MOODY'S ANALYTICS

Better Faster Decisions

Analytical Solutions to Multi-period Credit Portfolio Management: *A Macroeconomic Approach*

Juan M. Licari, Ph.D., Managing Director juan.licari@moodys.com

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Analytical Solutions to Multi-period Credit Portfolio Mgmt.

THE PROBLEM WE ARE SOLVING

Overcome the challenge of analyzing credit risk dynamically (multi-period) and integrate stress testing (credit risk, PPNR, provisions, mark-to-market) into a unified management framework.

MOTIVATION

Federal Reserve Bank of New York - Staff Report No. 696 – Nov 2014
Supervisory Stress Tests by Beverly Hirtle & Andreas Lehnert.

"The Basel Market Risk Amendment – finalized in 1995 – contained a provision encouraging the use of stress tests to augment the Value at Risk (VaR) measures of computing risk-weighted assets, the denominators in various measures of risk- based capital. VaR models consider the probability distribution of the value of a portfolio of assets. In principle VaR models can be thought of as the result of thousands of individual scenarios, weighted by their probability. In practice however the distributions are not tied to real-world variables other than the observed empirical distributions of the values of various assets."

Analytical Solutions to Multi-period Credit Portfolio Mgmt.

WHY IS THIS RELEVANT FOR BANKS?

(i) embed stress testing process into the banks' credit management practices (forecasting, provisioning, profitability, risk-based pricing, risk concentration, capital adequacy), (ii) dynamic risk and profitability assessment, (iii) optimal capital management, (iv) risk appetite.

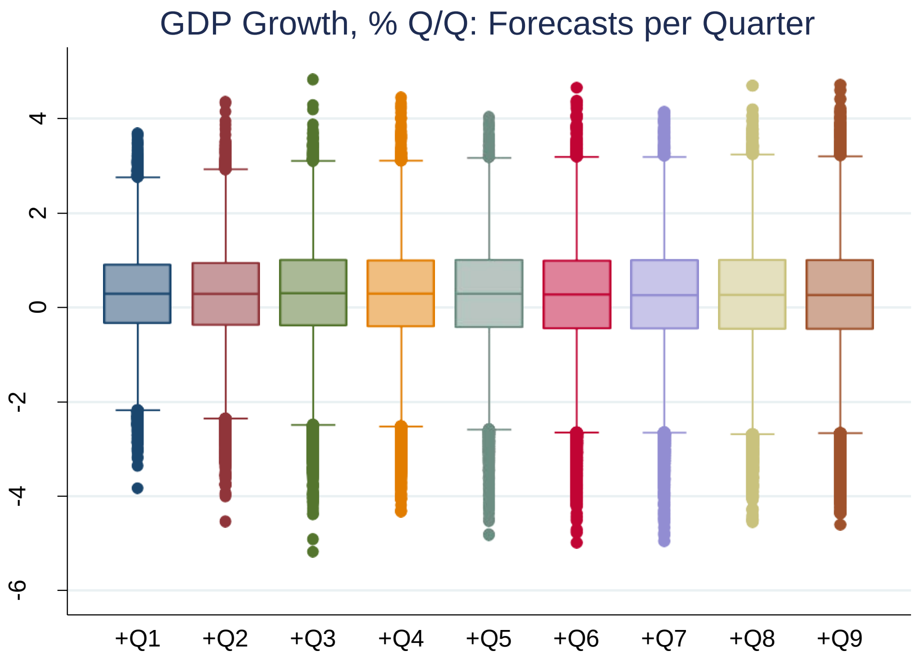
SPECIFICS OF WHAT WE DO

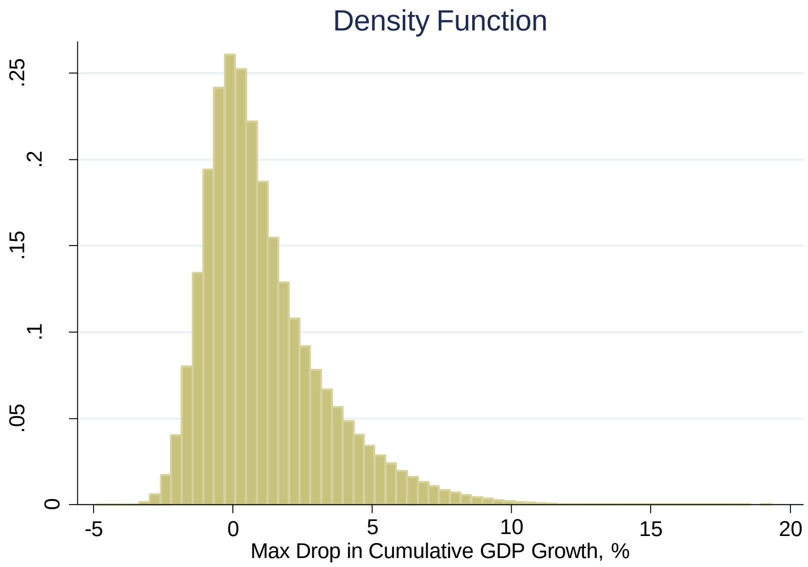
Compute analytical solutions to multi-period credit risk management.

1. Dynamic simulations of macro scenarios and probabilities for each path.
2. Multi-period risk parameter simulations (leveraging stress testing models) for conditional credit and mark-to-market parameter realizations.
3. Analytical calculation of (intra-period and cumulative) expected and unexpected losses, and the corresponding asset contributions.
4. Scenario-specific analysis: embedding ST into the framework.
5. Further applications: risk-based pricing, concentration and tail-risk analysis, analytical reverse stress testing, dynamic optimization, integrated mark-to-market and default risk assessment, enhanced Monte Carlo methods.

