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Multi-year projection of market-consistent liability valuations

Overview

Insurance groups, motivated by ORSA and wider business planning requirements, are increasingly interested in making medium-term forward projections of their regulatory and economic capital requirements across a range of future economic and business conditions. This paper presents the technical methodologies required to support this type of multi-year projection capability for market-consistent liability valuations, together with a case study that illustrates the applications of this capability in multi-year stochastic simulations, reverse stress testing and stress and scenario testing.

Contents

| | |
|--|----|
| 1. Introduction | 3 |
| 2. The LSMC method: from one year to multiple years | 4 |
| 3. Proxy fitting methodology and case study | 9 |
| 4. Proxy function validation | 10 |
| 5. Proxy function applications – Multi-year stochastic projections | 13 |
| 6. Proxy function applications - Reverse stress testing | 15 |
| 7. Proxy function applications – Stress testing | 17 |
| 8. Conclusions | 19 |
| References | 20 |

1. Introduction

The technical methodology for developing one-year projections of market-consistent liability values has undergone a considerable journey over the last five or so years. This work has been driven by the wide adoption of a 1-year Value-at-Risk measure of economic capital, which is based on the one-year variability of the market-consistent balance sheet, and regulators' demands for accurate projections that can pass rigorous out-of-sample validation testing. Through this period there has been an increasing recognition that the replicating portfolio and curve fitting methods that were used in early implementations of these 1-year VaR EC models can be materially limited in their ability to describe the one-year behavior of some forms of liabilities, particularly the complex options and guarantees that are found in long-term saving products. This has led to the adoption of more sophisticated and statistically efficient curve fitting approaches such as *Least Squares Monte Carlo*.

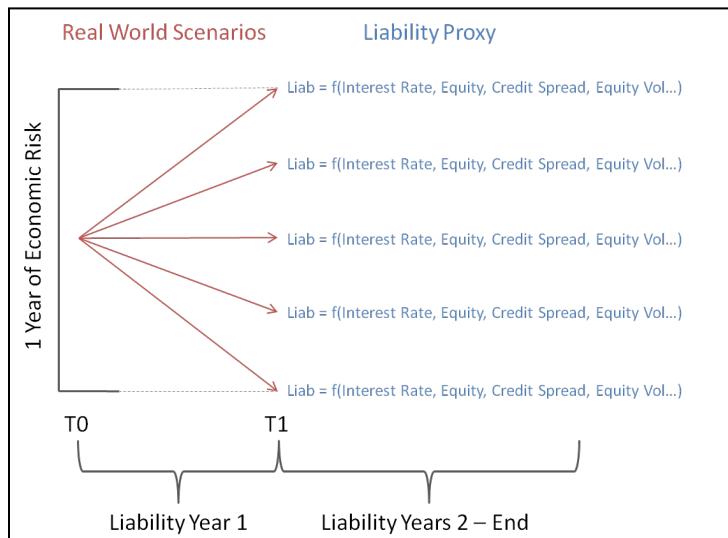
This work has been focused on the one-year projection of liabilities. Relatively little work has so far been done on the development of technical methods that can provide proxy functions for market-consistent liability valuations over multiple timesteps. Such a capability would be of considerable use in *the forward projection of economic capital requirements* (either in multi-year stress test scenarios or stochastic simulations). Fortunately, the more sophisticated techniques that have been developed for one-year liability proxy fitting for 1-year VaR assessment are quite naturally extendable to multi-year projection. This paper discusses how this can be done.

Section 2 develops the technical methodology for multi-timestep market-consistent liability valuation proxy fitting. Section 3 introduces a case study and applies the methodology described in section 2 to produce proxy functions for the behavior of the market-consistent liability valuation of a complex, path-dependent guarantee over a 10-year projection horizon. The validation of the performance of the proxy function over each of the next 10 years is presented in section 4. Sections 5, 6 and 7 then go on to explore the applications of the multi-timestep proxy functions in areas such as stochastic balance sheet projection, reverse stress testing and stress and scenario testing.

2. The LSMC method: from one year to multiple years

With the growing requirement for calculation of capital requirements based on a 1-year Value-at-Risk, many firms have developed models for projection of market-consistent liability values based on analytical proxy functions. These are formulas that estimate the change in the market-consistent value of the liabilities that arises over the 1-year projection horizon as a function of 1-year risk factor outcomes. This approach is illustrated in Figure 1 below.

Figure 1 1-year VaR of a market-consistent balance sheet: liability proxy function solution



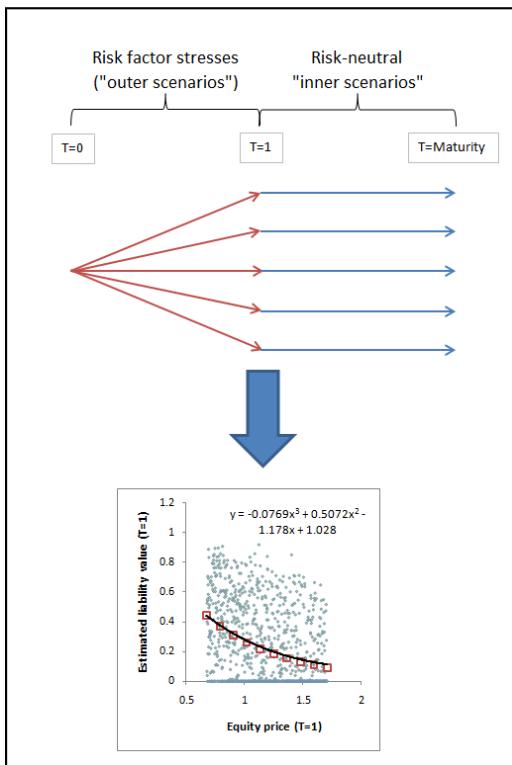
Various techniques have been proposed for describing and calibrating the proxy function, with 'curve fitting' emerging as the most popular of these. By 2012, more than two-thirds of firms implementing a liability proxy function framework were using a curve fitting approach to their 1-year VaR economic capital modeling implementation (KPMG, 2012). The curve fitting approach typically describes the liability proxy as a polynomial function of the risk factors. Calibration of this polynomial involves selecting its coefficients so as to best fit the actual liability value under a selection of stresses to the risk factors. In general, the actual liability value is estimated using risk-neutral Monte Carlo simulation techniques, and the function is fitted using Least Squares regression techniques. This general calibration approach has therefore become known as Least Squares Monte Carlo (LSMC).

When implementing LSMC, the user is faced with a couple of fundamental choices – how many risk factor stresses to use, and how many risk-neutral scenarios to use for valuation in each of these stresses. These choices are not independent – increasing both increases the overall computational requirements of the method. The total computational requirement can be written:

$$\text{Total computational requirement} = \text{number of stresses ("outer" scenarios)} \times \text{number of risk-neutral "inner scenarios"}$$

For a given computational budget, the user therefore has to decide how to allocate this between outer and inner scenarios. Recent research indicates that the optimal scenario allocation is to choose a relatively small number of inner scenarios and large number of outer scenarios (Koursaris, 2011; Cathcart, 2012). Indeed, for the purpose of projection of market-consistent value the optimal allocation may be as few as one inner scenario per outer (or two inners per outer if antithetic variables are used). This allocation is indicated in Figure 2, which shows the resulting fit for a European put option on an equity index in one year.

