach seems to be able to replicate the historical behaviour of the spread across maturities based on macro-economic fundamentals. The PCA-based

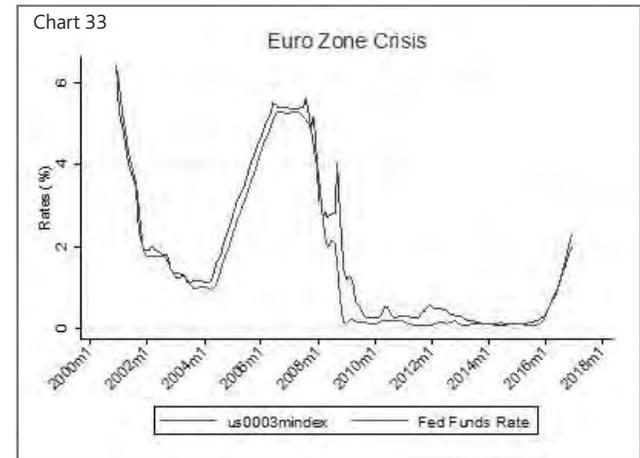
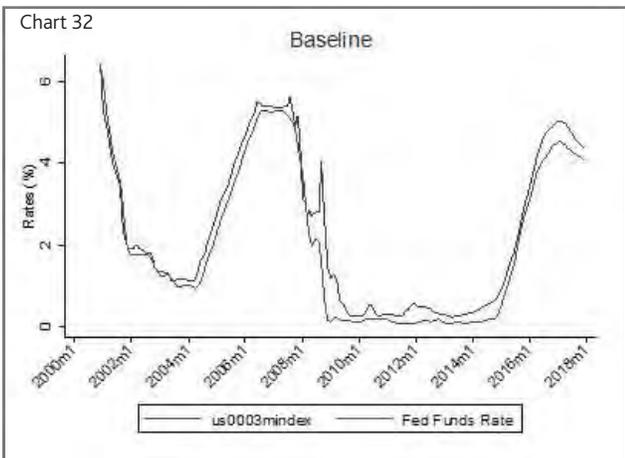
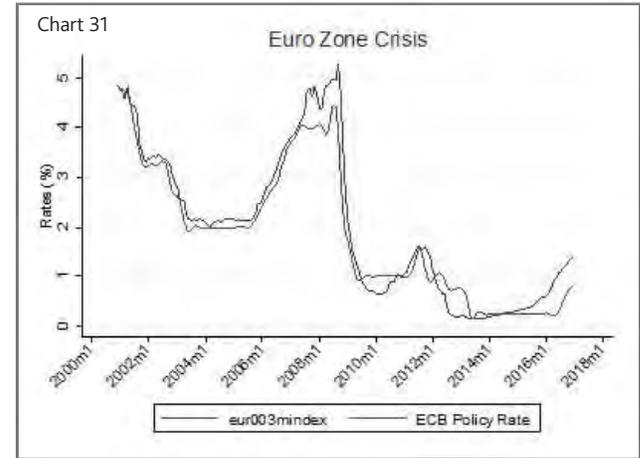
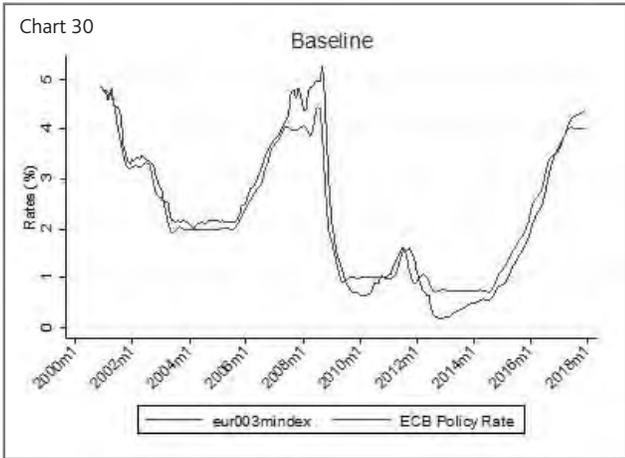
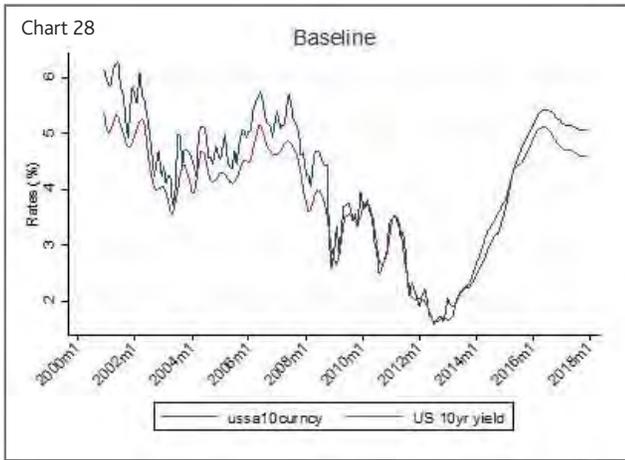
model forecasts a narrowing of the spread in the baseline scenario, which features a recovery of the economy, as the swap rates increase; that is, the swap curve becomes less steep. Under the more severe scenario, however, the spread is kept wide for the whole scenario horizon as indicated by the term premium; in other words, the curve remains quite steep for a long time (see Charts 22-25 on page 9).

