MOODY'S ANALYTICS

MODELING METHODOLOGY

Practical Considerations When Unifying Regulatory and Economic Capital in Investment Decisions

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Abstract

The degree to which an organization's regulatory capital is constraining impacts an investment's appeal. The more constraint on the organization, the more heavily an instrument's regulatory capital weighs down its appeal, with investments that are assigned higher regulatory capital being impacted more. This paper explores a method for measuring the extent to which an organization's regulatory capital is binding and calibrates the model introduced by Levy, Kaplin, Meng, and Zhang (2012), which unifies regulatory and economic capital in investment decisions. We then examine the impact of the regulatory capital requirement on investment decisions, based on the calibrated model. We find that the rank order of exposures' risk-return tradeoff in our sample portfolio changes substantially once the regulatory capital constraint is taken into consideration.

Table of Contents

1. Introduction

Since the financial crisis, the regulatory environment has forced financial institutions to adhere to ever-increasing capital standards. The new stress testing requirement and the advent of Basel III raises the need for better and perhaps less conventional management of capital to meet regulatory constraints. A central problem faced by banks is how to incorporate the impact of regulatory capital requirements into traditional economic-capital-styled risk management and risk-based pricing.

In a classical setting, investments are usually made in a way that optimize portfolios by weighing return and portfolio-referent risk through decision measures such as their EVA and RORAC. These measures are consistent with a setting where choices are free of regulatory constraints. This said, regulators typically require financial institutions to adhere to minimum capital standards. When investment decisions are impacted by these requirements, regulatory capital introduces an implicit cost that may vary across investments. Decision measures such as RORAC should thus take these requirements into consideration. Levy, Kaplin, Meng, and Zhang (2012) (LKMZ) propose a unified decision measure that incorporates regulatory capital requirements in the traditional economic framework underpinning EVA- and RORAC-style decision measures. In their model, the perceived expected spread of each investment is adjusted to reflect the regulatory capital constraint. The magnitude of adjustment is determined by the regulatory capital rate, also known as risk-weighted capital (RWC) of the instrument, as well as the degree to which the regulatory capital constraint is binding.

In this paper, an asset's RWC is presented in the context of Basel II or III-style requirement, but the overall framework is sufficiently general to cover a wide range of other regulatory requirements, such as stress testing capital requirements (e.g., CCAR). In the Basel setting, an asset's RWC is dependent on the characteristics of the instrument, such as PD, LGD, asset class, etc.

Meanwhile, the degree of the regulatory capital constraint is institution-specific, and, as regulatory capital is more constraining, the more taxing an investment's RWC, and the greater the adjustment to RORAC. In the LKMZ model, the binding degree is captured by a parameter $RegC_C$, which is latent.[1]

This paper proposes a calibration approach for $RegC_C$ under the LKMZ modeling framework. Intuitively, $RegC_C$ is linked to the distance between an institution's leverage when adhering to its regulatory limit, and its otherwise optimal leverage. Since leverage is observable, $RegC_C$ can be calibrated to its variation under different regulatory environments. In the paper, we estimate $RegC_C$ estimated for a pool of U.S. banks, and it is used to analyze a sample portfolio to demonstrate the impact of accounting for varying regulatory capital requirements.

The remainder of this paper is organized as follows: Section 2 provides an overview of the model proposed by LKMZ. Section 3 introduces the methodology, as well as the underlying theory we rely on to calibrate the parameter $RegC_C$. Section 4 examines the impact of the regulatory capital requirement on investment decisions. Section 5 concludes.

2. Overview of the Unified Regulatory and Economic Capital Model

In the traditional portfolio selection framework, an institution seeks to maximize its stakeholder's utility under a budget constraint. In the literature, an investor's optimal investment decision is usually captured by a CAPM-styled model such as the one used by LKMZ, where the lifetime quadratic utility is maximized by trading off consumption C_t (or dividends consumed) and investment at each period:

$$\max U(C) = E_0\left[\sum_t \gamma^t (C_t - bC_t^2)\right] \tag{1}$$

s.t.

$$C_t = \sum_j (P_{j,t} + CF_{j,t})N_{j,t-1} - D_{t-1}(1 + r_{D,t}) - \sum_j P_{j,t}N_{j,t} + D_t \tag{2}$$

where $P_{j,t}$ and $CF_{j,t}$ denote the price and cash flow of instrument j at time t per notional unit; $N_{j,t}$ denotes the notional amount held by the investor of instrument j at time t; D_t denotes the debt. It can be shown that under this modeling framework an investor can use a risk-adjusted return measure to determine whether and how much to invest in certain instruments. Common

[1] In LKMZ (2012), the $RegC_C$ parameter is denoted by C. We use a different notation here to avoid confusion between this parameter and consumption C_t.

choices of such measures include Sharpe Ratio (SR) and Return on Risk Adjusted Capital (RORAC). Formally, the SR for an instrument is defined as the ratio between the Expected Spread (ES) and the Risk Contribution (RC) of the instrument:

$$SR_{j,t} = \frac{ES_{j,t}}{RC_{j,t}} \quad (3)$$

where the ES of the instrument is defined as the difference between the expected return of the instrument and the borrowing rate for the investor:

$$ES_{j,t} = r_{j,t} - r_{D,t} \quad (4)$$

The investor can optimize the portfolio by increasing weight of investments with higher SR and reduce holdings of those with lower SR. Ranking instruments by SR is equivalent to sorting them by the RC-based RORAC measure, which is defined as

$$RORAC_{j,t} = \frac{ES_{j,t}}{CR_{j,t}} + r_{D,t} \quad (5)$$

where $CR_{j,t}$ is the capitalization rate of instrument j with the capital allocated across instruments through RC. Again, the general idea is that the higher the RORAC, the more attractive the instrument is in term of risk-return tradeoff.

Note that the RORAC measure is different from the traditional Return on Equity (ROE) measure, as it calculates the excess return on Economic Capital (EC) of the portfolio attributed to the corresponding instrument. This means that RORAC take into consideration of diversification, concentration, and other economic risks which are typically reflected in EC. However, RORAC does not consider the regulatory capital requirement which poses as an additional constraint on the investor portfolio optimization problem.

Regulatory authorities typically require a bank to post an adequate amount of capital for its Risk Weighted Asset (RWA). The regulatory capital rate RWC for each instrument is equal to

$$RWC = RWA \times Capital\ Adequacy\ Ratio \quad (6)$$

where the Capital Adequacy Ratio (CAR) is fixed at 8% in Basel II. Appendix B shows in detail how RWC is computed.