Figure 2 Illustration of LSMC applied to market-consistent value of a put option in one year

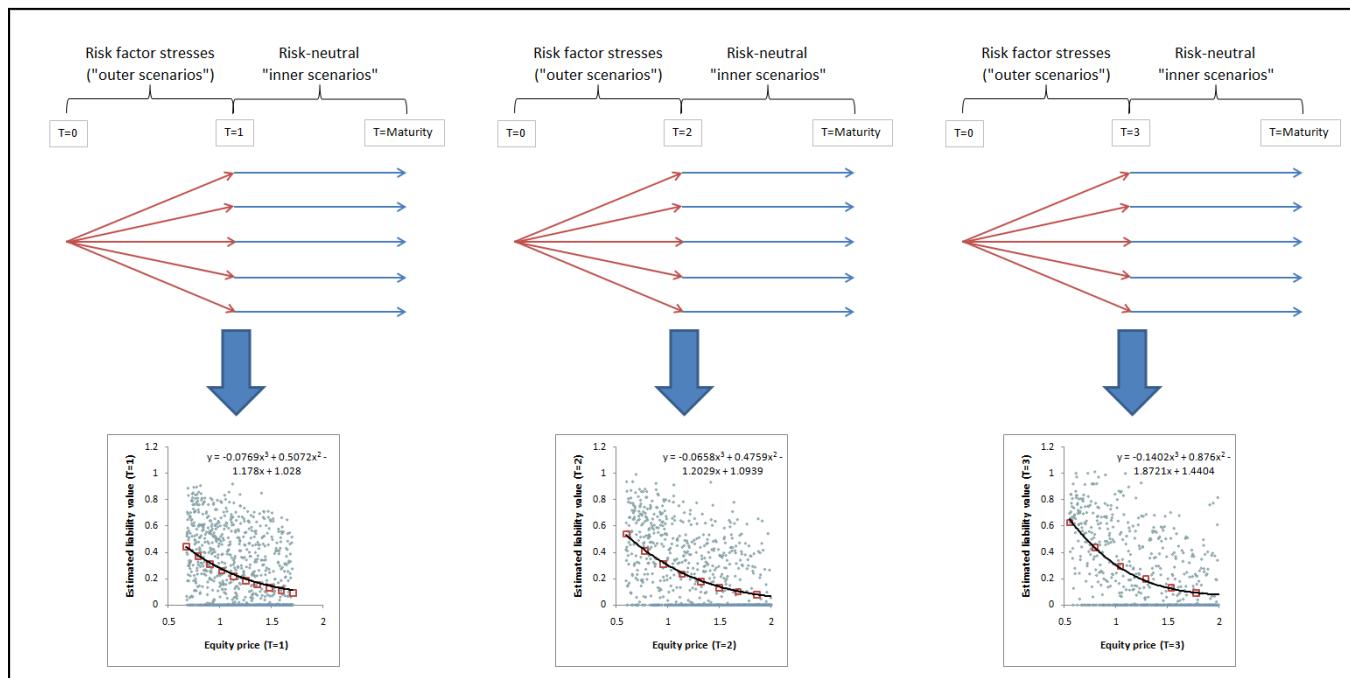


In this example, a cubic function of the equity risk factor is fitted and compared with actual values calculated using the Black-Scholes formula (indicated by the red symbols).

When calibrating the one year proxy function, outer and inner scenarios may be quite different in nature. While inner scenarios are used for valuation and therefore have to be risk-neutral, outer scenarios simply define the points at which we will perform the fit, and can be sampled from any distribution. In practice, we want to place the outer scenarios such that the resulting fitted proxy function provides an accurate approximation to the true valuation at particular points of interest. For 1-year VaR applications, it intuitively makes sense to place a number of stresses in the tails of the risk factor space, where capital requirements are usually greatest, and research indicates that sampling from a large uniform hypercube is usually superior to sampling from the 'real-world' distribution in such applications (Cathcart, 2012). In the 1-year VaR context, another difference between outer and inner scenarios is that outer scenarios define risk factors at a single point in time while (in general) inner scenarios define entire the path of risk factors up until the maturity of the final cash flow that is being valued.

The approach illustrated in Figure 2 can easily be generalised to *multiple years* - essentially we repeat for each projection time of interest. Figure 3 illustrates this process for projection of the put option at one, two and three years.

Figure 3 Illustration of LSMC applied to market-consistent value of a put option at years 1, 2, 3 (approach 1)



While this procedure works in principle, we note the following complications in the multi-year case:

1. Firstly, the computational demands are far greater. Since each valuation time essentially involves repeating the calculations required in the one time-step case, the computational requirement here scales almost¹ linearly with the number of timesteps.
2. In general, future liability values will be path-dependent i.e. depend on the entire path of risk factors up to the point of valuation. In principle, this could result in a huge number of risk factors describing, say, the proxy function for liability valuation at year-20 in a projection.

Fortunately, the risk factor path history can usually be efficiently summarized by a relatively small number of variables. For example, if we consider an Asian rather than vanilla European option, we can capture the path-dependency by extending the space of risk factors describing the multi-year proxy function to include the average equity price up to that point. This can be viewed as a rather straightforward extension of the one-year case.

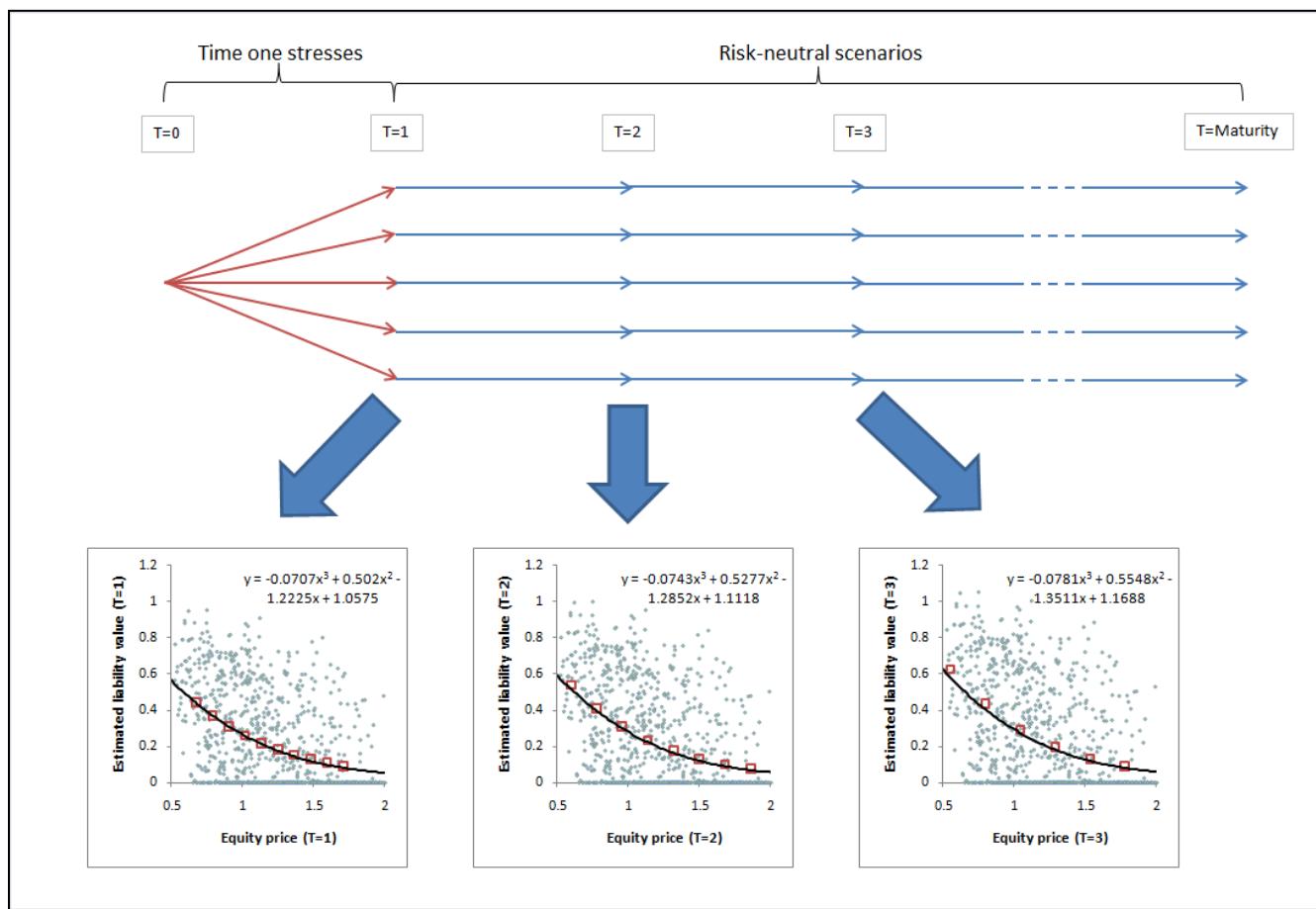
However, care must be taken in sampling from this risk factor space, as the additional variables describing path dependency may be (for structural reasons) strongly related to other risk factors in the proxy function. For example, in the case of the Asian option, we expect a strong positive correlation between the average equity price and the equity price at the time of valuation. In this case, uniform sampling may sample in areas which are highly unlikely, or indeed impossible to observe in the real world.