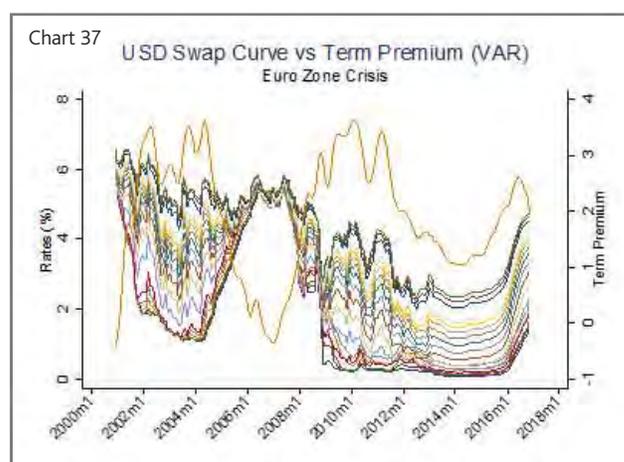
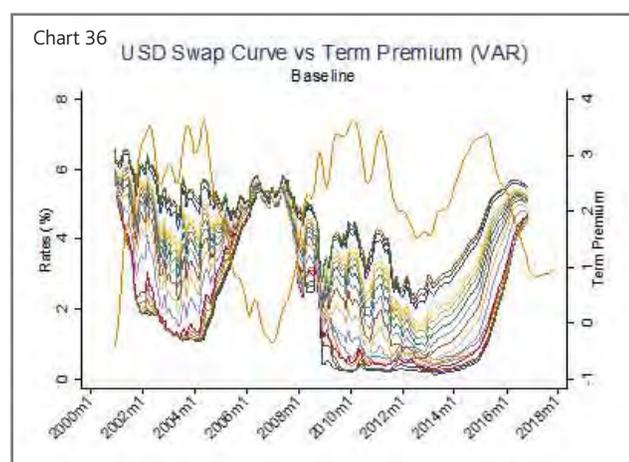
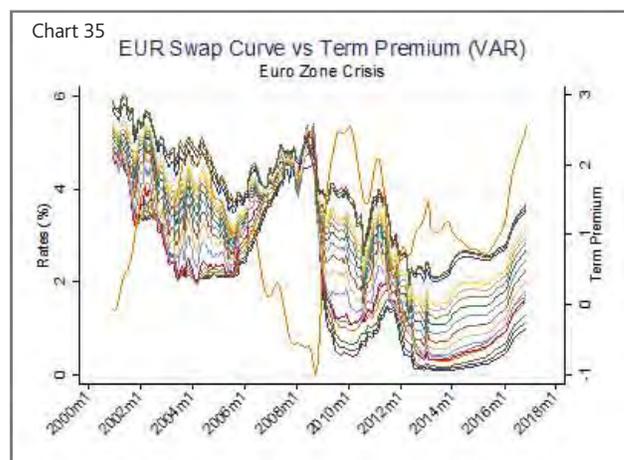
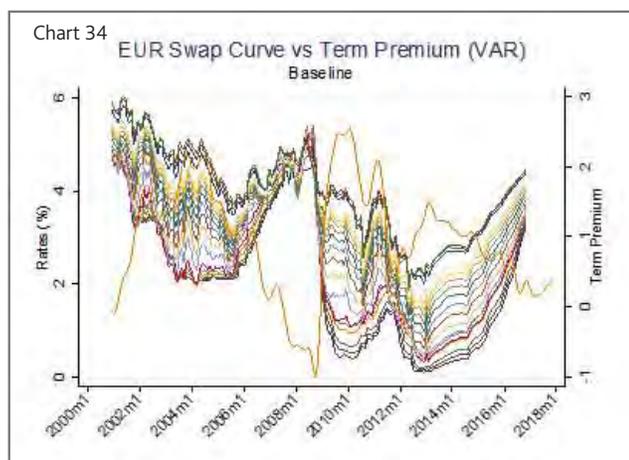
Our approach also seems to produce a fair alignment of the 10-year and three-month tenor points to the corresponding government yields (see Charts 26-33 on pages 9-10).