When regulatory capital requirement are constraining, LKMZ propose a Regulatory Capital Adjusted (RegC-Adjusted) RORAC measure. The unified measure is derived from the same modeling framework as represented by equations (1) and (2), with the additional regulatory capital constraint

$$Book\ Equity_t = \sum_i N_{i,t} - D_t \geq \sum_i N_{i,t} RWC_{i,t} \quad (7)$$

The corresponding RegC-Adjusted SR and RORAC for instrument j at time t is proven to be

$$\widehat{SR}_{j,t} = \frac{\widehat{ES}_{j,t}}{RC_{j,t}} \quad (8)$$

and

$$\widehat{RORAC}_{j,t} = \frac{\widehat{ES}_{j,t}}{\widehat{CR}_{j,t}} + r_{D,t} = \widehat{SR}_{j,t} \frac{\sum_j P_{j,t} \cdot N_{j,t} \cdot RC_j}{\widehat{CAP}_P} + r_D \quad (9)$$

where

$$\widehat{ES}_{j,t} = r_{j,t} - r_{D,t} \cdot f_{j,t} \quad (10)$$

and the adjusted EC (in dollar amount) for the entire portfolio $\widehat{CAP}_P$ is related to the unadjusted EC through the following equation:

$$\widehat{CAP}_P = CAP_P - \frac{ES_P - \widehat{ES}_P}{1 + r_D} MTM_P \quad (11)$$

The instrument-specific adjustment factor f_j is, in general, positive with the form

$$f_{j,t} = \frac{1-\left(1-\left(1-RWC_{P,t}\right)^{RegC_C}\right)\left(1-RWC_{j,t}\right)/P_{j,t}}{\left(1-RWC_{P,t}\right)^{RegC_C}} \quad (12)$$

At the portfolio level, the adjusted factor is

$$f_{P,t} = \frac{1-\left(1-\left(1-RWC_{P,t}\right)^{RegC_C}\right)\left(1-RWC_{P,t}\right)/P_{P,t}}{\left(1-RWC_{P,t}\right)^{RegC_C}} \quad (13)$$

The intuition behind these adjusted measures is that the regulatory capital requirement imposes an implicit tax, which makes assets less attractive and results in a lower perceived instrument ES.

The RegC-Adjusted RORAC/SR measure derived from the LKMZ model allows one to account for both economic risks and regulatory burden.

What remains is a practical approach to parameterize $RegC_{C,t}$ that accounts for the degree to which the regulatory capital constraint is binding, which is latent.

3. Calibration of $RegC_C$

Intuitively, $RegC_C$ measures the degree to which regulatory capital is binding. In general, when $RegC_C$ is larger, the adjustment factor is bigger, which means that the perceived ES is smaller. Said another way, the greater the effort needed to satisfy regulatory requirements, the higher the incurred cost. There is a link between reduced leverage and the cost regulatory capital imposes on investments. We describe the formal theoretical framework behind this link in Sub-section 3.1. Sub-section 3.2 provides numerical analysis for the relationship between $RegC_C$ and the deleverage induced by regulatory requirements. Sub-section 0 examines historical leverage time series of U.S. financial institutions. We then use these historical data to calibrate $RegC_C$ for a sample portfolio in Sub-section 3.4.

3.1 Modeling Framework

We now formalize the modeling framework under which $RegC_C$ is calibrated. We first establish the relationship between $RegC_C$ and the optimal leverage of an investor under the same modeling assumptions used by LKMZ. Specifically, leverage is defined as the ratio between total assets and total equity. Assuming the investor has one-unit of equity, leverage is equivalent to the total amount of asset.

THEOREM 1

Under the assumptions of the LKMZ model, $RegC_C$ has the following solution:

$$RegC_C = \frac{\ln\left(P_{P,t}-1+RWC_{P,t}\right)-\ln\left(P_{P,t}\cdot DelR\cdot\frac{ES_{P,t}}{r_D}+P_{P,t}-1+RWC_{P,t}\right)}{\ln\left(1-RWC_{P,t}\right)} \quad (14)$$

where the Deleverage Ratio (DelR) is defined as the relative decrease in optimal leverage level if the regulatory capital requirement is imposed:

$$DelR = 1-\frac{\hat{L}_t^*}{L_t^*} \quad (15)$$

PROOF

See Appendix C.

There are a number of approaches that can be used to calibrate DelR. Under a strict interpretation of the model, DelR represents the extent to which an institution needs to deleverage in order to adhere to its regulatory capital requirement. We can calculate DelR as:

$$DelR = 1-\frac{Target\ Capitalization\ Rate\ Under\ No\ Regulatory\ Constraint}{Target\ Capitalization\ Rate\ Under\ Regulatory\ Constraint} \quad (16)$$

In theory, the target capitalization rate is EC_P under no regulatory capital constraint, but is RWC_P under the constraint. In reality, the target capitalization rate under the regulatory constraint is likely to be higher than RWC_P, as organizations incorporate a buffer.

Alternatively, we can rely on historical L_t data. The idea is to explore leverage patterns during periods with relatively light regulatory capital requirements and compare them to periods with stringent capital requirements. We use this method to estimate DelR for a typical U.S. financial institution in Sub-section 0. First, we demonstrate the numerical relationship between $RegC_C$ and DelR, as well as provide additional intuition for their relationship in Sub-section 3.2.

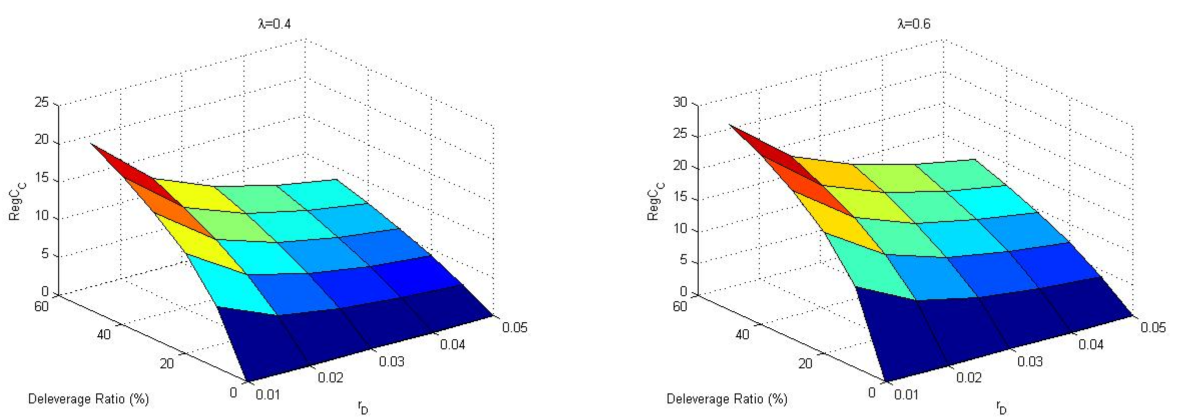
3.2 Relationship Between DelR and $RegC_C$

As captured by equation (14), the exact relationship between $RegC_C$ and DelR is dependent on the overall ES, RWC, the portfolio price, r_D, and λ; the relationship is portfolio- and parameter-specific. To better understand this relationship, we use the IACPM portfolio to explore the relationship between $RegC_C$ and DelR under different parameters.[2]

