

## MODELING METHODOLOGY

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## Multi-Period Capital Planning

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### Abstract

This paper proposes and illustrates a multi-period capital planning framework that can be used to calculate a portfolio's capital requirement over time and to determine the appropriate capital buffer level under various economic scenarios. Such analysis can help financial institutions gain a better understanding of credit portfolios' risk dynamics, allowing them to foresee and to prepare for potential increases in capital requirements resulting from economic shocks.

Using Moody's Analytics Stressed EL Calculator and RiskFrontier™, we implement the capital planning framework and analyze the impact of the 2015 CCAR scenarios on the capital requirement and capital buffer behaviors of a credit portfolio consisting of all U.S. public firms in CreditEdge™. Results show that capital buffer behaviors vary significantly under different scenarios and for various asset classes. We also observe that a proportion of the capital allocated to financial firms is always greater than their value proportion, and it increases for most periods under the Adverse and Severely Adverse Scenarios. This finding suggests that decreasing exposures in financial counterparties is more effective in terms of lowering the capital requirement when the economy undergoes stress.

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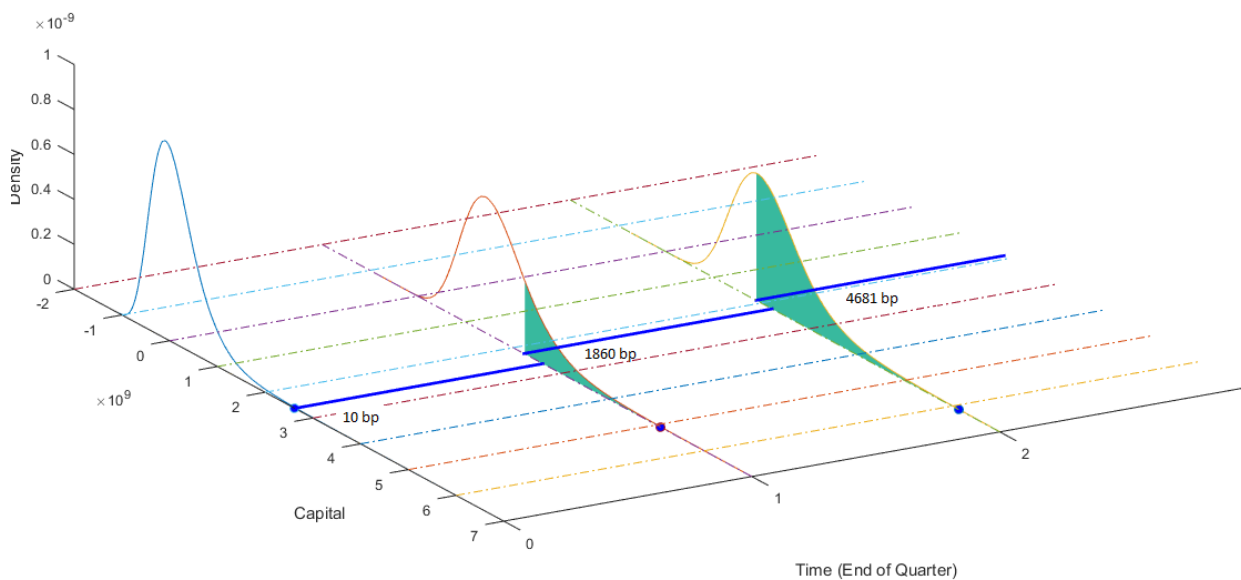
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## 1. Introduction

Institutions must set aside capital buffers to absorb credit portfolios losses. An important question they must ask is whether their capital buffers can withstand future economic shocks, and can they guarantee their buffers will provide portfolio loss protections at the necessary probability levels. In order to answer this question, institutions need to understand the dynamics of the portfolio's capital requirement (or the demand side of the proverbial capital planning coin) as well as the dynamics of the available capital buffer (the supply side).

As an example, Figure 1 illustrates how a credit portfolio's capital distribution can potentially change, given a negative economic shock.

**Figure 1** Capital distribution given economic shocks.



For this illustration, we assume that the financial organization rebalances its' portfolio at the beginning of each period to a certain representative portfolio, so that, at the beginning of each quarter, it begins with the exactly the same portfolio composition as in the previous period. While the composition of the representative portfolio remains the same at the start of each quarter, the capital buffer and the portfolio size will be affected by the losses. To account for these, for this analysis, we assume that the capital buffer and the portfolio size are reduced by the corresponding conditional expected loss for that quarter and for a given scenario. Additionally, if the economy worsens, we expect capital distribution to shift toward higher levels of losses, leading to a higher level of probability that the capital buffer will be insufficient to cover portfolio losses. Assuming the capital buffer equals 10bps probability at the beginning of first period (time 0), Figure 1 shows the probability of experiencing a loss exceeding the remaining capital buffer threshold over time for a representative portfolio. At the end of the first quarter, that probability is approximately 18.6%, and at the end of second quarter, it rises to almost 47%. These large probabilities indicate a significant increase in the economic capital requirement over time. Performing this type of analysis and having such information, institutions will be better equipped to adjust capital planning accordingly and to perform capital allocation more efficiently.

This paper examines the impact of 2015 CCAR scenarios on the capital requirement and the capital buffer behaviors of a portfolio consisting of U.S. public firms. The 2015 CCAR scenarios include projections of macroeconomic variables for 13 quarters. Based on these three scenarios — Baseline, Adverse, and Severely Adverse — we generate the portfolio's capital distribution dynamics under each of the CCAR scenarios and then calculate the capital requirement for 13 forward-analysis dates. An important part of the analysis is accounting for the scenarios between the analysis date and the forward-analysis date. We achieve this step by changing the inputs used for simulation, such as probability of default. Intuitively, if there is an adverse economic scenario up to the forward-analysis date, we would expect it to be reflected in poorer credit qualities and, thus, generate a higher probability of default. We use Moody's Analytics Stressed EL Calculator to obtain CCAR scenario-stressed PD term structures. We also use different market risk premiums and Zero-EDF rates consistent with CCAR scenarios. We then enter the corresponding sets of

inputs representing different forward-analysis dates for each of the scenarios into RiskFrontier to generate simulated capital distribution results. We also use RiskFrontier to produce the quarterly conditional expected losses for each of the scenarios.

The remainder of this paper is organized as follows: Section 2 discusses the methodology utilized to obtain capital distributions. Section 3.1 describes the portfolio composition. Section 3.2 describes the input data for simulations. Section 4.1 discusses the result of capital requirement and capital distribution dynamics. Section 4.2 compares the capital allocation between instruments from financial and nonfinancial firms. Section 5 concludes.

## 2. Methodology

To analyze the dynamics of a financial organization's portfolio across time under various stress scenarios, we utilize RiskFrontier's conditional simulation framework. For illustrative purposes, we assume a representative portfolio that constantly rebalances, i.e., whose composition remains constant over time, including maturities, cash flow parameters (coupon rate), and exposures. This assumption implies that if any of the instruments of the portfolio default, they will be replaced automatically at the beginning of the next period. While we choose this approach for illustrative simplicity, a similar approach can be applied to a portfolio with a particular evolution over the analysis period. We use the end of Q1 2015 (3/31/2015) as the portfolio analysis date. Our goal is to show how the capital distribution and the capital requirement change over time, when the U.S. economy experiences 13 quarters of 2015 CCAR scenarios.

For each of the 13 analysis dates, we use time- and stress-appropriate inputs for the RiskFrontier analysis. For each analysis date, we use Moody's Analytics Stressed EL Calculator to obtain a specific PD term structure that reflects the particular stressed scenario up to that date. Apart from PD term structures, market risk premiums, as well as interest rate curves, also vary with each forward-analysis date.