3. While the above procedure is conceptually straightforward, there may be implementation challenges in practice. In general, actuarial cash flow models are not designed in a way that easily allows the user to stress risk factors at some future point in time. Indeed, simply stressing risk factors at time one for the purpose of 1-year VaR has led to implementation challenges, with many firms adopting 'instantaneous' stresses (Morrison, 2013). While this may be an acceptable approximation at one year it will be increasingly inaccurate at longer horizons.

An alternative approach is illustrated below:

¹The scaling is not quite linear since the time to final cash flow gets shorter, and so run-time reduces, the longer the outer projection horizon.

Figure 4 Illustration of LSMC applied to market-consistent value of a put option at years 1, 2, 3 (approach 2)



In this case, the stresses at time two are defined by combining the stresses at time one with a risk-neutral path between time one and time two; stresses at time three are defined by combining the stresses at time one with a risk-neutral path between time one and time three, and so on.

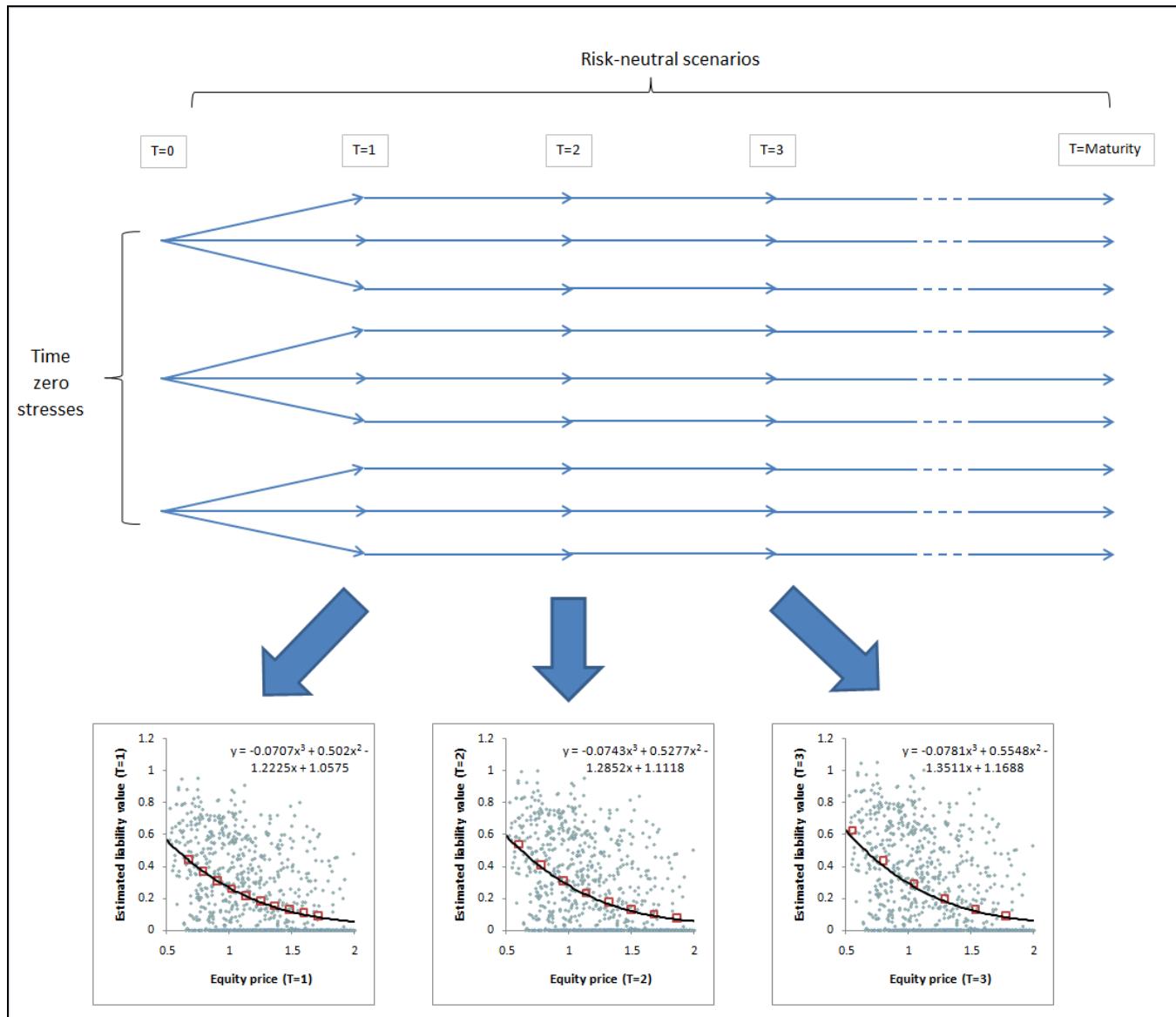
This addresses the problems with the previous approach:

- The same inner scenarios are used at all years so the computational requirement doesn't increase with the number of projection times. Indeed, the computational requirement is in principle no greater than for one-year proxy fitting.
- Risk factors at times greater than one year are sampled in such a way that they reflect any structural dependencies between them.
- In principle implementation is relatively straightforward, essentially just involving the same calculations as for one year LSMC.

We still have to decide how to pick the outer fitting scenarios at one year. For example, we could pick these from the uniform hypercube as discussed previously. Alternatively, we could pick these to be market-consistent, with the scenarios in Figure 4 simply representing a single set of market-consistent scenarios that have been used for time-0 valuation, and use the information in the cash flow results from that scenario set to produce proxy functions for all horizons in the lifetime of the product. This approach is particularly straightforward, but may not necessarily be the most efficient specification of outer scenarios for the purposes of proxy fitting – more robust fits may be produced by using outer scenarios that place more weight on the tails of the distribution.

In this research our fitting scenarios are chosen to be market-consistent, but with a number of stressed initial conditions, as illustrated in Figure 5.

Figure 5 Illustration of LSMC applied to market-consistent value of a put option at years 1, 2, 3 (approach 3)



This approach is still straightforward to implement, involving simply running a number of different risk-neutral simulations. Indeed many firms will already be carrying out such calculations, for example to calculate capital requirements using a 'stress and correlate' approach (such as the Solvency 2 standard formula). However, in contrast to simply using a 'base' market-consistent simulation, by choosing a number of different stresses we can put heavier weight on the tails of the risk factor space in a similar way to uniform sampling. An interesting question arises as to the optimal choice of time zero stresses and the optimal scenario budget between time zero stresses and risk-neutral scenarios. We will not explore this question further in this paper, and will simply choose a number of stresses which intuitively spans the space of risk factors well.