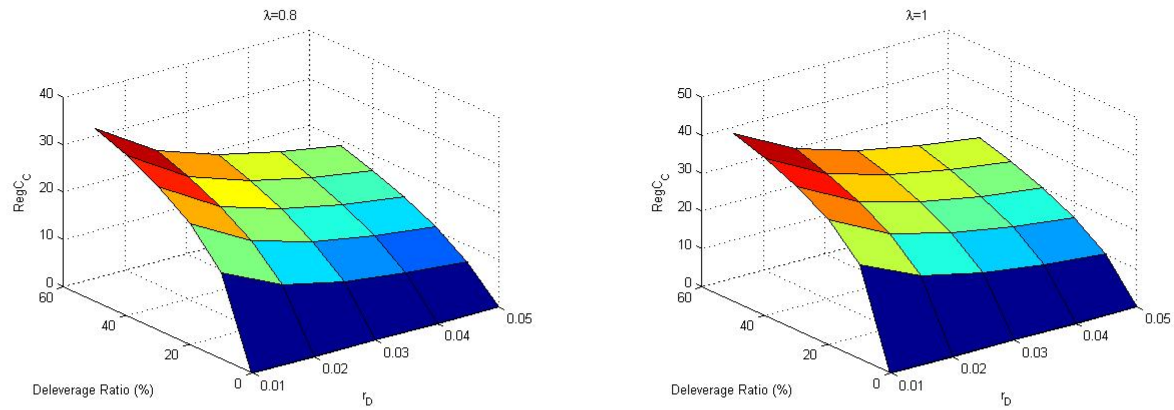
Figure 1 shows the value of $RegC_c$ as a function of DelR and the risk-free rate, assuming instrument RWCs are determined by the Basel II standardized approach with the rating to the RWC mapping recorded in Table 7, Appendix B. Note, $P_{P,t}$ and $ES_{P,H}$ values of the IACPM portfolio — two key inputs in equation (14) — are computed using Moody's Analytics RiskFrontier™ for each combination of λ and the risk-free rate. In the RiskFrontier setting, we use the "Lattice-Lattice" valuation method with 1,000,000 Monte Carlo simulation runs. The description of the valuation method can be found in Chapter 6 of the RF methodology document *Modeling Credit Portfolios* (2013).

First, notice $RegC_c$ is an increasing function of DelR. This finding is expected, as the larger the value of DelR, the larger the observed impact of the regulatory capital requirement and, naturally, the higher the adjustment needed to ensure the requirement is adhered to. Second, $RegC_c$ is a decreasing function of risk-free rate. This function results because the adjustment on ES is done by "inflating" the borrowing cost (assumed to be the risk-free rate), as seen in equation (10). Consequently, the larger the borrowing cost, the greater the adjustment on ES, and thus, the lower the $RegC_c$ needed to obtain the target DelR. Last, $RegC_c$ increases with λ, driven by the unadjusted ES being an increasing function ofλ. Given the same adjustment factor f_P and the risk-free rate, the larger the ES, the relative value of the adjustment becomes smaller, which leads to higher $RegC_c$ required to achieve a given DelR.

Figure 1 *$RegC_c$* as a Function of DelR and r_D (Standardized RWC)



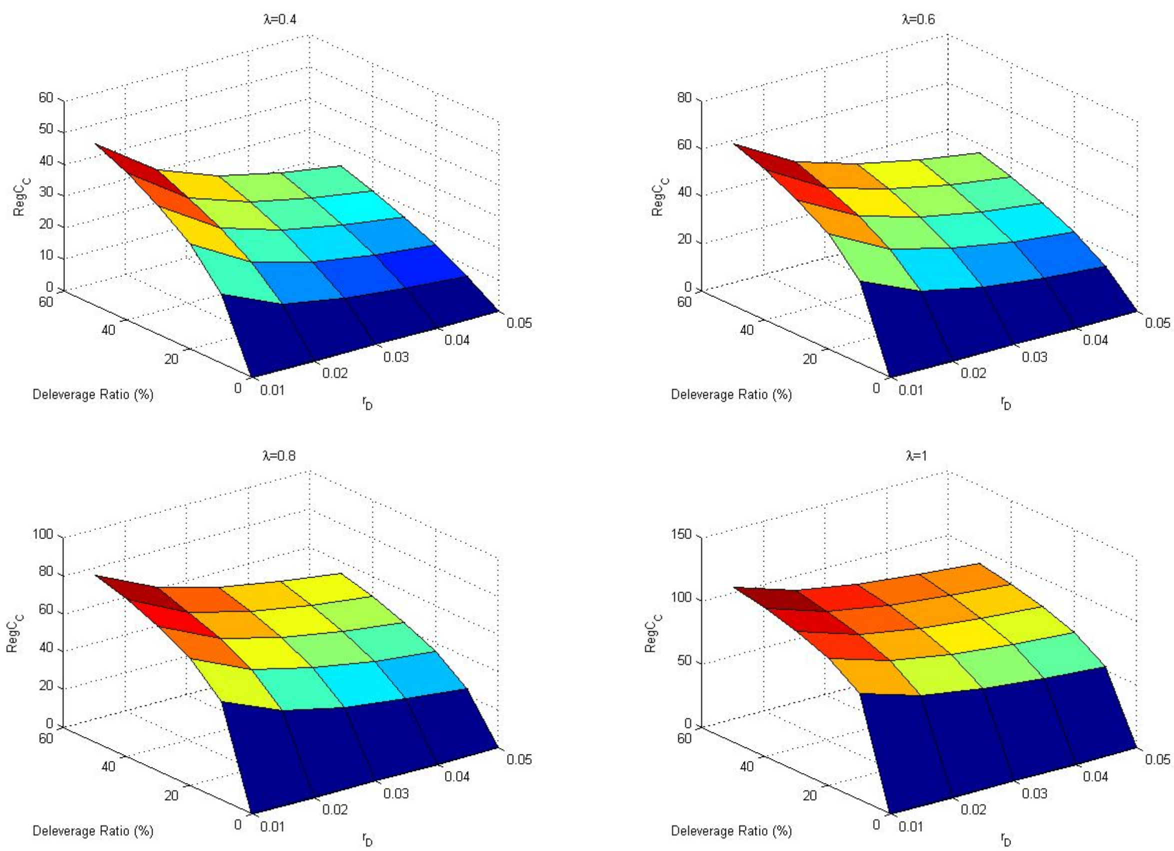
[2] The IACPM portfolio is created by IACPM and ISDA and is used in their study on the comparison of credit capital models. A detailed description of this portfolio can be seen in the Appendix.



This figure shows that the relationship between DelR, $RegC_C$ and $\boldsymbol{r_D}$ under different $\boldsymbol{\lambda}$ for the IACPM portfolio. The standardized approach under Basel II is used to determine RWC. Actual DelR is defined as the percentage decrease in leverage caused by the regulatory capital requirement.

Figure 2 demonstrates the same relationships as Figure 1, but under the advanced IRB approach. We can see that the general relationship between DelR, $RegC_C$, the risk-free rate, and λ remains the same. However, the magnitude of $RegC_C$ is higher than seen in Figure 1, i.e., the model-implied DelR is smaller given the same $RegC_C$. This finding is due to the fact that the overall portfolio RWC determined by the advanced IRB approach (4.07%) is significantly lower than that under the standardized approach (7.25%). The lower capital requirement means that it is easier for the bank to meet the constraint. In other words, the bank must make less adjustment to accommodate regulatory constraint in this case. As a result, the model-implied DelR is smaller.