Finally, results presented in Charts 34 through 37 suggest that modelling the PCA factors with a VAR or two separate ARIMA processes produces very similar results, which makes sense given that we did not include cross lags of the factors in the equations (see Charts 34-37 on page 11).

<sup>9</sup> As we mentioned before, since the principle components are independent, omitting additional components while leaving only two factors does not cause bias in the coefficient estimates.







## Conclusions

We have introduced a two-step modelling and stress-testing framework for the term structure of interest rates swaps that is able to generate forecasts that reflect two important features of the data: the dynamics of the spread across maturities and the alignment of the key swap rates tenor

points to their corresponding government yields. Modern models of the term structure of interest rates are designed to produce accurate projections only to some extent for a short time horizon, thus normally failing to replicate such behaviour in the data. We favor the extraction of factors via Principal Component Analysis, as

it helps reduce estimation biases and it is free from any structure or model imposition. PCA is also appropriate for reverse stress-testing, as it ensures that the mapping of a stress-testing process can be inverted. Future research will be directed to the modelling of dynamic loadings as a function of the economy.

### References

- A. Ang and M. Piazzesi, "A No-Arbitrage Vector Autoregression of Term Structure Dynamics With Macroeconomic and Latent Variables," *Journal of Monetary Economics* (2003): 50, 745–787.
- A. Carriero, G. Kapetanios and M. Marcellino, "Forecasting Government Bond Yields With Large Bayesian Vector Autoregressions," *Journal of Banking and Finance* (2012): 36, 2026–2047.
- J.H.E. Christensen, F.X. Diebold, and G.D. Rudebusch, "An Arbitrage-Free Generalized Nelson-Siegel Term Structure Model," *The Econometrics Journal* (2009): 12, 33–64.
- J. Cox, J.E. Ingersoll, and S.A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica* (1985): 53, 385–407.
- A. Dauwe and M. Moura, "Forecasting the term structure of the euro market using Principal Component Analysis," Insper working paper (2011): Insper Instituto de Ensino e Pesquisa.
- H. Dewachter and M. Lyrio, "Macro Factors and the Term Structure of Interest Rates," *Journal of Money, Credit and Banking* (2006): 38, 119–40.
- F.X. Diebold and C. Li, "Forecasting the Term Structure of Government Bond Yields," *Journal of Econometrics* (2006): 130, 337–364.
- F.X. Diebold, G.D. Rudebusch and B. Aruoba, "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach," *Journal of Econometrics* (2006): 131, 309–338.
- F.X. Diebold and G.D. Rudebusch, "Yield Curve Modeling and Forecasting: The Dynamic Nelson-Siegel Approach," *Econometrics Institute/Tinbergen Institute Lectures* (Princeton: Princeton University Press, 2013).
- G. Duffee, "Forecasting Interest Rates," forthcoming in the *Handbook of Economic Forecasting*, Vol. 2 (2012).
- S. Joslin, M. Pribsch and K. Singleton, "Risk Premiums in Dynamic Term Structure Models With Unspanned Macro Risks," Stanford University working paper (October 2012).
- C.R. Nelson and A.F. Siegel, "Parsimonious Modeling of Yield Curves," *Journal of Financial and Quantitative Analysis* (1987): 60, 473–489.
- M. Pooter, F. Ravazzolo, and D. van Dijk, "Term Structure Forecasting Using Macro Factors and Forecast Combination," *International Finance Papers 993*, Board of Governors of the (U.S.) Federal Reserve System (2010).
- G.D. Rudebusch and T. Wu, "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy," *The Economic Journal* (2008): 118, 906–926.
- G.D. Rudebusch, "Macro-Finance Models of Interest Rates and the Economy," Federal Reserve Bank of San Francisco working paper (2010).
- O. Simon, E. Pouliquen, O. Monso, G. Lalanne, C. Klein, H. Erkel-Rousse and P. Cabannes, "Overview of Mésange: A Macroeconomic Model for Practical Use," French National Institute of Statistics and Economic Studies working paper (2012): 451, 179–216.
- O. Vasicek, "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics* (1977): 5, 177–188.