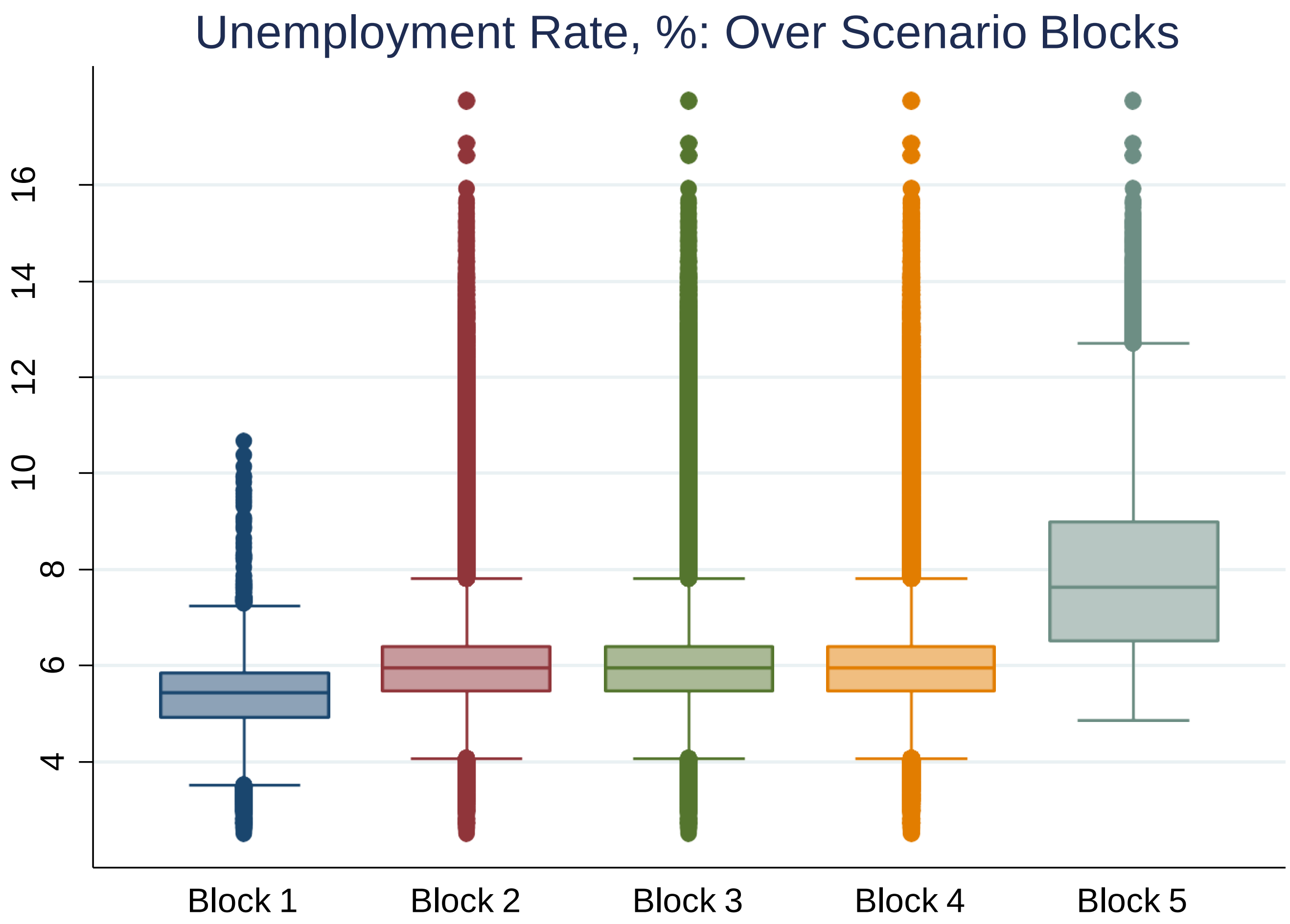
1 Dynamic Simulations of Macro Scenarios and Probabilities for Each Path

Severity of Scenarios



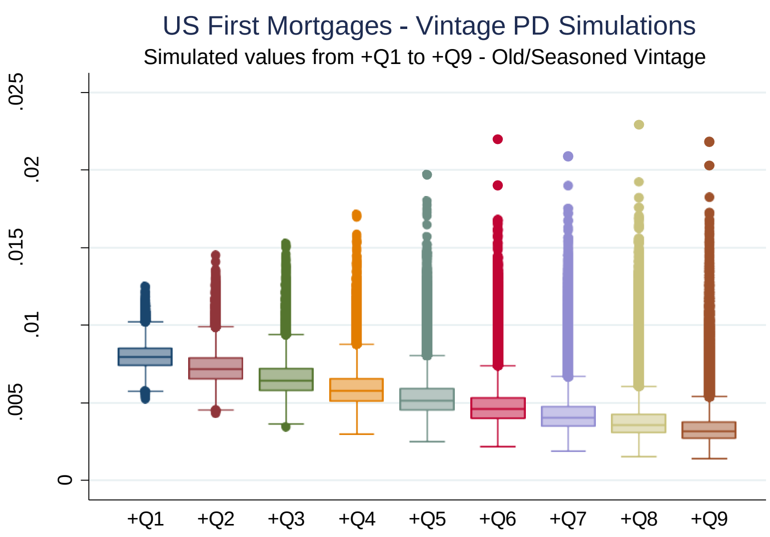


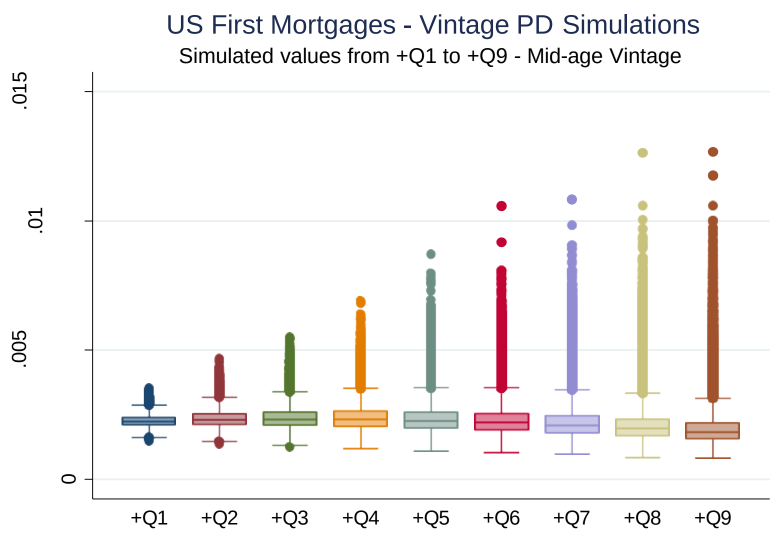
Severity of Scenarios (cont.)

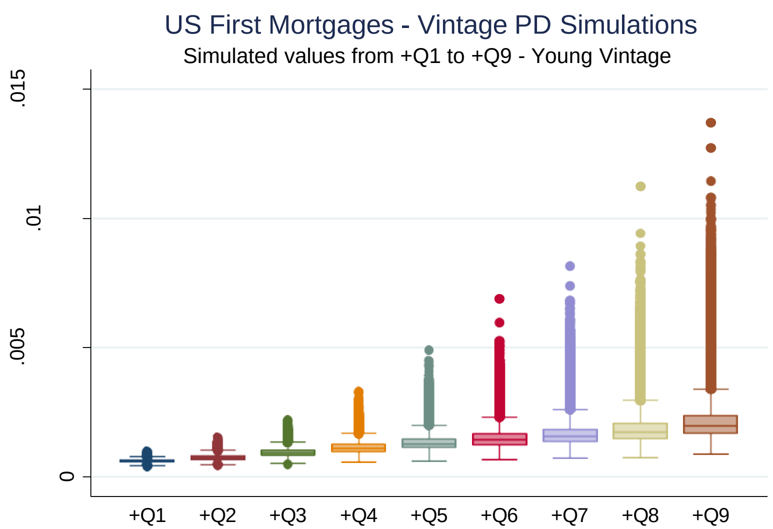


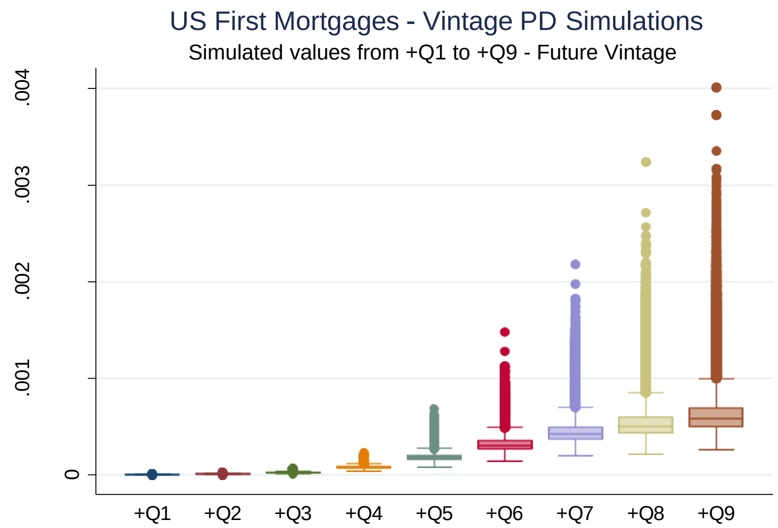
2 Dynamic Simulations of Credit Risk Parameters (Retail Credit as a Leading Example)