Figure 2 $RegC_C$ as a Function of DelR and $\boldsymbol{r_D}$ (Advanced IRB RWC)



This figure shows the relationship between $RegC_C$, DelR, and r_D under different λ for the IACPM portfolio. We use the advanced IRB approach under Basel II to determine RWC. Actual DelR is defined as the percentage decrease in leverage caused by the regulatory capital requirement.

3.3 Historical Leverage in the U.S.

We now explore the Consolidated Report of Condition and Income (Call Report) data to get an empirical estimate of DelR for U.S. banks. Since the common regulatory capital requirement focuses on Tier-One capital, we define leverage as the ratio between a bank's total asset and its Tier-One capital. Table1 reports the leverage of ten major U.S. commercial banks as well as the leverage of all regulated U.S. financial institutions in 2004 and 2014. On average, U.S. financial institutions reduced leverage by 15% since 2004, the year when Basel II was first introduced. The deleverage effect is significantly more pronounced among large U.S. commercial banks, whose deleverage ratio ranges from over 20% to under 40%. This phenomenon is expected as large U.S. financial institutions face stricter regulation, especially after the financial crisis.

Given the information in Table 1, we proceed by exploring deleverage ratios of 8%, 15%, and 30% when calibrating $RegC_C$.

Table 1 DELEVERAGE RATIO OF U.S. BANKS

BANK NAME	AVERAGE TIER 1 LEVERAGE IN 2004	TIER 1 LEVERAGE IN QTR1 2014	DELEVERAGE RATIO
Bank of America	16.3	11.6	28%
Bank of NY	N/A	14.5	N/A
Capital One	N/A	11.4	N/A
Citibank	16.5	10.6	36%
HSBC-U.S.	15.9	10.7	33%
JP Morgan	17.4	13.6	22%
PNC Bank	13.9	10.8	22%
TD Bank	N/A	14.6	N/A
U.S. Bank	17.2	11.3	34%
Wells Fargo	15.5	12.1	22%
All U.S. Financial Institutions (average)	12.8	10.9	15%
All European Banks*	17.2	11.5	33%

Leverage for ten large U.S. banks and all U.S. regulated financial institutions in 2004 and 2014. Leverage is defined as the ratio of total book assets and tier-one capital. The deleverage ratio is computed as the percentage decrease in leverage from 2004 to 2014. Data source: Call Report.
Leverage for all European banks is defined as the ratio of total book asset and total book equity of financial institutions (at Bank Holding Company level) operating mainly in Europe. Data source: Moody's CreditEdge.

3.4 Calibration of $RegC_C$

Since the value of DelR is dependent not only on $RegC_C$ but λ and risk-free rate as well, we need to fix the value of λ and risk-free rate in order to pinpoint the value of $RegC_C$ given DelR. In the following analysis, we set the value of λ to be 0.6, which is close to the average λ value estimated by Moody's from 2004 to 2014 [3]— the period during which DelR is computed. For simplicity, we assume a flat interest rate term structure and set the value of risk-free rate to be 2%, close to the average Zero-EDF rate during the same period.[4]

With an estimate of DelR, calibrate $RegC_C$ is straightforward. Figure 3 presents the relationship between DelR and $RegC_C$, highlighting values when DelR is 8%, 15%, and 30%. Notice that $RegC_C$ is higher when RWC is determined with the advanced IRB approach than with the standardized approach. This reflects the difference in RWC computed under the two approaches.

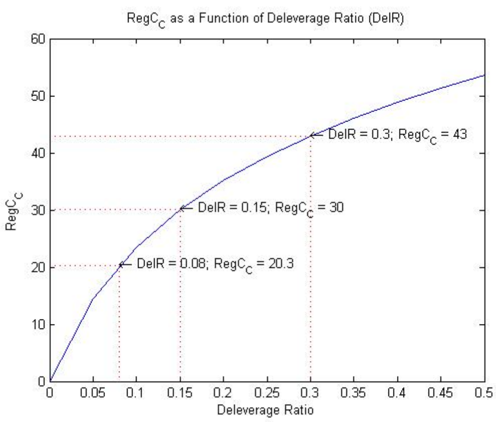
[3] Moody's computes bond price-implied daily λ values based on EDF9. The average value from January 2004 to March 2014 is 0.61.

[4] The average daily one-year Libor rate from January 2004 to March 2014 is 2.4%, which indicates an average Zero -EDF rate of around 2.3%.

Figure 3 Calibration of $RegC_C$

Advanced IRB RWC

Standardized RWC



This figure shows the relationship between DelR and $RegC_C$ when the risk-free rate is 2%, and λ is 0.6. In the left plot, the advanced IRB approach under Basel II is used to determine the value of RWC. In the right plot, the standardized approach is used. The value of $RegC_C$ is calibrated at three points — when DelR is 0.08, 0.15, and 0.3.

4. Instrument RORAC Under Regulatory Capital Constraint

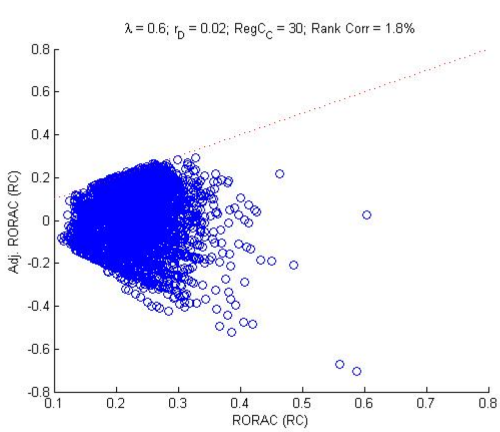
4.1 RegC-Adjusted RORAC

Once we obtain the value of $RegC_C$, we can compute RegC-Adjusted risk-return measures such as RORAC using equation (9). Figure 4 presents scatter plots of RegC-Adjusted RORAC against unadjusted RORAC when the target DelR is 15%. There appears to be little correlation between the adjusted and unadjusted measures. This observation is surprising as, in general, we expect positive correlation between the two measures. The reason behind the lack of correlation is that many of the instruments with high RORAC have low PDs (and thus ES as all the instruments are priced close to par) and even smaller RC. Under the regulatory capital requirement, the RegC-Adjusted ES of those instruments become negative. This means that these instruments would have negative-adjusted RORAC in sharp contrast to their large unadjusted RORAC. Intuitively, the regulatory capital requirement for high credit quality names is sufficiently onerous that the adverse impact of their regulatory capital burden makes their effective excess return negative. Figure 5 excludes these instruments and shows a positive correlation between adjusted and unadjusted RORAC among the remaining instruments.