## Appendix

### Macroeconomic scenarios

The macroeconomic model is constructed to perform forecasts and to simulate the impact of economic shocks on a country. The model consists of behavioural equations and identities that indicate the various interrelationships of the economic variables and portray the structure of an economy. These relationships and identities are estimated using econometric techniques.

The aggregate demand in the model is specified as the sum of consumption, investment, international trade and government spending. In turn, real per capita consumption is driven by real disposable income and real wealth. A measure of the wealth effect is included as gauged by movements in house prices and the stock market. Though real disposable income and real wealth are the long-term determinants of consumption, changes in the real interest rate account for short-run fluctuations in real consumption. Nominal interest rates are viewed as the opportunity cost of consumption. Finally, energy prices and inflation expectations also impact consumption decisions (see diagram below).

Gross domestic investment is divided into private investment and inventories. Fixed business investment plays an important role in both the demand and supply sides of the economy. In traditional multiplier theory, the level of investment depends on the change in expected output; investment changes will in turn stimulate further movements in output

through the multiplier effects. Following this approach, net investment is modelled as a function of changes in expected output and the cost of capital as proxied by an appropriate interest rate. Corporate cash flow and debt levels are also important determinants in the investment equations, and these are approximated by movements in the stock market.

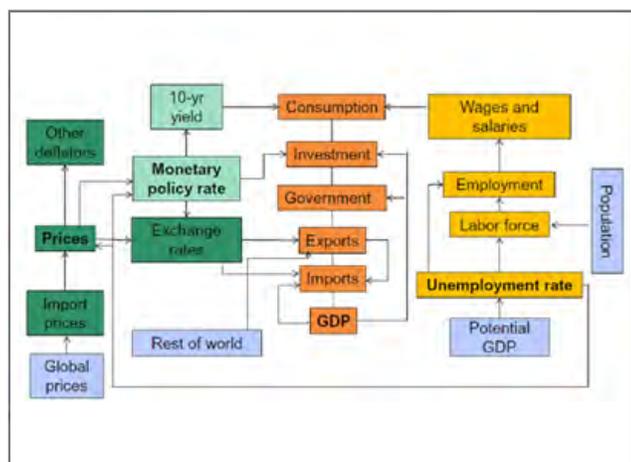
Government spending is modelled as a function of government revenue. Total government revenue is the sum of personal tax receipts, social insurance contributions, corporate profit tax receipts, and indirect tax receipts that are a function of total economic activity. The budget deficit is defined as the difference between government revenue and expenditure.

The international trade sector in the model represents the interactions between foreign and domestic prices, interest rates, exchange rates, and product flows. Export prices and volumes are determined by stochastic equations, while nominal trade flows are calculated as identities. The key determinants of a country's export volumes are relative prices and a weighted average of the GDP growth rate of trading partners, captured in a trade-weighted global GDP term. Weights are based on the geographic distribution of the country's exports. Meanwhile, real imports are determined by specific domestic spending categories and relative prices. Since the import content of exports is high, export volumes also are

GDP growth is determined exogenously using a Hodrick-Prescott filter that separates the long-term trend in GDP growth from business cycle activity. The unemployment depends on the difference between the growth rate of GDP and the exogenously determined potential GDP growth rate. The labor force in turn is a function of the working-age population in the country, while the level of employment is solved using the labor force and the unemployment rate. With these solved, total wages and salaries are determined as a function of the level of employment and the wage rate in the economy.

Firms set their prices with the prices of their inputs in mind and also adjust their prices in response to markets conditions. When looking at this process in terms of aggregate variables, prices tend to rise whenever GDP has been above potential and fall when it has been below potential. Consumer prices are the key price variable that is a part of the models' simultaneous core. Consumer prices are forecast based on the Phillips curve, which postulates a historical inverse relationship between the rate of unemployment and inflation in the economy. Producer prices are, in turn, driven by lagged consumer prices and import prices.