The generalisation to multiple timesteps also raises the question of how to treat time in the proxy function. In the single time-step case we describe the market-consistent value as a function of the relevant risk factors at that time:

$$\text{Market-consistent value} = f(\text{risk factors})$$

where the proxy function f is typically a polynomial function of the risk factors.

In the multiple timestep setting, market consistent value is a function not only of the risk factors, but also of time. We can think of incorporating time into the proxy function in two distinct ways:

» **'Local' fitting**

In this case, we separately calibrate different proxy functions for each time t :

$$\text{Market-consistent value}(t) = f_t(\text{risk factors})$$

A potential drawback with this fitting method is that certain coefficients in the polynomial may vary widely over time. Indeed, depending on the regression algorithm adopted, a particular polynomial term may appear at some times but not at others.

» **'Global' fitting**

In order to impose some consistency between proxy functions at different points in time, we can alternatively specify the coefficients of the above polynomials f_t as themselves being smooth functions of time. For example, we can assume that the coefficients are themselves polynomial functions.

Mathematically this can be written:

$$\text{Market-consistent value}(t) = f_{\text{global}}(\text{time, risk factors})$$

where f_{global} is a polynomial function depending not only on the risk factors but also explicitly on time, t . We can then perform a single fit of this 'global' function.

In the case study below we will consider both types of fitting approach and compare the relative quality of fit.

3. Proxy fitting methodology and case study

The application of LSMC to multi-year projection of market-consistent liability values will be illustrated using a case study. We consider projection of the market-consistent value of a policy with payout linked to the performance of a corporate bond portfolio, with a guarantee applied annually². The main assumptions are summarised below:

- » Assume an annual return of max (fund return – 1.5%, 2%) is credited to the policy account,

$$\text{i.e. Policy Account}(t) = \text{Policy Account}(t-1) \times (1 + \max(\text{fund return}(t-1), -1.5\%), 2\%)$$

$$\text{and Policy Account}(0) = \text{Fund Value}(0)$$

- » The underlying fund is a diversified portfolio of US corporate bonds. The bonds are assumed to be invested with a credit mix of 70% A-rated and 30% BBB-rated, and with a term of 8 years. The bonds' credit rating and term are assumed to be re-balanced annually.
- » The policyholder is assumed to exit the policy after ten years, and will receive the value of the policy account at that point.
- » No allowance is made for tax, mortality, expenses or lapses.

We note that the payout on this policy is path-dependent. The annual return credited to the policyholder has a year-on-year guarantee of 2%, and so the payout at year ten depends not just on the final value of the underlying investment fund, but on each annual return over the 10-year period.

The starting market-consistent value of the policy payout can be assessed using a set of market-consistent simulations for the joint behavior of US interest rates and corporate bond returns, calibrated to market prices at the valuation date. This was calculated using a standard B&H Economic Scenario Generator (B&H ESG) market-consistent calibration to end-December 2012 market prices, and the market-consistent value for the policy payout was found to be 119% of the starting fund value. We now focus on the estimation of how this market-consistent value of the policy payout will behave over a 10-year projection of the policy using multi-timestep proxy functions.

The proxy function has been calibrated using the approach illustrated in Figure 5. 10 stresses of a base market-consistent calibration were produced and 1,000 risk-neutral economic scenarios were generated for each stress, producing a total of 10,000 risk-neutral fitting scenarios. Scenarios were generated using the B&H ESG, with the LMMPlus model used for interest-rates, and

² Loyal readers will note that this is the same product example that was used in our previous research report on projecting CTE reserves (Morrison, Tadrowski, & Turnbull, One-year Projection of run-off conditional tail expectation (CTE) reserves, 2013).

G2 model used for corporate credit spreads and rating transitions. The base calibration is consistent with the market US Treasury curve, US corporate credit spreads and US swaption implied volatilities at end-December 2012. Stresses were applied to both the initial yield curve and the initial level of corporate credit spreads.

These scenarios were used to fit a proxy function depending on five risk factors:

» **Two factors representing the level of the risk-free yield curve**

In principle, the value of the policy depends on a large number of points on the yield curve. In practice, it is convenient to summarise the curve by a small number of variables. In this case study, we assume that the effect of the risk-free yield curve on the value of the policy can be well summarised by two risk factors: the yield curve 'level' (defined as the one year spot rate) and its slope (defined as the ten year spot rate minus the one year rate).

» **A variable representing changes in interest-rate volatility**

In the LMMPlus model used here, interest-rate volatility is assumed to be stochastic, with an explicit stochastic variable representing volatility. This variable is also explicitly included as a risk factor in the regression.

One factor representing the level of corporate credit spreads

Similarly to risk-free rates, in principle the policy value depends on a large number of credit spreads (of various maturities and credit ratings). However, for the G2 credit model used here, we know that the level of credit spreads can be completely summarised by a single spread, and in this case study we choose the 10-year spot spread corresponding to a corporate BBB rating.

» **The credited policy account at the time of valuation**

The product payout depends on future fund returns, captured by the risk-free rate, rate volatility and spread curve risk factors described above. However it also depends on the path of past fund returns.

At first sight this may appear problematic since, depending on the projection time, the value may depend on a large number of past fund returns. For example, the value of the policy in five years depends on five annual returns. This has the potential to significantly increase the dimension of the space of risk factors underlying the proxy function.

However, it should be noted that this path dependency is completely captured in a *single* factor: the current value of the policy account (i.e. the initial fund value rolled-up at the credited returns that have been accrued to the policy to-date using the credited return formula described above). At any future point in time, the current value of the policy account captures all required information about the impact of previous fund returns on the future payoff of the policy, and therefore on its value.

In summary, we assume that the market-consistent value of the policy at time t can be written:

$$\text{Market-consistent value of policy}(t) =$$

$$f_t(\text{yield curve level}, \text{yield curve slope}, \text{yield curve volatility}, \text{credit spread}, \text{policy account})$$

in the 'local' fitting case, and:

$$\text{Market-consistent value of policy}(t) =$$

$$f_{\text{global}}(\text{time}, \text{yield curve level}, \text{yield curve slope}, \text{yield curve volatility}, \text{credit spread}, \text{policy account})$$

in the 'global' fitting case.

In all cases, proxy functions were selected and fitted using the B&H Proxy Generator. When generating the proxy functions, the regression process considered variables up to a quadratic term in each individual risk factor and in cross-terms. For the local fitting approach, this resulted in functions which each had around 10 parameters that were statistically significant from zero. In the global fitting case, the fitted global function (where time is also a variable in the fitting process) had 32 significant parameters.

4. Proxy function validation

Having fitted proxy functions, we validate these under a selection of out-of-sample scenarios for the risk factors. These scenarios were selected as follows.

Firstly, 100,000 real-world scenarios were generated using the Barrie & Hibbert Economic Scenario Generator, with a 2-factor Black-Karasinski model used for interest-rates, and G2 model used for corporate credit spreads and rating transitions. Here we have used a real-world calibration, initialised to risk-free US Treasury and corporate credit spreads at end-December 2012.

A validation of the proxy function in all 100,000 real-world scenarios would be a computationally costly exercise, requiring full nested simulation. Our approach to validation involves selecting a subset of representative 200 scenarios consisting of 100 'random' scenarios, and an additional 100 'adverse' scenarios, in which the estimated market-consistent value of the policy significantly exceeds the value of the underlying fund. For each of these 200 validation scenarios, the 'actual' market-consistent value of the policy was estimated using 5,000 market-consistent scenarios and compared with that produced by the proxy functions.

Figures 6, 7 and 8 on page 12 show this comparison at years 1, 5 and 9 respectively.