Figure 4 Positive RegC-Adjusted RORAC vs. RORAC

Advanced IRB RWC

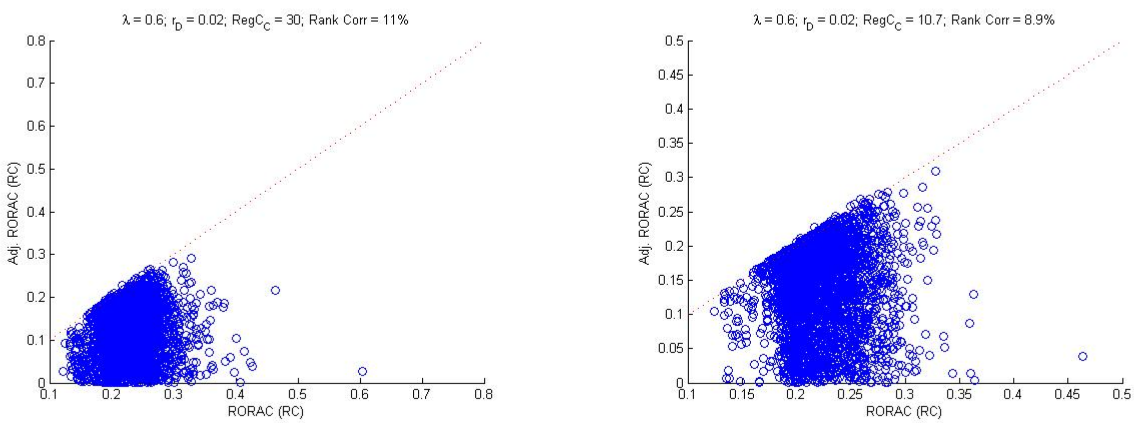
Standardized RWC



This figure shows the instrument RegC-Adjusted RORAC plotted against unadjusted RORAC. In the left plot, the advanced IRB approach under Basel II is used to determine the value of RWC. In the right plot, the standardized approach is used. In both plots, the value of $RegC_{C.}$ is chosen under DelR of 15%. The red dotted reference line is the 45-degree line.

Figure 5 Positive RegC-Adjusted RORAC vs. RORAC

Advanced IRB RWC Standardized RWC

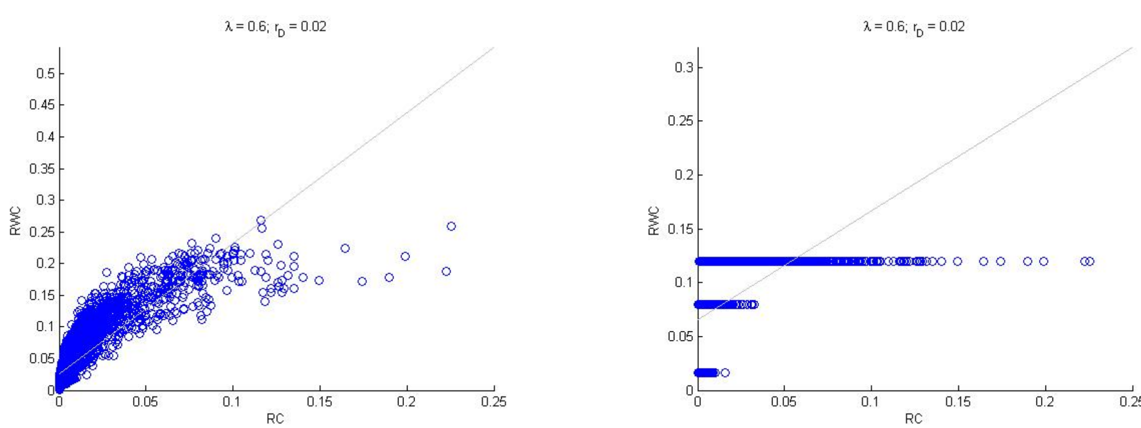


This figure shows instrument RegC-Adjusted RORAC plotted against unadjusted RORAC for positive RegC-Adjusted RORAC instruments. In the left plot, the advanced IRB approach under Basel II is used to determine RWC. In the right plot, the standardized approach is used. In both plots, the value of $RegC_{C.}$ is chosen under a DelR of 15%. The red dotted reference line is the 45-degree line.

The fact that some instruments with low PD but large RORAC have small or even negative RegC-Adjusted RORAC is worth further explanation. As explained previously, the regulatory capital requirement works as an additional cost/penalty on each investment. Since the required capitalization rate RWC does not fully reflect the risk profile of an instrument, it happens that safe instruments (instruments with small RC) tend to be relatively over-penalized when compared to more risky ones in the Basel framework. This phenomenon is illustrated in Figure 6, where we plot the instrument RWC against RC. It appears that, compared to the linear regression fitted values of RWC against RC, RWC in general is inflated for instruments with small RC and deflated for instruments with large RC. Consequently, safer instruments tend to have relatively unattractive RegC-Adjustment RORAC.

Figure 6 RWC vs. RC

Advanced IRB RWC Standardized RWC



This figure shows instrument RWC plotted against RC. In the left plot, the advanced IRB approach under Basel II is used to determine the value of RWC. In the right plot, the standardized approach is used. In both plots, the solid grey reference line is the linear regression fitted line.

4.2 Impact of Regulatory Capital on Investment Decision

Rank ordering instruments according to RORAC is a convenient way to understand which types of investments an organization should focus on. In this sub-section, we study how regulatory capital affects investment decisions by investigating its impact on the rank order of RORAC.

Table 2 presents the top-ten instruments with the highest unadjusted RORAC. Under no regulatory capital requirements, the portfolio's risk-return tradeoff can be improved by increasing investment weight on these instruments. However, it happens to be the case that all of them are relatively "safe" — they have very low PDs and EC. Since safe instruments are relatively over-penalized by the regulatory capital requirement, the rank of these instruments drops significantly according to RegC-Adjusted RORAC. In fact, some of the top instruments become some of the worst performers under the RegC-Adjusted measure. For example, instrument ID 2578 has the second highest unadjusted RORAC (59%) among all 6,000 instruments in the portfolio. However, once we take into consideration of the regulatory capital requirement, the same instrument is ranked last in the portfolio, with a negative 70% RegC-Adjusted RORAC.

Table 2 TOP-TEN INSTRUMENTS UNDER NO REGULATORY CAPITAL REQUIREMENT

INSTRUMENT ID	INDUSTRY CODE	RORAC (UNA)	RORAC (ADV)	RORAC (STD)	RANK (UNA)	RANK (ADV)	RANK (STD)	EDF	RSQ	EC
4765	N45	0.60	0.03	-1.43	1	3929	5878	0.01%	0.32	0.02%
2578	N37	0.59	-0.70	-2.11	2	6000	5963	0.02%	0.10	0.01%
5101	N51	0.56	-0.67	-5.37	3	5999	5999	0.01%	0.11	0.01%
4922	N12	0.49	-0.21	-0.84	4	5785	5605	0.02%	0.18	0.03%
5290	N61	0.46	0.22	0.04	5	118	3179	0.02%	0.55	0.09%
527	N07	0.45	-0.19	-0.89	6	5732	5668	0.02%	0.18	0.03%
4967	N12	0.43	-0.18	-0.64	7	5710	5416	0.02%	0.18	0.04%
5180	N51	0.43	0.04	-0.89	8	3758	5661	0.02%	0.30	0.03%
5809	N58	0.42	0.05	-0.27	9	3640	4757	0.02%	0.32	0.06%
4736	N44	0.42	-0.48	-1.69	10	5996	5928	0.02%	0.10	0.02%

Ten instruments (out of 6,000) with the highest unadjusted RORAC in the IACPM portfolio. Column "RORAC (UNA)" records the value of unadjusted RORAC of these instruments. For comparison, Columns "RORAC (ADV)" and "RORAC (STD)" show respectively the RegC-adjusted RORAC of these instruments under the advanced and standardized Basel II capital requirement. Column "Rank (UNA)" lists the ranking of these instruments in the portfolio by unadjusted RORAC. For comparison, Columns "Rank (ADV)" and "Rank (STD)" report the ranking under the RegC-Adjusted advanced and standardized Basel II RORAC respectively. The value of $RegC_c$ is chosen under DelR of 15%.