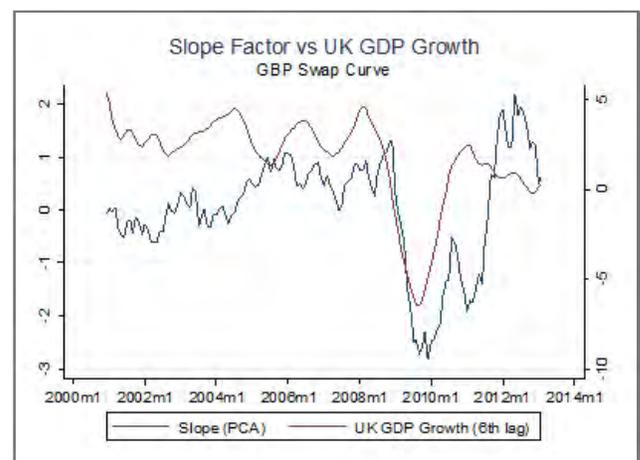
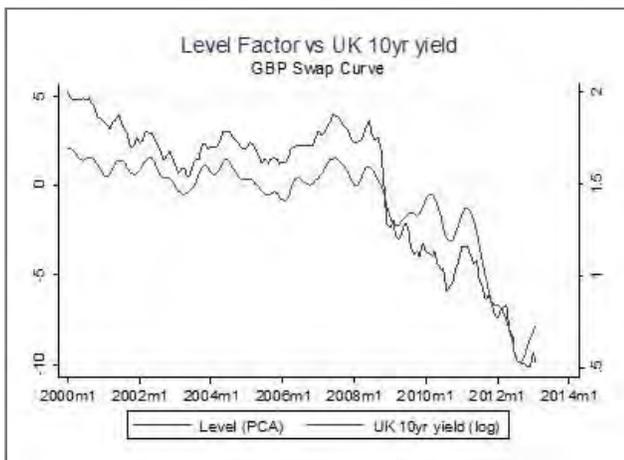
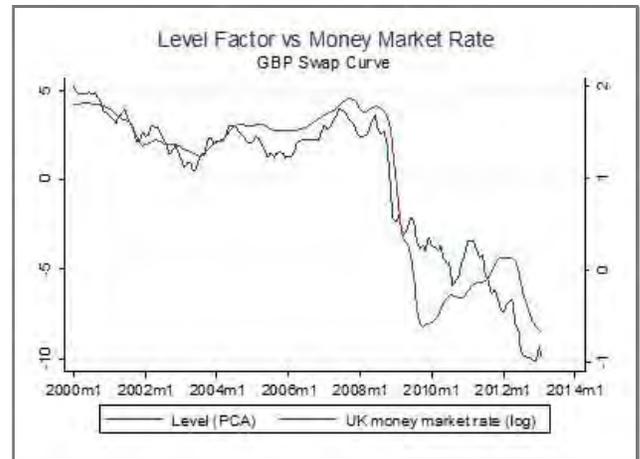
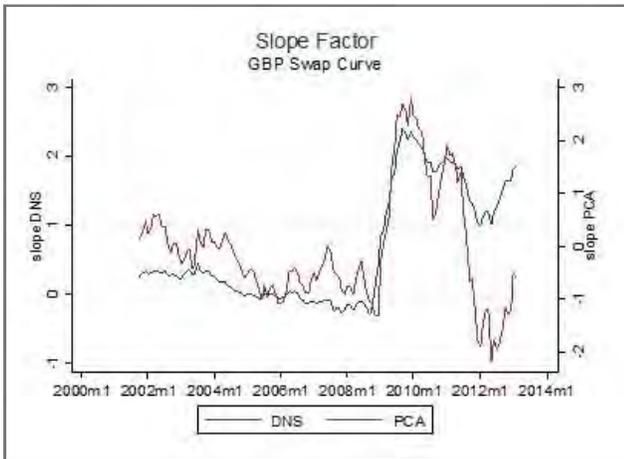
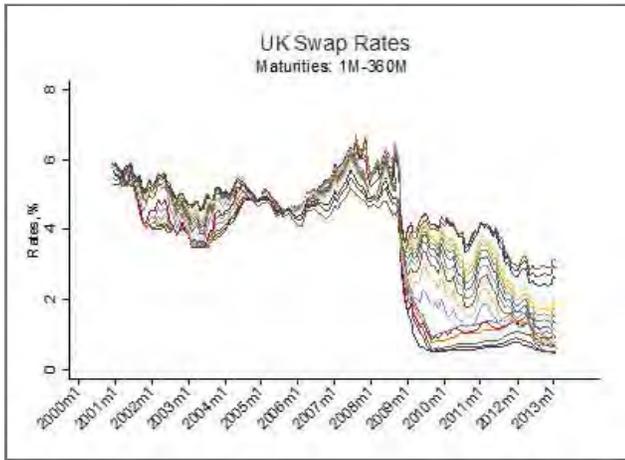
The financial sector of the model is composed of equations for money demand, and short- and long-term interest rates. The money demand equations are derived from portfolio theory, in which the demand for cash depends on the level of income, the expected level of transactions, and the opportunity cost of holding liquid assets as opposed to other interest-earning instruments. The key short-term rate in the model is the central bank's policy rate, which is modeled using a Taylor rule such as reaction function. The most important long-term interest rate is the 10-year bond yield, which is a function of factors closely followed by bond investors. These include indicators of current economic conditions, expectations of the future national budget deficit, and the monetary policy rate. These factors are pivotal in determining inflation expectations of bond investors, making them relevant to the long-term interest rate forecast.