US First Mortgage Portfolio – Quarterly Vintages







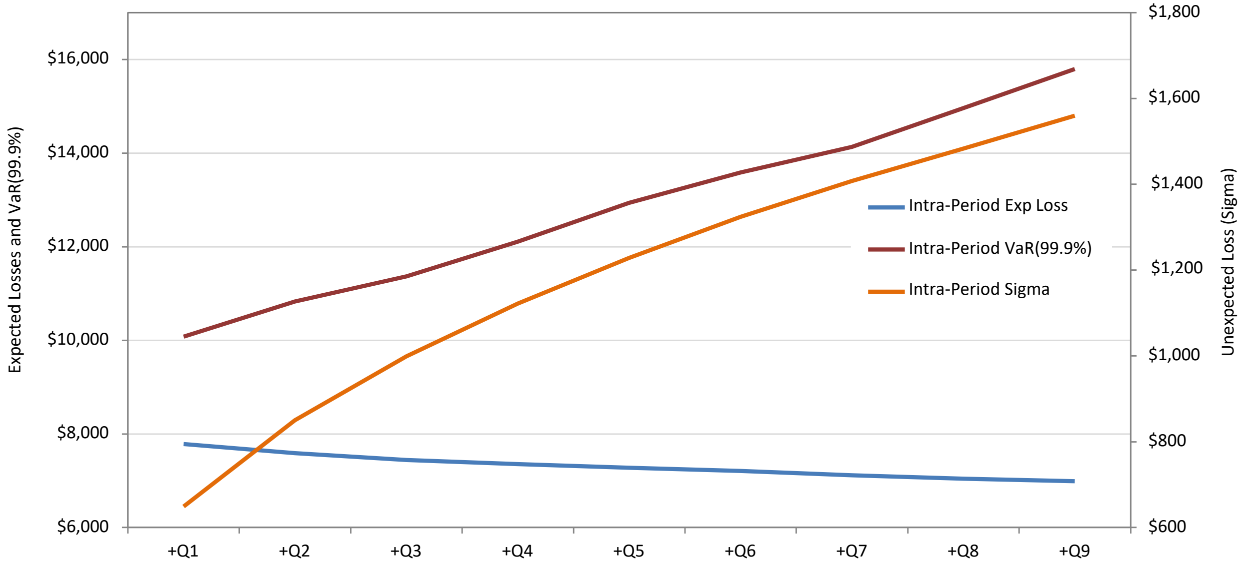


3 Analytical Calculation Of (Intra-period and Cumulative) Expected and Unexpected Losses, Asset Contributions

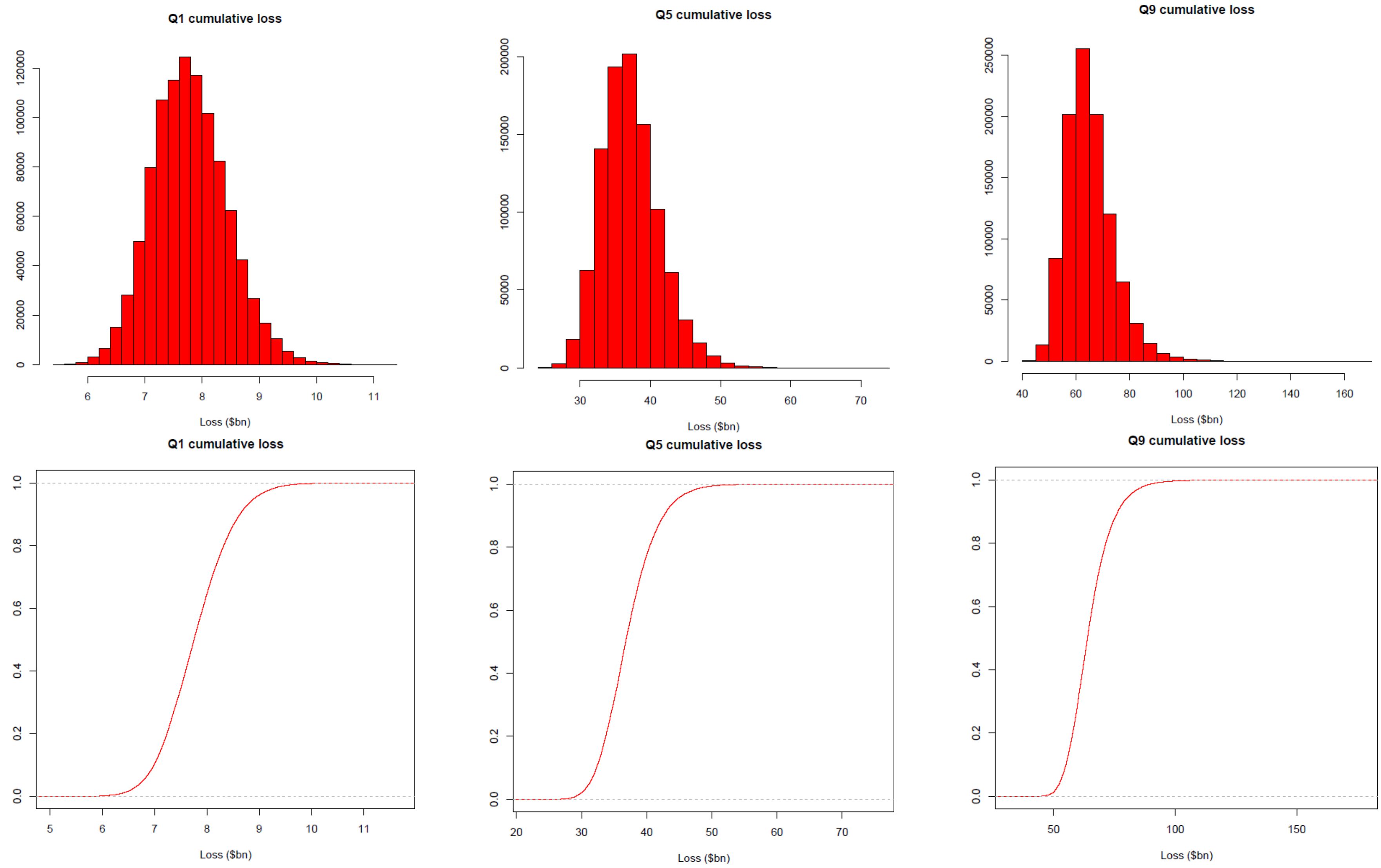
US First Mortgage Portfolio – Loss Metrics