Table 3 and Table 4 list the top-ten instruments by RegC-Adjusted RORAC in the IACPM portfolio when the target DelR is 15%. In the two tables, instrument RWC's are determined by the advanced IRB approach and standardized approach, respectively. While the top-ten instruments with highest unadjusted RORAC fare very poorly once the capital requirement is imposed, the top-ten instruments under the regulatory capital requirement appear to have relatively attractive risk-return tradeoff, even if the capital requirement is removed. All of these instruments' unadjusted RORAC are in the top quintile of the portfolio with most of them in the top decile.

Table 3 TOP-TEN INSTRUMENTS UNDER ADVANCED IRB BASEL II CAPITAL REQUIREMENT

INSTRUMENT ID	INDUSTRY CODE	RORAC (UNA)	RORAC (ADV)	RORAC (STD)	RANK (UNA)	RANK (ADV)	RANK (STD)	EDF	RSQ	EC
2038	N53	0.33	0.29	0.31	104	1	1	0.18%	0.26	4.75%
1802	N53	0.30	0.28	0.28	265	2	4	0.18%	0.35	4.77%
2085	N02	0.32	0.27	0.29	151	3	2	0.18%	0.29	4.13%
535	N07	0.26	0.27	0.26	927	4	16	0.18%	0.59	9.67%
1455	N40	0.26	0.26	0.27	1016	5	8	0.02%	0.47	3.89%
496	N07	0.28	0.26	0.26	486	6	18	0.18%	0.43	5.18%
1848	N02	0.31	0.26	0.24	158	7	78	0.18%	0.41	2.17%
5030	N11	0.32	0.26	0.25	131	8	25	1.27%	0.62	3.97%
285	N47	0.27	0.26	0.27	786	9	9	0.02%	0.34	2.07%
5184	N61	0.28	0.26	0.28	587	10	5	0.02%	0.29	1.74%

Ten instruments (out of 6,000) with the highest RegC-Adjusted RORAC in the IACPM portfolio assuming the advanced approach under Basel II is adopted to determine RWC. The values of corresponding RegC-Adjusted RORAC are recorded in Column "RORAC(ADV)". For comparison, Columns "RORAC (UNA)" and "RORAC (STD)" show respectively the unadjusted RORAC and the RegC-Adjusted RORAC under the standardized Basel II capital requirement. Column "Rank (ADV)" lists the ranking of these instruments in the portfolio by RegC-Adjusted RORAC under the advanced Basel II capital requirement. For comparison, Columns "Rank (UNA)" and "Rank (STD)" report the ranking under unadjusted and the RegC-Adjusted standardized Basel II RORAC respectively. In all cases, the risk-free rate is set to be 2%, and λ is 0.6. Whenever applicable, the value of $RegC_C$ is chosen such that the model implied DelR is 15%.

Table 4 TOP-TEN INSTRUMENTS UNDER STANDARDIZED BASEL II CAPITAL REQUIREMENT

INSTRUMENT ID	INDUSTRY CODE	RORAC (UNA)	RORAC (ADV)	RORAC (STD)	RANK (UNA)	RANK (ADV)	RANK (STD)	EDF	RSQ	EC
2038	N53	0.33	0.29	0.31	104	1	1	0.18%	0.26	4.75%
2085	N02	0.32	0.27	0.29	151	3	2	0.18%	0.29	4.13%
2556	N37	0.28	0.25	0.28	439	16	3	25.50%	0.10	19.55%
1802	N53	0.30	0.28	0.28	265	2	4	0.18%	0.35	4.77%
5184	N61	0.28	0.26	0.28	587	10	5	0.02%	0.29	1.74%
2555	N37	0.28	0.25	0.27	528	19	6	25.50%	0.10	21.88%
4817	N12	0.28	0.22	0.27	462	87	7	0.02%	0.18	1.14%
1455	N40	0.26	0.26	0.27	1016	5	8	0.02%	0.47	3.89%
285	N47	0.27	0.26	0.27	786	9	9	0.02%	0.34	2.07%
5028	N49	0.26	0.26	0.26	936	11	10	0.02%	0.52	2.69%

The ten instruments (out of 6,000) with the highest RegC-Adjusted RORAC in the IACPM portfolio assuming the standardized approach under Basel II is adopted to determine RWC. The values of corresponding RegC-Adjusted RORAC are recorded in Column "RORAC(STD)". For comparison, Columns "RORAC (UNA)" and "RORAC (ADV)" show respectively the unadjusted RORAC and the RegC-Adjusted RORAC under the advanced Basel II capital requirement. Column "Rank (STD)" lists the ranking of these instruments in the portfolio by RegC-Adjusted RORAC under the standardized Basel II capital requirement. For comparison, Columns "Rank (UNA)" and "Rank (ADV)" report the ranking under unadjusted and RegC-Adjusted RORAC advanced Basel IIRORAC respectively. In all cases, the risk-free rate is set to be 2%, and λ is 0.6. Whenever applicable, the value of $RegC_C$ is chosen such that the model implied DelR is 15%.

In determining the RegC-Adjusted RORAC in the analysis above, we calibrate the value of $RegC_C$ under DelR of 15%. We find that either reducing by half or doubling the target DelR to 8% or 30% does not change the ranking of instrument RegC-Adjusted RORAC substantially. Table 5 and Table 6 show the Spearman's rank correlations of instrument RegC-Adjusted RORAC under different target DelRs. If the advanced IRB approach is used to compute RWC, the RegC-Adjusted RORAC under 15% target DelR has a 96% rank correlation with that under 8% target DelR. The corresponding correlation is 98% between the cases of 15% and 30% target DelRs. Also, no matter which approach is used to determine RWC, the correlation between RegC-Adjusted RORAC and unadjusted RORAC (i.e. target DelR is 0%) is small if not negative. This correlation also decreases as the target DelR increases as expected. Similar phenomenon can be observed if the RWC is calculated under the standardized approach.

Table 5 SPEARMAN'S RANK CORRELATION MATRIX OF INSTRUMENT RORAC (STANDARDIZED BASEL II)

DELR	0	0.08	0.15	0.3
0	100%	27%	2%	-14%
0.08	27%	100%	96%	89%
0.15	2%	96%	100%	98%
0.3	-14%	89%	98%	100%

Spearman's rank correlation matrix of instrument RegC-adjusted RORAC under different target DelR: 0%, 8%, 15%, and 30%. Note that setting DelR to 0% is the equivalent to assuming no RegC constraint. In this case, unadjusted RORACs are used. In cases where a positive target DelR is used, the advanced IRB approach under Basel II is adopted to determine RWC.