In-line with traditional capital budgeting, for RiskFrontier analysis, we set the horizon to one year. We use book and lattice valuation methods for the analysis date and the horizon, respectively. We perform two types of RiskFrontier analyses, conditional and unconditional, depending on whether we assume knowledge regarding the given economic scenarios beyond the forward-analysis date or not. For example, if the forward-analysis date is the end of Q2, and we are interested in calculating the one-year horizon economic capital for Q3–Q6, then both the conditional and the unconditional RiskFrontier analysis inputs, such as PDs and LGDs, are the same, and they depend on scenarios in Q1 and Q2. The difference is that, for unconditional RF analysis, there is no conditioning on the economic scenarios for the period Q3–Q6, while conditional analysis further takes into account the expected one-year scenario over Q3–Q6.<sup>1</sup>

To account for the losses that a credit risk portfolio experiences over the lifetime of a stress scenario, we calculate the conditional expected losses for each quarter and adjust by these amounts the capital and the portfolio notional remaining at the beginning of the following period.

<sup>1</sup>For more details on conditional RiskFrontier analysis, please see Moody's Analytics, "Modeling Credit Portfolios: RiskFrontier Methodology," Fifth Edition, 2015.

### 3. Data

To illustrate the methodology (and for our analysis), we use representative data from various Moody's Analytics databases.

#### 3.1 Portfolio Statistics

The representative portfolio contains 9,818 instruments, with a total exposure of \$24.77 billion dollars USD. The underlying 4,909 obligors represent the entire universe of U.S. public companies in the CreditEdge dataset.<sup>2</sup> Each obligor possesses two instruments (both term bullet loans), one with a maturity of one year, the other with a maturity of seven years. The commitment amount for a one-year loan is proportional to the underlying obligor's current liability, while the commitment amount for a seven-year loan is proportional to the obligor's long-term liability. We extract the LGD and the LGD variance parameter for most obligors from Moody's Analytics LossCalc™ 3.0 as of 03/31/2015 under the assumption that instruments are senior unsecured loans. LGD related information of a small number of obligors is not available in LossCalc 3.0. In these cases, we use the average values computed based on corresponding variables of all other obligors. The portfolio has an exposure-weighted, average one-year PD of 76bp, a maturity of 4.5 years, an LGD of 37%, and an R-square of 36.36%.

All instruments in the portfolio have annual coupon payments. To study the effects of the coupon type, we analyze two types of portfolios, one with floating coupons and one with fixed coupons. We obtain the floating rates and fixed rates using RiskFrontier's mark-to-par feature for the representative portfolio, on the initial analysis date. We then apply the same coupon rates to all scenarios and forward-analysis dates.

#### 3.2 Simulation Inputs

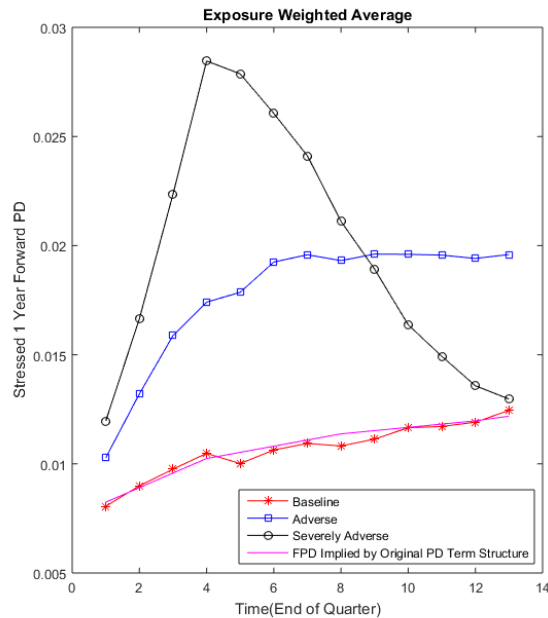
We calculate instrument-level forward PDs for all the forward-analysis dates, based on the output from Moody's Analytics Stressed EL Calculator.<sup>3</sup> Variable selection in Pospisil, Kaplin, Levy, and Patel (2014) suggests there are four macroeconomic variables most related with U.S. counterparties, and, thus, they should be used for stress testing: U.S. Unemployment, U.S. Equity, U.S. VIX, and U.S. BBB spread. Therefore, for this paper, we use only the above four variables to obtain stressed forward PDs and to implement conditional RiskFrontier Analysis.

Figure 2 shows the exposure-weighted average of the one-year forward PDs that are conditional on a given stress scenario macrovariables realizations up to that forward date – for example, the Q4 number represents the weighted average of one-year forward PDs, or covering the period from end of Q4 up to the end of Q8, that are conditional on the scenario realizations up to the end of Q4. The horizontal axis represents forward date, and the graph illustrates how the one-year PD changes over time under different economic scenarios. In Figure 2, the baseline forward PDs are in-line with the forward PDs implied by the original PD term structures, as one would generally expect.

<sup>2</sup> There are a few exclusions. Please see Appendix A in Xu and Levy (2015).

<sup>3</sup> Please see the Appendix for a detailed description of the procedure.

Figure 2 Exposure-weighted average of the one-year forward PDs conditional on the realized stress scenarios.



PDs from the Adverse Scenario first increase, then level off, while PDs from the Severely Adverse Scenario soar sharply at the beginning and then peak at the end of Q4. Beginning during the fifth quarter, the term structure begins to decrease, and PDs become smaller than those from the Adverse Scenario after the ninth quarter. Although it may seem counterintuitive to observe PDs from the Severely Adverse Scenario dropping below those from the Adverse Scenario, this pattern can be explained by the stressed macroeconomic variable paths, representing the CCAR scenarios plotted on the graphs in Figure 3.

For U.S. Equity and U.S. VIX, during the last few quarters, the levels are better in the Severely Adverse Scenario than in the Adverse Scenario. These levels help explain the observation that PDs from the Severely Adverse Scenario become smaller than those from the Adverse Scenario beginning in the ninth quarter. For the Severely Adverse Scenario, barring Unemployment Rate, all variables show signs of recovery beginning in the fifth quarter. This finding explains why we see a peak in PDs at the end of Q4 for the Severely Adverse Scenario. Following Pospisil, Kaplin, Levy, and Patel (2014), we map the original macroeconomic variables into standard normal space, presented in Figure 3. Standard normal shocks, shown in Figure 4, make it easier to compare the magnitude of the shocks across time and between variables. For example, Figure 3 shows that the most severe shocks occurred during the first quarter under the Severely Adverse Scenario for U.S. VIX and U.S. BBB Spread, both with a standard shock above three. Recovery is most noticeable for U.S. Equity and U.S. VIX beginning in the fifth quarter.

Figure 3 Macrovariable dynamics under the 2015 CCAR scenarios.