Figure 6 Market-consistent value of policy at year 1: Proxy vs actual value in 200 validation scenarios

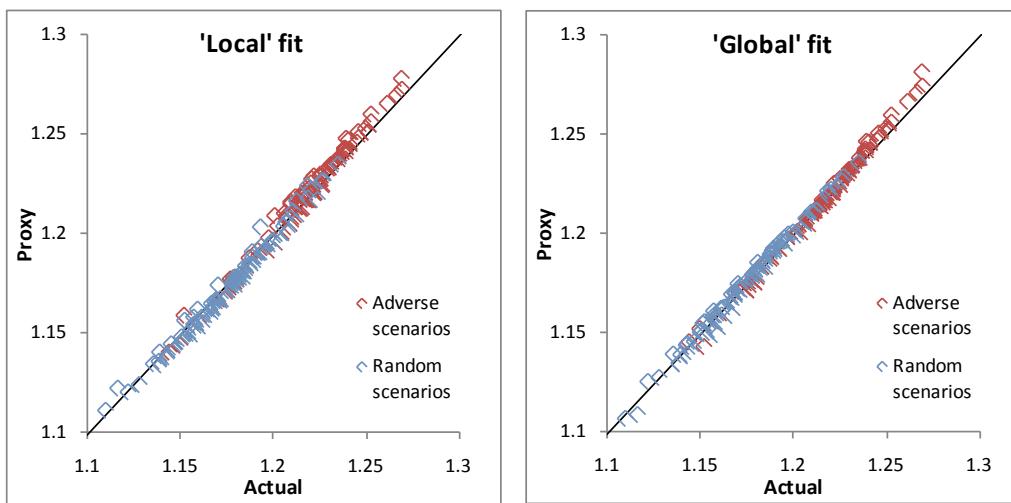


Figure 7 Market-consistent value of policy at year 5: Proxy vs actual value in 200 validation scenarios

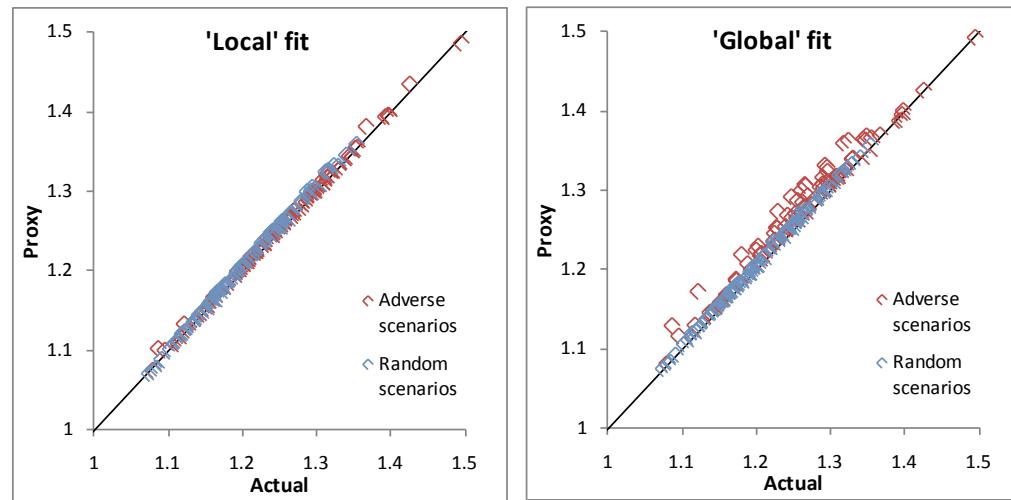
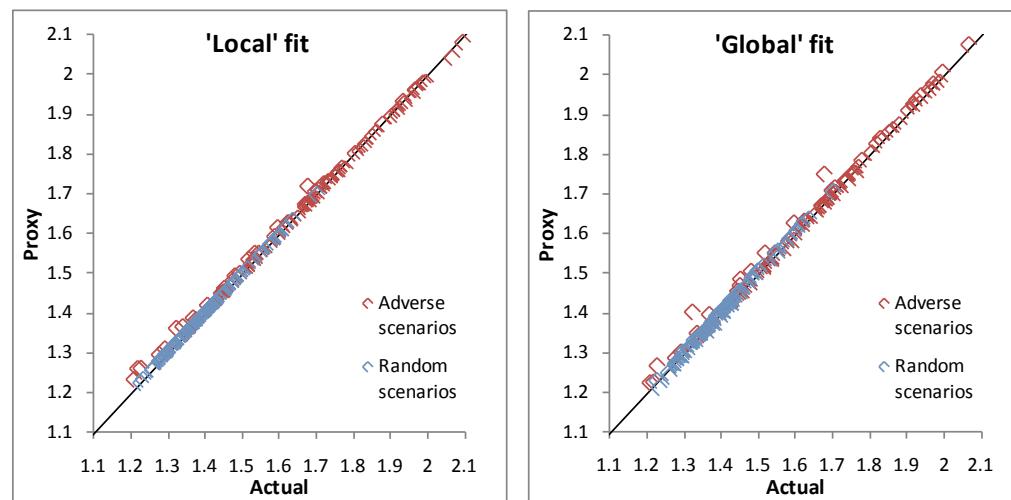


Figure 8 Market-consistent value of policy at year 9: Proxy vs actual value in 200 validation scenarios



We can see that a good fit is obtained in most random and adverse scenarios, at all times considered. In general, local proxy functions give a more accurate fit than the global proxy function, which is expected given that the global proxy function takes a more constrained functional form than the local functions. However, the global approach is arguably easier to implement – the regression fit process only needs to be done once rather than nine times; and a single function may provide more insight into how the market-consistent value of the policy behaves as an explicit function of time.

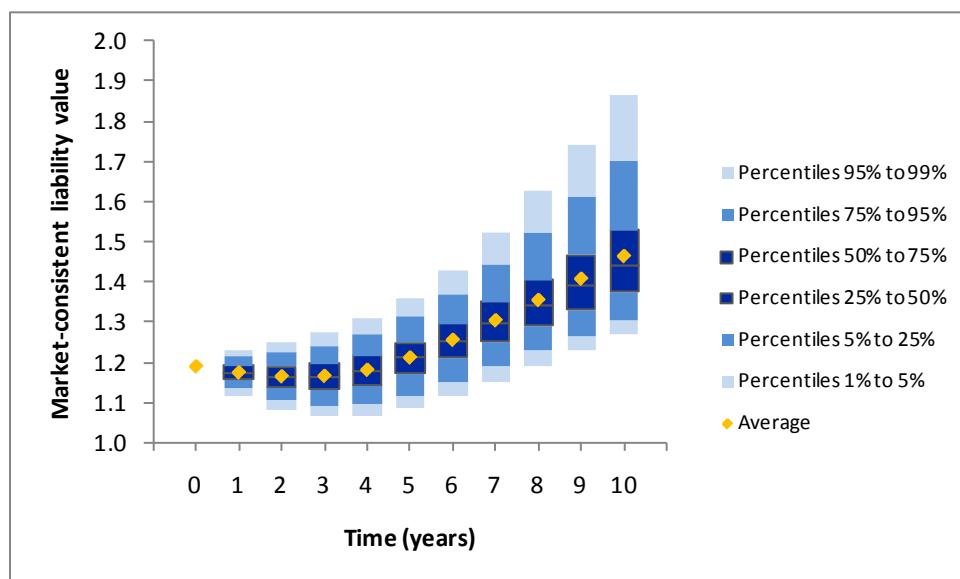
Having now fitted and validated proxy functions that describe the market-consistent value of the policy at all times, the remainder of the paper considers what these functions³ can tell us about the future behavior of the market-consistent value of the product payout.