	Intra-Period Exp Loss	Intra-Period Sigma	Intra-Period VaR(99.9%)	Cumulative Exp Loss	Cumulative Sigma	Cumulative VaR(99.9%)
+Q1	$ 7,784.9558	$ 648.5309	$ 10,078.3132	$ 7,784.9558	$ 648.5309	$ 10,078.3132
+Q2	$ 7,587.3311	$ 850.0719	$ 10,828.4443	$ 15,372.2869	$ 1,364.6327	$ 20,248.9755
+Q3	$ 7,442.0702	$ 999.0752	$ 11,365.3219	$ 22,814.3572	$ 2,193.1532	$ 30,983.0449
+Q4	$ 7,354.4725	$ 1,121.8841	$ 12,104.1981	$ 30,168.8297	$ 3,118.9971	$ 42,307.3145
+Q5	$ 7,274.9933	$ 1,228.1673	$ 12,933.1039	$ 37,443.8230	$ 4,124.4656	$ 54,375.8396
+Q6	$ 7,206.7848	$ 1,323.8555	$ 13,583.7713	$ 44,650.6077	$ 5,207.1490	$ 66,748.3775
+Q7	$ 7,113.0817	$ 1,407.5700	$ 14,128.6149	$ 51,763.6895	$ 6,352.9917	$ 79,727.7836
+Q8	$ 7,041.5606	$ 1,483.2394	$ 14,961.4969	$ 58,805.2500	$ 7,553.4249	$ 93,484.9177
+Q9	$ 6,989.0686	$ 1,559.7431	$ 15,795.7796	$ 65,794.3186	$ 8,805.0694	$ 107,915.7781

Intra-Period Expected Losses, σ and VaR(99.9%) (Millions of USD)



US First Mortgage Portfolio – Loss Metrics



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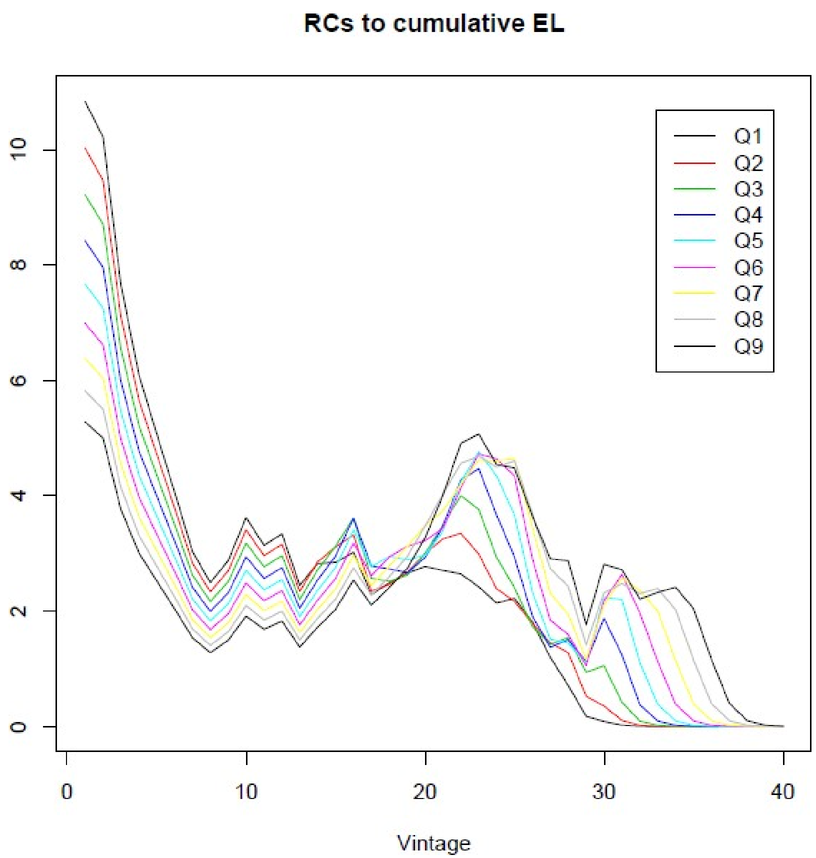
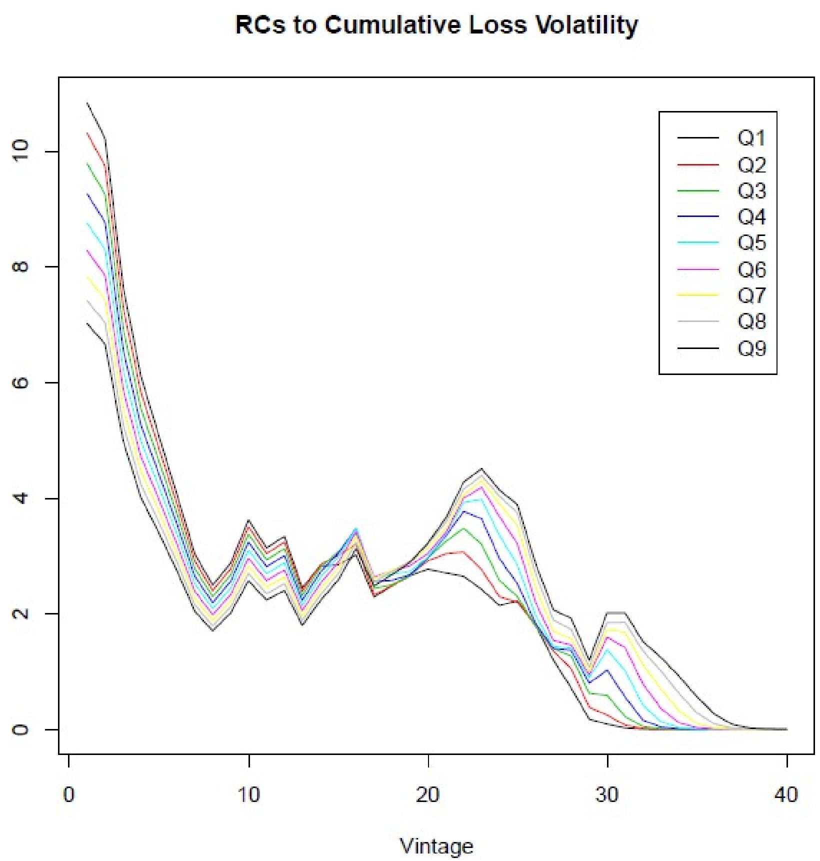
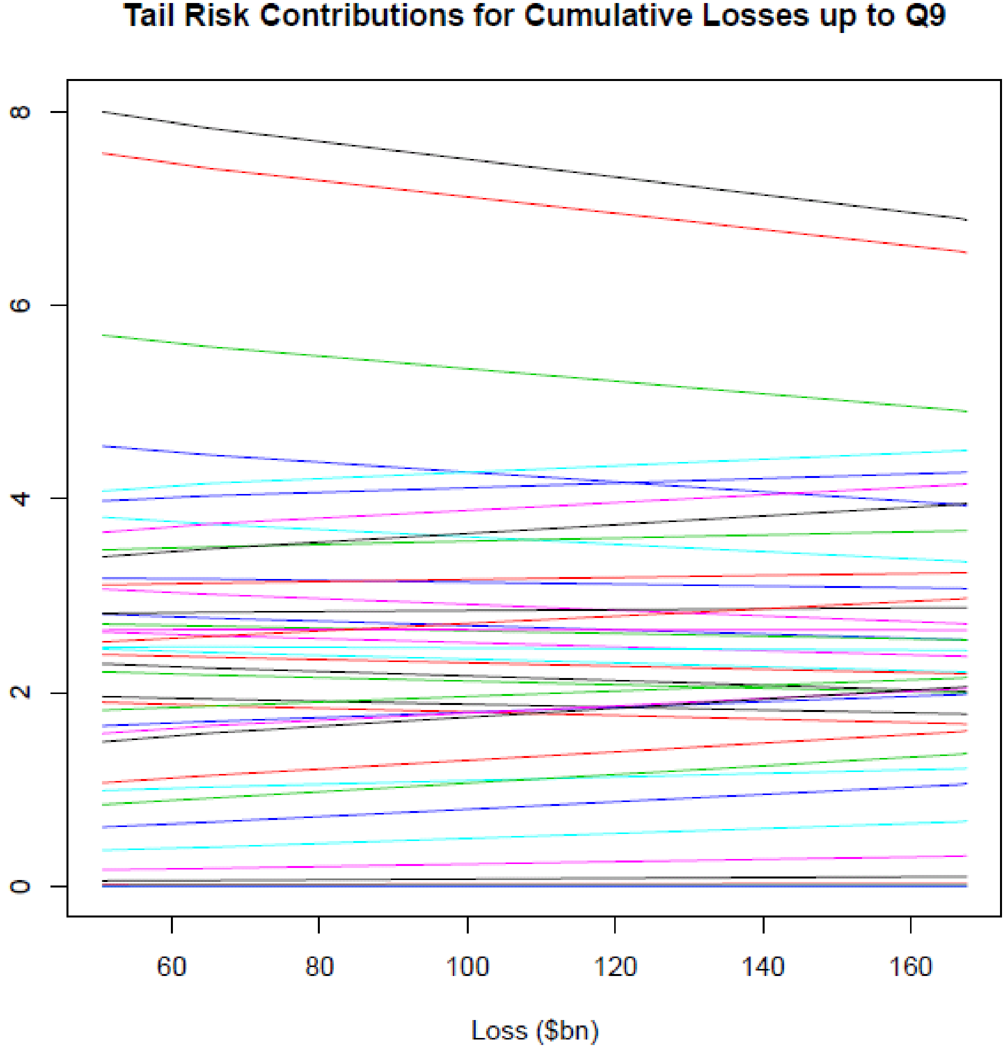