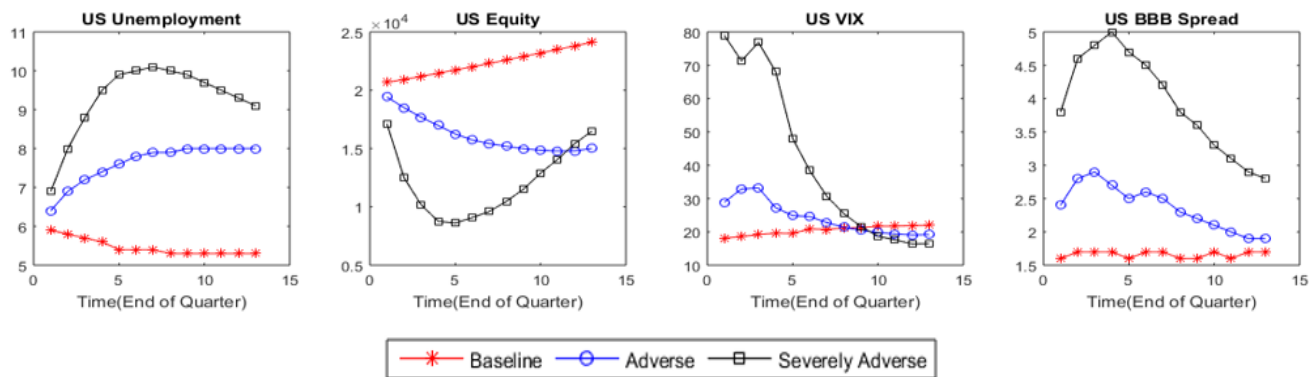
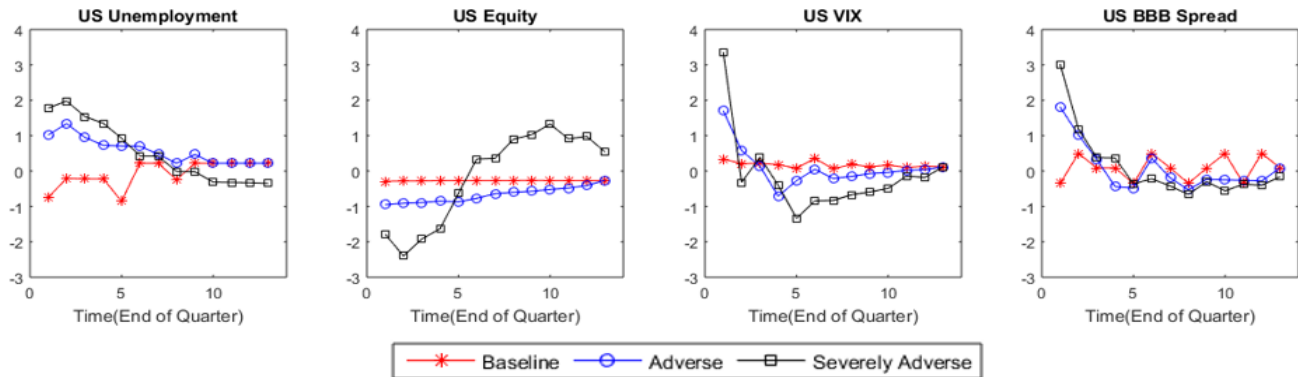


Figure 4 Standardized shocks for macrovariables under the 2015 CCAR scenarios.



For our analysis, the Zero-EDF curves are both discount and reference curves (for the floating-rate portfolio). We construct different Zero-EDF curves for different forward-analysis dates, based on the treasury yields assumed in the CCAR scenarios. More specifically, we obtain the 3-month, 5-year, and 10-year Zero-EDF rates by adjusting treasury yields upward by 0.28%, 0.21%, and 0.08%, respectively. Figure 5 shows how the yields change over time. By interpolating the yields, we can obtain Zero-EDF rate curves for different analysis dates, as shown in Figure 6. In each graph, the dotted line is the Zero-EDF rate on analysis date. The rest of the curves represent interest rate curves utilized on different forward-analysis dates. We can see that the difference between the yield curves on the analysis date and the yield curves for the forward-analysis dates are the largest for the Adverse Scenario and the smallest for the Severely Adverse Scenario. We show later that this difference greatly impacts the fixed-rate portfolio's value.

Figure 5 Zero-EDF rates under the three 2015 CCAR scenarios.

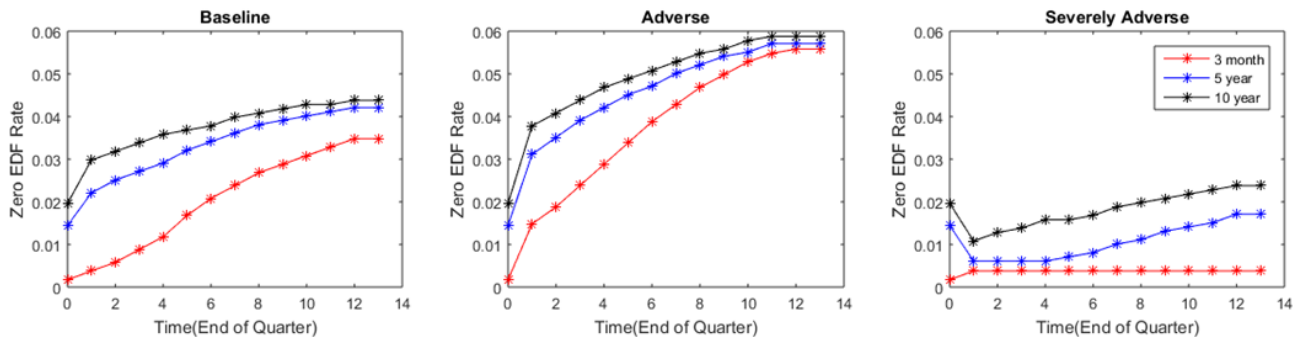
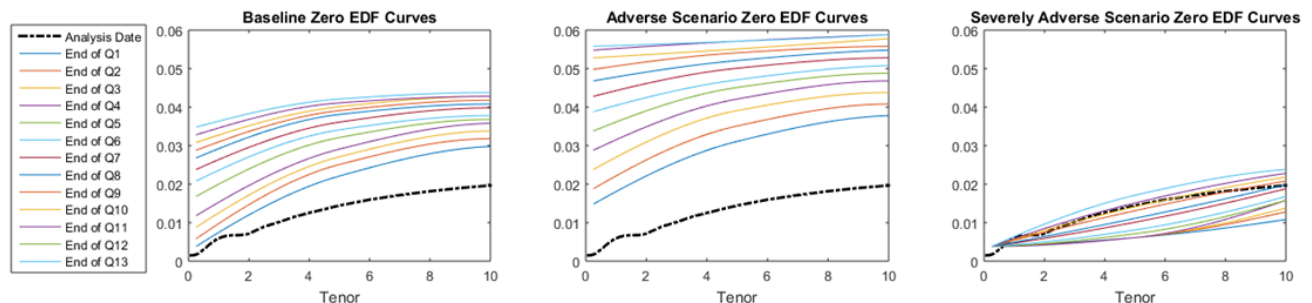
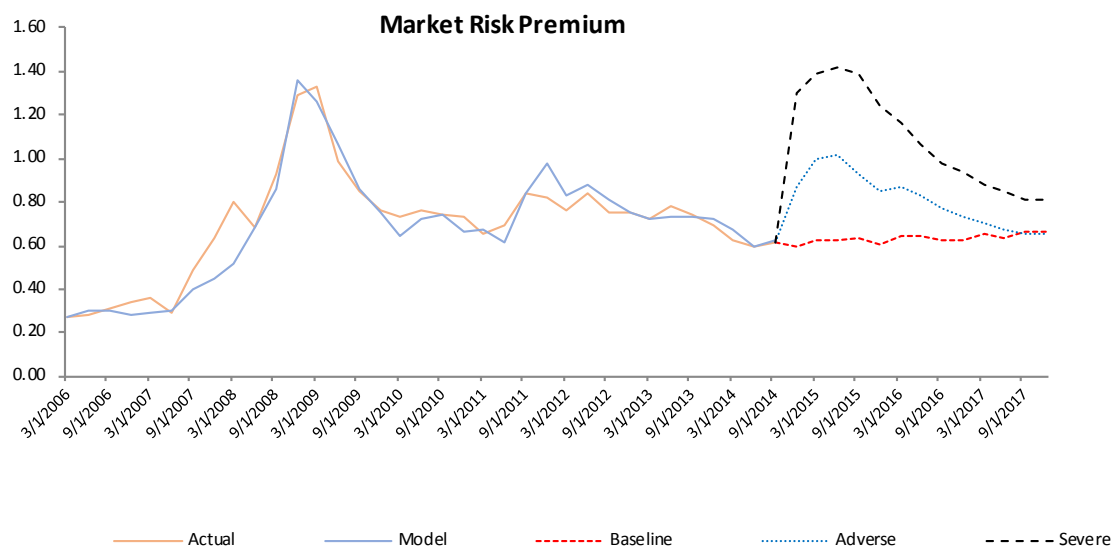


Figure 6 Zero-EDF term structures under the three 2015 CCAR scenarios.