5. Proxy function applications: multi-year stochastic projections

The multi-timestep proxy function provides the means to efficiently implement multi-year real-world stochastic projections of the market-consistent balance sheet. This can provide the firm with probabilistic estimates of the likelihoods of different scenarios emerging over time, and a measure of the impact that these scenarios will have on the market-consistent balance sheet (with full allowance for the path-dependency and non-linearities that can be found in long-term product guarantees). Such analysis could form an important element of the forward projection of economic capital, for example, for the purposes of ORSA.

Figure 9 below shows the probability distribution for the market-consistent liability value in each of the next 10 years, as implied by the multi-timestep liability proxy function and the standard end-2012 real-world calibration of the B&H ESG (using 10,000 10-year scenarios).

Figure 9 Probability distributions for market-consistent liability value at every timestep



In Figure 9, the proxy function has been used to estimate the market-consistent liability value in each of the 10,000 real-world scenarios in years 1-9. The time-0 value is calculated directly using a set of market-consistent scenarios. The year-10 value is simply the product payout, and so can be calculated directly in each real-world scenario. From the crediting rate formula we know that the product payout at year 10 cannot be less than 1.02^{10} (1.22). The 1st percentile of the policy payout (i.e. the year-10 liability value) is 1.27 – even in a 1-in-100 scenario, there is at least one year where the fund earns more than the guaranteed minimum.

The upper tails of the above distributions are not necessarily the scenarios that cause most difficulty for the insurer – indeed, many of these scenarios will be where asset returns have been strongest and will actually be the most profitable for the insurer. To understand the insurer's risk exposure, we need to consider how the market-consistent liability value is projected to behave relative to the asset portfolio value. Figure 10 shows the probability distributions produced for the liability value net of the asset

³ In the examples that follow, the multi-timestep proxy function produced by the 'global' fitting method is used throughout.

fund value. Again the proxy function has been used for the liability value; the asset portfolio value can be directly calculated for each real-world scenario.

Figure 10 Probability distributions for (market-consistent liability value – fund value) at every timestep

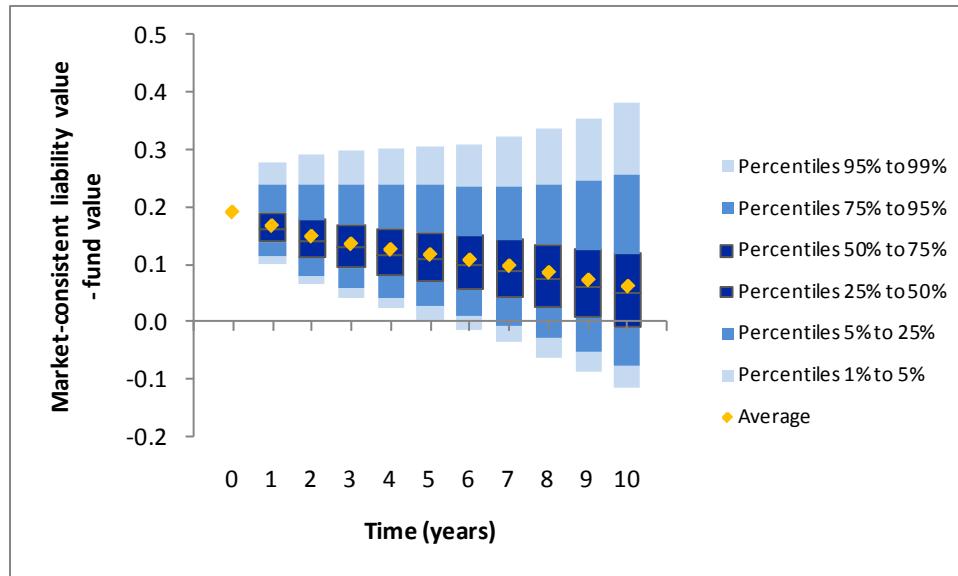


Figure 10 shows that the net liability value of the product will, on average, fall over the life of the policy. This is intuitive – the product's guarantees are not hedged but are backed by assets that are invested in risky assets (corporate bonds). These assets, on average, earn a risk premium in excess of the risk-free rate, and this results in assets, on average, growing faster than the market-consistent liability values. However, this effect can only take us so far: the downside risk in the asset portfolio, and, more importantly, the path-dependency in the liabilities, means that the policy payout exceeds the year-10 asset fund value in approximately 2 of every 3 real-world scenarios.

The real-world simulation output can also be used to identify the type of economic scenario that is associated with significant market-consistent balance sheet deficits. The reverse stress testing concept will be addressed more fully in the following section, but we first show how the simulation output for the full probability distribution can be used to gain insight into the behavior of the balance sheet. Figure 11 jointly plots the simulated values for the market-consistent deficit at year 5 together with the value for BBB-rated credit spreads that arose in the simulation. Figure 12 plots the same deficits, but this time with the 10-year Treasury yield.

Figure 11 Credit spreads and (market-consistent value of policy – fund value); at year 5

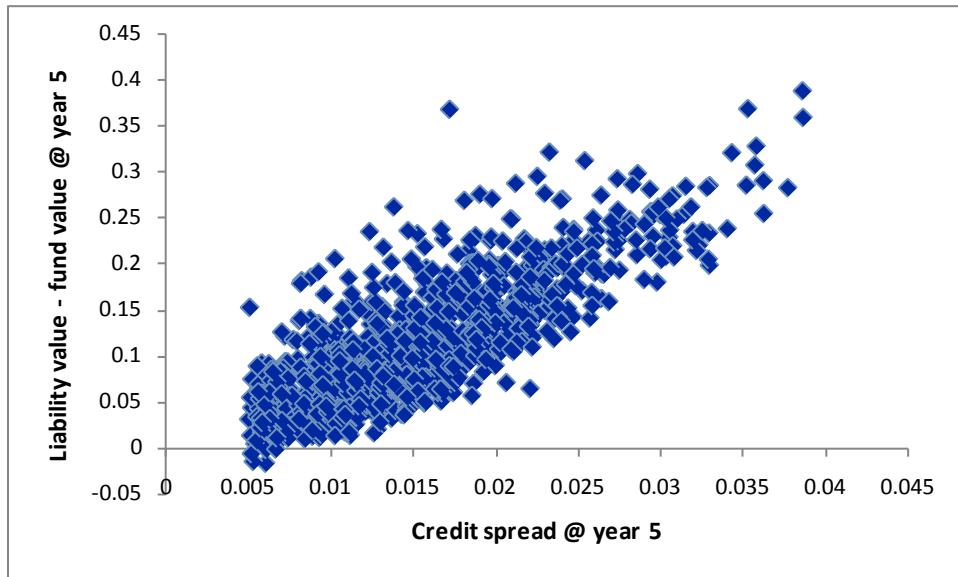
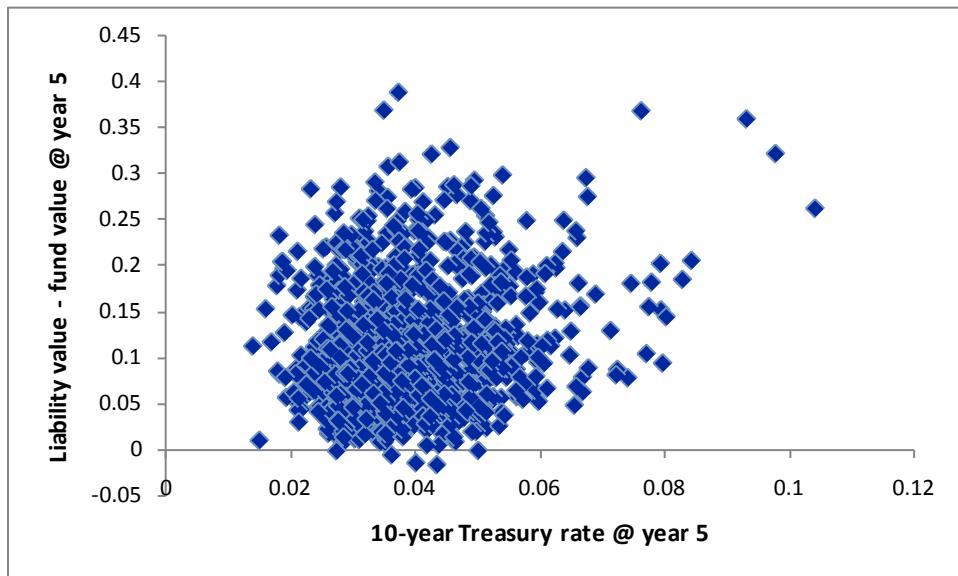