US First Mortgages – Tail-Risk Contributions



4 Scenario-specific Analysis: Embedding ST Into Dynamic Portfolio Analysis

US First Mortgages – Cumulative Scenario Losses

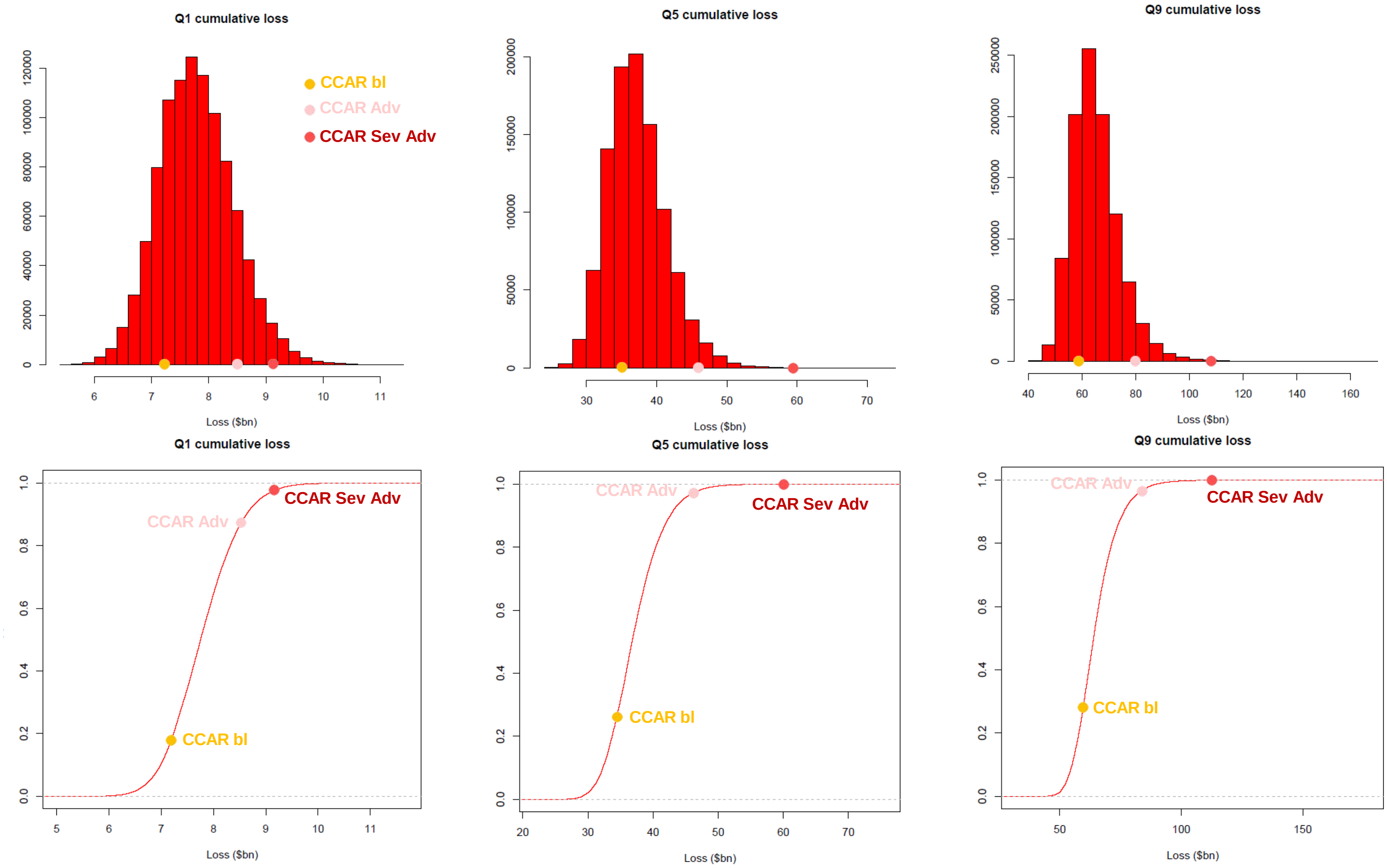
Scenario-Specific Cumulative Losses over time (MCC output)