We also use different market risk premiums consistent with each CCAR scenario.<sup>4</sup> Figure 7 shows the projected market risk premiums.

Figure 7 Actual, Modeled, and Forecasted market risk premiums under the 2015 CCAR Scenarios.



<sup>4</sup> Please see Mangla, et al., (2015) for a detailed description of its construction.

## 4. Results

This section presents analysis results for the representative portfolio.

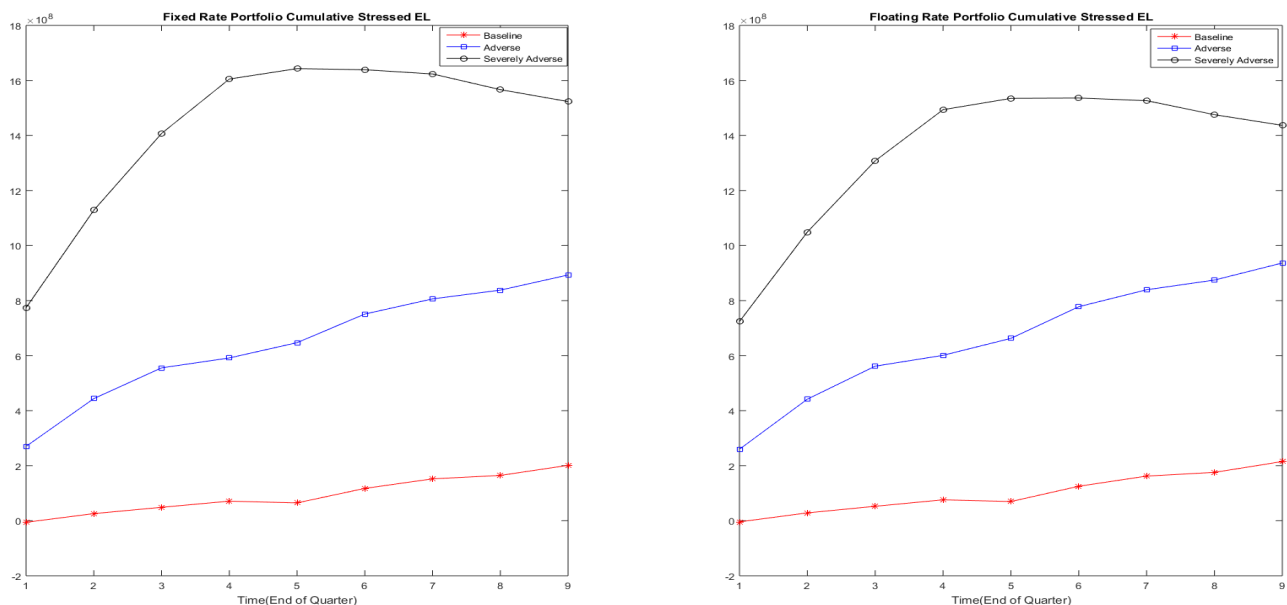
### 4.1 Capital Planning

In this section our primary focus is the capital supply and how it is affected by the stress scenarios over their lifetime. Here we illustrate how one can utilize economic capital distribution dynamics generated from RiskFrontier, together with quarterly expected losses, to conduct multi-period equity capital planning and to forecast changes to the capital supply. To do so, we assume that an equity-based capital buffer equal to 10bp portfolio economic capital is available to the organization on the analysis date, or time 0, and here we also assume that the capital buffer is reduced at the end of each quarter by that quarter's conditional expected loss. The same reduction affects the portfolio's size as well. The objective is to see whether the capital buffer can withstand (and how well it can do that) different economic shocks in the future.

There are two main components in this analysis. First, we expect economic capital distributions to change with the economic shocks. For example, if a severe economic scenario is realized, we expect economic capital distribution to shift upward for the same portfolio, as illustrated in Figure 1. In this case, even if the capital buffer available at the beginning of each quarter is kept constant, and the portfolio under consideration remains the same, the probability of breaching that threshold increases.

Second, we expect to realize losses at the end of each quarter.<sup>5</sup> The expected losses have two effects on the analysis. One is that expected loss decreases the capital buffer available at the beginning of each quarter. The other is that portfolio size (in terms of notional) shrinks accordingly.<sup>6</sup> Figure 8 depicts the stressed cumulative expected losses for the representative portfolio under fixed- and floating-rate coupon assumptions. Note, while the cumulative expected losses are the largest for the Severely Adverse Scenario, the downward-sloping curve during the later quarters corresponds to the partial recovery implied by the CCAR 2015 Severely Adverse Stress Scenario. This behavior might seem to be counterintuitive. When only accrual losses are taken into account, they will always stay positive, due to the fact that the cumulative PDs will not decrease as time passes, even when the economy starts to improve. In our case, we measure conditional expected loss between the conditional expected value and the unconditional promised value of an instrument. Thus, the observed non-monotonic behavior is due to the fact that, during the second half of that scenario, the assumed macroeconomic shocks are sufficiently high for the conditional value, given no default to increase, so much that, in expectation, we observe conditional expected value higher than the unconditional promised value, leading to negative conditional expected loss for that period.

Figure 8 Cumulative Stressed Expected Losses.



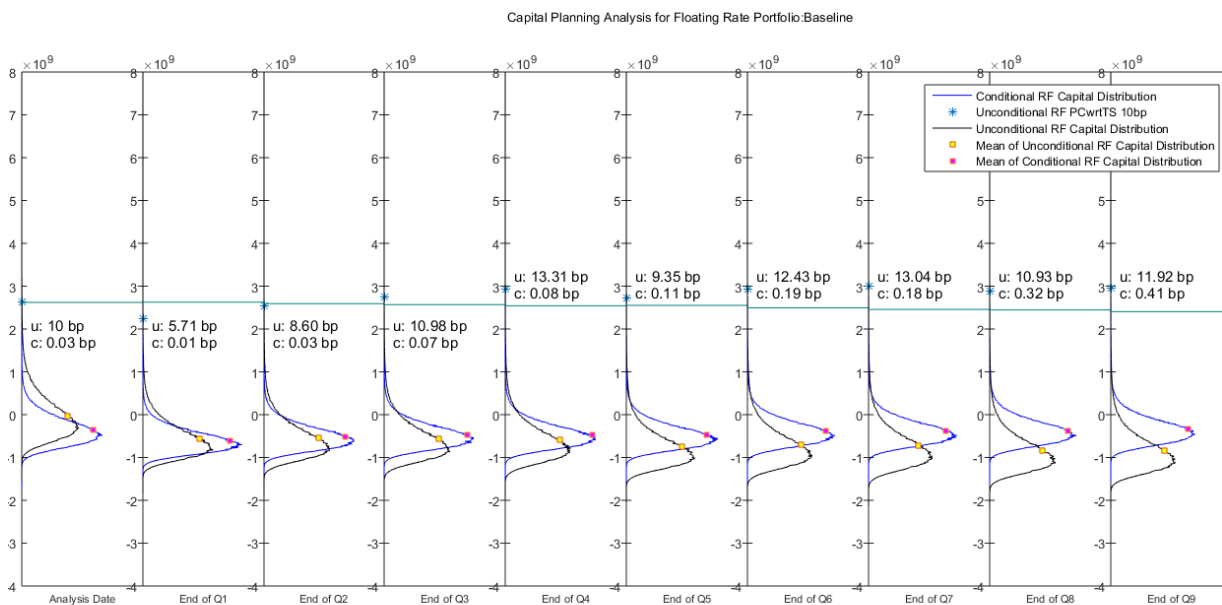
<sup>5</sup> We use quarterly stressed expected loss calculated by RiskFrontier in this analysis.

<sup>6</sup> We assume the portfolio composition (proportion allocated to different instruments) does not change.