Figure 12 10-year Treasury yield and (market-consistent value of policy – fund value); at year 5



These scatterplots suggest that increases in credit spreads are particularly problematic for the insurer. There is also a weaker exposure to increases in risk-free interest rates. Given the annual return guarantees in the product, it is unsurprising that increases in bond yields result in losses to the insurer. The greater sensitivity to credit spreads than to risk-free yields is because higher risk-free yields have some offsetting impact on the market-consistent liability valuation, whereas no such effect arises with credit spreads (as the market-consistent liability valuation is risk-neutral and hence uses risk-free yields in discounting).

6. Proxy function applications: reverse stress testing

The real-world simulation results presented above in section 5 can be used to identify *reverse stress tests*, i.e. to find the scenarios that cause greatest stress to the insurer's balance sheet. As an example, we consider in this section how 5-year reverse stress tests can be identified from the real-world simulation output by ranking these simulations by their year-5 market-consistent deficit (i.e.

market-consistent liability value – asset fund value) and inspecting the scenarios with the largest deficits. In our illustrative example we only consider a single scenario. However, in a ranking of simulation output, there may be considerable variation in the simulated circumstances that have led to losses, so a 'real-life' implementation should consider a number of these scenarios in a reverse stress test study. Figures 13 and 14 below show the 5-year economic scenario path and balance sheet path produced by the worst ranking scenario in our example.

Figure 13 Economic scenario path in reverse stress test (worst ranking scenario for market-consistent deficit at year-5)

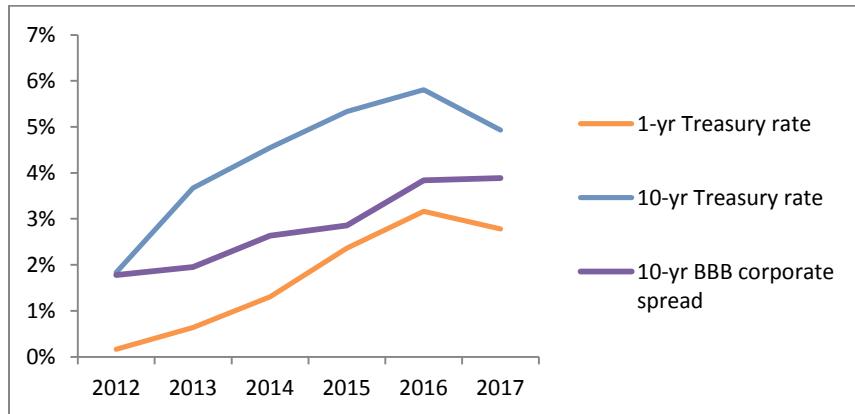
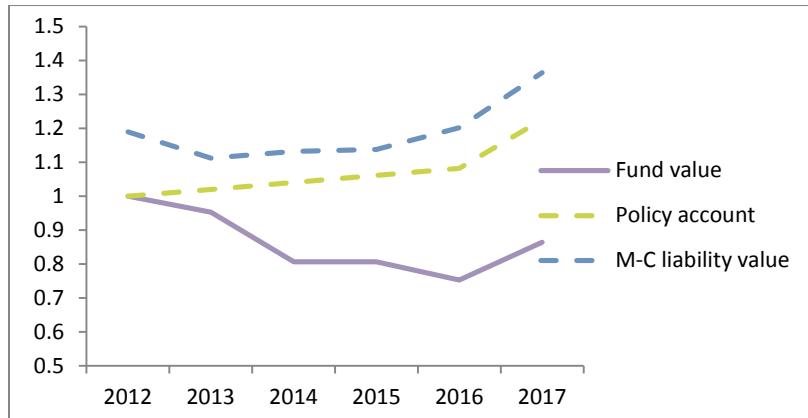


Figure 14 Asset and liability value behavior in reverse stress test



The economic scenario in Figure 13 is consistent with the insights we gained from the scatterplots in section 5. Over the 5-year projection horizon considered for the reverse stress test, BBB credit spreads more than double (note that the real-world simulation model assumes credit spreads of various quality are highly correlated, so A spreads will tend to move in a similar way). This impact is compounded by the path that the Treasury yield curve takes over the 5-year projection, with the 10-year Treasury yield some 300 basis points higher than its starting value after five years.

Figure 14 shows that the impact on the fund value is strongly negative, with the value after 4 years some 25% below the starting value. The 2% minimum guaranteed return keeps the policy account value steadily increasing throughout this period, and the guaranteed return is applied in each of the first four years of the projection. In year 5, a strong asset return is finally earned, but this is passed through to the crediting rate (less 1.5%), so offers little relief – the irreparable damage has already been done. After 5 years, the credited account is 1.23, the market-consistent liability value is 1.36 (the difference between these two numbers reflects the time value in the future annual guaranteed returns), and the asset fund value is 0.86. You may note that this year-5 market-consistent deficit of 0.50 is 'off the chart' of the year-5 probability distribution shown in Figure 10. This is because this reverse stress test scenario is the very worst scenario produced out of all 10,000 real-world scenarios.

7. Proxy function applications: stress testing

The proxy function applications discussed in sections 5 and 6 were focused on the use of multi-year real-world simulation modeling that had been enabled by the availability of the multi-timestep proxy functions. This section considers a different type of application – how proxy functions can be used to estimate the balance sheet associated with a number of multi-timestep deterministic stress test projections. It could be argued that the computational demands of fully implementing market-consistent valuations within a handful of deterministic scenarios are sufficiently manageable as to make proxy functions redundant. However, we believe there are still many significant benefits to the use of robust proxy functions in stress testing such as:

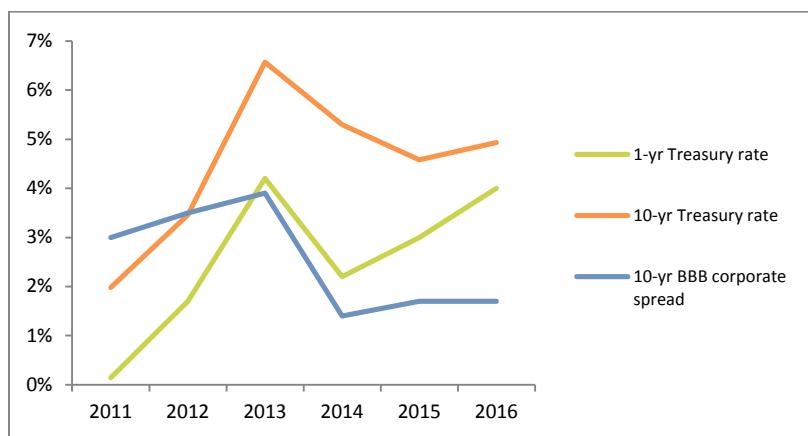
- *Speedy assessment of the impact of what-if scenarios.* Proxy functions enable the impact of what-if scenarios to be measured immediately. For example, in the course of an ORSA forward solvency projection, it is likely a senior manager or regulator will ask how the market-consistent balance sheet behaves if the stress test involved a slightly different path for credit spreads or interest rates or equities, etc. Proxy functions allow these questions to be answered in minutes instead of weeks.
- The multi-timestep projection will require *the estimation of future capital requirements as well as the value of the market-consistent balance sheet.* The Stress-and-Correlate approach to 1-year VaR is based on multiple market-consistent valuations, so the computational burden of a multi-timestep *capital* projection can be many times that of a multi-timestep market-consistent balance sheet projection.