Table 6 SPEARMAN'S RANK CORRELATION MATRIX OF INSTRUMENT RORAC (STANDARDIZED BASEL II)

DELR	0	0.08	0.15	0.3
0	100%	10%	-3%	-11%
0.08	10%	100%	98%	96%
0.15	-3%	98%	100%	99%
0.3	-11%	96%	99%	100%

Spearman's rank correlation matrix of instrument RegC-adjusted RORAC under different target DelR: 0%, 8%, 15%, and 30%. Note that setting DelR to 0% is equivalent to assuming no RegC constraint. In this case, unadjusted RORACs are used. In cases where a positive target DelR is used, the standardized approach under Basel II is adopted to determine RWC.

5. Conclusion

This paper examines how investment decisions should be made in the presence of the regulatory capital requirement based on the model introduced by Levy, Kaplin, Meng, and Zhang (2012). We propose a method to calibrate $RegC_C$, the latent parameter that specifies the extent to which the regulatory capital constraint is binding in the LKMZ model. The method is based on a framework that links $RegC_C$ to the leverage of a bank. We then use historical leverage data of U.S. banks to calibrate $RegC_C$ for a sample portfolio and investigate how the regulatory capital requirement affects investment decisions. We find that the relative attractiveness of each instrument measured in terms of risk-return tradeoff changes considerably once the regulatory capital requirement is imposed.

Appendix A Description of the Sample Portfolio – IACPM Portfolio

The $100 billion test portfolio is comprised of two floating rate term loans, to each of 3,000 obligors, across a diverse set of industries (643 NAICS codes, 60 industry sectors by Moody's definition), and seven countries dispersed along eight whole-grade rating buckets and varying LGD values. Exposure amounts vary from $1MM to $1.250MM, and tenors ranged from six months to seven years. "R-squares" (the degree to which obligors exhibit systematic vs. idiosyncratic risk) varied from 10% to 65%. Contractual spreads over a risk-free rate are chosen so that the mark-to-market value of the exposures at time zero, relative to specified required market spreads, would be approximately par. General characteristics of the portfolio are summarized below

Portfolio Size

Exposures: 6,000

Portfolio Size: $100 Billion

Obligors

Number of Obligors: 3,000

Rating Scheme: eight ratings buckets

Credit rating: Average = BBB

Industry Classifications: 60 Moody's Analytics industries, *643 NAICS Codes (6 digit)*

Countries: seven countries

Facilities

Facility Type: 100% Term Loans

Fixed vs. Floating: 100% Floating Rate

Exposure Distribution by Facility

Mean: $16.7 million

Standard Deviation: $101.7 million

Minimum: $1 million

Maximum: $1.250 million

Tenor Distribution by Facility

Mean: 2.5 years

Standard Deviation: 1.7 years

Minimum: six months

Maximum: seven years

Appendix B Determining RWC

According to the Basel Committee on Banking Supervision (2006), RWC is usually dependent on the instrument's risk profile, such as asset class and rating. For example, there are two common methods for computing RWC for each instrument under Basel II: the standardized approach and the advanced Internal Rating Based (IRB) approach.[5] With the standardized approach, the RWC is determined through a mapping table with instrument rating from External Credit Rating Agencies. Different mapping tables are used for different asset classes. Table 7 lists such a mapping table for three asset classes.

Table 7 RWC MAPPING TABLE UNDER BASEL II STANDARDIZED APPROACH

RATING	EDF LOWER BOUND (%)	EDF UPPER BOUND (%)	RWC FOR CORPORATES	RWC FOR BANKS	RWC FOR RETAILS
AAA	0.0100	0.0126	0.016	0.016	0.060
AA+	0.0126	0.0178	0.016	0.016	0.060
AA	0.0178	0.0214	0.016	0.016	0.060
AA-	0.0214	0.0245	0.016	0.016	0.060
A+	0.0245	0.0280	0.040	0.040	0.060
A	0.0280	0.0322	0.040	0.040	0.060
A-	0.0322	0.0370	0.040	0.040	0.060
BBB+	0.0370	0.0425	0.080	0.040	0.060
BBB	0.0425	0.0519	0.080	0.040	0.060
BBB-	0.0519	0.0673	0.080	0.040	0.060
BB+	0.0673	0.0871	0.080	0.080	0.060
BB	0.0871	0.1307	0.080	0.080	0.060
BB-	0.1307	0.2271	0.080	0.080	0.060
B+	0.2271	0.3946	0.120	0.080	0.060
B	0.3946	0.8655	0.120	0.080	0.060
B-	0.8655	2.3963	0.120	0.080	0.060
CCC+	2.3963	6.6347	0.120	0.120	0.060
CCC	6.6347	11.7708	0.120	0.120	0.060
CCC-	11.7708	14.2667	0.120	0.120	0.060
CC	14.2667	19.6570	0.120	0.120	0.060
C	19.6570	35.0000	0.120	0.120	0.060
D	35.0000	35.0000	0.120	0.120	0.060

This table reports the mapping table used in the Basel II standardized approach to determine the RWC under each rating category for three asset classes: Corporates, Banks, and Retails. The range of EDF associated with each rating is the actual range specified by Moody's as of December 2014. The EDF values are provided here for reference only, as they do not affect the actual value of RWC once the instrument rating is known under the standardized approach.

Under the advanced IRB approach, the banks use internal estimates of PD, EAD, and downturn LGD to compute RWC with the formula

$$RWC = \left[LGD \times N\left[(1-R)^{-0.5} \times G(PD) + \left(\frac{R}{1-R}\right)^{0.5} \times G(0.999)\right] - PD \times LGD\right] \times \frac{1+(M-2.5) \times b(PD)}{1-1.5 \times b(PD)} \quad (17)$$

where R measures the asset correlation of the instrument with the systemic factor and is a function of PD, the asset class, and size. The maturity M is capped by five years. Specifically, for corporate exposures

[5] In total, there are three approaches specified by Basel II to calculate RWA. In addition to the two approaches listed in the main text, the third approach is the Foundation IRB approach. It is very similar to the advanced IRB approach with the main distinction that certain input parameters such as LGD should be prescribed by the regulator rather than estimated by the bank itself. Because of this similarity, we omit the Foundation IRB approach from our discussion.

$$R = 0.12 \times \frac{1-\exp(-50 \times PD)}{1-\exp(-50)} + 0.24 \times \left(1 - \frac{1-\exp(-50 \times PD)}{1-\exp(-50)}\right) - 0.04 \times \left(1 - \frac{S-5}{45}\right) \quad (18)$$

where the third term in equation (18) is the size adjustment with size bounded between 5 and 50 (million euro).
For large corporate, the size adjustment is 0, and the value of R is bounded between 0.12 and 0.24. For retail exposures (excluding residential mortgages and qualifying revolving retail exposures),[6]

$$R = 0.03 \times \frac{1-\exp(-35 \times PD)}{1-\exp(-35)} + 0.16 \times \left(1 - \frac{1-\exp(-35 \times PD)}{1-\exp(-35)}\right) \quad (19)$$

The function $b(\cdot)$ is used in the maturity adjustment and has the form:

$$b(PD) = (0.11852 - 0.05478 \times \log(PD))^2 \quad (20)$$

If we use Moody's rating to PD mapping as of December 2014, compared to the standardized approach, the advanced IRB approach appears to assign relatively lower RWC to instruments with a good rating but relatively higher RWC to instruments with a poor rating. Table 8 illustrates this finding, where we compare the RWC values for large corporates under the two different Basel II approaches.