We begin our analysis with the portfolio that consists of floating rate instruments. Figure 9 illustrates the capital planning analysis for the floating rate portfolio under the Baseline Scenario. The graph uses 10 plots, each representing the capital distribution for a particular analysis date (corresponding to the beginning of a quarter). The first is the capital distribution for the actual analysis date (3/31/2015). The second is for the forward-analysis date at the end of Q1, and the third is for the forward-analysis date at the end of Q2, etc. In each of the plots, the horizontal axis represents the density of the distribution, and the vertical axis represents the capital level. The line segment in each of the plots represents the capital buffer threshold at the beginning of the period.

For the analysis date, the capital buffer threshold equals the 10bp capital level. Starting at the beginning of Q2 (end of Q1), the capital buffer threshold decreases over time by the amount of cumulative expected loss up to that period. We conduct both unconditional and conditional RiskFrontier (RF) analysis and calculate the probability of the capital level exceeding the capital buffer threshold in each case, given by "u" and "c" respectively. For the Baseline Scenario, the capital requirement remains relatively stable. For unconditional RF capital distributions, probabilities of exceeding the capital buffer threshold are always around 10bp. For the conditional RF capital distributions, probabilities are much smaller than 10bp (less than 1bp).

**Figure 9** Capital planning analysis for the Floating-Rate Portfolio under the Baseline Scenario.



In comparison, the probabilities in the Adverse Scenario, illustrated in Figure 10, have considerably larger probabilities. Under unconditional RF analysis, the probability is always greater than 10bp. It is the largest at the end of Q3, at 2.47%. For conditional RF analysis, the mean of the capital distribution is greater than that from the unconditional RF analysis. However, the capital requirement is smaller beginning at the end of Q1. This occurs because, even under stressed scenarios, PDs can decrease for low PD (high quality) firms, thinning the tail of the distribution around the area of 10bp. Note, while the available capital buffer decreases over time as the blue line steps down monotonically during the analysis, the corresponding probabilities are non-monotonic—reflecting multidimensional effects that stresses have on the portfolios and the corresponding capital distributions.

Figure 10 Capital planning analysis for the Floating-Rate Portfolio under the Adverse Scenario.

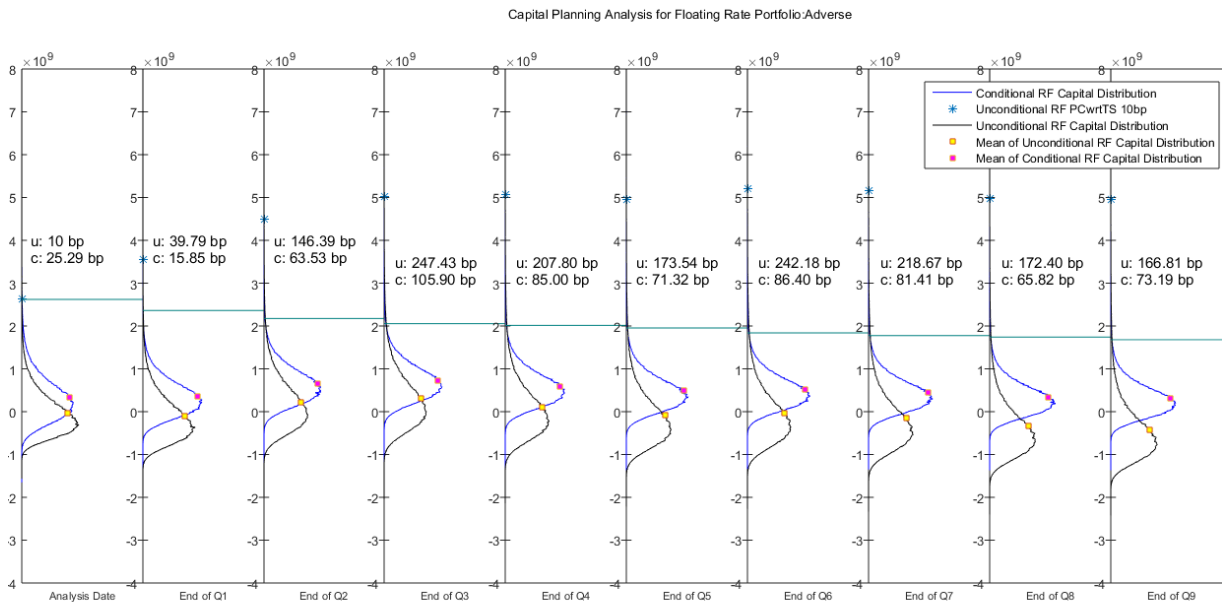
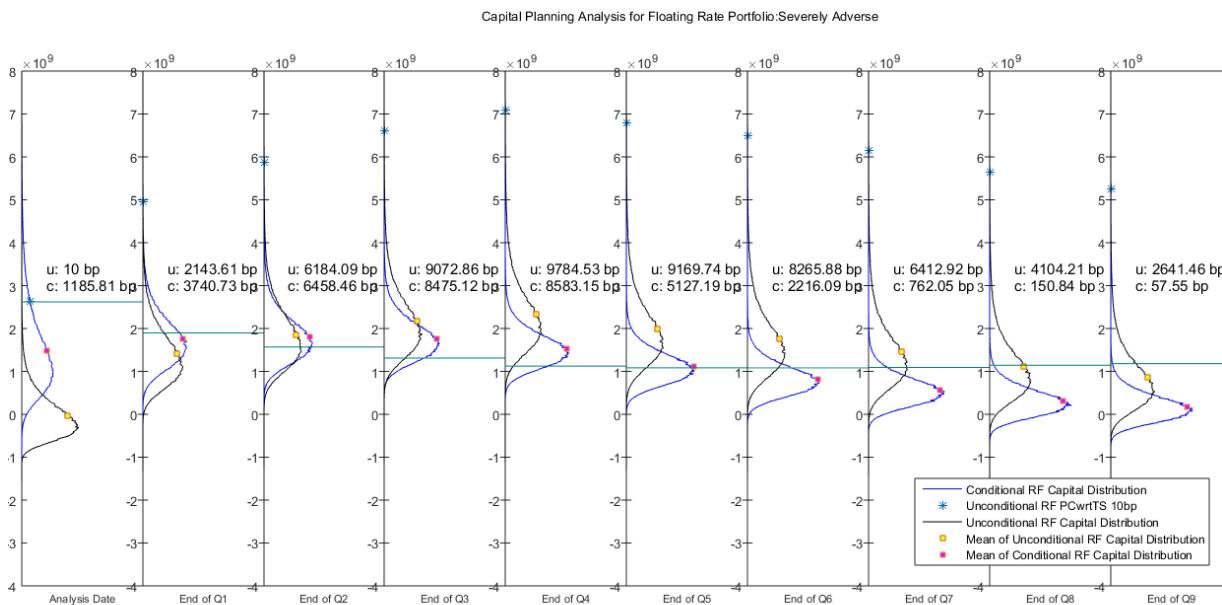


Figure 11 illustrates the case of the Severely Adverse Scenario for the floating-rate portfolio. For unconditional RF capital distributions, probability is highest at the end of Q4, about 97.8%. This finding is consistent with the stressed variables' paths, shown in Figure 4, and there is a clear sign of recovery beginning in Q5. For conditional RF capital distribution, probability is the largest at the same time, almost 86%. Note, the conditional probabilities that the remaining capital buffer will be insufficient to cover the outstanding portfolio are higher than the unconditional ones during the beginning of the analysis and lower later. This trait occurs because conditional analysis on the analysis date (at time 0) already takes into account severe shocks from Q1 to Q4, which describe the phases of major economic deterioration. But as the recovery phases begin, the conditional analysis starts to incorporate them, leading to decreases in the capital insufficiency levels.