	CCAR Baseline	CCAR Adverse	CCAR Severely Adverse	ECCA's Baseline	ECCA's s1	ECCA's s2	ECCA's s3	ECCA's s4	ECCA's s5	ECCA's s6
+Q1	$ 7,232.9020	$ 8,498.6685	$ 9,173.4777	$ 7,355.2682	$ 7,355.6346	$ 7,355.6346	$ 7,355.6346	$ 7,355.6346	$ 7,355.6346	$ 7,355.5408
+Q2	$ 14,257.5338	$ 17,940.8468	$ 20,399.0835	$ 14,300.3248	$ 14,199.2785	$ 14,793.8070	$ 16,178.1381	$ 16,609.8792	$ 14,721.8985	$ 14,749.7852
+Q3	$ 21,091.9610	$ 27,563.2185	$ 32,994.9114	$ 20,960.1226	$ 20,464.3961	$ 22,081.8591	$ 26,916.6141	$ 28,335.3669	$ 21,865.6050	$ 22,007.2487
+Q4	$ 27,816.9351	$ 37,047.0213	$ 46,296.8187	$ 27,437.8960	$ 26,544.2066	$ 29,248.4545	$ 38,484.9117	$ 41,403.1589	$ 28,975.0811	$ 30,088.6691
+Q5	$ 34,406.5024	$ 46,157.3768	$ 59,486.5298	$ 33,796.6555	$ 32,525.2174	$ 36,293.6484	$ 49,833.8217	$ 54,890.3760	$ 35,995.7029	$ 39,871.8138
+Q6	$ 40,858.0799	$ 54,983.3008	$ 71,969.6092	$ 40,029.5324	$ 38,479.7494	$ 43,129.8369	$ 60,706.4973	$ 68,013.2971	$ 42,893.2674	$ 50,664.9719
+Q7	$ 47,216.9322	$ 63,658.3171	$ 84,099.8995	$ 46,270.8379	$ 44,358.1512	$ 49,707.3865	$ 70,766.6219	$ 80,577.3767	$ 49,723.6893	$ 61,452.2176
+Q8	$ 53,421.8923	$ 72,154.3765	$ 95,528.2296	$ 52,484.7535	$ 50,315.5577	$ 56,019.5191	$ 80,329.6951	$ 92,670.4991	$ 56,512.7693	$ 72,040.5363
+Q9	$ 59,602.6835	$ 80,626.3743	$ 106,408.0386	$ 58,606.4104	$ 56,232.3471	$ 62,215.5086	$ 89,370.3160	$ 104,633.0555	$ 63,265.6943	$ 82,196.9864

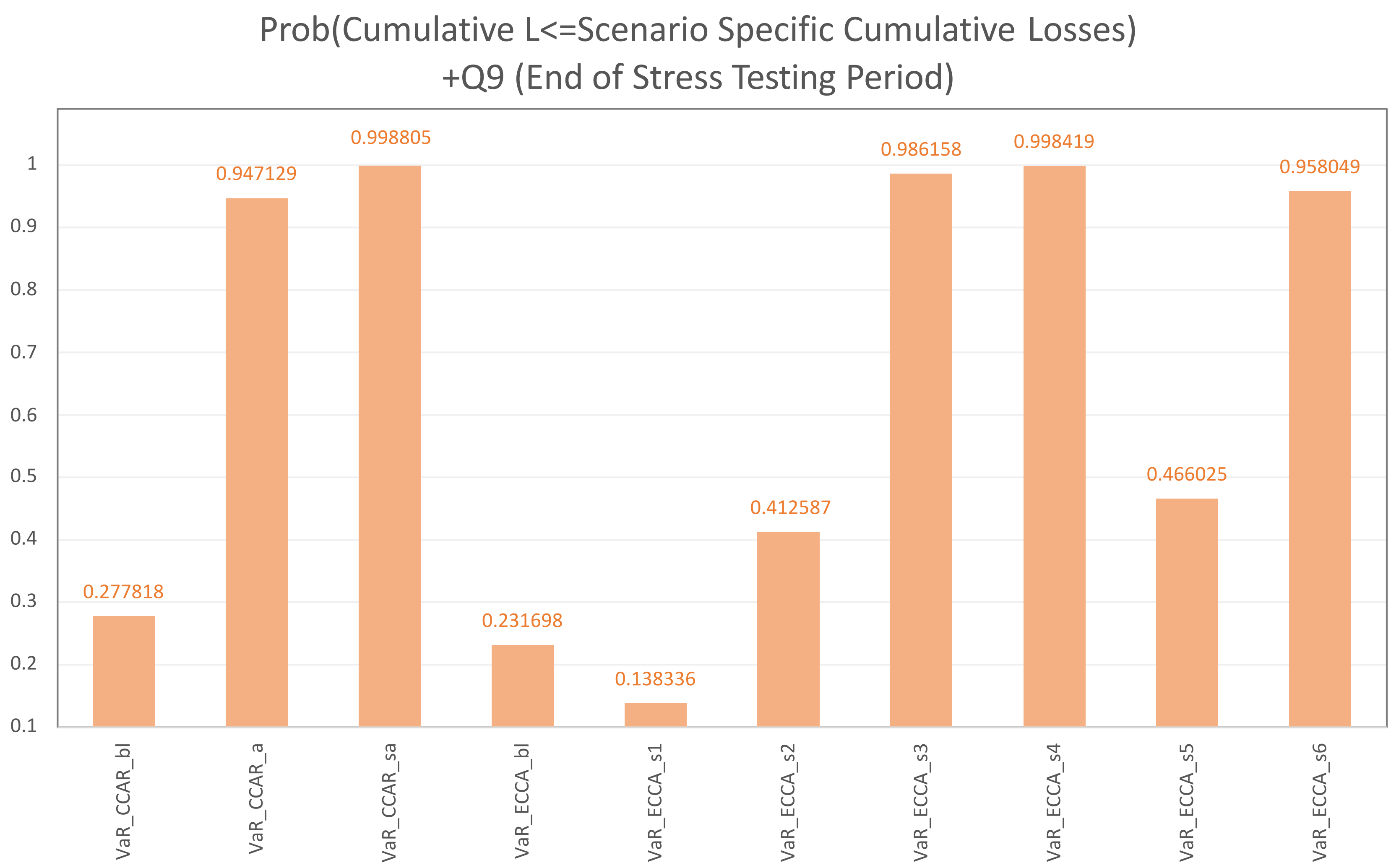
Prob(Cumulative Losses <= Scenario-Specific Cumulative Losses)

	CCAR Baseline	CCAR Adverse	CCAR Severely Adverse	ECCA's Baseline	ECCA's s1	ECCA's s2	ECCA's s3	ECCA's s4	ECCA's s5	ECCA's s6
+Q1	0.19926	0.86432	0.97673	0.26567	0.26586	0.26586	0.26586	0.26586	0.26586	0.26582
+Q2	0.21753	0.96094	0.99924	0.22824	0.20338	0.36408	0.74231	0.83049	0.34415	0.35199
+Q3	0.23275	0.97534	0.99978	0.21042	0.14411	0.41393	0.96075	0.98751	0.37204	0.39988
+Q4	0.24821	0.97694	0.99993	0.20638	0.11976	0.43719	0.98946	0.99834	0.39955	0.54948
+Q5	0.26141	0.97201	0.99989	0.20775	0.11394	0.45008	0.99376	0.99921	0.41742	0.76821
+Q6	0.26556	0.96541	0.99980	0.20765	0.11485	0.44924	0.99370	0.99927	0.42910	0.88858
+Q7	0.27193	0.95873	0.99963	0.21521	0.11952	0.44080	0.99188	0.99920	0.44197	0.93348
+Q8	0.27540	0.95260	0.99932	0.22232	0.12918	0.42279	0.98954	0.99886	0.45349	0.95168
+Q9	0.27782	0.94713	0.99881	0.23170	0.13834	0.41259	0.98616	0.99842	0.46603	0.95805