As an example of how these macro stresses might be established, Moody's Analytics' economic experts regularly publish economic forecasts for a wide range of both global and regional economic variables under a range of alternative macro-economic scenarios. In 2012, these were⁴:

- Scenario 1: Stronger Near-Term Rebound;
- Scenario 2: Slower Near-Term Growth;
- Scenario 3: Double-Dip Recession;
- Scenario 4: Protracted Slump;
- Scenario 5: Below-Trend Long-Term Growth;
- Scenario 6: Oil Shock, Dollar Crash, Inflation.

In each of these scenarios, 5-year paths for a range of financial market and economic variables are specified, together with an estimate of the probabilistic severity of the scenario and an intuitive description of the background circumstances that determine the scenario outcome. Figure 15 shows the 5-year paths⁵ for 1-year Treasury rates, 10-year Treasury rates and BBB corporate bond spreads in the Oil Shock, Dollar Crash, Inflation Scenario.

Figure 15 ECCA macroeconomic stress test scenario – Oil shock, US Dollar crash, Inflation



⁴ See, for example, "[U.S. Macroeconomic Outlook: Alternative Scenarios](#)", Moody's Analytics, April 2012.

⁵ Note that these stress tests were specified with end-2011 start points, and so have a different start date to the simulation output presented in section 5.

You can see that this scenario produces very significant increases in the Treasury curve over the first two years of the stress test (the 10-year Treasury rate triples over the first two years). After this shock period, Treasury yields then revert to a more stable position. Credit spreads fall over the 5-year projection, presumably because high inflation is substantially eroding the nominal value of firms' debt. Figure 16 shows the asset and liability values projected for our case study insurance product. Again, the market-consistent liability values are projected using the proxy function developed in section 3.

Figure 16 Asset and liability projection using multi-timestep proxy function – ECCA Oil shock / USD crash / inflation scenario

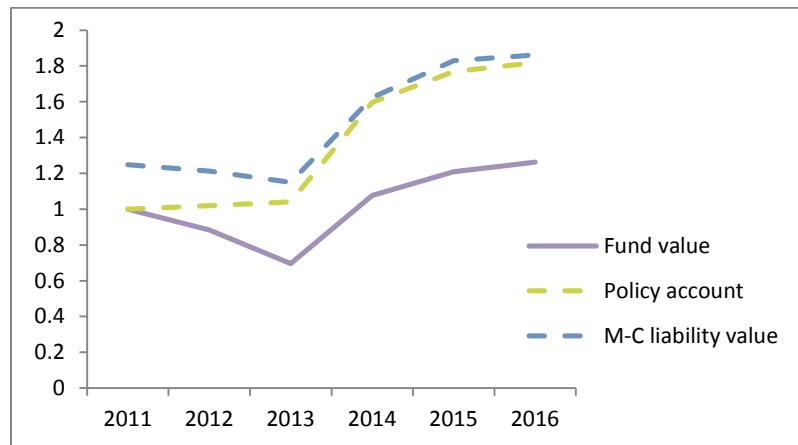
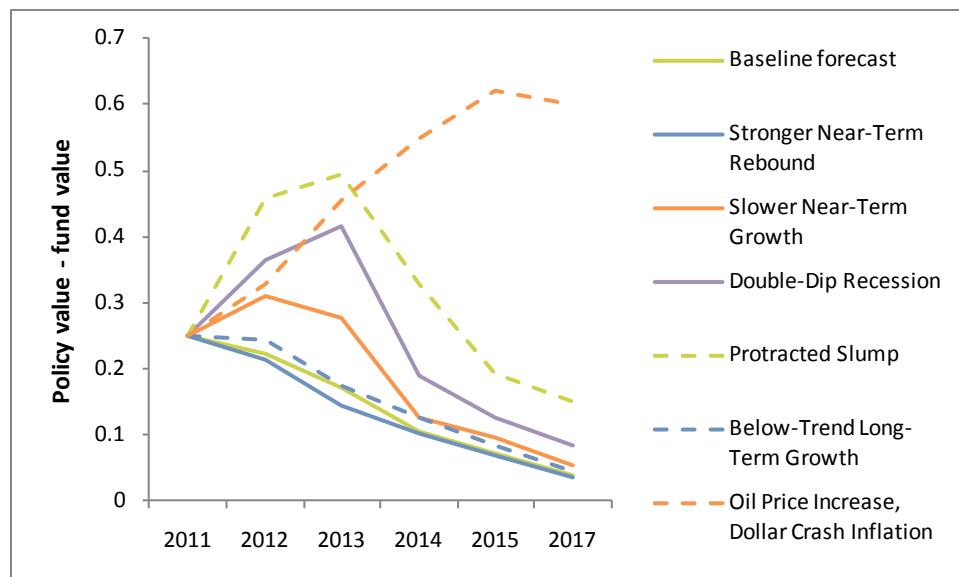


Figure 16 shows that the exceptional increases in bond yields over the first two years of the stress test projection result in a fall in the asset fund of more than 30% by the end of year-2. The product receives its 2% minimum guaranteed return in each of these years, so that the gap between the asset portfolio value and the credited policy account is 35% of the starting fund value. Fund returns are very strong in the subsequent three years, but, interestingly, the gap between the asset value and the liability value keeps on growing. This is because the credited return is applied to the size of the credited account, which is proportionally 50% bigger than the asset portfolio value after two years. So, even after the 150 basis point haircut, the credited account grows faster than the asset portfolio.

This is an interesting example of one of the types of path-dependency that exists in this form of product. The result is financial Armageddon, as highlighted in Figure 17. This shows the market-consistent balance sheet deficit projection over all 6 of the above stress test scenarios, as well as the baseline forecast.

Figure 17 Asset and liability projection using multi-timestep proxy function – 7 ECCA scenarios



Interestingly, the 5-year projection of the market-consistent deficit in the Oil Shock, Dollar crash, Inflation scenario lies well outside the 5-year probability distribution in Figure 10. The stochastic analysis in section 5 suggested that the 99th percentile value of the market-consistent deficit at year 5 was 0.30, whereas this stress test produced a value after 5 years of around 0.60. The stochastic model implies the probability of such an extreme path for interest rates emerging is extremely low. This is a good example of how stress testing and stochastic simulation can provide alternative insights into the measurement of risk. It is ultimately a matter of judgment whether this tells us that the stress testing is identifying feasible event risks that are beyond the capabilities of the stochastic model to identify; or whether the stress test scenario is vanishingly improbable.

8. Conclusions

This paper has described and demonstrated how the more statistically sophisticated proxy fitting approaches for 1-year VaR can be naturally extended to produce accurate proxy functions for multi-year projection of market-consistent liability valuations. The computational implementation requirements of the multi-timestep fitting approach developed in this paper are very similar to those required for single-timestep proxy fitting. The quality of fit, as measured using out-of-sample validation testing, was shown to be consistently high throughout the ten-year projection horizon considered in our case study.

The capability to efficiently produce robust and accurate proxy functions for market-consistent liability valuation behavior across a wide range of multi-timestep, multi-risk-factor scenarios can significantly enhance firms' forward solvency projection analytics. We believe this can be extremely useful for firms who are calculating economic or regulatory capital based on a market-consistent balance sheet, and who need to consider the medium-term behavior of their solvency requirements as part of ORSA and other business planning requirements. The case study presented in this paper highlighted how the multi-timestep proxy functions could be used to facilitate medium-term stress testing, reverse stress testing and stochastic projection of the market-consistent balance sheet.

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