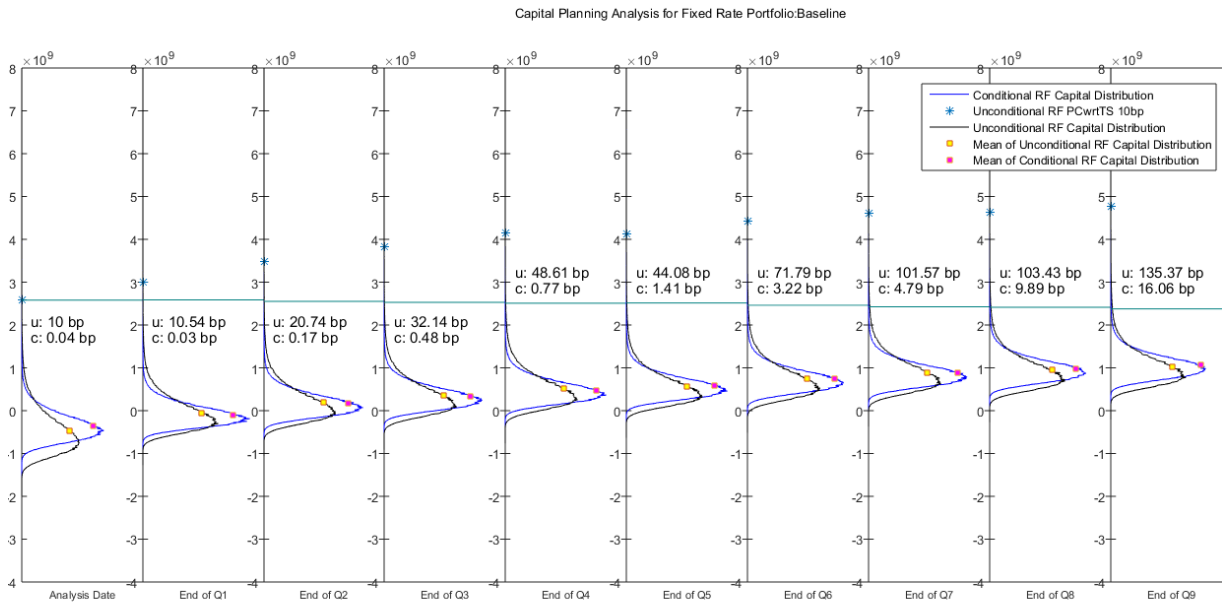
Figure 11 Capital planning analysis for the Floating-Rate Portfolio under the Severely Adverse Scenario.



For comparison, we also look at the distribution dynamics for the fixed-rate portfolio, depicted in Figures 12–14. The distribution dynamics are very similar to those of the floating-rate portfolio under all three scenarios. However, due to the CCAR 2015

assumptions on rate dynamics, the probabilities of exceeding the capital buffer threshold are much higher for the fixed-rate portfolio under the Adverse Scenario, reaching almost 100% at the end of Q9. Note that, unlike for the floating rate portfolio, the probabilities that the capital will be insufficient are lower under the Severely Adverse Scenario for the fixed-rate portfolio, reflecting the hedging nature of the floating rates, and for the CCAR assumptions on the interest rates under the Adverse and Severely Adverse Scenarios, as illustrated in Figure 6.

**Figure 12** Capital planning analysis for the Fixed-Rate Portfolio under the Baseline Scenario.



**Figure 13** Capital planning analysis for the Fixed-Rate Portfolio under the Adverse Scenario.

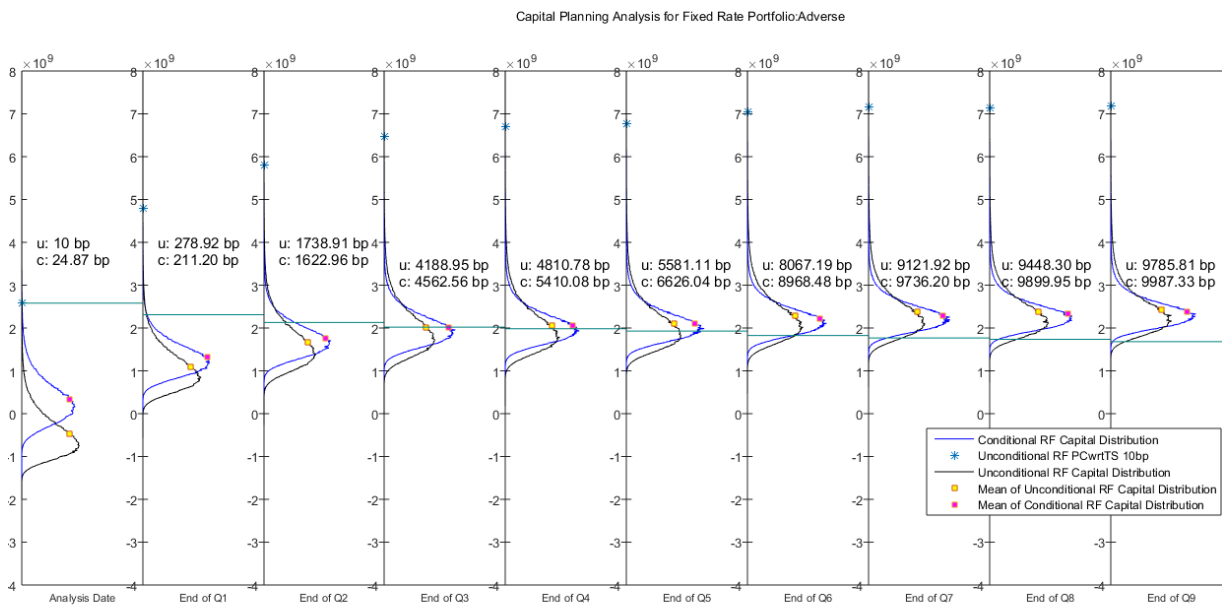
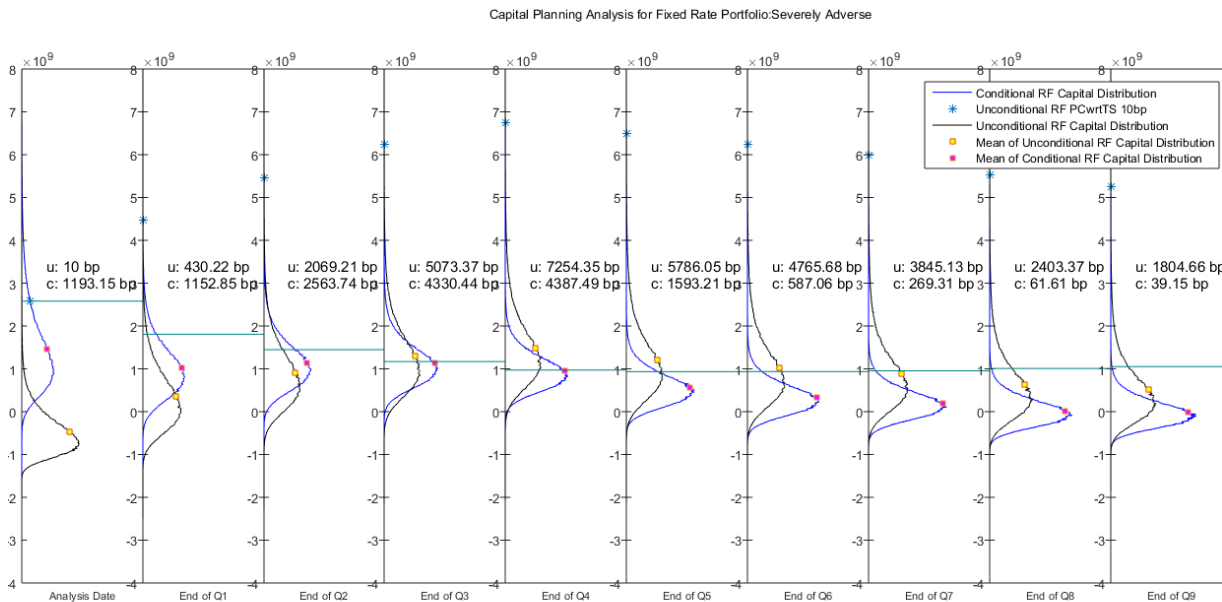
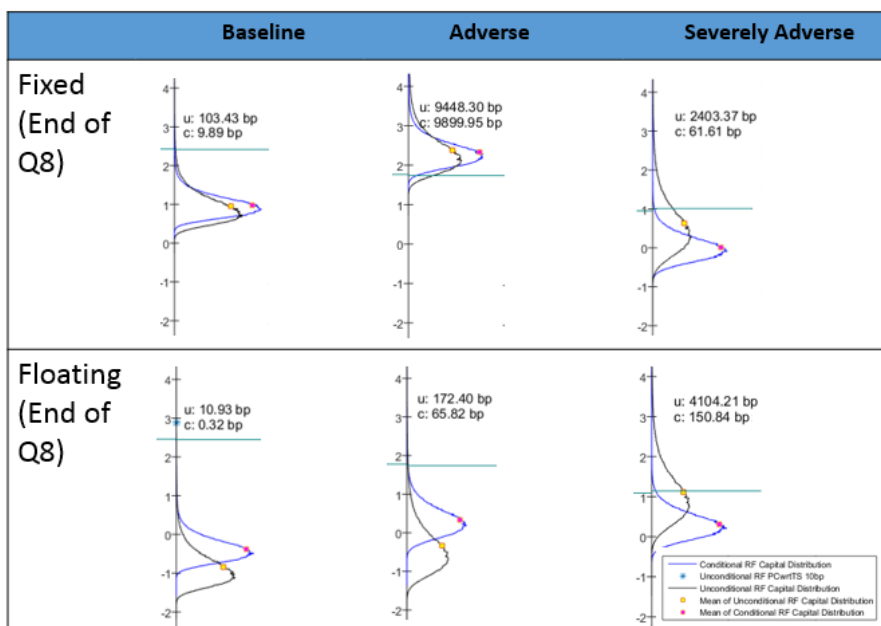