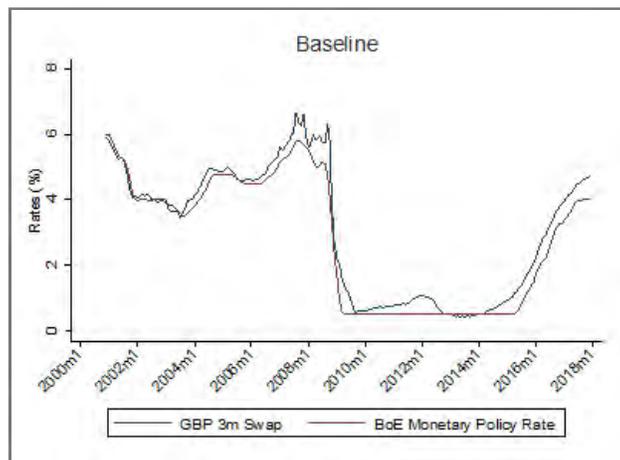
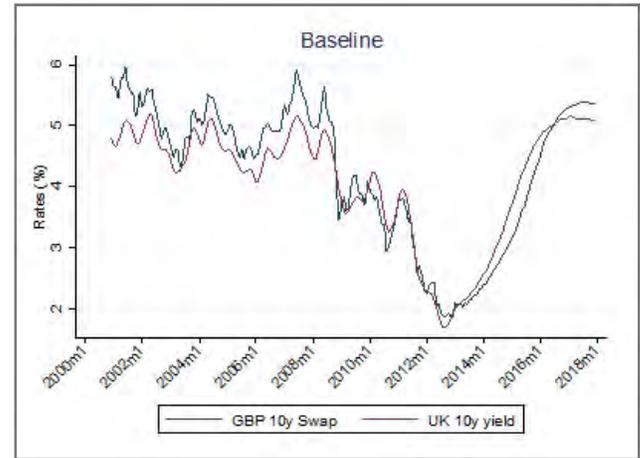
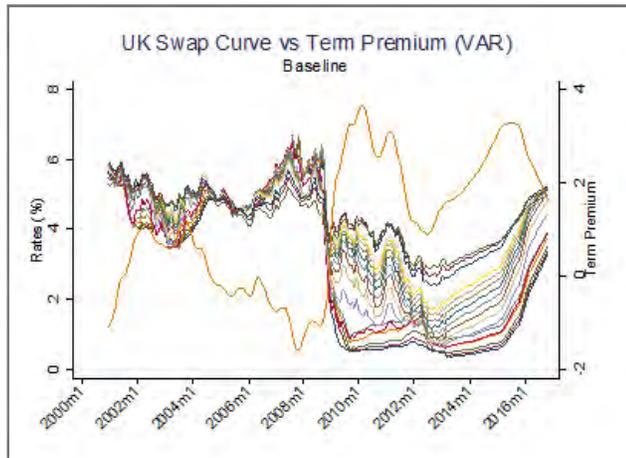
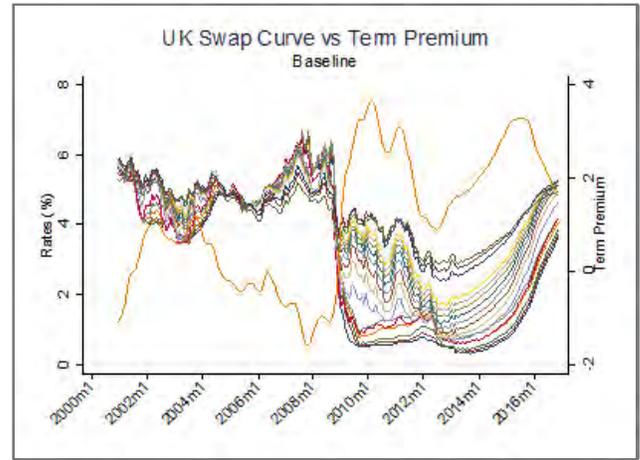
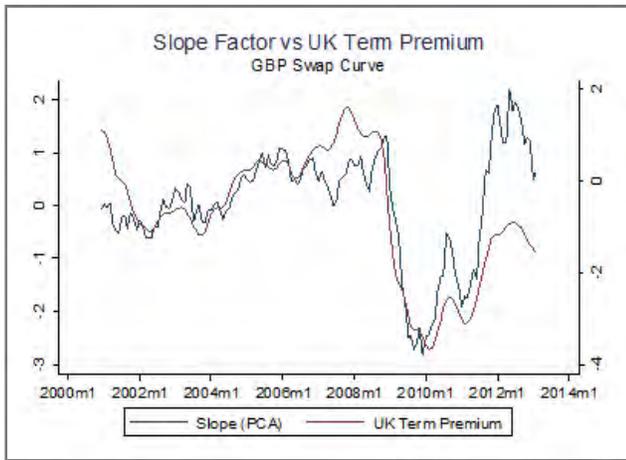
used as an explanatory variable. Import prices are captured via the exchange rate converted into local currency.