Table 8 COMPARISON OF RWC FOR LARGE CORPORATES UNDER STANDARDIZED AND ADVANCED IRB APPROACH

RATING	EDF LOWER BOUND (%)	EDF UPPER BOUND (%)	STD RWC	ADV RWC (1 YEAR)	ADV RWC (3 YEAR)	ADV RWC (5 YEAR)
AAA	0.0100	0.0126	0.016	0.004	0.011	0.018
AA+	0.0126	0.0178	0.016	0.005	0.013	0.021
AA	0.0178	0.0214	0.016	0.006	0.015	0.023
AA-	0.0214	0.0245	0.016	0.007	0.016	0.025
A+	0.0245	0.0280	0.040	0.008	0.017	0.027
A	0.0280	0.0322	0.040	0.009	0.019	0.029
A-	0.0322	0.0370	0.040	0.010	0.020	0.031
BBB+	0.0370	0.0425	0.080	0.011	0.022	0.033
BBB	0.0425	0.0519	0.080	0.012	0.024	0.037
BBB-	0.0519	0.0673	0.080	0.015	0.028	0.042
BB+	0.0673	0.0871	0.080	0.018	0.033	0.048
BB	0.0871	0.1307	0.080	0.024	0.041	0.058
BB-	0.1307	0.2271	0.080	0.035	0.055	0.076
B+	0.2271	0.3946	0.120	0.049	0.072	0.096
B	0.3946	0.8655	0.120	0.073	0.100	0.127
B-	0.8655	2.3963	0.120	0.109	0.136	0.162
CCC+	2.3963	6.6347	0.120	0.158	0.183	0.208
CCC	6.6347	11.7708	0.120	0.200	0.224	0.249
CCC-	11.7708	14.2667	0.120	0.215	0.239	0.262
CC	14.2667	19.6570	0.120	0.237	0.259	0.280
C	19.6570	35.0000	0.120	0.250	0.267	0.283
D	35.0000	35.0000	0.120	0.250	0.267	0.283

In this table, we compare the RWC of large corporates under two different Basel II approaches: the standardized approach and the advanced IRB approach. Column STD RWC shows the standardized RWC value for each rating categories. Columns ADV RWC (n Year) show the RWC under the advanced IRB approach for instruments with n-year maturity. The upper bound of EDF level is used as the PD input in the calculation of ADV RWC for each rating category. The downturn LGD is set to be 0.6 in all cases. The range of EDF levels associated with each rating is the actual range specified by Moody's as of December 2014.

[6] We set the value of R as a constant at 0.15 for residential mortgages and 0.04 for qualifying revolving retail exposures.

Appendix C Proof of Theorems

PROOF OF THEOREM 1

The investor aims to maximize the utility function:

$$U_t = E_t(R_{t+1} - b\sigma_P^2) \quad (21)$$

where R_{t+1} and σ_P^2 measure the net return and constant variance of the overall portfolio. Parameter b measures the investor's risk-aversion level. It follows that

$$R_{t+1} = (1 - L_t)r_D + L_t r_{P,t+1} \quad (22)$$

where $L_t r_{P,t+1}$is the total return on assets and $(1 - L_t)r_D$ is the total cost of borrowing. The variance of the investor's portfolio is

$$\sigma_R^2 = L_t^2\sigma_P^2 \quad (23)$$

Substitute equations (22) and (23) into the utility function, we have

$$U_t = (1 - L_t)r_D + L_t E_t(r_{P,t+1}) - bL_t^2\sigma_P^2 \quad (24)$$

The determine the optimal L_t, we solve the FOC

$$\frac{\partial U_t}{\partial L_t} = -r_D + E_t(r_{P,t+1}) - 2bL_t = 0 \quad (25)$$

The solution is

$$L_t^* = \frac{E_t(r_{P,t+1} - r_D)}{2b\sigma_P^2} = \frac{ES_{P,H}}{2b\sigma_P^2} \quad (26)$$

This implies that the optimal leverage under the regulatory capital constraint is

$$\widehat{L_t^*} = \frac{\widehat{ES_{P,H}}}{2b\sigma_P^2} = \frac{ES_{P,H} - r_D(f_{P,t} - 1)}{2b\sigma_P^2} \quad (27)$$

Note, DelR is defined as

$$DelR := 1 - \frac{\widehat{L}_t^*}{L_t^*} = 1 - \frac{ES_{P,H} - r_D(f_{P,t} - 1)}{ES_{P,H}} = \frac{r_D(f_{P,t} - 1)}{ES_{P,H}} \quad (28)$$

Rearrange this equation and solve $f_{P,t}$, we have

$$f_{P,t} = DelR \cdot \frac{ES_{P,t}}{r_D} + 1 \quad (29)$$

Substitute the value of $f_{P,t}$ with equation (13), we have

$$\frac{1 - \left(1 - (1 - RWC_P)^{RegC_C} \cdot \frac{(1 - RWC_P)}{P_{P,t}}\right)}{(1 - RWC_P)^{RegC_C}} = DelR \cdot \frac{ES_{P,t}}{r_D} + 1 \quad (30)$$

Define $X = (1 - RWC_P)^{RegC_C}$ and substitute it into equation (30), we have

$$\frac{1 - \left(1 - X \cdot \frac{(1 - RWC_P)}{P_{P,t}}\right)}{X} = DelR \cdot \frac{ES_{P,t}}{r_D} + 1 \quad (31)$$

Solve for X, we have

$$X = \frac{P_{P,t} - 1 + RWC_P}{P_{P,t} \cdot DelR \cdot \frac{ES_{P,t}}{r_D} + P_{P,t} - 1 + RWC_P} \quad (32)$$

Plug the expression of X back into equation (32), we have

$$(1 - RWC_P)^{RegC_C} = \frac{P_{P,t} - 1 + RWC_P}{P_{P,t} \cdot DelR \cdot \frac{ES_{P,t}}{r_D} + P_{P,t} - 1 + RWC_P} \quad (33)$$

Solve for $RegC_C$, we have

$$RegC_C = \frac{\ln\left(P_{P,t} - 1 + RWC_{P,t}\right) - \ln\left(P_{P,t} \cdot DelR \cdot \frac{ES_{P,t}}{r_D} + P_{P,t} - 1 + RWC_{P,t}\right)}{\ln\left(1 - RWC_{P,t}\right)} \quad (34)$$

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