Figure 14 Capital planning analysis for the Fixed-Rate Portfolio under the Severely Adverse Scenario.



Apart from comparing capital distributions on different forward-analysis dates for a single portfolio under a specific scenario, we can also compare capital distributions between portfolios and across scenarios and associated capital buffers. For example, Figure 15 provides an illustration for a forward-analysis date at the end of Q8. It is clear from the graph that, for the fixed-rate portfolio, probabilities that the capital buffer will be insufficient to cover losses are largest under the Adverse Scenario, while for the floating-rate portfolio, those are largest under the Severely Adverse Scenario. This finding suggests that portfolios with various asset classes, and, as an implication, with hedging or without, can react to the same scenario very differently and, thus, require different capital plans. Furthermore, by comparing results for the same portfolio across all possible scenarios, we have an idea of the range of capital levels required for the portfolio in the future.

Figure 15 Comparison of capital distributions across scenarios and between the floating- and fixed-rate portfolios.

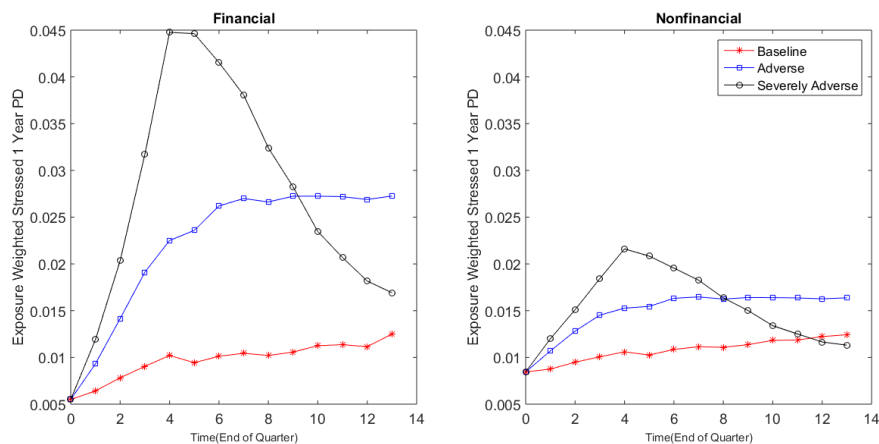


## 4.2 Financial and Non-financial Comparison

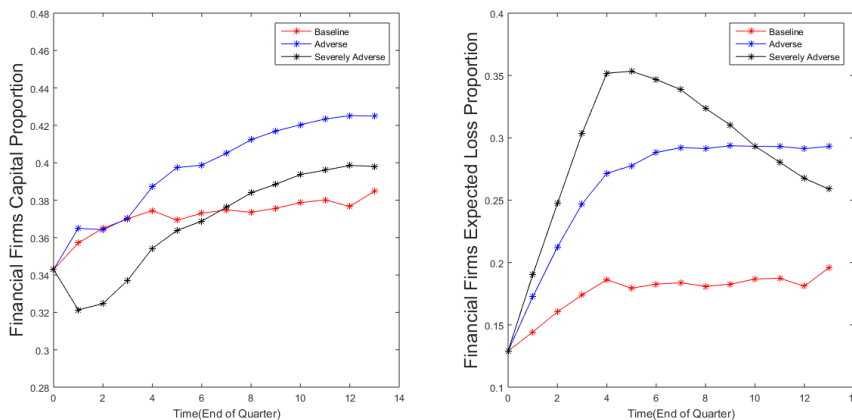
In addition to understanding how the total capital distributions and available capital buffers evolve over time, it is also important to understand capital allocation across segments within a portfolio. When a significant increase in capital requirements is expected for a portfolio, as in the case of the fixed rate-portfolio under the Adverse Scenario illustrated in the previous section, institutions may need to adjust their portfolio composition and decrease exposures. In these cases, capital allocation can shed light on how to effectively change the portfolio composition.

In this section, as an illustration for the above point, we compare two sectors that comprise the overall portfolio—financial and non-financial counterparties' sub-portfolios. Typically, financial firms are more exposed to systematic risk. They have a higher, exposure-weighted average RSQ of 0.431 compared to an RSQ of 0.335 for non-financial firms. This trait results in higher a sensitivity of stressed PDs to the stressed scenarios for financial firms. From Figure 16, we see that the analysis-date average PD for financial firms is smaller, around 50bp, compared to 8bp for non-financial firms. However, as the economy deteriorates, such as in the Severely Adverse Scenario, financial firms' PDs reach as high as 4.5% at the end of Q4, compared to only 2.4% for non-financial firms. The higher sensitivity of financial firm PDs also leads to a higher responsiveness of expected losses to the scenarios, as demonstrated in Figure 17 (right panel).

**Figure 16** Weighted-average, one-year forward PDs conditional on a given stress scenario up to the quarter for financial and non-financial sub-portfolios.



**Figure 17** Proportions of capital and expected loss for financial sub-portfolio under the 2015 CCAR Scenarios.



The graph on the left-hand side of Figure 17 shows the capital proportion of instruments from financial firms. In our portfolio, 23% instruments are from financial firms, which account for 29% of the portfolio's value. We also see from Figure 17 that the analysis date capital allocated to financial firm is above 34% and, thus, higher than their value proportion. Furthermore, under the Adverse and Severely Adverse Scenarios, the capital proportion of financial firms increases during most of the periods. This finding may suggest that, when adjusting the portfolio composition in order to decrease the capital requirement, it is more effective to decrease exposure to financial counterparties.