The supply side of the macro model describes the economy's capabilities for producing output. The labor market and the potential GDP growth rate make up the supply side in the model. Potential

Additional charts



# ANALYSIS >> Modelling and Stressing the Interest Rates Swap Curve



## About the Authors

### Juan M. Licari

Juan M. Licari is a senior director at Moody's Analytics. Dr. Licari is a member of the Credit Analytics group and specializes in financial economics. Juan leads consulting projects with major industry players, builds econometric tools to model credit phenomena, and has implemented several stress-testing platforms to quantify portfolio risk exposure. He has a leading role in the development and implementation of credit solutions and is actively involved in communicating these to the market. Dr. Licari holds a PhD and an MA in economics from the University of Pennsylvania and graduated summa cum laude from the National University of Cordoba in Argentina.

### Olga Loiseau-Aslanidi

Olga Loiseau-Aslanidi is an assistant director in the Moody's Analytics Prague office. As part of the Credit Analytics team, she designs and implements macroeconometric models for stress-testing and forecasting. Before joining Moody's Analytics, Olga worked for an economic consultancy firm, taught several economics courses, and performed academic research focused on macroeconomics of emerging markets. Olga received her PhD and MA in economics from CERGE-EI, a joint workplace of the Center for Economic Research and Graduate Education of Charles University in Prague and the Economics Institute of the Academy of Sciences of the Czech Republic.

### Jose Suarez-Lledo

Jose Suarez-Lledo is a director at Moody's Analytics based in London. As part of the Credit Analytics team he designs retail and corporate credit models as well as macro-econometric models for key economic and financial variables. Before joining Moody's Analytics, Jose held a research position at the Universidad Autonoma de Barcelona, where he developed models for illiquid financial markets and the dynamics of asset prices and credit. Jose holds a PhD and an MA in Economics from the University of Pennsylvania.

### More on Global Economies from Moody's Analytics

#### MOODY'S CREDITCYCLE™

Integrating regional economics with industry-leading consumer credit forecasting & stress testing.

[www.economy.com/mcc](http://www.economy.com/mcc)

#### STRESS TESTING

Meet regulatory requirements, evaluate the impact of shocks, expose vulnerabilities, and develop strategic business plans.