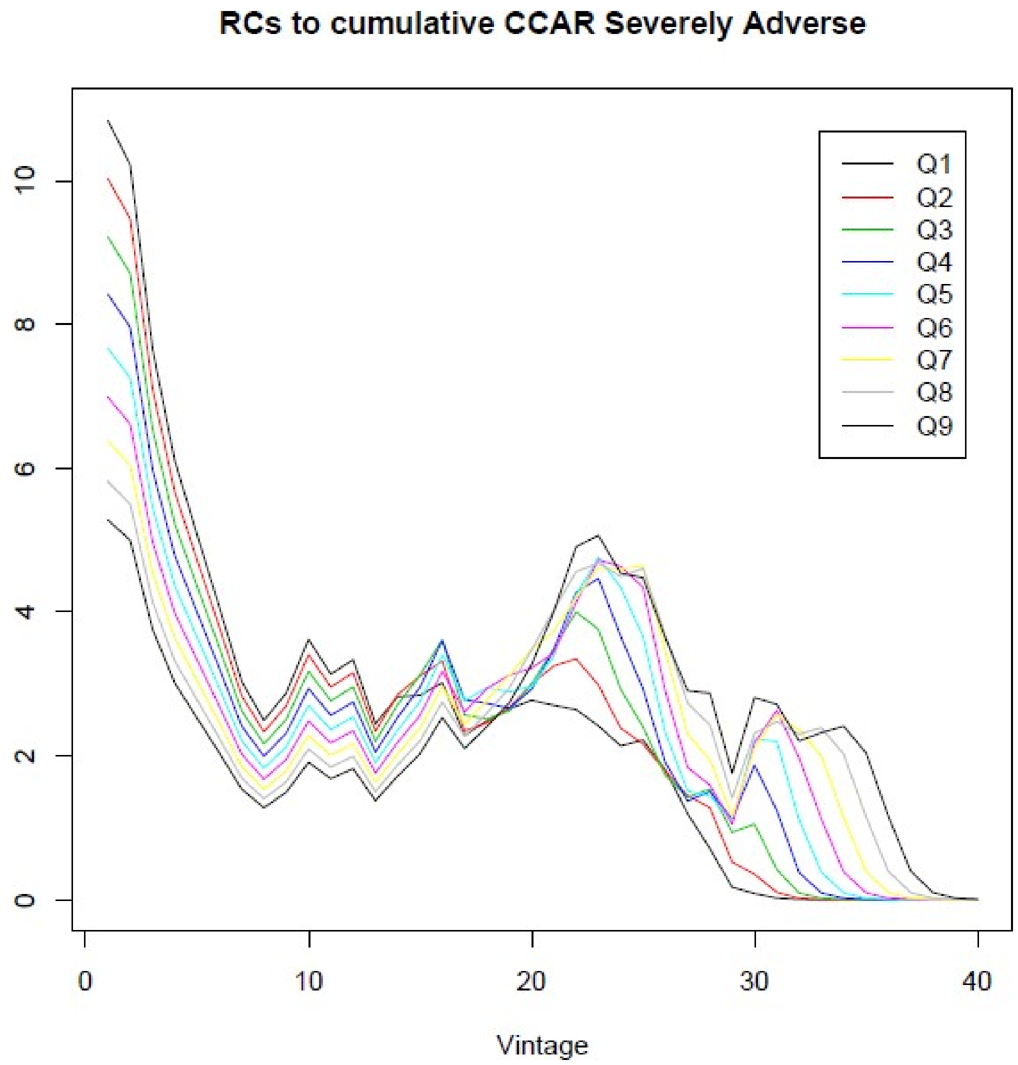
US First Mortgages – Cumulative Scenario Losses



US First Mortgages – Cumulative Scenario Losses



US First Mortgages – Cumulative Scenario Losses



5 Practical Applications: RAROC, Optimization, Optimal Importance Sampling and Reverse Stress Testing

Risk Adjusted Pricing (RAROC)

Analytical calculation of the portfolio volatility up to the new deal’s maturity allows us to instantaneously price.

The Sharpe ratio of the new portfolio with the new deal (or deals) should be larger than without:

$$\frac{\mu^* - r}{\sigma(L^*)} > \frac{\mu - r}{\sigma(L)} \Rightarrow \text{BUY}$$

Portfolio Optimisation

What is the portfolio composition n_i that minimises the portfolio loss volatility given a level of expected loss (and hence return) $EL = L$?

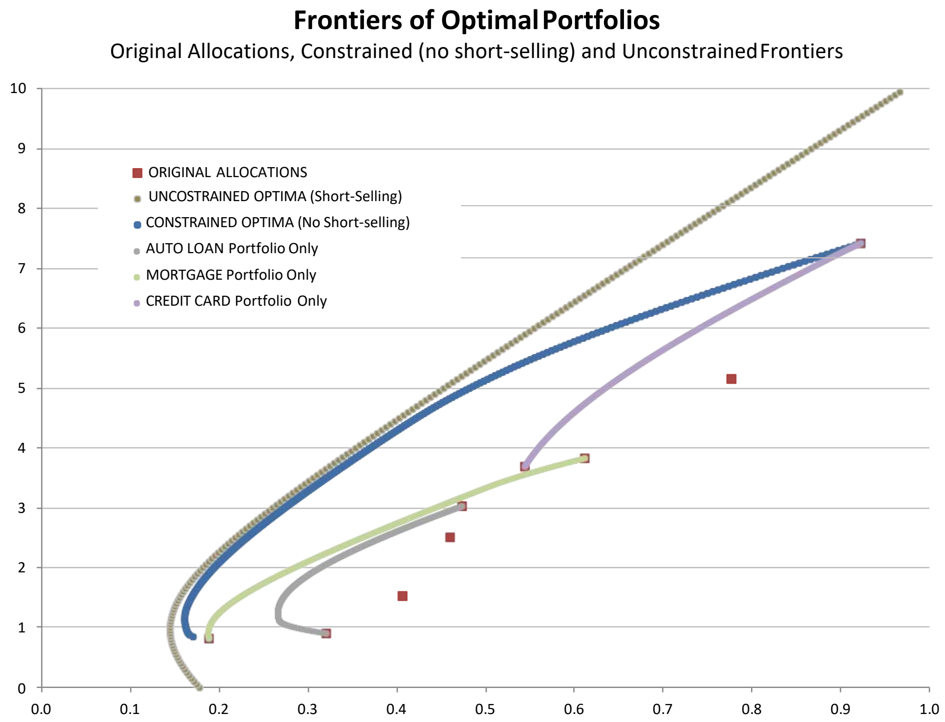
Using the Lagrange multipliers:

$$\Lambda(n_i, \lambda) = \sigma(L;\ n_i) + \lambda(EL(n_i) - L)$$

Thus:

$$n_i C_i + \lambda EL_i^* = 0$$

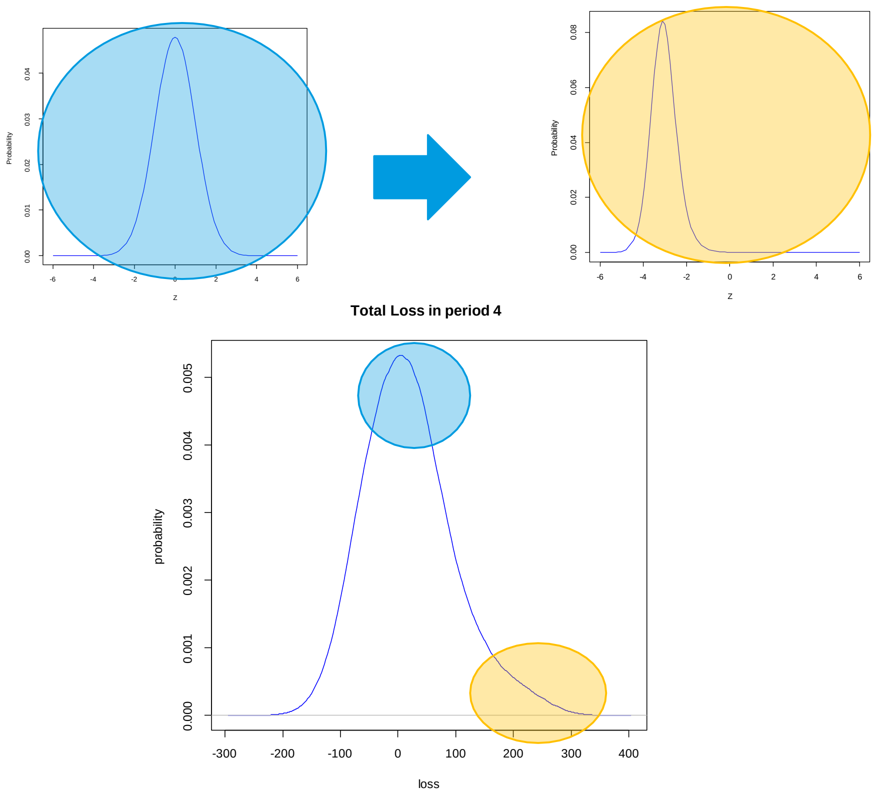
$$\sum_{i=1}^{N} n_i EL_i^* - L = 0$$



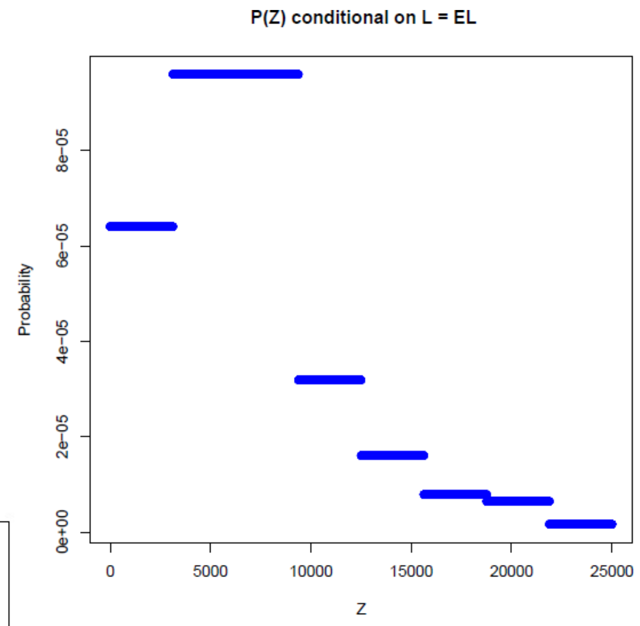
Extend the current framework to study DYNAMIC OPTIMISATION (infinite horizon) ***Recursive Dynamic Programming*** (Bellman Equations) and the study of the optimal solutions to the underlying stochastic difference equations

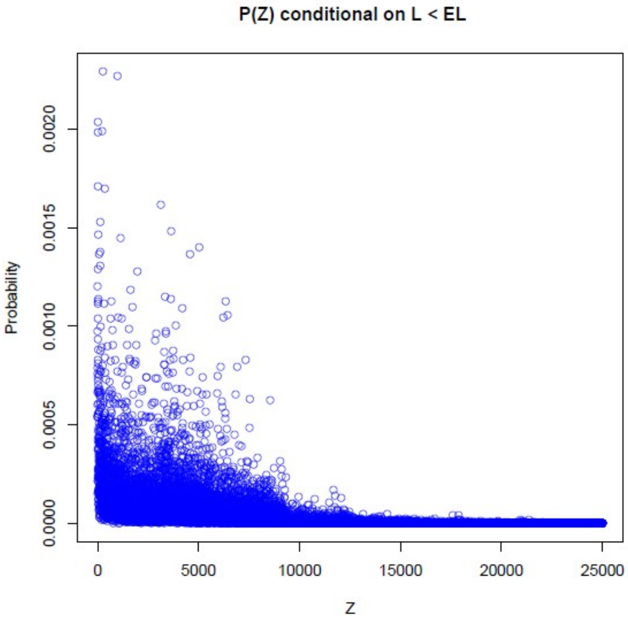
Optimal Importance Sampling

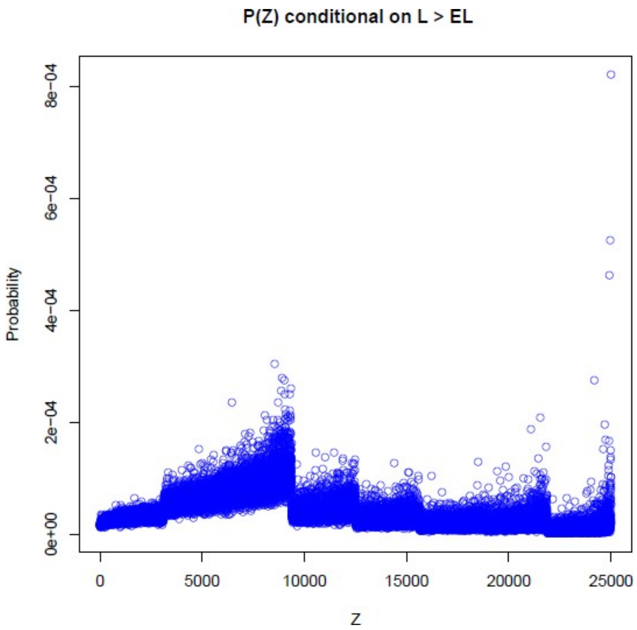
Instead of drawing random numbers with unconditional probability from conditional distribution w_z draw from conditional $w_z(\alpha)$. After the simulation reweigh the loss distribution using $\text{weight} = \frac{w}{w_z(\alpha)}$



Analytical Reverse Stress Testing







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Contact Us: Economics & Business Analytics Offices

West Chester, EBA-HQ
+1.610.235.5299
121 North Walnut Street, Suite 500
West Chester PA 19380
USA

New York, Corporate-HQ
+1.212.553.1653
7 World Trade Center, 14th Floor
250 Greenwich Street
New York, NY 10007
USA

London
+44.20.7772.5454
One Canada Square
Canary Wharf
London E14 5FA
United Kingdom

Toronto
416.681.2133
200 Wellington Street West, 15th Floor
Toronto ON M5V 3C7
Canada

Prague
+420.22.422.2929
Washingtonova 17
110 00 Prague 1
Czech Republic

Sydney
+61.2.9270.8111
Level 10
1 O'Connell Street
Sydney, NSW, 2000
Australia

Singapore
+65.6511.4400
6 Shenton Way
#14-08 OUE Downtown 2
Singapore 068809

Shanghai
+86.21.6101.0172
Unit 2306, Citigroup Tower
33 Huayuanshiqiao Road
Pudong New Area, 200120
China

help@economy.com

www.economy.com