## 5. Summary

This paper proposes and illustrates a multi-period capital planning framework, which can be used to calculate capital requirements for a financial organization's portfolio over time under various economic stress scenarios and to determine appropriate capital buffer level(s), while analyzing how well a chosen capital buffer will withstand future macroeconomic shocks. This type of analysis can help institutions gain a better understanding of their credit portfolios' risk dynamics. The methodology allows institutions to foresee any capital requirement increases due to severe economic shocks and to prepare for it accordingly. If expecting a significant capital increase, the institution can either structure a higher capital buffer level today or make a plan to obtain adequate resources overtime. With this forward-looking perspective, institutions are less likely to be caught short of capital due to a stressed economy.

To illustrate this methodology, we analyze the impact of the 2015 CCAR scenarios on the capital requirements of two representative credit portfolios, both of which consist of all U.S. public firms in CreditEdge, one with fixed-rate coupons and the other with floating-rate coupons. The main determinants of the capital requirements for the floating-rate portfolio are the stressed probabilities of default and the trend of the capital requirement to mimic the stressed PDs under all three scenarios. As expected, the stressed PDs and the capital requirement are relatively flat under the Baseline Scenario, and they increase consistently under the Adverse Scenario. While, as expected, probabilities that a capital buffer will be insufficient to cover portfolio losses under the Severely Adverse Scenario are greater than under the Adverse Scenario, it should be noted that the stressed PDs and probabilities of capital buffer insufficiency increase relatively sharply during the first year and peak at the end of Q4, after which, they decrease. Although counterintuitive, these patterns fall in-line with the assumption of strong recovery for the CCAR 2015 Severely Adverse Scenario. In addition to these observations, for the fixed-rate portfolio, the capital requirement is also impacted greatly by the difference between the projected interest rates for forward-analysis dates and the actual interest rates on analysis date, as the time value of money becomes an important component of the portfolio value. This situation calls for either an increase in the capital buffer at the initial analysis date or a change in portfolio composition to lower the capital requirement.

An analysis of capital allocation across segments within the portfolio can shed light on how to effectively change the portfolio composition, if necessary. For example, results for the analyzed representative portfolios show that the proportion of capital allocated to financial firms is always greater than their value proportion, and it increases in most of the periods under both the Adverse and Severely Adverse Scenarios. This finding suggests that decreasing exposures in financial counterparties is more effective in terms of lowering the capital requirement when the economy is stressed, providing guidance on how a portfolio's composition can be managed for more efficient capital planning.

## Appendix A

The stressed PD term structures entered in RiskFrontier for each of the forward-analysis dates are stressed forward PDs. These PDs should be stressed up to the forward-analysis date of interest in order to reflect the scenarios assumed between the two dates.

The first step requires obtaining stressed quarterly PDs using Moody's Analytics Stressed EL Calculator, which allows users to specify the number of quarters to be stressed. For example, to obtain the stressed PDs for the forward-analysis date as the end of Q2, we must set the number of the stressed quarters to two. Since we have 13 forward-analysis dates, we must run the Stressed EL Calculator 13 times.

After obtaining quarterly PDs from the Stressed EL Calculator, we must convert the quarterly PDs to forward PDs using the following equations, which calculate the forward PD (of term equal to  $L - K$ ) for the  $K^{th}$  forward-analysis date.  $D_t$  is the time of default, and  $PDQ_i$  represents the quarterly PD for quarter  $i$ .

$$\begin{aligned}
 FPD(K, L) &= P(D_t < L \mid D_t > K) \\
 &= 1 - \frac{S(L)}{S(K)} \\
 &= 1 - \frac{1 - (PDQ_1 + \dots + PDQ_L)}{1 - (PDQ_1 + \dots + PDQ_K)} \\
 &= \frac{PDQ_{K+1} + \dots + PDQ_L}{1 - (PDQ_1 + \dots + PDQ_K)} \\
 &= 1 - \frac{1 - (PDQ_1 + \dots + PDQ_L)}{1 - (PDQ_1 + \dots + PDQ_K)} \\
 &= \frac{\sum_{i=K+1}^L PDQ_i}{1 - \sum_{j=1}^K PDQ_j}
 \end{aligned}$$

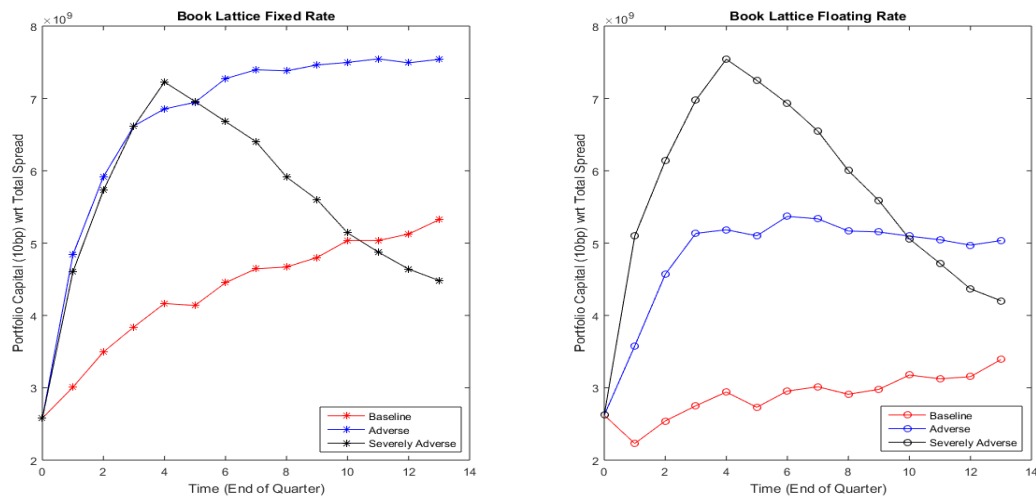
The forward PD is then annualized to obtain input PDs for RiskFrontier.

$$\text{Annualized Forward PD} = 1 - (1 - FPD(K, L))^{\frac{4}{L-K}}$$

## Appendix B

In this appendix, we analyze the effects that stress scenarios might have on the capital demand or the amount of capital required to cover the portfolio losses up to a certain probability level. To provide the common basis for the comparison in this part of the study, we assume that the portfolio is rebalanced to the initial, or time 0, size and composition at the beginning of each quarter. This assumption essentially implies that, at the end of each preceding quarter, there is a capital injection that brings the portfolio to its initial state. While this makes the analysis exogenous, as it relies on the non-specified sources of additional capital and such presumption might not be particular useful for real-life applications, it will allow us to study the pure effects of conditional PDs and other parameters on the portfolio capital distributions and economic capital requirements. Figure 18 presents the economic capital required to cover losses at 10bp, with respect to Total Spread, calculated for both the floating rate and fixed-rate portfolios at various time points under the three scenarios. In the case of the floating-rate portfolio (right-hand graph), the interest rate effect is small. In this case, the portfolio's capital demand pattern is driven primarily by stressed PDs. Comparing Figure 18 to Figure 2, we see that the shape of the portfolio capital mimics the shape of the stressed PDs under all three CCAR scenarios.

**Figure 18** Capital demand under stress scenarios — Dynamics of portfolio capital at 10bp under the 2015 CCAR scenarios.



In the case of the fixed-rate portfolio, the economic capital dynamics look similar to the floating-rate portfolio under the Severely Adverse Scenario. However, the portfolio economic capital is much larger under both the Baseline Scenario and the Adverse Scenario. This finding is due mainly to the interest rate assumptions in the CCAR scenarios (and, thus, the Zero-EDF rates constructed). The much higher capital of the fixed-rate portfolio under the Baseline and Adverse Scenarios is due to the jump in interest rate curves for the forward-analysis dates, as illustrated in Figure 8. A large interest rate increase after the analysis date results in higher losses in the fixed-rate portfolio and, thus, higher capital. On the other hand, capital is smaller for the fixed-rate portfolio when projected future Zero-EDF rates are smaller than its analysis date level, as in the first few quarters under the Severely Adverse Scenario.

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