[www.economy.com/stresstesting](http://www.economy.com/stresstesting)

U.S./CANADA  
+1 866.275.3266

EMEA  
+44 (0) 20.7772.1646

ASIA/PACIFIC  
+61 2 9270 8111

OTHER LOCATIONS  
+1 610.235.5299

Email: [help@economy.com](mailto:help@economy.com)

# About Moody's Analytics

## Economic & Consumer Credit Analytics

Moody's Analytics helps capital markets and credit risk management professionals worldwide respond to an evolving marketplace with confidence. Through its team of economists, Moody's Analytics is a leading independent provider of data, analysis, modeling and forecasts on national and regional economies, financial markets, and credit risk.

Moody's Analytics tracks and analyzes trends in consumer credit and spending, output and income, mortgage activity, population, central bank behavior, and prices. Our customized models, concise and timely reports, and one of the largest assembled financial, economic and demographic databases support firms and policymakers in strategic planning, product and sales forecasting, credit risk and sensitivity management, and investment research. Our customers include multinational corporations, governments at all levels, central banks and financial regulators, retailers, mutual funds, financial institutions, utilities, residential and commercial real estate firms, insurance companies, and professional investors.

Our web periodicals and special publications cover every U.S. state and metropolitan area; countries throughout Europe, Asia and the Americas; the world's major cities; and the U.S. housing market and other industries. From our offices in the U.S., the United Kingdom, the Czech Republic and Australia, we provide up-to-the-minute reporting and analysis on the world's major economies.

Moody's Analytics added Economy.com to its portfolio in 2005. Now called Economic & Consumer Credit Analytics, this arm is based in West Chester PA, a suburb of Philadelphia, with offices in London, Prague and Sydney. More information is available at [www.economy.com](http://www.economy.com).

---

© 2013, Moody's Analytics, Inc. and/or its licensors and affiliates (together, "Moody's"). All rights reserved. ALL INFORMATION CONTAINED HEREIN IS PROTECTED BY COPYRIGHT LAW AND NONE OF SUCH INFORMATION MAY BE COPIED OR OTHERWISE REPRODUCED, REPACKAGED, FURTHER TRANSMITTED, TRANSFERRED, DISSEMINATED, REDISTRIBUTED OR RESOLD, OR STORED FOR SUBSEQUENT USE FOR ANY PURPOSE, IN WHOLE OR IN PART, IN ANY FORM OR MANNER OR BY ANY MEANS WHATSOEVER, BY ANY PERSON WITHOUT MOODY'S PRIOR WRITTEN CONSENT. All information contained herein is obtained by Moody's from sources believed by it to be accurate and reliable. Because of the possibility of human and mechanical error as well as other factors, however, all information contained herein is provided "AS IS" without warranty of any kind. Under no circumstances shall Moody's have any liability to any person or entity for (a) any loss or damage in whole or in part caused by, resulting from, or relating to, any error (negligent or otherwise) or other circumstance or contingency within or outside the control of Moody's or any of its directors, officers, employees or agents in connection with the procurement, collection, compilation, analysis, interpretation, communication, publication or delivery of any such information, or (b) any direct, indirect, special, consequential, compensatory or incidental damages whatsoever (including without limitation, lost profits), even if Moody's is advised in advance of the possibility of such damages, resulting from the use of or inability to use, any such information. The financial reporting, analysis, projections, observations, and other information contained herein are, and must be construed solely as, statements of opinion and not statements of fact or recommendations to purchase, sell, or hold any securities. NO WARRANTY, EXPRESS OR IMPLIED, AS TO THE ACCURACY, TIMELINESS, COMPLETENESS, MERCHANTABILITY OR FITNESS FOR ANY PARTICULAR PURPOSE OF ANY SUCH OPINION OR INFORMATION IS GIVEN OR MADE BY MOODY'S IN ANY FORM OR MANNER WHATSOEVER. Each opinion must be weighed solely as one factor in any investment decision made by or on behalf of any user of the information contained herein, and each such user must accordingly make its own study and evaluation prior to investing.

## CONTACT US

For further information contact us at a location below:

### U.S./CANADA

+1.866.275.3266

### EMEA

+44.20.7772.5454 London  
+420.224.222.929 Prague

### ASIA/PACIFIC

+852.3551.3077

### OTHER LOCATIONS

+1.610.235.5299

Email us: [help@economy.com](mailto:help@economy.com)  
Or visit us: [www.economy.com](http://www.economy.com)

Copyright © 2013, Moody's Analytics, Inc. All Rights Reserved.